MAA HL

Test on Sequences and Binomial theorem

by Christos Nikolaidis

Date: 17 January 2020

Marks: /40

Paper 1: without GDC

Name of student: _____

1. [Maximum mark: 5]

The **eleventh** term of an arithmetic sequence is 69 while the **sum** of the first 3 terms is 45. Find the third term.

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Turn over

2. [Maximum mark: 5]

Expand $\left(\sqrt{3}-\sqrt{2}\right)^4$ and express the result in the form $a+b\sqrt{6}$

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3. [Maximum mark: 5]

(a) Find $12C4$ [2]

(b) Prove that

$$2020C19 = 2019C19 + 2019C18$$ [3]

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Turn over

4. [Maximum mark: 7]

Let $x \in R$, $|x| < 1$ and

$$A = 1 + x + x^2 + x^3 + \cdots$$

(a) Show that

$$1 + x^2 + x^4 + x^6 + \cdots = \frac{A^2}{2A - 1}$$ [5]

(b) Find the value of x if $A = 5$. [2]

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5. [Maximum mark: 6]

Find $\sum_{n=0}^{\infty} \frac{1+3^n}{5^n}$

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Turn over

6. [Maximum mark: 5]

The coefficient of x^4 is twice the coefficient of x^2 in the expansion of $\left(x^2 + a\right)^n$.

Show that $n - 4a = 1$.

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7. [Maximum mark: 7]

The **7^{th}** term, the **11^{th}** term and the **23^{rd}** term of an arithmetic sequence are the first three terms of a geometric sequence.

(a) Find the common ratio of the geometric sequence. [5]

(b) Find the **5^{th} term** of the arithmetic sequence. [2]

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Paper 2: with GDC

Name of student: _____

1. [Maximum mark: 5]

The **constant** term in the expansion of $\left(\frac{9}{x^3} - ax\right)^8$ is 145152. Find the possible values of a.

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Turn over

2. [Maximum mark: 6]

The sum of the first 100 terms of an arithmetic sequence is 15250 while the sum of the next 100 terms is 45250. Find the sum of the next 100 terms.

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3. [Maximum mark: 6]

(a) Express $10x^2 - 19x + 6$ in the form $(ax - b)(cx - d)$, where a, b, c, d are positive integers. [1]

(b) **Hence** or otherwise find the coefficient of x^2 in the expansion of $(10x^2 - 19x + 6)^6$. [5]

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4. [Maximum mark: 5]

Let

$$S = 27 + 32 + 37 + 42 + \cdots + 362$$

(a) Express the sum S in the form $\sum_{k=1}^{n}(ak+b)$, where a, b, n are integers to be determined. [4]

(b) Hence find the value of S. [1]

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5. [Maximum mark: 6]

(a) Find the sum of all integers between 300 and 800 which are multiples of 6. [4]

(b) Find the sum of all integers between 300 and 800 which are **not** multiples of 6. [2]

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6. [Maximum mark: 6]

The sum of the first n terms of an arithmetic sequence is given by

$$S_n = 104n - 4n^2$$

(a) Find the **second** term. [2]

(b) Find the sum of the **positive** terms of the sequence. [4]

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7. [Maximum mark: 6]

Fileas invests 800€ at 8% per year (compounded yearly).

Filomila invests 760€ at 8% per year compounded **quarterly**.

(a) Find whether Fileas or Filomila would receive more money after 10 years. [3]

(b) Filomila would receive more money that Fileas after n **complete** years.

Find the minimum value of n. [3]

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