Topic 3 Part 1 *[449 marks]*

1a. Find all values of x for $0.1 \leqslant x \leqslant 1$ such that $\sin(\pi x^{-1}) = 0$. *[2 marks]*

1b. Find $\int_{\frac{1}{n+1}}^{\frac{1}{n}} \pi x^{-2} \sin(\pi x^{-1}) \mathrm{d}x$, showing that it takes different integer values when n is even and when n is odd. *[3 marks]*

1c. Evaluate $\int_{0.1}^{1} |\pi x^{-2} \sin(\pi x^{-1})| \mathrm{d}x$. *[2 marks]*

2a. Express $4x^2 - 4x + 5$ in the form $a(x-h)^2 + k$ where a, h, $k \in \mathbb{Q}$. *[2 marks]*

2b. The graph of $y = x^2$ is transformed onto the graph of $y = 4x^2 - 4x + 5$. Describe a sequence of transformations that does this, making the order of transformations clear. *[3 marks]*

The function *f* is defined by $f(x) = \frac{1}{4x^2 - 4x + 5}$.

2c. Sketch the graph of $y = f(x)$. *[2 marks]*

2d. Find the range of *f*. *[2 marks]*

2e. By using a suitable substitution show that $\int f(x) \mathrm{d}x = \frac{1}{4} \int \frac{1}{u^2+1} \mathrm{d}u$. *[3 marks]*

2f. Prove that $\int_{1}^{3.5} \frac{1}{4x^2 - 4x + 5} \mathrm{d}x = \frac{\pi}{16}$. *[7 marks]*

3a. Given that

$\arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right) = \arctan\left(\frac{1}{p}\right)$, where

$p \in \mathbb{Z}^+$, find p.

[3 marks]

3b. Hence find the value of

$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)$.

[3 marks]

4a. Use the identity

$\cos 2\theta = 2\cos^2\theta - 1$ to prove that

$\cos \frac{1}{2}x = \sqrt{\frac{1+\cos x}{2}},\ 0 \leqslant x \leqslant \pi$.

[2 marks]

4b. Find a similar expression for

$\sin \frac{1}{2}x,\ 0 \leqslant x \leqslant \pi$.

[2 marks]

4c. Hence find the value of

$\int_0^{\frac{\pi}{2}} \left(\sqrt{1+\cos x} + \sqrt{1-\cos x}\,\right) \mathrm{d}x$.

[4 marks]

5. The triangle ABC is equilateral of side 3 cm. The point D lies on [BC] such that BD = 1 cm. Find

$\cos \mathrm{D\hat{A}C}$.

[5 marks]

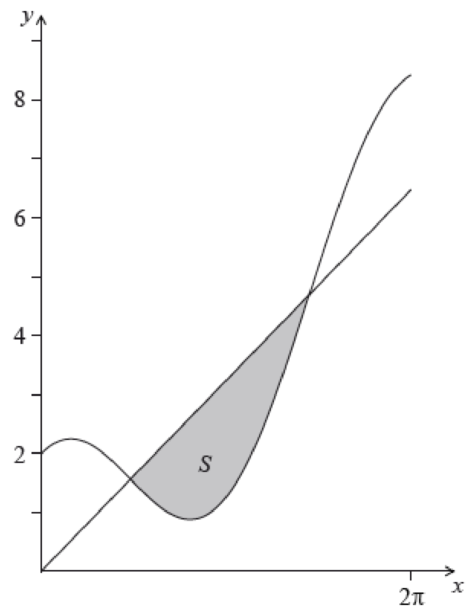
6. Given that

$\sin x + \cos x = \frac{2}{3}$, find

$\cos 4x$.

[6 marks]

The shaded region S is enclosed between the curve
$y = x + 2\cos x$, for
$0 \leqslant x \leqslant 2\pi$, and the line
$y = x$, as shown in the diagram below.



7a. Find the coordinates of the points where the line meets the curve. *[3 marks]*

7b. The region
S is rotated by
2π about the
x-axis to generate a solid. *[5 marks]*
(i) Write down an integral that represents the volume
V of the solid.
(ii) Find the volume
V.

8a. Sketch the graph of
$y = \left|\cos\left(\frac{x}{4}\right)\right|$ for
$0 \leqslant x \leqslant 8\pi$. *[2 marks]*

8b. Solve
$\left|\cos\left(\frac{x}{4}\right)\right| = \frac{1}{2}$ for
$0 \leqslant x \leqslant 8\pi$. *[3 marks]*

9. The first three terms of a geometric sequence are
$\sin x,\ \sin 2x$ and
$4\sin x\cos^2 x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$. *[7 marks]*
(a) Find the common ratio r.
(b) Find the set of values of x for which the geometric series
$\sin x + \sin 2x + 4\sin x\cos^2 x + \ldots$ converges.
Consider
$x = \arccos\left(\frac{1}{4}\right),\ x > 0$.
(c) Show that the sum to infinity of this series is
$\frac{\sqrt{15}}{2}$.

Consider the following functions:

$h(x) = \arctan(x),\ x \in \mathbb{R}$

$g(x) = \frac{1}{x}$,
$x \in \mathbb{R}$,
$x \neq 0$

10a. Sketch the graph of $y = h(x)$. [2 marks]

10b. Find an expression for the composite function $h \circ g(x)$ and state its domain. [2 marks]

10c. Given that $f(x) = h(x) + h \circ g(x)$, [7 marks]

(i) find $f'(x)$ in simplified form;

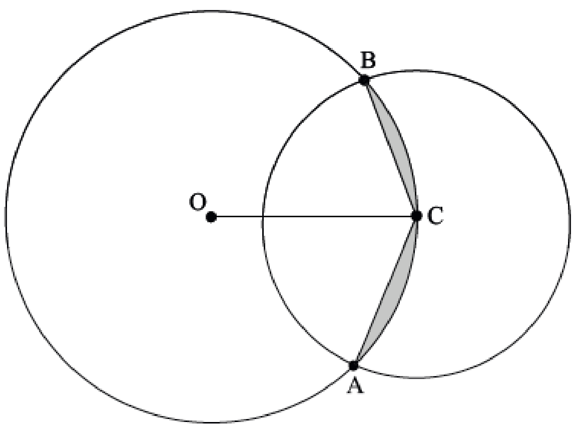
(ii) show that $f(x) = \frac{\pi}{2}$ for $x > 0$.

10d. Nigel states that f is an odd function and Tom argues that f is an even function. [3 marks]

(i) State who is correct and justify your answer.

(ii) Hence find the value of $f(x)$ for $x < 0$.

11. The following diagram shows two intersecting circles of radii 4 cm and 3 cm. The centre C of the smaller circle lies on the circumference of the bigger circle. O is the centre of the bigger circle and the two circles intersect at points A and B. [6 marks]



Find:

(a) $\mathrm{B\hat{O}C}$;

(b) the area of the shaded region.

12. (a) Prove the trigonometric identity [8 marks]

$\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$.

(b) Given

$f(x) = \sin(x + \frac{\pi}{6})\sin(x - \frac{\pi}{6})$, $x \in [0,\, \pi]$, find the range of

f.

(c) Given

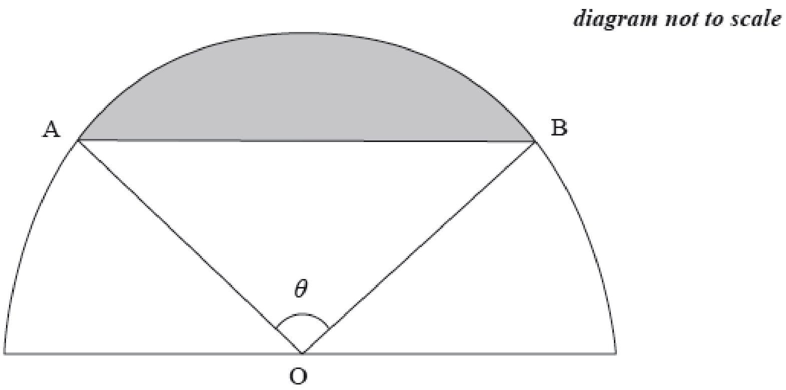
$g(x) = \csc(x + \frac{\pi}{6})\csc(x - \frac{\pi}{6})$, $x \in [0,\, \pi]$, $x \neq \frac{\pi}{6}$, $x \neq \frac{5\pi}{6}$, find the range of

g.

The diagram below shows a semi-circle of diameter 20 cm, centre O and two points A and B such that

$\mathrm{A\hat{O}B} = \theta$, where

θ is in radians.



13a. Show that the shaded area can be expressed as [2 marks]

$50\theta - 50\sin\theta$.

13b. Find the value of [3 marks]

θ for which the shaded area is equal to half that of the unshaded area, giving your answer correct to four significant figures.

14a. (i) Show that $(1 + \mathrm{i}\tan\theta)^n + (1 - \mathrm{i}\tan\theta)^n = \frac{2\cos n\theta}{\cos^n\theta}, \quad \cos\theta \neq 0.$ [10 marks]

(ii) Hence verify that $\mathrm{i}\tan\frac{3\pi}{8}$ is a root of the equation $(1+z)^4 + (1-z)^4 = 0, \quad z \in \mathbb{C}.$

(iii) State another root of the equation $(1+z)^4 + (1-z)^4 = 0, \quad z \in \mathbb{C}.$

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14b. (i) Use the double angle identity $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ to show that $\tan\frac{\pi}{8} = \sqrt{2} - 1$. *[13 marks]*

(ii) Show that $\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$.

(iii) Hence find the value of $\int_0^{\frac{\pi}{8}} \frac{2\cos 4x}{\cos^2 x}\mathrm{d}x$.

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Compactness is a measure of how compact an enclosed region is.

The compactness,
C, of an enclosed region can be defined by $C = \frac{4A}{\pi d^2}$, where
A is the area of the region and
d is the maximum distance between any two points in the region.

For a circular region, $C = 1$.

Consider a regular polygon of
n sides constructed such that its vertices lie on the circumference of a circle of diameter
x units.

15a. If $n > 2$ and even, show that $C = \frac{n}{2\pi}\sin\frac{2\pi}{n}$. *[3 marks]*

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15b. If $n > 1$ and odd, it can be shown that $C = \frac{n \sin \frac{2\pi}{n}}{\pi\left(1+\cos \frac{\pi}{n}\right)}$. *[4 marks]*

Find the regular polygon with the least number of sides for which the compactness is more than 0.99.

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Consider the triangle PQR where $\mathrm{Q\hat{P}R} = 30^\circ$, $\mathrm{PQ} = (x+2)\ \mathrm{cm}$ and $\mathrm{PR} = (5-x)^2\ \mathrm{cm}$, where $-2 < x < 5$.

16a. Show that the area, $A\ \mathrm{cm}^2$, of the triangle is given by $A = \frac{1}{4}(x^3 - 8x^2 + 5x + 50)$. *[2 marks]*

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16b. (i) State $\frac{\mathrm{d}A}{\mathrm{d}x}$. *[3 marks]*

(ii) Verify that $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ when $x = \frac{1}{3}$.

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16c. (i) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence justify that $x = \frac{1}{3}$ gives the maximum area of triangle PQR. *[7 marks]*

(ii) State the maximum area of triangle PQR.

(iii) Find QR when the area of triangle PQR is a maximum.

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In triangle ABC,

$3 \sin B + 4 \cos C = 6$ and

$4 \sin C + 3 \cos B = 1$.

17a. Show that $\sin(B + C) = \frac{1}{2}$. *[6 marks]*

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17b. Robert conjectures that $\mathrm{C\hat{A}B}$ can have two possible values. *[5 marks]*

Show that Robert's conjecture is incorrect by proving that $\mathrm{C\hat{A}B}$ has only one possible value.

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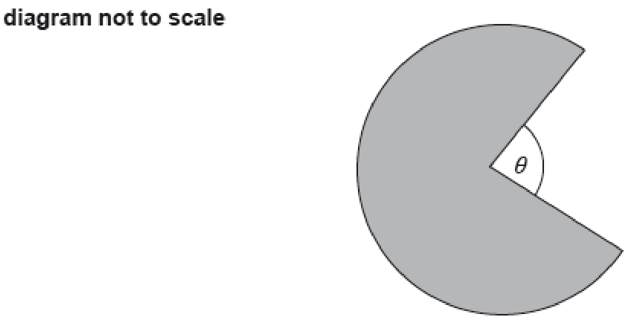
The logo, for a company that makes chocolate, is a sector of a circle of radius 2 cm, shown as shaded in the diagram. The area of the logo is $3\pi \text{ cm}^2$.



18a. Find, in radians, the value of the angle θ, as indicated on the diagram. *[3 marks]*

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18b. Find the total length of the perimeter of the logo. *[2 marks]*

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19. Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$. *[6 marks]*

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In triangle ABC, $\mathrm{BC} = \sqrt{3}\ \mathrm{cm}$, $\mathrm{A\hat{B}C} = \theta$ and $\mathrm{B\hat{C}A} = \frac{\pi}{3}$.

20a. Show that length $\mathrm{AB} = \frac{3}{\sqrt{3}\cos\theta + \sin\theta}$. *[4 marks]*

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20b. Given that AB has a minimum value, determine the value of θ for which this occurs. *[4 marks]*

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Consider the functions $f(x) = \tan x,\ 0 \leq\ x < \frac{\pi}{2}$ and $g(x) = \frac{x+1}{x-1},\ x \in \mathbb{R},\ x \neq 1$.

21a. Find an expression for $g \circ f(x)$, stating its domain. *[2 marks]*

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21b. Hence show that $g \circ f(x) = \frac{\sin x+\cos x}{\sin x-\cos x}$. *[2 marks]*

21c. Let $y = g \circ f(x)$, find an exact value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on the graph of $y = g \circ f(x)$ where $x = \frac{\pi}{6}$, expressing your answer in the form $a + b\sqrt{3},\ a,\ b \in \mathbb{Z}$. *[6 marks]*

21d. Show that the area bounded by the graph of $y = g \circ f(x)$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{6}$ is $\ln\left(1 + \sqrt{3}\right)$. *[6 marks]*

22. A triangle ABC has $\hat{A} = 50°$, $\mathrm{AB} = 7\ \mathrm{cm}$ and $\mathrm{BC} = 6\ \mathrm{cm}$. Find the area of the triangle given that it is smaller than $10\ \mathrm{cm}^2$. *[6 marks]*

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23a. (i) Use the binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$. *[6 marks]*

(ii) Hence use De Moivre's theorem to prove

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta.$$

(iii) State a similar expression for $\cos 5\theta$ in terms of $\cos\theta$ and $\sin\theta$.

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Let $z = r(\cos\alpha + \mathrm{i}\sin\alpha)$, where α is measured in degrees, be the solution of $z^5 - 1 = 0$ which has the smallest positive argument.

23b. Find the value of r and the value of α. *[4 marks]*

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23c. Using (a) (ii) and your answer from (b) show that $16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$. *[4 marks]*

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23d. Hence express *[5 marks]*

$\sin 72^\circ$ in the form $\frac{\sqrt{a+b\sqrt{c}}}{d}$ where $a,\ b,\ c,\ d \in \mathbb{Z}$.

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In triangle ABC, $\mathrm{AB} = 5\ \mathrm{cm}$, $\mathrm{BC} = 12\ \mathrm{cm}$ and $\mathrm{A\hat{B}C} = 100^\circ$.

24a. Find the area of the triangle. *[2 marks]*

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24b. Find AC. *[2 marks]*

Farmer Bill owns a rectangular field, 10 m by 4 m. Bill attaches a rope to a wooden post at one corner of his field, and attaches the other end to his goat Gruff.

25a. Given that the rope is 5 m long, calculate the percentage of Bill's field that Gruff is able to graze. Give your answer correct to the nearest integer. *[4 marks]*

25b. Bill replaces Gruff's rope with another, this time of length $a,\ 4 < a < 10$, so that Gruff can now graze exactly one half of Bill's field. *[4 marks]*

Show that a satisfies the equation

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40.$$

25c. Find the value of a. *[2 marks]*

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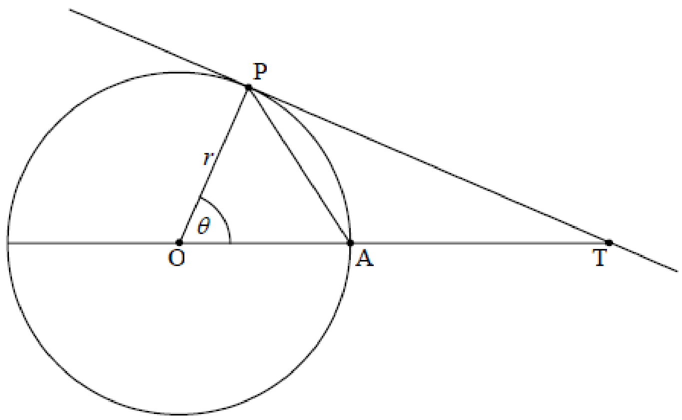
The diagram shows a tangent, (TP) , to the circle with centre O and radius r . The size of

$\text{P}\hat{\text{O}}\text{A}$ is

θ radians.



26a. Find the area of triangle AOP in terms of r and θ . *[1 mark]*

26b. Find the area of triangle POT in terms of r and θ . *[2 marks]*

26c. Using your results from part (a) and part (b), show that $\sin\theta < \theta < \tan\theta$. *[2 marks]*

27a. (i) Sketch the graphs of $y = \sin x$ and $y = \sin 2x$, on the same set of axes, for $0 \leqslant x \leqslant \frac{\pi}{2}$. *[9 marks]*

(ii) Find the x-coordinates of the points of intersection of the graphs in the domain $0 \leqslant x \leqslant \frac{\pi}{2}$.

(iii) Find the area enclosed by the graphs.

27b. Find the value of [8 marks]

$\int_0^1 \sqrt{\frac{x}{4-x}}\mathrm{d}x$ using the substitution

$x = 4\sin^2\theta$.

27c. The increasing function f satisfies [8 marks]

$f(0) = 0$ and

$f(a) = b$, where

$a > 0$ and

$b > 0$.

(i) By reference to a sketch, show that

$\int_0^a f(x)\mathrm{d}x = ab - \int_0^b f^{-1}(x)\mathrm{d}x$.

(ii) **Hence** find the value of

$\int_0^2 \arcsin\left(\frac{x}{4}\right)\mathrm{d}x$.

28. The points P and Q lie on a circle, with centre O and radius 8 cm, such that [5 marks]

$\mathrm{P\hat{O}Q} = 59^\circ$.

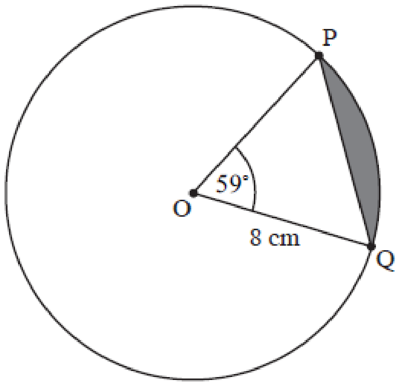


diagram not to scale

Find the area of the shaded segment of the circle contained between the arc PQ and the chord [PQ].

29. The vertices of an equilateral triangle, with perimeter P and area A , lie on a circle with radius r . Find an expression for [6 marks]

$\frac{P}{A}$ in the form

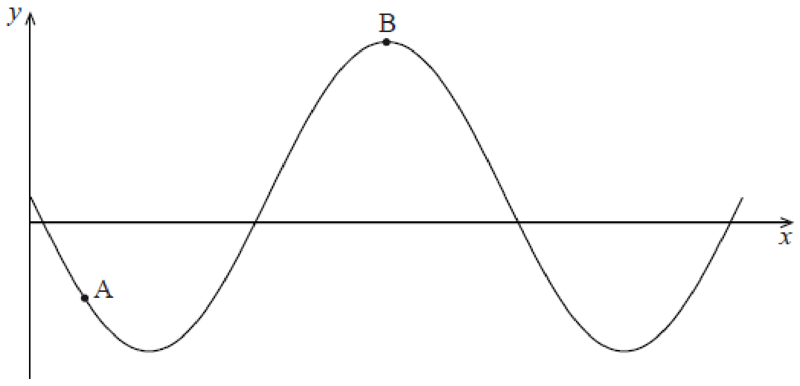
$\frac{k}{r}$, where

$k \in \mathbb{Z}^+$.

30. The diagram below shows a curve with equation [5 marks]

$y = 1 + k\sin x$, defined for

$0 \leqslant x \leqslant 3\pi$.



The point

A $\left(\frac{\pi}{6}, -2\right)$ lies on the curve and

B$(a,\ b)$ is the maximum point.

(a) Show that $k = -6$.

(b) Hence, find the values of a and b .

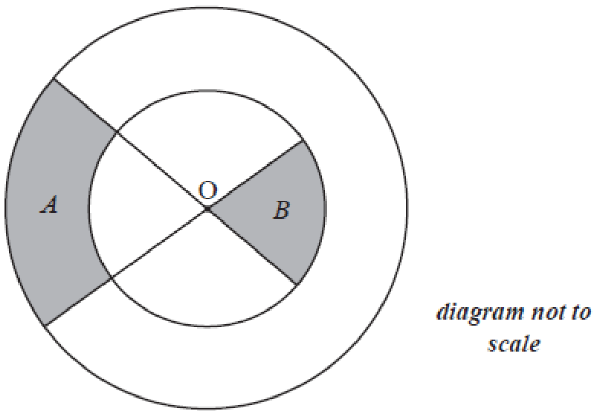
31. (a) Show that [5 marks]

$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$.

(b) Hence, or otherwise, find the value of

$\arctan(2) + \arctan(3)$.

32. The diagram below shows two straight lines intersecting at O and two circles, each with centre O. The outer circle has radius R [5 marks] and the inner circle has radius r .



Consider the shaded regions with areas A and B . Given that

$A : B = 2 : 1$, find the **exact** value of the ratio

$R : r$.

33. A triangle has sides of length *[8 marks]*

$(n^2 + n + 1)$,

$(2n + 1)$ and

$(n^2 - 1)$ where

$n > 1$.

(a) Explain why the side

$(n^2 + n + 1)$ must be the longest side of the triangle.

(b) Show that the largest angle,

θ, of the triangle is

$120°$.

34. (a) Show that *[20 marks]*

$\sin 2nx = \sin((2n + 1)x)\cos x - \cos((2n + 1)x)\sin x$.

(b) **Hence** prove, by induction, that

$$\cos x + \cos 3x + \cos 5x + \ldots + \cos((2n - 1)x) = \frac{\sin 2nx}{2\sin x},$$

for all

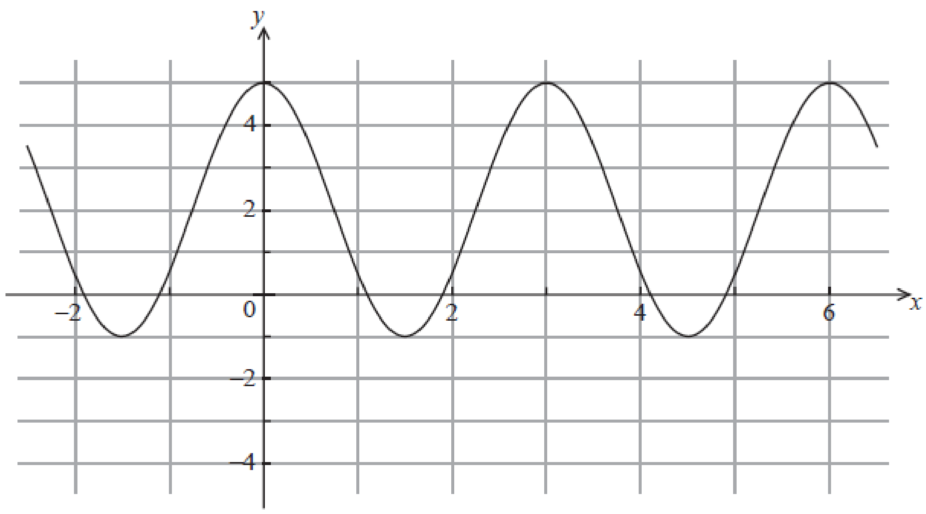
$n \in \mathbb{Z}^+$, $\sin x \neq 0$.

(c) Solve the equation

$\cos x + \cos 3x = \frac{1}{2}$, $0 < x < \pi$.

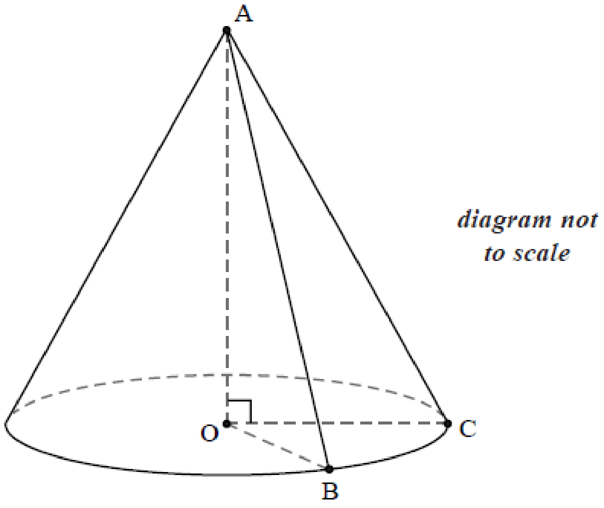
35. The graph below shows *[4 marks]*

$y = a\cos(bx) + c$.



Find the value of a, the value of b and the value of c.

36. In the right circular cone below, O is the centre of the base which has radius 6 cm. The points B and C are on the circumference of the base of the cone. The height AO of the cone is 8 cm and the angle $B\hat{O}C$ is 60°. *[6 marks]*



Calculate the size of the angle $B\hat{A}C$.

37. Points A, B and C are on the circumference of a circle, centre O and radius [11 marks]

r . A trapezium OABC is formed such that AB is parallel to OC, and the angle

$A\hat{O}C$ is

θ ,

$\frac{\pi}{2} \leqslant \theta \leqslant \pi$.

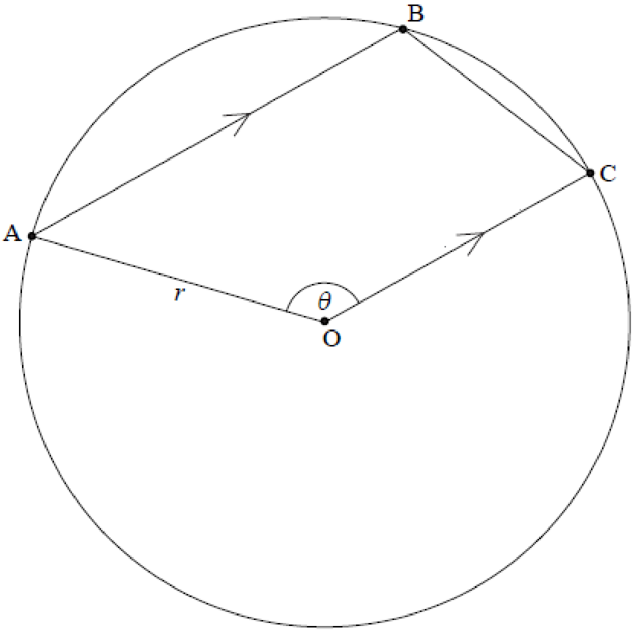


diagram not to scale

(a) Show that angle

$B\hat{O}C$ is

$\pi - \theta$.

(b) Show that the area, T, of the trapezium can be expressed as

$$T = \frac{1}{2}r^2 \sin\theta - \frac{1}{2}r^2 \sin 2\theta.$$

(c) (i) Show that when the area is maximum, the value of

θ satisfies

$$\cos\theta = 2\cos 2\theta.$$

(ii) **Hence** determine the maximum area of the trapezium when $r = 1$.

(Note: It is not required to prove that it is a maximum.)

38. If x satisfies the equation [6 marks]

$\sin\left(x + \frac{\pi}{3}\right) = 2\sin x \sin\left(\frac{\pi}{3}\right)$, show that

$11\tan x = a + b\sqrt{3}$, where a, b

$\in \mathbb{Z}^+$.

39. Consider the triangle ABC where *[6 marks]*

$B\hat{A}C = 70^\circ$, AB = 8 cm and AC = 7 cm. The point D on the side BC is such that

$\frac{BD}{DC} = 2$.

Determine the length of AD.

40. The interior of a circle of radius 2 cm is divided into an infinite number of sectors. The areas of these sectors form a geometric sequence with common ratio k. The angle of the first sector is *[12 marks]*

θ radians.

(a) Show that

$\theta = 2\pi(1-k)$.

(b) The perimeter of the third sector is half the perimeter of the first sector.

Find the value of k and of

θ.

41. Triangle ABC has AB = 5 cm, BC = 6 cm and area 10 *[6 marks]*

cm^2.

(a) Find

$\sin \hat{B}$.

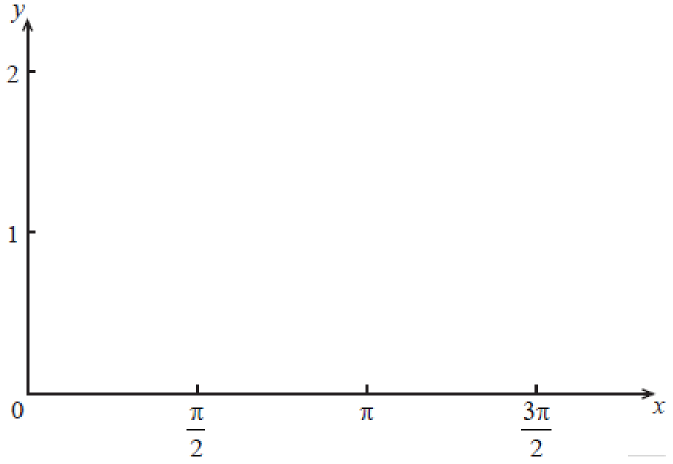
(b) **Hence**, find the two possible values of AC, giving your answers correct to two decimal places.

Given that

$f(x) = 1 + \sin x,\ 0 \leqslant x \leqslant \frac{3\pi}{2}$,

42a. sketch the graph of *[1 mark]*

f;

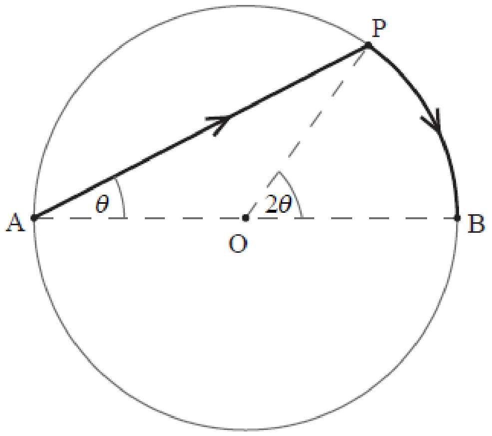


42b. show that *[1 mark]*

$(f(x))^2 = \frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x$;

42c. find the volume of the solid formed when the graph of f is rotated through 2π radians about the x-axis. *[4 marks]*

The diagram below shows a circular lake with centre O, diameter AB and radius 2 km.



Jorg needs to get from A to B as quickly as possible. He considers rowing to point P and then walking to point B. He can row at 3 km h^{-1} and walk at 6 km h^{-1}. Let $\text{P}\hat{\text{A}}\text{B} = \theta$ radians, and t be the time in hours taken by Jorg to travel from A to B.

43a. Show that $t = \frac{2}{3}(2\cos\theta + \theta)$. *[3 marks]*

43b. Find the value of θ for which $\frac{\text{d}t}{\text{d}\theta} = 0$. *[2 marks]*

43c. What route should Jorg take to travel from A to B in the least amount of time? Give reasons for your answer. *[3 marks]*

44a. Given that $\arctan\frac{1}{2} - \arctan\frac{1}{3} = \arctan a,\ a \in \mathbb{Q}^{+}$, find the value of a. *[3 marks]*

44b. Hence, or otherwise, solve the equation $\arcsin x = \arctan a$. *[2 marks]*

45a. Show that $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$. *[2 marks]*

45b. Hence find the value of $\cot\frac{\pi}{8}$ in the form $a+b\sqrt{2}$, where $a,b \in \mathbb{Z}$. *[3 marks]*

46a. Write down the expansion of $(\cos\theta + \mathrm{i}\sin\theta)^3$ in the form $a+\mathrm{i}b$, where a and b are in terms of $\sin\theta$ and $\cos\theta$. *[2 marks]*

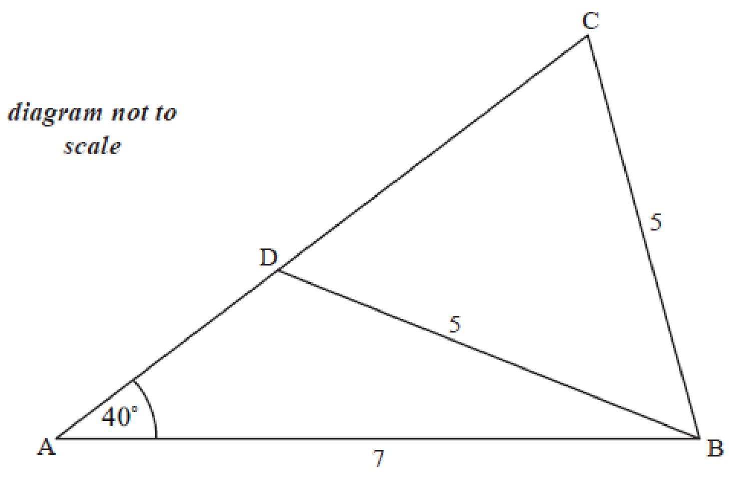
46b. Hence show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. *[3 marks]*

46c. Similarly show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. *[3 marks]*

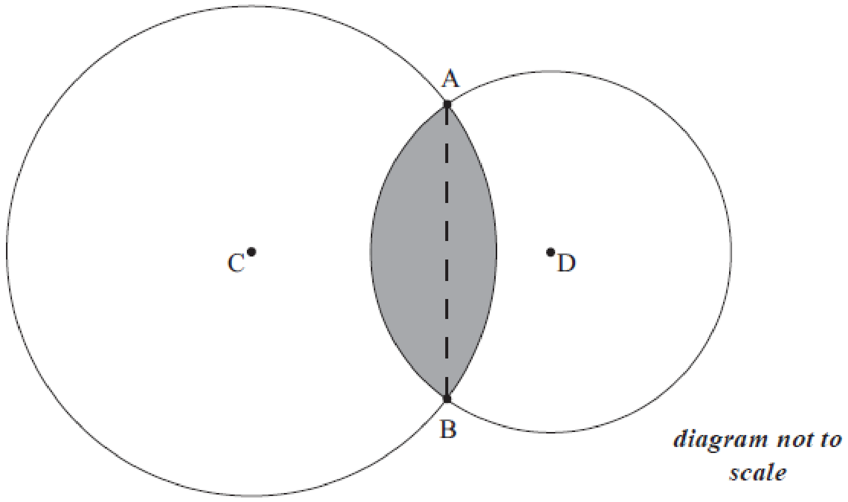
46d. **Hence** solve the equation $\cos 5\theta + \cos 3\theta + \cos\theta = 0$, where $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. *[6 marks]*

46e. By considering the solutions of the equation $\cos 5\theta = 0$, show that $\cos\frac{\pi}{10} = \sqrt{\frac{5+\sqrt{5}}{8}}$ and state the value of $\cos\frac{7\pi}{10}$. *[8 marks]*

47. Given ΔABC, with lengths shown in the diagram below, find the length of the line segment [CD]. *[5 marks]*

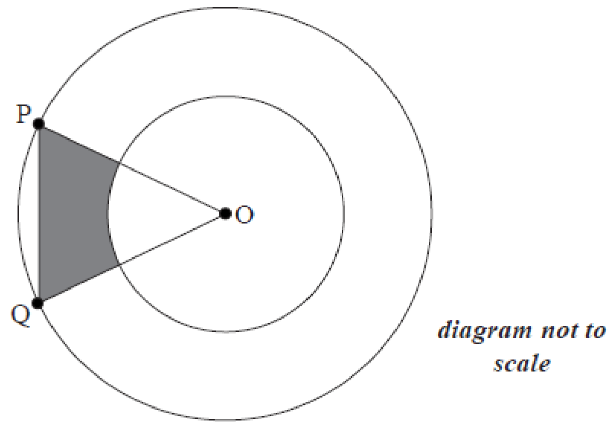


48. The radius of the circle with centre C is 7 cm and the radius of the circle with centre D is 5 cm. If the length of the chord [AB] is 9 cm, find the area of the shaded region enclosed by the two arcs AB. *[7 marks]*



The diagram below shows two concentric circles with centre O and radii 2 cm and 4 cm.

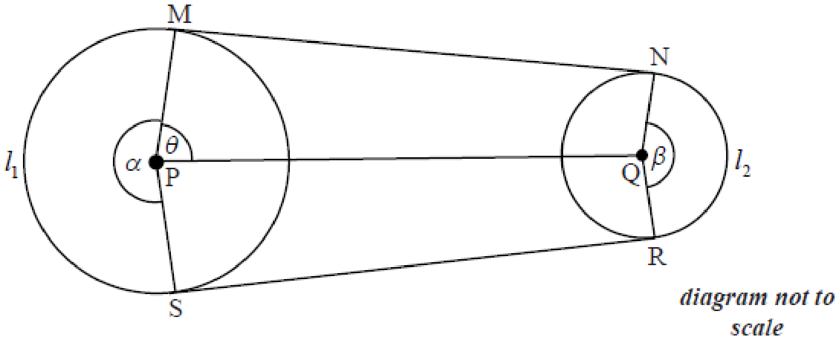
The points P and Q lie on the larger circle and
$\mathrm{P\hat{O}Q} = x$, where
$0 < x < \frac{\pi}{2}$.



49. (a) Show that the area of the shaded region is $8\sin x - 2x$. *[7 marks]*

(b) Find the maximum area of the shaded region.

Two non-intersecting circles C_1 , containing points M and S , and C_2 , containing points N and R, have centres P and Q where PQ $= 50$. The line segments [MN] and [SR] are common tangents to the circles. The size of the reflex angle MPS is
α, the size of the obtuse angle NQR is
β , and the size of the angle MPQ is
θ . The arc length MS is
l_1 and the arc length NR is
l_2 . This information is represented in the diagram below.



The radius of C_1 is
x , where
$x \geqslant 10$ and the radius of C_2 is
10.

50. (a) Explain why
$x < 40$. *[18 marks]*

(b) Show that cosθ = x −10 50.

(c) (i) Find an expression for MN in terms of
x .

(ii) Find the value of
x that maximises MN.

(d) Find an expression in terms of
x for

(i)
α ;

(ii)
β .

(e) The length of the perimeter is given by
$l_1 + l_2 + \text{MN} + \text{SR}$.

(i) Find an expression,
$b(x)$, for the length of the perimeter in terms of
x .

(ii) Find the maximum value of the length of the perimeter.

(iii) Find the value of
x that gives a perimeter of length
200.