

Topic 6 Part 4 [317 marks]

1a. [2 marks]

Markscheme

$$\int (1 + \tan^2 x) dx = \int \sec^2 x dx = \tan x (+c) \quad \mathbf{M1A1}$$

[2 marks]

Examiners report

Some correct answers but too many candidates had a poor approach and did not use the trig identity.

1b. [3 marks]

Markscheme

$$\int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx \quad \mathbf{M1A1}$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} (+c) \quad \mathbf{A1}$$

Note: Allow integration by parts followed by trig identity.

Award **M1** for parts, **A1** for trig identity, **A1** final answer.

[3 marks]

Total [5 marks]

Examiners report

Same as (a).

2a.

[4 marks]

Markscheme

$$f : x \rightarrow y = \frac{3x-2}{2x-1} \quad f^{-1} : y \rightarrow x$$

$$y = \frac{3x-2}{2x-1} \Rightarrow 3x - 2 = 2xy - y \quad \mathbf{M1}$$

$$\Rightarrow 3x - 2xy = -y + 2 \quad \mathbf{M1}$$

$$x(3 - 2y) = 2 - y$$

$$x = \frac{2-y}{3-2y} \quad \mathbf{A1}$$

$$\left(f^{-1}(y) = \frac{2-y}{3-2y} \right)$$

$$f^{-1}(x) = \frac{2-x}{3-2x} \quad \left(x \neq \frac{3}{2} \right) \quad \mathbf{A1}$$

Note: x and y might be interchanged earlier.

Note: First **M1** is for interchange of variables second **M1** for manipulation

Note: Final answer must be a function of x

[4 marks]

Examiners report

Well done. Only a few candidates confused inverse with derivative or reciprocal.

2b.

[2 marks]

Markscheme

$$\frac{3x-2}{2x-1} = A + \frac{B}{2x-1} \Rightarrow 3x - 2 = A(2x - 1) + B$$

equating coefficients $3 = 2A$ and $-2 = -A + B$ (**M1**)

$$A = \frac{3}{2} \text{ and } B = -\frac{1}{2} \quad \mathbf{A1}$$

Note: Could also be done by division or substitution of values.

[2 marks]

Examiners report

Not enough had the method of polynomial division.

2c.

[1 mark]

Markscheme

$$\int f(x)dx = \frac{3}{2}x - \frac{1}{4}\ln|2x - 1| + c \quad \mathbf{A1}$$

Note: accept equivalent e.g.

$$\ln|4x - 2|$$

[1 mark]

Total [7 marks]

Examiners report

Reasonable if they had an answer to (b) (follow through was given) usual mistakes with not allowing for the derivative of the bracket.

3.

[7 marks]

Markscheme

$$\frac{du}{dx} = e^x \quad (\mathbf{A1})$$

EITHER

$$\text{integral is } \int \frac{e^x}{(e^x+3)^2+2^2} dx \quad \mathbf{M1A1}$$

$$= \frac{1}{u^2+2^2} du \quad \mathbf{M1A1}$$

Note: Award **M1** only if the integral has completely changed to one in u .

Note: du needed for final **A1**

OR

$$e^x = u - 3$$

$$\text{integral is } \int \frac{1}{(u-3)^2+6(u-3)+13} du \quad \mathbf{M1A1}$$

Note: Award **M1** only if the integral has completely changed to one in u .

$$= \int \frac{1}{u^2+2^2} du \quad \mathbf{M1A1}$$

Note: In both solutions the two method marks are independent.

THEN

$$= \frac{1}{2} \arctan\left(\frac{u}{2}\right) (+c) \quad (\mathbf{A1})$$

$$= \frac{1}{2} \arctan\left(\frac{e^x+3}{2}\right) (+c) \quad \mathbf{A1}$$

Total [7 marks]

Examiners report

Many good complete answers. Some did not realise it was arctan. Some had poor understanding of the method.

4a.

[2 marks]

Markscheme

$$\frac{dy}{dx} = 1 \times e^{3x} + x \times 3e^{3x} = (e^{3x} + 3xe^{3x}) \quad \mathbf{M1A1}$$

[2 marks]

Examiners report

Well done.

4b.

[7 marks]

Markscheme

let $P(n)$ be the statement $\frac{d^n y}{dx^n} = n3^{n-1}e^{3x} + x3^n e^{3x}$

prove for $n = 1$ **M1**

LHS of $P(1)$ is $\frac{dy}{dx}$ which is $1 \times e^{3x} + x \times 3e^{3x}$ and

RHS is $3^0 e^{3x} + x3^1 e^{3x}$ **R1**

as $LHS = RHS$, $P(1)$ is true

assume $P(k)$ is true and attempt to prove $P(k+1)$ is true **M1**

assuming $\frac{d^k y}{dx^k} = k3^{k-1}e^{3x} + x3^k e^{3x}$

$$\frac{d^{k+1}y}{dx^{k+1}} = \frac{d}{dx} \left(\frac{d^k y}{dx^k} \right) \quad (\mathbf{M1})$$

$$= k3^{k-1} \times 3e^{3x} + 1 \times 3^k e^{3x} + x3^k \times 3e^{3x} \quad \mathbf{A1}$$

$$= (k+1)3^k e^{3x} + x3^{k+1} e^{3x} \quad (\text{as required}) \quad \mathbf{A1}$$

Note: Can award the **A** marks independent of the **M** marks

since $P(1)$ is true and $P(k)$ is true $\Rightarrow P(k+1)$ is true

then (by

PMI), $P(n)$ is true ($\forall n \in \mathbb{Z}^+$) **R1**

Note: To gain last **R1** at least four of the above marks must have been gained.

[7 marks]

Examiners report

The logic of an induction proof was not known well enough. Many candidates used what they had to prove rather than differentiating what they had assumed. They did not have enough experience in doing Induction proofs.

4c.

[5 marks]

$$e^{3x} + x \times 3e^{3x} = 0 \Rightarrow 1 + 3x = 0 \Rightarrow x = -\frac{1}{3}$$

$$\left(-\frac{1}{3}, -\frac{1}{3e}\right)$$

$$\frac{d^2 y}{dx^2} = 2 \times 3e^{3x} + x \times 3^2 e^{3x}$$

$$x = -\frac{1}{3}, \frac{d^2 y}{dx^2} > 0$$

Examiners report

Good, some forgot to test for min/max, some forgot to give the y value.

4d.

[5 marks]

Markscheme

$$\frac{d^2y}{dx^2} = 2 \times 3e^{3x} + x \times 3^2 e^{3x} \quad \mathbf{A1}$$

$$2 \times 3e^{3x} + x \times 3^2 e^{3x} = 0 \Rightarrow 2 + 3x = 0 \Rightarrow x = -\frac{2}{3} \quad \mathbf{M1A1}$$

$$\text{point is } \left(-\frac{2}{3}, -\frac{2}{3e^2}\right) \quad \mathbf{A1}$$

x	$-\frac{2}{3}$
$\frac{d^2y}{dx^2}$	$-ve \quad 0 \quad +ve$

since the curvature does change (concave down to concave up) it is a point of inflection $\mathbf{R1}$

Note: Allow 3^{rd} derivative is not zero at $-\frac{2}{3}$

[5 marks]

Examiners report

Again quite good, some forgot to check for change in curvature and some forgot the y value.

5a.

[4 marks]

Markscheme

$$\text{attempt to differentiate } f(x) = x^3 - 3x^2 + 4 \quad \mathbf{M1}$$

$$f'(x) = 3x^2 - 6x \quad \mathbf{A1}$$

$$= 3x(x - 2)$$

(Critical values occur at)

$$x = 0, x = 2 \quad (\mathbf{A1})$$

$$\text{so } f \text{ decreasing on } x \in]0, 2[\quad (\text{or } 0 < x < 2) \quad \mathbf{A1}$$

[4 marks]

Examiners report

[N/A]

5b.

[3 marks]

Markscheme

$$f''(x) = 6x - 6 \quad (\mathbf{A1})$$

$$\text{setting } f''(x) = 0 \quad \mathbf{M1}$$

$$\Rightarrow x = 1$$

$$\text{coordinate is } (1, 2) \quad \mathbf{A1}$$

[3 marks]

Total [7 marks]

Examiners report

[N/A]

6.

[6 marks]

Markscheme

any attempt at integration by parts **M1**

$$u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \quad \textbf{(A1)}$$

$$\frac{dv}{dx} = x^3 \Rightarrow v = \frac{x^4}{4} \quad \textbf{(A1)}$$

$$= \left[\frac{x^4}{4} \ln x \right]_1^2 - \int_1^2 \frac{x^3}{4} dx \quad \textbf{A1}$$

Note: Condone absence of limits at this stage.

$$= \left[\frac{x^4}{4} \ln x \right]_1^2 - \left[\frac{x^4}{16} \right]_1^2 \quad \textbf{A1}$$

Note: Condone absence of limits at this stage.

$$= 4 \ln 2 - \left(1 - \frac{1}{16} \right) \quad \textbf{A1}$$

$$= 4 \ln 2 - \frac{15}{16} \quad \textbf{AG}$$

[6 marks]

Examiners report

[N/A]

7a.

[4 marks]

Markscheme

any attempt to use sine rule **M1**

$$\frac{AB}{\sin \frac{\pi}{3}} = \frac{\sqrt{3}}{\sin \left(\frac{2\pi}{3} - \theta \right)} \quad \textbf{A1}$$

$$= \frac{\sqrt{3}}{\sin \frac{2\pi}{3} \cos \theta - \cos \frac{2\pi}{3} \sin \theta} \quad \textbf{A1}$$

Note: Condone use of degrees.

$$= \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta} \quad \textbf{A1}$$

$$\frac{AB}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta}$$

$$\therefore AB = \frac{3}{\sqrt{3} \cos \theta + \sin \theta} \quad \textbf{AG}$$

[4 marks]

Examiners report

[N/A]

7b.

[4 marks]

Markscheme

METHOD 1

$$(AB)' = \frac{-3(-\sqrt{3} \sin \theta + \cos \theta)}{(\sqrt{3} \cos \theta + \sin \theta)^2} \quad \mathbf{M1A1}$$

$$\text{setting } (AB)' = 0 \quad \mathbf{M1}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6} \quad \mathbf{A1}$$

METHOD 2

$$AB = \frac{\sqrt{3} \sin \frac{\pi}{3}}{\sin\left(\frac{2\pi}{3} - \theta\right)}$$

$$AB \text{ minimum when } \sin\left(\frac{2\pi}{3} - \theta\right) \text{ is maximum} \quad \mathbf{M1}$$

$$\sin\left(\frac{2\pi}{3} - \theta\right) = 1 \quad \mathbf{(A1)}$$

$$\frac{2\pi}{3} - \theta = \frac{\pi}{2} \quad \mathbf{M1}$$

$$\theta = \frac{\pi}{6} \quad \mathbf{A1}$$

METHOD 3

$$\text{shortest distance from } B \text{ to } AC \text{ is perpendicular to } AC \quad \mathbf{R1}$$

$$\theta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} \quad \mathbf{M1A2}$$

[4 marks]

Total [8 marks]

Examiners report

[N/A]

8.

[8 marks]

Markscheme

EITHER

$$x = \arctan t \quad (\mathbf{M1})$$

$$\frac{dx}{dt} = \frac{1}{1+t^2} \quad \mathbf{A1}$$

OR

$$t = \tan x$$

$$\frac{dt}{dx} = \sec^2 x \quad (\mathbf{M1})$$

$$= 1 + \tan^2 x \quad \mathbf{A1}$$

$$= 1 + t^2$$

THEN

$$\sin x = \frac{t}{\sqrt{1+t^2}} \quad (\mathbf{A1})$$

Note: This **A1** is independent of the first two marks

$$\int \frac{dx}{1+\sin^2 x} = \int \frac{\frac{dt}{1+t^2}}{1+\left(\frac{t}{\sqrt{1+t^2}}\right)^2} \quad \mathbf{M1A1}$$

Note: Award **M1** for attempting to obtain integral in terms of t and dt

$$= \int \frac{dt}{(1+t^2)+t^2} = \int \frac{dt}{1+2t^2} \quad \mathbf{A1}$$

$$= \frac{1}{2} \int \frac{dt}{\frac{1}{2}+t^2} = \frac{1}{2} \times \frac{1}{\frac{1}{\sqrt{2}}} \arctan\left(\frac{t}{\frac{1}{\sqrt{2}}}\right) \quad \mathbf{A1}$$

$$= \frac{\sqrt{2}}{2} \arctan(\sqrt{2} \tan x) (+c) \quad \mathbf{A1}$$

[8 marks]

Examiners report

[N/A]

9a.

[2 marks]

Markscheme

$$g \circ f(x) = \frac{\tan x + 1}{\tan x - 1} \quad \mathbf{A1}$$

$$x \neq \frac{\pi}{4}, 0 \leq x < \frac{\pi}{2} \quad \mathbf{A1}$$

[2 marks]

Examiners report

[N/A]

Markscheme

$$\frac{\tan x+1}{\tan x-1} = \frac{\frac{\sin x}{\cos x}+1}{\frac{\sin x}{\cos x}-1} \quad \textbf{M1A1}$$

$$= \frac{\sin x+\cos x}{\sin x-\cos x} \quad \textbf{AG}$$

[2 marks]

Examiners report

[N/A]

Markscheme

METHOD 1

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2} \quad \textbf{M1(A1)}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2\sin x\cos x-\cos^2 x-\sin^2 x)-(2\sin x\cos x+\cos^2 x+\sin^2 x)}{\cos^2 x+\sin^2 x-2\sin x\cos x}$$

$$= \frac{-2}{1-\sin 2x}$$

Substitute $\frac{\pi}{6}$ into any formula for $\frac{\mathrm{d}y}{\mathrm{d}x}$ **M1**

$$\frac{-2}{1-\sin \frac{\pi}{3}}$$

$$= \frac{-2}{1-\frac{\sqrt{3}}{2}} \quad \textbf{A1}$$

$$= \frac{-4}{2-\sqrt{3}}$$

$$= \frac{-4}{2-\sqrt{3}}\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right) \quad \textbf{M1}$$

$$= \frac{-8-4\sqrt{3}}{1} = -8-4\sqrt{3} \quad \textbf{A1}$$

METHOD 2

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(\tan x-1)\sec^2 x-(\tan x+1)\sec^2 x}{(\tan x-1)^2} \quad \textbf{M1A1}$$

$$= \frac{-2\sec^2 x}{(\tan x-1)^2} \quad \textbf{A1}$$

$$= \frac{-2\sec^2 \frac{\pi}{6}}{\left(\tan \frac{\pi}{6}-1\right)^2} = \frac{-2\left(\frac{4}{3}\right)}{\left(\frac{1}{\sqrt{3}}-1\right)^2} = \frac{-8}{(1-\sqrt{3})^2} \quad \textbf{M1}$$

Note: Award **M1** for substitution $\frac{\pi}{6}$.

$$\frac{-8}{(1-\sqrt{3})^2} = \frac{-8}{(4-2\sqrt{3})}\frac{(4+2\sqrt{3})}{(4+2\sqrt{3})} = -8-4\sqrt{3} \quad \textbf{M1A1}$$

[6 marks]

Examiners report

[N/A]

9d.

[6 marks]

Markscheme

$$\text{Area} \left| \int_0^{\frac{\pi}{6}} \frac{\sin x + \cos x}{\sin x - \cos x} dx \right| \quad \mathbf{M1}$$

$$= \left| \left[\ln |\sin x - \cos x| \right]_0^{\frac{\pi}{6}} \right| \quad \mathbf{A1}$$

Note: Condone absence of limits and absence of modulus signs at this stage.

$$= \left| \ln \left| \sin \frac{\pi}{6} - \cos \frac{\pi}{6} \right| - \ln |\sin 0 - \cos 0| \right| \quad \mathbf{M1}$$

$$= \left| \ln \left| \frac{1}{2} - \frac{\sqrt{3}}{2} \right| - 0 \right|$$

$$= \left| \ln \left(\frac{\sqrt{3}-1}{2} \right) \right| \quad \mathbf{A1}$$

$$= -\ln \left(\frac{\sqrt{3}-1}{2} \right) = \ln \left(\frac{2}{\sqrt{3}-1} \right) \quad \mathbf{A1}$$

$$= \ln \left(\frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \right) \quad \mathbf{M1}$$

$$= \ln(\sqrt{3} + 1) \quad \mathbf{AG}$$

[6 marks]

Total [16 marks]

Examiners report

[N/A]

10.

[4 marks]

Markscheme

$$\int_{-1}^1 \pi \left(e^{-x^2} \right)^2 dx \quad \left(\int_{-1}^1 \pi e^{-2x^2} dx \quad \text{or} \quad \int_0^1 2\pi e^{-2x^2} dx \right) \quad (\mathbf{M1})(\mathbf{A1})(\mathbf{A1})$$

Note: Award **M1** for integral involving the function given; **A1** for correct limits; **A1** for π and $\left(e^{-x^2} \right)^2$

$$= 3.758249 \dots = 3.76 \quad \mathbf{A1}$$

[4 marks]

Examiners report

Most candidates answered this question correctly. Those candidates who attempted to manipulate the function or attempt an integration wasted time and obtained 3/4 marks. The most common errors were an extra factor '2' and a fourth power when attempting to square the function. Many candidates wrote down the correct expression but not all were able to use their calculator correctly.

11.

[5 marks]

Markscheme

$$V = 200\pi r^2 \quad \mathbf{(A1)}$$

Note: Allow $V = \pi h r^2$ if value of h is substituted later in the question.

EITHER

$$\frac{dV}{dt} = 200\pi 2r \frac{dr}{dt} \quad \mathbf{M1A1}$$

Note: Award **M1** for an attempt at implicit differentiation.

$$\text{at } r = 2 \text{ we have } 30 = 200\pi 4 \frac{dr}{dt} \quad \mathbf{M1}$$

OR

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dr}} \quad \mathbf{M1}$$

$$\frac{dV}{dr} = 400\pi r \quad \mathbf{M1}$$

$$r = 2 \text{ we have } \frac{dV}{dr} = 800\pi \quad \mathbf{A1}$$

THEN

$$\frac{dr}{dt} = \frac{30}{800\pi} \quad \left(= \frac{3}{80\pi} = 0.0119 \right) \text{ (cm s}^{-1}\text{)} \quad \mathbf{A1}$$

[5 marks]

Examiners report

This question was well understood and a large percentage appreciated the need for implicit differentiation although some candidates did not recognise the need to treat h as a constant till late in the question. A number of candidates found the answer $\frac{3\pi}{80}$ instead of $\frac{3}{80\pi}$ due to a basic incorrect use of the GDC.

12.

[4 marks]

Markscheme

$$f'(x) = 3x^2 + e^x \quad \mathbf{A1}$$

Note: Accept labelled diagram showing the graph $y = f'(x)$ above the x-axis;
do not accept unlabelled graphs nor graph of $y = f(x)$.

EITHER

this is always > 0 **R1**

so the function is (strictly) increasing **R1**

and thus $1 - 1$ **A1**

OR

this is always > 0 (accept $\neq 0$) **R1**

so there are no turning points **R1**

and thus $1 - 1$ **A1**

Note: **A1** is dependent on the first **R1**.

[4 marks]

Examiners report

The differentiation was normally completed correctly, but then a large number did not realise what was required to determine the type of the original function. Most candidates scored 1/4 and wrote explanations that showed little or no understanding of the relation between first derivative and the given function. For example, it was common to see comments about horizontal and vertical line tests but applied to the incorrect function. In term of mathematical language, it was noted that candidates used many terms incorrectly showing no knowledge of the meaning of terms like 'parabola', 'even' or 'odd' (or no idea about these concepts).

13.

[7 marks]

Markscheme

$$x = 0 \Rightarrow y = 1 \quad (\mathbf{A1})$$

$$y'(0) = 1.367879 \dots \quad (\mathbf{M1})(\mathbf{A1})$$

Note: The exact answer is $y'(0) = \frac{e+1}{e} = 1 + \frac{1}{e}$.

so gradient of normal is $\frac{-1}{1.367879 \dots}$ ($= -0.731058 \dots$) **(M1)(A1)**

equation of normal is $y = -0.731058 \dots x + c$ **(M1)**

gives $y = -0.731x + 1$ **A1**

Note: The exact answer is $y = -\frac{e}{e+1}x + 1$.

Accept $y - 1 = -0.731058 \dots (x - 0)$

[7 marks]

Examiners report

Surprisingly many candidates ignored that fact that paper 2 is a calculator paper, attempted an algebraic approach and wasted lots of time. Candidates that used the GDC were in general successful and achieved 7/7. A number of candidates either found the equation of the tangent or used the positive reciprocal for the normal and many did not find the value of y corresponding to $f(0)$.

14a. [6 marks]

Markscheme

(i) $a(t) = \frac{dv}{dt} = -10 \text{ (ms}^{-2}\text{)}$ **A1**

(ii) $t = 10 \Rightarrow v = -100 \text{ (ms}^{-1}\text{)}$ **A1**

(iii) $s = \int -10t dt = -5t^2 (+c)$ **M1A1**

$s = 1000$ for $t = 0 \Rightarrow c = 1000$ **(M1)**

$s = -5t^2 + 1000$ **A1**

at $t = 10$, $s = 500 \text{ (m)}$ **AG**

Note: Accept use of definite integrals.

[6 marks]

Examiners report

Parts (i) and (ii) were well answered by most candidates.

In (iii) the constant of integration was often forgotten. Most candidates calculated the displacement and then used different strategies, mostly incorrect, to remove the negative sign from -500 .

14b. [1 mark]

Markscheme

$\frac{dt}{dv} = \frac{1}{(-10-5v)}$ **A1**

[1 mark]

Examiners report

Surprisingly part (b) was not well done as the question stated the method. Many candidates simply wrote down $\frac{dv}{dt}$ while others seemed unaware that $\frac{dv}{dt}$ was the acceleration.

Markscheme

METHOD 1

$$t = \int \frac{1}{-10-5v} dv = -\frac{1}{5} \ln(-10-5v)(+c) \quad \mathbf{M1A1}$$

Note: Accept equivalent forms using modulus signs.

$$t = 10, v = -100$$

$$10 = -\frac{1}{5} \ln(490) + c \quad \mathbf{M1}$$

$$c = 10 + \frac{1}{5} \ln(490) \quad \mathbf{A1}$$

$$t = 10 + \frac{1}{5} \ln 490 - \frac{1}{5} \ln(-10-5v) \quad \mathbf{A1}$$

Note: Accept equivalent forms using modulus signs.

$$t = 10 + \frac{1}{5} \ln\left(\frac{98}{-2-v}\right) \quad \mathbf{AG}$$

Note: Accept use of definite integrals.

METHOD 2

$$t = \int \frac{1}{-10-5v} dv = -\frac{1}{5} \int \frac{1}{2+v} dv = -\frac{1}{5} \ln|2+v|(+c) \quad \mathbf{M1A1}$$

Note: Accept equivalent forms.

$$t = 10, v = -100$$

$$10 = -\frac{1}{5} \ln|-98| + c \quad \mathbf{M1}$$

Note: If $\ln(-98)$ is seen do not award further A marks.

$$c = 10 + \frac{1}{5} \ln 98 \quad \mathbf{A1}$$

$$t = 10 + \frac{1}{5} \ln 98 - \frac{1}{5} \ln|2+v| \quad \mathbf{A1}$$

Note: Accept equivalent forms.

$$t = 10 + \frac{1}{5} \ln\left(\frac{98}{-2-v}\right) \quad \mathbf{AG}$$

Note: Accept use of definite integrals.

[5 marks]

Examiners report

Part (c) was not always well done as it followed from (b) and at times there was very little to allow follow through. Once again some candidates started with what they were trying to prove. Among the candidates that attempted to integrate many did not consider the constant of integration properly.

14d. [2 marks]

Markscheme

$$5(t - 10) = \ln \frac{98}{(-2-v)}$$

$$\frac{2+v}{98} = -e^{-5(t-10)} \quad \textbf{(M1)}$$

$$v = -2 - 98e^{-5(t-10)} \quad \textbf{A1}$$

[2 marks]

Examiners report

In part (d) many candidates ignored the answer given in (c) and attempted to manipulate different expressions.

14e. [5 marks]

Markscheme

$$\frac{ds}{dt} = -2 - 98e^{-5(t-10)}$$

$$s = -2t + \frac{98}{5}e^{-5(t-10)} (+k) \quad \textbf{M1A1}$$

$$\text{at } t = 10, s = 500 \Rightarrow 500 = -20 + \frac{98}{5} + k \Rightarrow k = 500.4 \quad \textbf{M1A1}$$

$$s = -2t + \frac{98}{5}e^{-5(t-10)} + 500.4 \quad \textbf{A1}$$

Note: Accept use of definite integrals.

[5 marks]

Examiners report

Part (e) was poorly answered: the constant of integration was often again forgotten and some inappropriate uses of Physics formulas assuming that the acceleration was constant were used. There was unclear thinking with the two sides of an equation being integrated with respect to different variables.

14f. [2 marks]

Markscheme

$$t = 250 \text{ for } s = 0 \quad \textbf{(M1)A1}$$

[2 marks]

Total [21 marks]

Examiners report

Although part (e) was often incorrect, some follow through marks were gained in part (f).

15a. [4 marks]

Markscheme

EITHER

$$y = \ln(x - a) + b = \ln(5x + 10) \quad \textbf{(M1)}$$

$$y = \ln(x - a) + \ln c = \ln(5x + 10)$$

$$y = \ln(c(x - a)) = \ln(5x + 10) \quad \textbf{(M1)}$$

OR

$$y = \ln(5x + 10) = \ln(5(x + 2)) \quad \textbf{(M1)}$$

$$y = \ln(5) + \ln(x + 2) \quad \textbf{(M1)}$$

THEN

$$a = -2, b = \ln 5 \quad \textbf{A1A1}$$

Note: Accept graphical approaches.

Note: Accept $a = 2, b = 1.61$

[4 marks]

Examiners report

[N/A]

15b. [2 marks]

Markscheme

$$V = \pi \int_e^{2e} [\ln(5x + 10)]^2 dx \quad \textbf{(M1)}$$

$$= 99.2 \quad \textbf{A1}$$

[2 marks]

Total [6 marks]

Examiners report

[N/A]

16a. [3 marks]

Markscheme

attempt at implicit differentiation **M1**

$$2x - 5x \frac{dy}{dx} - 5y + 2y \frac{dy}{dx} = 0 \quad \textbf{A1A1}$$

Note: **A1** for differentiation of $x^2 - 5xy$, **A1** for differentiation of y^2 and 7.

$$2x - 5y + \frac{dy}{dx}(2y - 5x) = 0$$

$$\frac{dy}{dx} = \frac{5y-2x}{2y-5x} \quad \textbf{AG}$$

[3 marks]

16b. [4 marks]

Markscheme

$$\frac{dy}{dx} = \frac{5 \times 1 - 2 \times 6}{2 \times 1 - 5 \times 6} = \frac{1}{4} \quad \mathbf{A1}$$

$$\text{gradient of normal} = -4 \quad \mathbf{A1}$$

$$\text{equation of normal } y = -4x + c \quad \mathbf{M1}$$

substitution of (6, 1)

$$y = -4x + 25 \quad \mathbf{A1}$$

Note: Accept $y - 1 = -4(x - 6)$

[4 marks]

Examiners report

[N/A]

16c. [8 marks]

Markscheme

$$\text{setting } \frac{5y-2x}{2y-5x} = 1 \quad \mathbf{M1}$$

$$y = -x \quad \mathbf{A1}$$

substituting into original equation $\mathbf{M1}$

$$x^2 + 5x^2 + x^2 = 7 \quad (\mathbf{A1})$$

$$7x^2 = 7$$

$$x = \pm 1 \quad \mathbf{A1}$$

points (1, -1) and (-1, 1) $(\mathbf{A1})$

$$\text{distance} = \sqrt{8} \quad (= 2\sqrt{2}) \quad (\mathbf{M1})\mathbf{A1}$$

[8 marks]

Total [15 marks]

Examiners report

[N/A]

17a. [3 marks]

$$s = \int (9t - 3t^2) dt = \frac{9}{2}t^2 - t^3 (+c)$$

$$t = 0, s = 3 \Rightarrow c = 3$$

$$t = 4 \Rightarrow s = 11$$

$$s = 3 + \int_0^4 (9t - 3t^2) dt$$

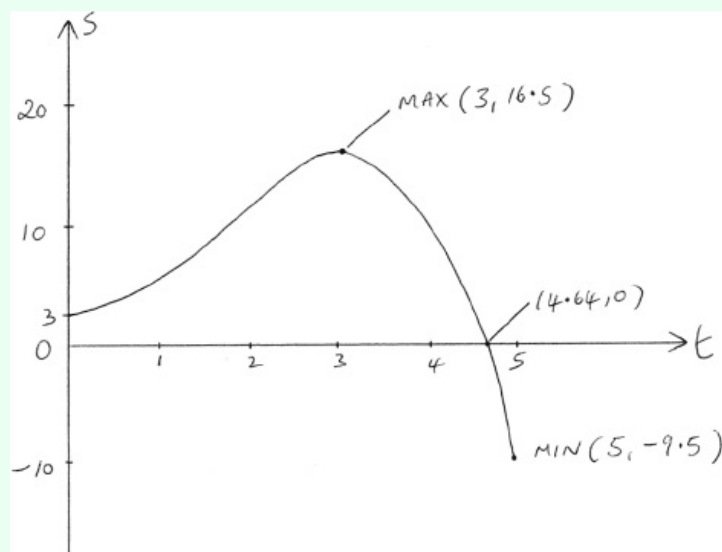
$$s = 11$$

Examiners report

[N/A]

17b. [5 marks]

Markscheme



correct shape over correct domain **A1**

maximum at (3, 16.5) **A1**

t intercept at 4.64, s intercept at 3 **A1**

minimum at (5, -9.5) **A1**

[5 marks]

Examiners report

[N/A]

17c. [3 marks]

Markscheme

$$-9.5 = a + b \cos 2\pi$$

$$16.5 = a + b \cos 3\pi \quad (\mathbf{M1})$$

Note: Only award **M1** if two simultaneous equations are formed over the correct domain.

$$a = \frac{7}{2} \quad \mathbf{A1}$$

$$b = -13 \quad \mathbf{A1}$$

[3 marks]

Examiners report

[N/A]

17d. [4 marks]

Markscheme

at t_1 :

$$3 + \frac{9}{2}t^2 - t^3 = 3 \quad (M1)$$

$$t^2 \left(\frac{9}{2} - t \right) = 0$$

$$t_1 = \frac{9}{2} \quad A1$$

$$\text{solving } \frac{7}{2} - 13 \cos \frac{2\pi t}{5} = 3 \quad (M1)$$

$$\text{GDC} \Rightarrow t_2 = 6.22 \quad A1$$

Note: Accept graphical approaches.

[4 marks]

Total [15 marks]

Examiners report

[N/A]

18a. [2 marks]

Markscheme

$$\cos x = 2\cos^2 \frac{1}{2}x - 1$$

$$\cos \frac{1}{2}x = \pm \sqrt{\frac{1+\cos x}{2}} \quad M1$$

positive as

$$0 \leq x \leq \pi \quad R1$$

$$\cos \frac{1}{2}x = \sqrt{\frac{1+\cos x}{2}} \quad AG$$

[2 marks]

Examiners report

[N/A]

18b. [2 marks]

Markscheme

$$\cos 2\theta = 1 - 2\sin^2 \theta \quad (M1)$$

$$\sin \frac{1}{2}x = \sqrt{\frac{1-\cos x}{2}} \quad A1$$

[2 marks]

Examiners report

[N/A]

18c. [4 marks]

Markscheme

$$\begin{aligned} & \sqrt{2} \int_0^{\frac{\pi}{2}} \cos \frac{1}{2}x + \sin \frac{1}{2}x dx \quad \text{AI} \\ &= \sqrt{2} \left[2 \sin \frac{1}{2}x - 2 \cos \frac{1}{2}x \right]_0^{\frac{\pi}{2}} \quad \text{AI} \\ &= \sqrt{2}(0) - \sqrt{2}(0 - 2) \quad \text{AI} \\ &= 2\sqrt{2} \quad \text{AI} \end{aligned}$$

[4 marks]

Examiners report

[N/A]

19. [6 marks]

Markscheme

(a)

$$x = 1 \quad \text{AI}$$

[1 mark]

(b) AI for point $(-4, 0)$

AI for $(0, -4)$

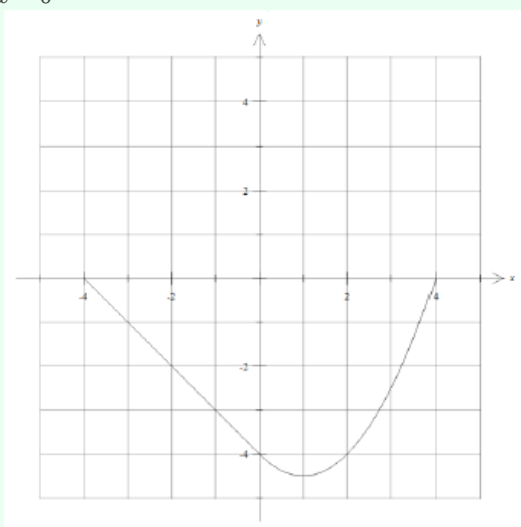
AI for min at

$x = 1$ in approximately the correct place

AI for $(4, 0)$

AI for shape including continuity at

$x = 0$



[5 marks]

Total [6 marks]

Examiners report

[N/A]

20.

[6 marks]

Markscheme

$$\frac{dv}{ds} = 2s^{-3} \quad \text{MIAI}$$

Note: Award **MI** for $2s^{-3}$ and **AI** for the whole expression.

$$a = v \frac{dv}{ds} \quad (\text{MI})$$

$$a = -\frac{1}{s^2} \times \frac{2}{s^3} \left(= -\frac{2}{s^5} \right) \quad (\text{AI})$$

when

$$s = \frac{1}{2}, a = -\frac{2}{(0.5)^5} (= -64) \text{ (ms}^{-2}\text{)} \quad \text{MIAI}$$

Note: **MI** is for the substitution of 0.5 into their equation for acceleration.

Award **MIA0** if

$s = 50$ is substituted into the correct equation.

[6 marks]

Examiners report

[N/A]

21.

[9 marks]

Markscheme

(a) **METHOD 1**

$$\frac{2x}{1+x^4} + \frac{2y}{1+y^4} \frac{dy}{dx} = 0 \quad \text{MIAIAI}$$

Note: Award **MI** for implicit differentiation, **AI** for LHS and **AI** for RHS.

$$\frac{dy}{dx} = -\frac{x(1+y^4)}{y(1+x^4)} \quad \text{AI}$$

METHOD 2

$$y^2 = \tan\left(\frac{\pi}{4} - \arctan x^2\right) \\ = \frac{\tan \frac{\pi}{4} - \tan(\arctan x^2)}{1 + \left(\tan \frac{\pi}{4}\right)(\tan(\arctan x^2))} \quad (\text{MI})$$

$$= \frac{1-x^2}{1+x^2} \quad \text{AI}$$

$$2y \frac{dy}{dx} = \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} \quad \text{MI}$$

$$2y \frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = -\frac{2x}{y(1+x^2)^2} \quad \text{AI}$$

$$\left(= \frac{2x\sqrt{1+x^2}}{\sqrt{1-x^2}(1+x^2)^2} \right)$$

[4 marks]

(b)

$$y^2 = \tan\left(\frac{\pi}{4} - \arctan \frac{1}{2}\right) \quad (\text{MI})$$

$$= \frac{\tan \frac{\pi}{4} - \tan\left(\arctan \frac{1}{2}\right)}{1 + \left(\tan \frac{\pi}{4}\right)\left(\tan\left(\arctan \frac{1}{2}\right)\right)} \quad (\text{MI})$$

Note: The two **MI**s may be awarded for working in part (a).

$$= \frac{1-\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3} \quad \text{AI}$$

$$y = -\frac{1}{\sqrt{3}} \quad \text{AI}$$

substitution into

$$\frac{dy}{dx} \\ = \frac{4\sqrt{6}}{9} \quad \text{AI}$$

Note: Accept

$$\frac{8\sqrt{3}}{9\sqrt{2}} \text{ etc.}$$

[5 marks]**Total [9 marks]**

Examiners report

[N/A]

22a.

[2 marks]

Markscheme

$$f'(x) = \frac{x \times \frac{1}{x} - \ln x}{x^2} \quad \text{MIAI}$$

$$= \frac{1-\ln x}{x^2} \quad \text{AG}$$

[2 marks]

Examiners report

[N/A]

22b. [3 marks]

Markscheme

$$\frac{1-\ln x}{x^2} = 0 \text{ has solution}$$

$$x = e \quad \text{MIAI}$$

$$y = \frac{1}{e} \quad \text{AI}$$

hence maximum at the point

$$\left(e, \frac{1}{e}\right)$$

[3 marks]

Examiners report

[N/A]

22c. [5 marks]

Markscheme

$$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - 2x(1-\ln x)}{x^4} \quad \text{MIAI}$$

$$= \frac{2\ln x - 3}{x^3}$$

Note: The *MIAI* should be awarded if the correct working appears in part (b).

point of inflexion where

$$f''(x) = 0 \quad \text{MI}$$

so

$$x = e^{\frac{3}{2}}, y = \frac{3}{2}e^{-\frac{3}{2}} \quad \text{AIAI}$$

C has coordinates

$$\left(e^{\frac{3}{2}}, \frac{3}{2}e^{-\frac{3}{2}}\right)$$

[5 marks]

Examiners report

[N/A]

22d. [4 marks]

Markscheme

$$f(1) = 0 \quad \text{AI}$$

$$f'(1) = 1 \quad (\text{AI})$$

$$y = x + c \quad (\text{MI})$$

through (1, 0)

equation is

$$y = x - 1 \quad \text{AI}$$

[4 marks]

Examiners report

[N/A]

22e.

[7 marks]

Markscheme

METHOD 1

area

$$= \int_1^e x - 1 - \frac{\ln x}{x} dx \quad MIAIAI$$

Note: Award **MI** for integration of difference between line and curve, **AI** for correct limits, **AI** for correct expressions in either order.

$$\int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} (+c) \quad (MI)AI$$

$$\int (x - 1) dx = \frac{x^2}{2} - x (+c) \quad AI$$

$$= \left[\frac{1}{2}x^2 - x - \frac{1}{2}(\ln x)^2 \right]_1^e$$

$$= \left(\frac{1}{2}e^2 - e - \frac{1}{2} \right) - \left(\frac{1}{2} - 1 \right)$$

$$= \frac{1}{2}e^2 - e \quad AI$$

METHOD 2

area = area of triangle

$$- \int_1^e \frac{\ln x}{x} dx \quad MIAI$$

Note: **AI** is for correct integral with limits and is dependent on the **MI**.

$$\int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} (+c) \quad (MI)AI$$

area of triangle

$$= \frac{1}{2}(e - 1)(e - 1) \quad MIAI$$

$$\frac{1}{2}(e - 1)(e - 1) - \left(\frac{1}{2} \right) = \frac{1}{2}e^2 - e \quad AI$$

[7 marks]

Examiners report

[N/A]

23a.

[2 marks]

Markscheme

$$1 - 2(2) = -3 \text{ and}$$

$$\frac{3}{4}(2 - 2)^2 - 3 = -3 \quad AI$$

both answers are the same, hence f is continuous (at $x = 2$) **RI**

Note: **RI** may be awarded for justification using a graph or referring to limits. Do not award **A0RI**.

[2 marks]

Examiners report

[N/A]

23b.

[4 marks]

Markscheme

reflection in the y-axis

$$f(-x) = \begin{cases} 1 + 2x, & x \geq -2 \\ \frac{3}{4}(x+2)^2 - 3, & x < -2 \end{cases} \quad (MI)$$

Note: Award *MI* for evidence of reflecting a graph in y-axis.

translation

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$g(x) = \begin{cases} 2x - 3, & x \geq 0 \\ \frac{3}{4}x^2 - 3, & x < 0 \end{cases} \quad (MI)A1A1$$

Note: Award (*MI*) for attempting to substitute $(x - 2)$ for x , or translating a graph along positive x -axis.

Award *A1* for the correct domains (this mark can be awarded independent of the *MI*).

Award *A1* for the correct expressions.

[4 marks]

Examiners report

[N/A]

24.

[7 marks]

Markscheme

$$x = a \sec \theta$$

$$\frac{dx}{d\theta} = a \sec \theta \tan \theta \quad (A1)$$

new limits:

$$x = a\sqrt{2} \Rightarrow \theta = \frac{\pi}{4} \text{ and}$$

$$x = 2a \Rightarrow \theta = \frac{\pi}{3} \quad (A1)$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{a \sec \theta \tan \theta}{a^3 \sec^3 \theta \sqrt{a^2 \sec^2 \theta - a^2}} d\theta \quad MI$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos^2 \theta}{a^3} d\theta \quad A1$$

using

$$\cos^2 \theta = \frac{1}{2}(\cos 2\theta + 1) \quad MI$$

$$\frac{1}{2a^3} \left[\frac{1}{2} \sin 2\theta + \theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} \text{ or equivalent} \quad A1$$

$$= \frac{1}{4a^3} \left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3} - 1 - \frac{\pi}{2} \right) \text{ or equivalent} \quad A1$$

$$= \frac{1}{24a^3} (3\sqrt{3} + \pi - 6) \quad AG$$

[7 marks]

Examiners report

[N/A]

25a. [2 marks]

Markscheme

(a)

$$f'(x) = \frac{(x^2+1)-2x(x+1)}{(x^2+1)^2} \left(= \frac{-x^2-2x+1}{(x^2+1)^2} \right) \quad M1A1$$

[2 marks]

Examiners report

[N/A]

25b. [1 mark]

Markscheme

$$\frac{-x^2-2x+1}{(x^2+1)^2} = 0$$

$$x = -1 \pm \sqrt{2} \quad A1$$

[1 mark]

Examiners report

[N/A]

25c. [3 marks]

Markscheme

$$f''(x) = \frac{(-2x-2)(x^2+1)^2 - 2(2x)(x^2+1)(-x^2-2x+1)}{(x^2+1)^4} \quad A1A1$$

Note: Award **A1** for

$(-2x-2)(x^2+1)^2$ or equivalent.

Note: Award **A1** for

$-2(2x)(x^2+1)(-x^2-2x+1)$ or equivalent.

$$= \frac{(-2x-2)(x^2+1) - 4x(-x^2-2x+1)}{(x^2+1)^3}$$

$$= \frac{2x^3+6x^2-6x-2}{(x^2+1)^3} \quad A1$$

$$\left(= \frac{2(x^3+3x^2-3x-1)}{(x^2+1)^3} \right)$$

[3 marks]

Examiners report

[N/A]

25d. [4 marks]

Markscheme

recognition that

$(x - 1)$ is a factor **(R1)**

$$(x - 1)(x^2 + bx + c) = (x^3 + 3x^2 - 3x - 1) \quad \textbf{M1}$$

$$\Rightarrow x^2 + 4x + 1 = 0 \quad \textbf{A1}$$

$$x = -2 \pm \sqrt{3} \quad \textbf{A1}$$

Note: Allow long division / synthetic division.

[4 marks]

Examiners report

[N/A]

25e. [6 marks]

Markscheme

$$\int_{-1}^0 \frac{x+1}{x^2+1} dx \quad \textbf{M1}$$

$$\int \frac{x+1}{x^2+1} dx = \int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx \quad \textbf{M1}$$

$$= \frac{1}{2} \ln(x^2 + 1) + \arctan(x) \quad \textbf{A1A1}$$

$$= \left[\frac{1}{2} \ln(x^2 + 1) + \arctan(x) \right]_{-1}^0 = \frac{1}{2} \ln 1 + \arctan 0 - \frac{1}{2} \ln 2 - \arctan(-1) \quad \textbf{M1}$$

$$= \frac{\pi}{4} - \ln \sqrt{2} \quad \textbf{A1}$$

[6 marks]

Examiners report

[N/A]

26. [6 marks]

Markscheme

use of the quotient rule or the product rule **M1**

$$C'(t) = \frac{(3+t^2) \times 2 - 2t \times 2t}{(3+t^2)^2} \quad \left(= \frac{6-2t^2}{(3+t^2)^2} \right) \quad \text{or} \quad \frac{2}{3+t^2} - \frac{4t^2}{(3+t^2)^2} \quad \textbf{A1A1}$$

Note: Award **A1** for a correct numerator and **A1** for a correct denominator in the quotient rule, and **A1** for each correct term in the product rule.

attempting to solve $C'(t) = 0$ for t **(M1)**

$$t = \pm \sqrt{3} \quad (\text{minutes}) \quad \textbf{A1}$$

$$C(\sqrt{3}) = \frac{\sqrt{3}}{3} \quad (\text{mg l}^{-1}) \quad \text{or equivalent.} \quad \textbf{A1}$$

[6 marks]

Examiners report

This question was generally well done. A significant number of candidates did not calculate the maximum value of C .

27.

[6 marks]

Markscheme

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad \mathbf{A1}$$

$$dx = 2(u-1)du$$

Note: Award the **A1** for any correct relationship between dx and du .

$$\int \frac{\sqrt{x}}{1+\sqrt{x}} dx = 2 \int \frac{(u-1)^2}{u} du \quad (\mathbf{M1})\mathbf{A1}$$

Note: Award the **M1** for an attempt at substitution resulting in an integral only involving u .

$$= 2 \int u - 2 + \frac{1}{u} du \quad (\mathbf{A1})$$

$$= u^2 - 4u + 2 \ln u (+C) \quad \mathbf{A1}$$

$$= x - 2\sqrt{x} - 3 + 2 \ln(1 + \sqrt{x}) (+C) \quad \mathbf{A1}$$

Note: Award the **A1** for a correct expression in x , but not necessarily fully expanded/simplified.

[6 marks]

Examiners report

Many candidates worked through this question successfully. A significant minority either made algebraic mistakes with the substitution or tried to work with an integral involving both x and u .

28a.

[2 marks]

Markscheme

$$p'(3) = f'(3)g(3) + g'(3)f(3) \quad (\mathbf{M1})$$

Note: Award **M1** if the derivative is in terms of x or 3.

$$= 2 \times 4 + 3 \times 1$$

$$= 11 \quad \mathbf{A1}$$

[2 marks]

Examiners report

This was a problem question for many candidates. Some quite strong candidates, on the evidence of their performance on other questions, did not realise that ‘composite functions’ and ‘functions of a function’ were the same thing, and therefore that the chain rule applied.

28b.

[4 marks]

Markscheme

$$h'(x) = g'(f(x)) f'(x) \quad (\mathbf{M1})(\mathbf{A1})$$

$$h'(2) = g'(1) f'(2) \quad \mathbf{A1}$$

$$= 4 \times 4$$

$$= 16 \quad \mathbf{A1}$$

[4 marks]

Total [6 marks]

Examiners report

This was a problem question for many candidates. Some quite strong candidates, on the evidence of their performance on other questions, did not realise that ‘composite functions’ and ‘functions of a function’ were the same thing, and therefore that the chain rule applied.

29a. [4 marks]

Markscheme

(i) $x = e^{3y+1}$ **M1**

Note: The **M1** is for switching variables and can be awarded at any stage.

Further marks do not rely on this mark being awarded.

taking the natural logarithm of both sides and attempting to transpose **M1**

$(f^{-1}(x)) = \frac{1}{3}(\ln x - 1)$ **A1**

(ii) $x \in \mathbb{R}^+$ or equivalent, for example $x > 0$. **A1**

[4 marks]

Examiners report

Generally very well done, even by candidates who had shown considerable weaknesses elsewhere on the paper.

29b. [5 marks]

Markscheme

$\ln x = \frac{1}{3}(\ln x - 1) \Rightarrow \ln x - \frac{1}{3}\ln x = -\frac{1}{3}$ (or equivalent) **M1A1**

$\ln x = -\frac{1}{2}$ (or equivalent) **A1**

$x = e^{-\frac{1}{2}}$ **A1**

coordinates of P are $\left(e^{-\frac{1}{2}}, -\frac{1}{2}\right)$ **A1**

[5 marks]

Examiners report

Generally very well done, even by candidates who had shown considerable weaknesses elsewhere on the paper.

29c. [3 marks]

Markscheme

coordinates of Q are $(1, 0)$ seen anywhere **A1**

$\frac{dy}{dx} = \frac{1}{x}$ **M1**

at Q , $\frac{dy}{dx} = 1$ **A1**

$y = x - 1$ **AG**

[3 marks]

Examiners report

Generally very well done, even by candidates who had shown considerable weaknesses elsewhere on the paper.

29d.

[5 marks]

Markscheme

let the required area be A

$$A = \int_1^e x - 1 dx - \int_1^e \ln x dx \quad \mathbf{M1}$$

Note: The **M1** is for a difference of integrals. Condone absence of limits here.

attempting to use integration by parts to find $\int \ln x dx$ (**M1**)

$$= \left[\frac{x^2}{2} - x \right]_1^e - [x \ln x - x]_1^e \quad \mathbf{A1A1}$$

Note: Award **A1** for $\frac{x^2}{2} - x$ and **A1** for $x \ln x - x$.

Note: The second **M1** and second **A1** are independent of the first **M1** and the first **A1**.

$$= \frac{e^2}{2} - e - \frac{1}{2} \left(= \frac{e^2 - 2e - 1}{2} \right) \quad \mathbf{A1}$$

[5 marks]

Examiners report

A productive question for many candidates, but some didn't realise that a difference of areas/integrals was required.

Markscheme

(i) **METHOD 1**

consider for example $h(x) = x - 1 - \ln x$

$$h(1) = 0 \quad \text{and} \quad h'(x) = 1 - \frac{1}{x} \quad \textbf{(A1)}$$

as $h'(x) \geq 0$ for $x \geq 1$, then $h(x) \geq 0$ for $x \geq 1$ **R1**

as $h'(x) \leq 0$ for $0 < x \leq 1$, then $h(x) \geq 0$ for $0 < x \leq 1$ **R1**

so $g(x) \leq x - 1, x \in \mathbb{R}^+$ **AG**

METHOD 2

$$g''(x) = -\frac{1}{x^2} \quad \textbf{A1}$$

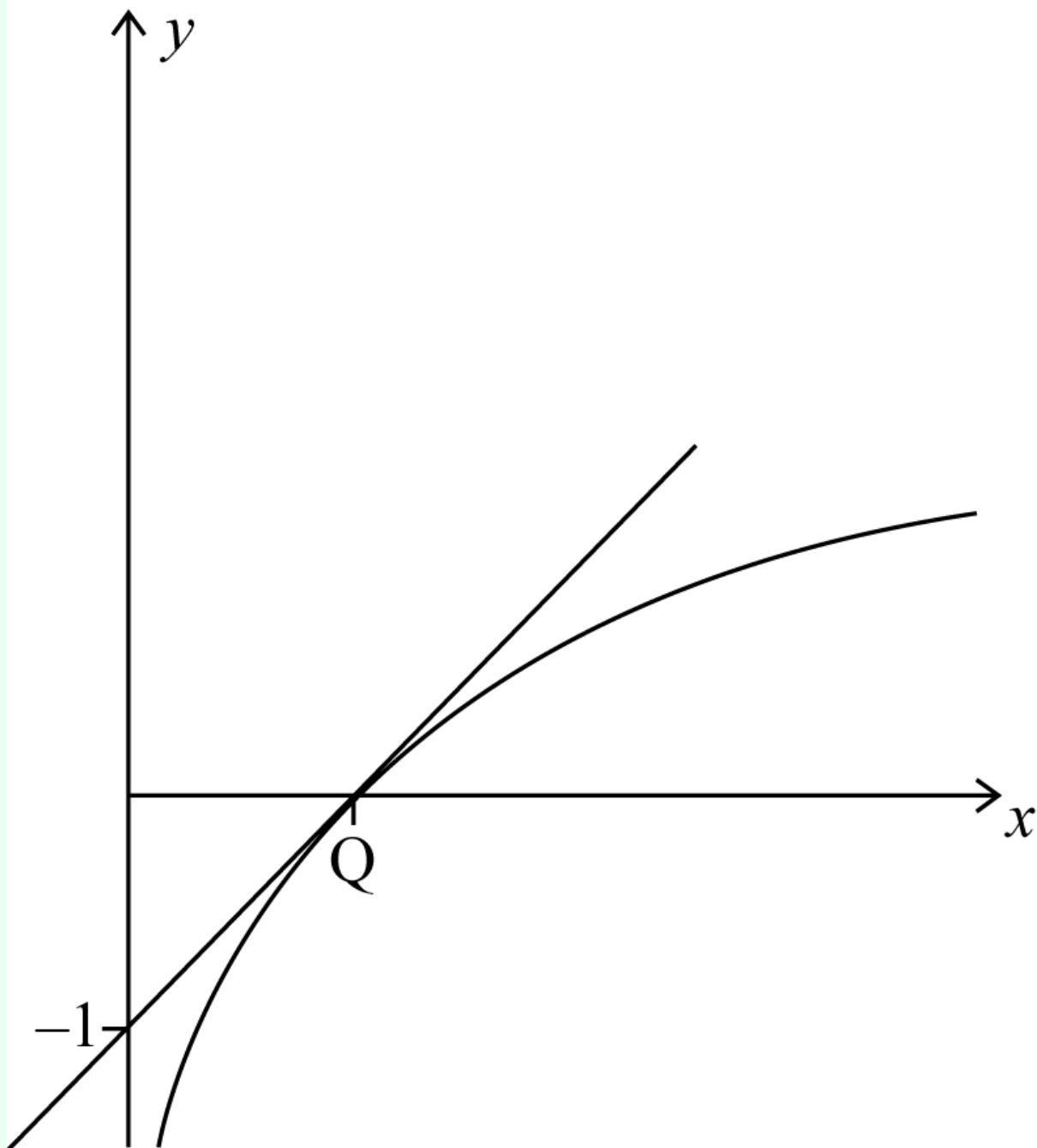
$g''(x) < 0$ (concave down) for $x \in \mathbb{R}^+$ **R1**

the graph of $y = g(x)$ is below its tangent ($y = x - 1$ at $x = 1$) **R1**

so $g(x) \leq x - 1, x \in \mathbb{R}^+$ **AG**

Note: The reasoning may be supported by drawn graphical arguments.

METHOD 3



clear correct graphs of $y = x - 1$ and $\ln x$ for $x > 0$ **A1A1**

statement to the effect that the graph of $\ln x$ is below the graph of its tangent at $x = 1$ **R1AG**

(ii) replacing x by $\frac{1}{x}$ to obtain $\ln\left(\frac{1}{x}\right) \leq \frac{1}{x} - 1 \left(= \frac{1-x}{x}\right)$ **M1**

$-\ln x \leq \frac{1}{x} - 1 \left(= \frac{1-x}{x}\right)$ **(A1)**

$\ln x \geq 1 - \frac{1}{x} \left(= \frac{x-1}{x}\right)$ **A1**

so $\frac{x-1}{x} \leq g(x)$, $x \in \mathbb{R}^+$ **AG**

[6 marks]

Total [23 marks]

Examiners report

(i) Many candidates adopted a graphical approach, but sometimes with unconvincing reasoning.

(ii) Poorly answered. Many candidates applied the suggested substitution only to one side of the inequality, and then had to fudge the answer.

Markscheme

METHOD 1

attempt to set up (diagram, vectors) **(M1)**

correct distances $x = 15t$, $y = 20t$ **(A1) (A1)**

the distance between the two cyclists at time t is $s = \sqrt{(15t)^2 + (20t)^2} = 25t$ (km) **A1**

$$\frac{ds}{dt} = 25 \text{ (km h}^{-1}\text{)} \quad \mathbf{A1}$$

hence the rate is independent of time **AG**

METHOD 2

attempting to differentiate $x^2 + y^2 = s^2$ implicitly **(M1)**

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2s \frac{ds}{dt} \quad \mathbf{(A1)}$$

the distance between the two cyclists at time t is $\sqrt{(15t)^2 + (20t)^2} = 25t$ (km) **(A1)**

$$2(15t)(15 + 2(20t)(20)) = 2(25t) \frac{ds}{dt} \quad \mathbf{M1}$$

Note: Award **M1** for substitution of correct values into their equation involving $\frac{ds}{dt}$.

$$\frac{ds}{dt} = 25 \text{ (km h}^{-1}\text{)} \quad \mathbf{A1}$$

hence the rate is independent of time **AG**

METHOD 3

$$s = \sqrt{x^2 + y^2} \quad \mathbf{(A1)}$$

$$\frac{ds}{dt} = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{\sqrt{x^2 + y^2}} \quad \mathbf{(M1)(A1)}$$

Note: Award **M1** for attempting to differentiate the expression for s .

$$\frac{ds}{dt} = \frac{(15t)(15) + (20t)(20)}{\sqrt{(15t)^2 + (20t)^2}} \quad \mathbf{M1}$$

Note: Award **M1** for substitution of correct values into their $\frac{ds}{dt}$.

$$\frac{ds}{dt} = 25 \text{ (km h}^{-1}\text{)} \quad \mathbf{A1}$$

hence the rate is independent of time **AG**

[5 marks]

Examiners report

Reasonably well done. Most successful candidates determined that $s = 25t \Rightarrow \frac{ds}{dt} = 25$ from $x = 15t$ and $y = 20t$. A number of candidates did not use calculus while a few candidates correctly used implicit differentiation.

31a. [2 marks]

Markscheme

$$3 - \frac{t}{2} = 0 \Rightarrow t = 6 \text{ (s)} \quad \textbf{(M1)(A1)}$$

Note: Award **A0** if either $t = -0.236$ or $t = 4.24$ or both are stated with $t = 6$.

[2 marks]

Examiners report

Part (a) was not done as well as expected. A large number of candidates attempted to solve $5 - (t - 2)^2 = 0$ for t . Some candidates attempted to find when the particle's acceleration was zero.

31b. [5 marks]

Markscheme

let

d be the distance travelled before coming to rest

$$d = \int_0^4 5 - (t - 2)^2 dt + \int_4^6 3 - \frac{t}{2} dt \quad \textbf{(M1)(A1)}$$

Note: Award **M1** for two correct integrals even if the integration limits are incorrect. The second integral can be specified as the area of a triangle.

$$d = \frac{47}{3} \quad (= 15.7) \text{ (m)} \quad \textbf{(A1)}$$

attempting to solve $\int_6^T \left(\frac{t}{2} - 3\right) dt = \frac{47}{3}$ (or equivalent) for T **M1**

$$T = 13.9 \text{ (s)} \quad \textbf{A1}$$

[5 marks]

Total [7 marks]

Examiners report

Most candidates had difficulty with part (b) with a variety of errors committed. A significant proportion of candidates did not understand what was required. Many candidates worked with indefinite integrals rather than with definite integrals. Only a small percentage of candidates started by correctly finding the distance travelled by the particle before coming to rest. The occasional candidate made adroit use of a GDC and found the correct value of t by finding where the graph of $\int_0^4 5 - (t - 2)^2 dt + \int_4^x 3 - \frac{t}{2} dt$ crossed the horizontal axis.

32a. [2 marks]

Markscheme

use of $A = \frac{1}{2}qr \sin \theta$ to obtain $A = \frac{1}{2}(x + 2)(5 - x)^2 \sin 30^\circ$ **M1**

$$= \frac{1}{4}(x + 2)(25 - 10x + x^2) \quad \textbf{A1}$$

$$A = \frac{1}{4}(x^3 - 8x^2 + 5x + 50) \quad \textbf{AG}$$

[2 marks]

Examiners report

This question was generally well done. Parts (a) and (b) were straightforward and well answered.

32b.

[3 marks]

Markscheme

(i) $\frac{dA}{dx} = \frac{1}{4}(3x^2 - 16x + 5) = \frac{1}{4}(3x - 1)(x - 5)$ **A1**

(ii) **METHOD 1**

EITHER

$$\frac{dA}{dx} = \frac{1}{4}\left(3\left(\frac{1}{3}\right)^2 - 16\left(\frac{1}{3}\right) + 5\right) = 0$$
 M1A1

OR

$$\frac{dA}{dx} = \frac{1}{4}\left(3\left(\frac{1}{3}\right) - 1\right)\left(\left(\frac{1}{3}\right) - 5\right) = 0$$
 M1A1

THEN

so $\frac{dA}{dx} = 0$ when $x = \frac{1}{3}$ **AG**

METHOD 2

solving $\frac{dA}{dx} = 0$ for x **M1**

$$-2 < x < 5 \Rightarrow x = \frac{1}{3}$$
 A1

so $\frac{dA}{dx} = 0$ when $x = \frac{1}{3}$ **AG**

METHOD 3

a correct graph of $\frac{dA}{dx}$ versus x **M1**

the graph clearly showing that $\frac{dA}{dx} = 0$ when $x = \frac{1}{3}$ **A1**

so $\frac{dA}{dx} = 0$ when $x = \frac{1}{3}$ **AG**

[3 marks]

Examiners report

This question was generally well done. Parts (a) and (b) were straightforward and well answered.

32c.

[7 marks]

$$\frac{d^2A}{dx^2} = \frac{1}{2}(3x - 8)$$

$$x = \frac{1}{3}, \frac{d^2A}{dx^2} = -3.5 (< 0)$$

$$x = \frac{1}{3} \qquad \qquad \qquad PQR$$

$$A_{\max} = \frac{343}{27} (= 12.7) \text{ (cm}^2\text{)}$$

$$PQ = \frac{7}{3} \text{ (cm)} \qquad PR = \left(\frac{14}{3}\right)^2 \text{ (cm)}$$

$$QR^2 = \left(\frac{7}{3}\right)^2 + \left(\frac{14}{3}\right)^4 - 2\left(\frac{7}{3}\right)\left(\frac{14}{3}\right)^2 \cos 30^\circ$$

$$= 391.702 \dots$$

$$QR = 19.8 \text{ (cm)}$$

Examiners report

This question was generally well done. Parts (c) (i) and (ii) were also well answered with most candidates correctly applying the second derivative test and displaying sound reasoning skills.

Part (c) (iii) required the use of the cosine rule and was reasonably well done. The most common error committed by candidates in attempting to find the value of QR was to use $PR = \frac{14}{3}$ (cm) rather than $PR = \left(\frac{14}{3}\right)^2$ (cm). The occasional candidate used $\cos 30^\circ = \frac{1}{2}$.

33a. [3 marks]

Markscheme

attempting to use $V = \pi \int_a^b x^2 dy$ **(M1)**

attempting to express x^2 in terms of y ie $x^2 = 4(y + 16)$ **(M1)**

for $y = h$, $V = 4\pi \int_0^h y + 16 dy$ **A1**

$$V = 4\pi \left(\frac{h^2}{2} + 16h \right) \quad \mathbf{AG}$$

[3 marks]

Examiners report

This question was done reasonably well by a large proportion of candidates. Many candidates however were unable to show the required result in part (a). A number of candidates seemingly did not realize how the container was formed while other candidates attempted to fudge the result.

33b. [3 marks]

Markscheme

EITHER

the depth stabilizes when $\frac{dV}{dt} = 0$ ie $8.5 - \frac{250\sqrt{h}}{\pi(h+16)} = 0$ **R1**

attempting to solve $8.5 - \frac{250\sqrt{h}}{\pi(h+16)} = 0$ for h **(M1)**

OR

the depth stabilizes when $\frac{dh}{dt} = 0$ ie $\frac{1}{4\pi(h+16)} \left(8.5 - \frac{250\sqrt{h}}{\pi(h+16)} \right) = 0$ **R1**

attempting to solve $\frac{1}{4\pi(h+16)} \left(8.5 - \frac{250\sqrt{h}}{\pi(h+16)} \right) = 0$ for h **(M1)**

THEN

$$h = 5.06 \text{ (cm)} \quad \mathbf{A1}$$

[3 marks]

Total [16 marks]

Examiners report

In part (c), a pleasing number of candidates realized that the water depth stabilized when either $\frac{dV}{dt} = 0$ or $\frac{dh}{dt} = 0$, sketched an appropriate graph and found the correct value of h . Some candidates misinterpreted the situation and attempted to find the coordinates of the local minimum of their graph.

Markscheme

$$f(0) = 0 \quad \mathbf{A1}$$

$$f'(x) = -e^{-x} \cos x - e^{-x} \sin x + 1 \quad \mathbf{M1A1}$$

$$f'(0) = 0 \quad \mathbf{(M1)}$$

$$f''(x) = 2e^{-x} \sin x \quad \mathbf{A1}$$

$$f''(0) = 0$$

$$f^{(3)}(x) = -2e^{-x} \sin x + 2e^{-x} \cos x \quad \mathbf{A1}$$

$$f^{(3)}(0) = 2$$

$$\text{the first non-zero term is } \frac{2x^3}{3!} \quad \left(= \frac{x^3}{3} \right) \quad \mathbf{A1}$$

Note: Award no marks for using known series.

[7 marks]

Examiners report

Most students had a good understanding of the techniques involved with this question. A surprising number forgot to show $f(0) = 0$. Some candidates did not simplify the second derivative which created extra work and increased the chance of errors being made.