Topic 2 Part 4 *[47 marks]*

1a. *[1 mark]*

Markscheme

sum $= \frac{45}{9}$, product $= \frac{40}{9}$ ***A1***

[1 mark]

Examiners report

[N/A]

1b. *[6 marks]*

Markscheme

it follows that

$3\alpha = \frac{45}{9}$ and

$\alpha(\alpha^2 - \beta^2) = \frac{40}{9}$ ***A1A1***

solving,

$\alpha = \frac{5}{3}$ ***A1***

$\frac{5}{3}\left(\frac{25}{9} - \beta^2\right) = \frac{40}{9}$ ***M1***

$\beta = (\pm)\frac{1}{3}$ ***A1***

the other two roots are 2,

$\frac{4}{3}$ ***A1***

[6 marks]

Examiners report

[N/A]

2a. *[3 marks]*

Markscheme

$f(-x) = 2\cos(-x) + (-x)\sin(-x)$ ***M1***

$= 2\cos x + x\sin x \quad (= f(x))$ ***A1***

therefore f is even ***A1***

[3 marks]

Examiners report

[N/A]

2b. *[2 marks]*

Markscheme

$f'(x) = -2\sin x + \sin x + x\cos x \quad (= -\sin x + x\cos x)$ ***A1***

$f''(x) = -\cos x + \cos x - x\sin x \quad (= -x\sin x)$ ***A1***

so

$f''(0) = 0$ ***AG***

[2 marks]

Examiners report

[N/A]

2c. *[2 marks]*

Markscheme

John's statement is incorrect because

either; there is a stationary point at (0, 2) and since *f* is an even function and therefore symmetrical about the *y*-axis it must be a maximum or a minimum

or;

$f''(x)$ is even and therefore has the same sign either side of (0, 2) ***R2***

[2 marks]

Examiners report

[N/A]

3a. *[3 marks]*

Markscheme

(i)

$f'(x) = \mathrm{e}^x - \mathrm{e}x^{\mathrm{e}-1}$ ***A1***

(ii) by inspection the two roots are 1, e ***A1A1***

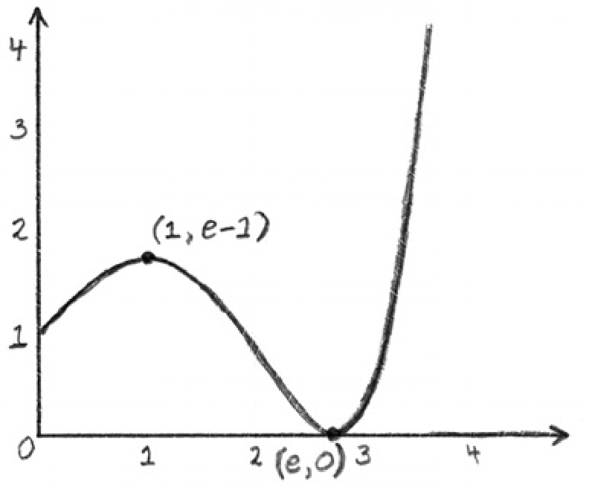
[3 marks]

Examiners report

[N/A]

3b. *[3 marks]*

Markscheme



A3

Note: Award *A1* for maximum, *A1* for minimum and *A1* for general shape.

[3 marks]

Examiners report

[N/A]

3c. *[1 mark]*

Markscheme

from the graph:

$\mathrm{e}^x > x^\mathrm{e}$ for all

$x > 0$ except $x = \mathrm{e}$ ***R1***

putting

$x = \pi$, conclude that

$\mathrm{e}^\pi > \pi^\mathrm{e}$ ***AG***

[1 mark]

Examiners report

[N/A]

4a.

[6 marks]

Markscheme

f continuous

$\Rightarrow \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x)$ ***M1***

$4a + 2b = 8$ ***A1***

$f'(x) = \begin{cases} 2, & x < 2 \\ 2ax + b, & 2 < x < 3 \end{cases}$ ***A1***

f' continuous $\Rightarrow \lim\limits_{x \to 2^-} f'(x) = \lim\limits_{x \to 2^+} f'(x)$

$4a + b = 2$ ***A1***

solve simultaneously ***M1***

to obtain $a = -1$ and $b = 6$ ***A1***

[6 marks]

Examiners report

[N/A]

4b.

[3 marks]

Markscheme

for

$x \leqslant 2,\ f'(x) = 2 > 0$ ***A1***

for

$2 < x < 3,\ f'(x) = -2x + 6 > 0$ ***A1***

since

$f'(x) > 0$ for all values in the domain of f, f is increasing ***R1***

therefore one-to-one ***AG***

[3 marks]

Examiners report

[N/A]

4c.

[5 marks]

Markscheme

$x = 2y - 1 \Rightarrow y = \frac{x+1}{2}$ ***M1***

$x = -y^2 + 6y - 5 \Rightarrow y^2 - 6y + x + 5 = 0$ ***M1***

$y = 3 \pm \sqrt{4 - x}$

therefore

$f^{-1}(x) = \begin{cases} \frac{x+1}{2}, & x \leqslant 3 \\ 3 - \sqrt{4 - x}, & 3 < x < 4 \end{cases}$ ***A1A1A1***

Note: Award ***A1*** for the first line and ***A1A1*** for the second line.

[5 marks]

Examiners report

[N/A]

5. *[6 marks]*

Markscheme

$f(2) = 8 + 4a + 2b - 4 = 0$ ***M1***

$\Rightarrow 4a + 2b = -4$ ***A1***

$f(1) = 1 + a + b - 4 = -6$ ***M1***

$\Rightarrow a + b = -3$ ***A1***

solving,

$a = 1,\ b = -4$ ***A1A1***

[6 marks]

Examiners report

[N/A]

6. *[6 marks]*

Markscheme

vertical asymptote

$x = -4 \Rightarrow -4b + c = 0$ ***M1***

horizontal asymptote

$y = -2 \Rightarrow \frac{1}{b} = -2$ ***M1***

$b = -\frac{1}{2}$ and $c = -2$ ***A1A1***

$1 = \frac{\frac{2}{3}+a}{-\frac{1}{2}\times\frac{2}{3}-2}$ ***M1***

$a = -3$ ***A1***

[6 marks]

Examiners report

[N/A]