Topic 4 Part 1 *[437 marks]*

1a. *[3 marks]*

Markscheme

$\left|\overrightarrow{OA}\right| = \left|\overrightarrow{CB}\right| = \left|\overrightarrow{OC}\right| = \left|\overrightarrow{AB}\right| = 6$ (therefore a rhombus) ***A1A1***

Note: Award ***A1*** for two correct lengths, ***A2*** for all four.

Note: Award ***A1A0*** for

$\overrightarrow{OA} = \overrightarrow{CB} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$, $\overrightarrow{OC} = \overrightarrow{AB} = \begin{pmatrix} 0 \\ -\sqrt{24} \\ \sqrt{12} \end{pmatrix}$

if no magnitudes are shown.

$\overrightarrow{OA} \cdot \overrightarrow{OC} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -\sqrt{24} \\ \sqrt{12} \end{pmatrix} = 0$ (therefore a square) ***A1***

Note: Other arguments are possible with a minimum of three conditions.

[3 marks]

Examiners report

[N/A]

1b. *[1 mark]*

Markscheme

$M\left(3,\ -\frac{\sqrt{24}}{2},\ \frac{\sqrt{12}}{2}\right)\ \left(=\left(3,\ -\sqrt{6},\ \sqrt{3}\right)\right)$ ***A1***

[1 mark]

Examiners report

[N/A]

1c. *[3 marks]*

Markscheme

METHOD 1

$\overrightarrow{OA} \times \overrightarrow{OC} = \begin{pmatrix}6\\0\\0\end{pmatrix} \times \begin{pmatrix}0\\-\sqrt{24}\\\sqrt{12}\end{pmatrix} = \begin{pmatrix}0\\-6\sqrt{12}\\-6\sqrt{24}\end{pmatrix} \left(= \begin{pmatrix}0\\-12\sqrt{3}\\-12\sqrt{6}\end{pmatrix}\right)$ ***M1A1***

Note: Candidates may use other pairs of vectors.

equation of plane is

$-6\sqrt{12}y - 6\sqrt{24}z = d$

any valid method showing that

$d = 0$ ***M1***

$\Pi : y + \sqrt{2}z = 0$ ***AG***

METHOD 2

equation of plane is

$ax + by + cz = d$

substituting O to find

$d = 0$ ***(M1)***

substituting two points (A, B, C or M) ***M1***

eg

$6a = 0, \ -\sqrt{24}b + \sqrt{12}c = 0$ ***A1***

$\Pi : y + \sqrt{2}z = 0$ ***AG***

[3 marks]

Examiners report

[N/A]

1d. *[3 marks]*

Markscheme

$r = \begin{pmatrix}3\\-\sqrt{6}\\\sqrt{3}\end{pmatrix} + \lambda \begin{pmatrix}0\\1\\\sqrt{2}\end{pmatrix}$ ***A1A1A1***

Note: Award ***A1*** for ***r*** =, ***A1A1*** for two correct vectors.

[3 marks]

Examiners report

[N/A]

1e. *[3 marks]*

Markscheme

Using $y = 0$ to find λ ***M1***

Substitute their λ into their equation from part (d) ***M1***

D has coordinates

$(3, 0, 3\sqrt{3})$ ***A1***

[3 marks]

Examiners report

[N/A]

1f. *[3 marks]*

Markscheme

λ for point E is the negative of the λ for point D ***(M1)***

Note: Other possible methods may be seen.

E has coordinates $\left(3,\ -2\sqrt{6},\ -\sqrt{3}\right)$ ***A1A1***

Note: Award ***A1*** for each of the y and z coordinates.

[3 marks]

Examiners report

[N/A]

1g. *[6 marks]*

Markscheme

(i)

$\overrightarrow{\mathrm{DA}} \cdot \overrightarrow{\mathrm{DO}} = \begin{pmatrix} 3 \\ 0 \\ -3\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 0 \\ -3\sqrt{3} \end{pmatrix} = 18$ ***M1A1***

$\cos \mathrm{O\hat{D}A} = \frac{18}{\sqrt{36}\sqrt{36}} = \frac{1}{2}$ ***M1***

hence $\mathrm{O\hat{D}A} = 60^\circ$ ***A1***

Note: Accept method showing OAD is equilateral.

(ii) OABCDE is a regular octahedron (accept equivalent description) ***A2***

Note: ***A2*** for saying it is made up of 8 equilateral triangles

Award ***A1*** for two pyramids, ***A1*** for equilateral triangles.

(can be either stated or shown in a sketch – but there must be clear indication the triangles are equilateral)

[6 marks]

Examiners report

[N/A]

2. *[6 marks]*

Markscheme

(a)

$\overrightarrow{PR} = \boldsymbol{a} + \boldsymbol{b}$ ***A1***

$\overrightarrow{QS} = \boldsymbol{b} - \boldsymbol{a}$ ***A1***

[2 marks]

(b)

$\overrightarrow{PR} \cdot \overrightarrow{QS} = (\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{b} - \boldsymbol{a})$ ***M1***

$= |\boldsymbol{b}|^2 - |\boldsymbol{a}|^2$ ***A1***

for a rhombus $|\boldsymbol{a}| = |\boldsymbol{b}|$ ***R1***

hence $|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2 = 0$ ***A1***

Note: Do not award the final ***A1*** unless ***R1*** is awarded.

hence the diagonals intersect at right angles ***AG***

[4 marks]

Total [6 marks]

Examiners report

[N/A]

3a. *[2 marks]*

Markscheme

direction vector $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} -1 \\ -3 \\ 5 \end{pmatrix}$ ***A1***

$r = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or $r = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or equivalent ***A1***

Note: Do not award final ***A1*** unless '$r = $ K' (or equivalent) seen.

Allow FT on direction vector for final ***A1***.

[2 marks]

Examiners report

[N/A]

3b. *[5 marks]*

Markscheme

both lines expressed in parametric form:

L_1:

$x = 1 + t$

$y = 3t$

$z = 4 - 5t$

L_2:

$x = 1 + 3s$

$y = -2 + s$

$z = -2s + 1$ ***M1A1***

Notes: Award ***M1*** for an attempt to convert L_2 from Cartesian to parametric form.

Award ***A1*** for correct parametric equations for L_1 and L_2.

Allow ***M1A1*** at this stage if same parameter is used in both lines.

attempt to solve simultaneously for *x* and *y*: ***M1***

$1 + t = 1 + 3s$

$3t = -2 + s$

$t = -\frac{3}{4}, s = -\frac{1}{4}$ ***A1***

substituting both values back into *z* values respectively gives $z = \frac{31}{4}$ and $z = \frac{3}{2}$ so a contradiction ***R1***

therefore L_1 and L_1 are skew lines ***AG***

[5 marks]

Examiners report

[N/A]

3c. *[4 marks]*

Markscheme

finding the cross product:

$\begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ ***(M1)***

$= -\boldsymbol{i} - 13\boldsymbol{j} - 8\boldsymbol{k}$ ***A1***

Note: Accept $\boldsymbol{i} + 13\boldsymbol{j} + 8\boldsymbol{k}$

$-1(0) - 13(1) - 8(-2) = 3$ ***(M1)***

$\Rightarrow -x - 13y - 8z = 3$ or equivalent ***A1***

[4 marks]

Examiners report

[N/A]

3d. *[7 marks]*

Markscheme

(i)

$(\cos\theta =)\dfrac{\begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}\bullet\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{k^2+1+1}\times\sqrt{1+1}}$ ***M1***

Note: Award ***M1*** for an attempt to use angle between two vectors formula.

$\frac{\sqrt{3}}{2} = \frac{k+1}{\sqrt{2(k^2+2)}}$ ***A1***

obtaining the quadratic equation

$4(k+1)^2 = 6(k^2+2)$ ***M1***

$k^2 - 4k + 4 = 0$

$(k-2)^2 = 0$

$k = 2$ ***A1***

Note: Award ***M1A0M1A0*** if $\cos 60°$ is used $(k = 0$ or $k = -4)$.

(ii)

substituting into the equation of the plane Π_2:

$r = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

$3 + 2\lambda + \lambda = 12$ ***M1***

$\lambda = 3$ ***A1***

point P has the coordinates:

(9, 3, –2) ***A1***

Notes: Accept $9\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ and $\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$.

Do not allow FT if two values found for k.

[7 marks]

Examiners report

[N/A]

4a. *[4 marks]*

Markscheme

$\overrightarrow{CA} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ ***(A1)***

$\overrightarrow{CB} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ ***(A1)***

Note: If $\overrightarrow{AC}$ and $\overrightarrow{BC}$ found correctly award ***(A1) (A0)***.

$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} i & j & k \\ 1 & -2 & -1 \\ 2 & 0 & 1 \end{vmatrix}$ ***(M1)***

$\begin{pmatrix} -2 \\ -3 \\ \ldots \end{pmatrix}$ ***A1***

[4 marks]

$\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

Examiners report

Part a) proved an easy start, though a few (weaker) candidates still believe $\overrightarrow{\mathrm{CA}}$ ~~to~~ to be $\overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OA}}$

4b. *[3 marks]*

Markscheme

METHOD 1

$\frac{1}{2}\left|\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}\right|$ ***(M1)***

$\sqrt{(-2)^2 + (-3)^2 + 4^2}$ ***(A1)***

$= \frac{\sqrt{29}}{2}$ ***A1***

METHOD 2

attempt to apply $\frac{1}{2}|\mathrm{CA}||\mathrm{CB}|\sin C$ ***(M1)***

$\mathrm{CA}.\mathrm{CB} = \sqrt{5}.\sqrt{6}\cos C \Rightarrow \cos C = \frac{1}{\sqrt{30}} \Rightarrow \sin C = \frac{\sqrt{29}}{\sqrt{30}}$ ***(A1)***

area $= \frac{\sqrt{29}}{2}$ ***A1***

[3 marks]

Examiners report

Part b) was an easy 3 marks and incorrect answers were rare.

4c. *[3 marks]*

Markscheme

METHOD 1

$\mathbf{r}.\begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix}$ ***M1A1***

$\Rightarrow -2x - 3y + 4z = -2$ ***A1***

$\Rightarrow 2x + 3y - 4z = 2$ ***AG***

METHOD 2

$-2x - 3y + 4z = d$

substituting a point in the plane ***M1A1***

$\mathrm{d} = -2$ ***A1***

$\Rightarrow -2x - 3y + 4z = -2$

$\Rightarrow 2x + 3y - 4z = 2$ ***AG***

Note: Accept verification that all 3 vertices of the triangle lie on the given plane.

[3 marks]

Examiners report

Part c) was answered well, though reasoning sometimes seemed sparse, especially given that this was a ‘show that’ question.

4d. *[5 marks]*

Markscheme

METHOD 1

$\begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ 2 & 3 & -4 \\ 4 & -1 & -1 \end{vmatrix} = \begin{pmatrix} -7 \\ -14 \\ -14 \end{pmatrix}$ ***M1A1***

$z = 0 \Rightarrow y = 0,\ x = 1$ ***(M1)(A1)***

$\mathrm{n} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$L_1\!: \mathrm{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ ***A1***

Note: Do not award the final ***A1*** if $\mathrm{r} =$ is not seen.

METHOD 2

eliminate 1 of the variables, *eg x* ***M1***

$-7y + 7z = 0$ ***(A1)***

introduce a parameter ***M1***

$\Rightarrow z = \lambda$,

$y = \lambda,\ x = 1 + \frac{\lambda}{2}$ or equivalent ***(A1)***

$\mathrm{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ ***A1***

Note: Do not award the final ***A1*** if $\mathrm{r} =$ is not seen.

METHOD 3

$z = t$ ***M1***

write *x* and *y* in terms of $t \Rightarrow 4x - y = 4 + t,\ 2x + 3y = 2 + 4t$ or equivalent ***A1***

attempt to eliminate *x* or *y* ***M1***

expressed in parameters x, y, z

$\Rightarrow z = t$,

$y = t,\ x = 1 + \frac{t}{2}$ or equivalent ***A1***

$\mathrm{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ ***A1***

Note: Do not award the final ***A1*** if $\mathrm{r} =$ is not seen.

[5 marks]

Examiners report

Part d) proved more challenging, despite being a very standard question. Many candidates gained only 2 marks, either through correctly calculating the direction vector, or by successfully eliminating one of the variables. A number of clear fully correct solutions were seen, though the absence of ‘$\boldsymbol{r}$ =’ is still prevalent, and candidates might be reminded of the correct form for the vector equation of a line.

4e. *[3 marks]*

Markscheme

METHOD 1

direction of the line is perpendicular to the normal of the plane

M1A1

$\begin{pmatrix} 16 \\ \alpha \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 0$

A1

$16 + 2\alpha - 6 = 0 \Rightarrow \alpha = -5$

METHOD 2

solving line/plane simultaneously

M1A1

$16(1+\lambda) + 2\alpha\lambda - 6\lambda = \beta$

$16 + (10 + 2\alpha)\lambda = \beta$

A1

$\Rightarrow \alpha = -5$

METHOD 3

M1

$\begin{vmatrix} 2 & 3 & -4 \\ 4 & -1 & -1 \\ 16 & \alpha & -3 \end{vmatrix} = 0$

A1

$2(3+\alpha) - 3(-12+16) - 4(4\alpha+16) = 0$

A1

$\Rightarrow \alpha = -5$

METHOD 4

attempt to use row reduction on augmented matrix ***M1***

to obtain

A1

$\left(\begin{array}{ccc|c} 2 & 3 & -4 & 2 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & \alpha+5 & \beta-16 \end{array}\right)$

A1

$\Rightarrow \alpha = -5$

[3 marks]

Examiners report

Part e) proved a puzzle for most, though an attempt to use row reduction on an augmented matrix seemed to be the choice way for most successful candidates.

4f. *[2 marks]*

Markscheme

A1

$\alpha = -5$

A1

$\beta \neq 16$

[2 marks]

Examiners report

Only the very best were able to demonstrate a complete understanding of intersecting planes and thus answer part f) correctly.

5a. *[3 marks]*

Markscheme

attempting to form

M1

$(3\cos\theta + 6)(\cos\theta - 2) + 7(1 + \sin\theta) = 0$

A1

$3\cos^2\theta - 12 + 7\sin\theta + 7 = 0$

M1

$3\left(1 - \sin^2\theta\right) + 7\sin\theta - 5 = 0$

AG

$3\sin^2\theta - 7\sin\theta + 2 = 0$

[3 marks]

Examiners report

Part (a) was very well done. Most candidates were able to use the scalar product and

$\cos^2\theta = 1 - \sin^2\theta$

to show the required result.

5b. *[3 marks]*

Markscheme

attempting to solve algebraically (including substitution) or graphically for $\sin\theta$ ***(M1)***

$\sin\theta = \frac{1}{3}$ ***(A1)***

$\theta = 0.340\ (= 19.5^\circ)$ ***A1***

[3 marks]

Examiners report

Part (b) was reasonably well done. A few candidates confused 'smallest possible positive value' with a minimum function value. Some candidates gave $\theta = 0.34$ as their final answer.

6.

[7 marks]

Markscheme

(a) **METHOD 1**

for P on L_1, $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} -5-\lambda \\ -3+2\lambda \\ 2+2\lambda \end{pmatrix}$

require $\begin{pmatrix} -5-\lambda \\ -3+2\lambda \\ 2+2\lambda \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = 0$ (or equivalent) ***M1A1***

$5+\lambda-6+4\lambda+4+4\lambda = 0$ ***A1***

$\lambda = -\frac{1}{3}$ ***A1***

$\therefore \overrightarrow{\mathrm{OP}} = \begin{pmatrix} -\frac{14}{3} \\ -\frac{11}{3} \\ \frac{4}{3} \end{pmatrix}$ ***A1***

$L_2: r = \mu \begin{pmatrix} -14 \\ -11 \\ 4 \end{pmatrix}$

Note: Do not award the final ***A1*** if $r =$ is not seen.

[5 marks]

METHOD 2

Calculating either $\left|\overrightarrow{\mathrm{OP}}\right|^2$ or $\left|\overrightarrow{\mathrm{OP}}\right|$ ***A1***

$\left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{(-5-\lambda)^2 + (-3+2\lambda)^2 + (2+2\lambda)^2} = \sqrt{9\lambda^2+6\lambda+38}$

Solving either $\frac{d}{d\lambda}\left(\left|\overrightarrow{\mathrm{OP}}\right|^2\right) = 0$ or $\frac{d}{d\lambda}\left(\left|\overrightarrow{\mathrm{OP}}\right|\right) = 0$ for λ ***M1A1***

$\lambda = -\frac{1}{3}$ ***A1***

$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} -\frac{14}{3} \\ -\frac{11}{3} \\ \frac{4}{3} \end{pmatrix}$ ***A1***

$L_2: r = \mu \begin{pmatrix} -14 \\ -11 \\ 4 \end{pmatrix}$

Note: Do not award the final ***A1*** if $r =$ is not seen.

[5 marks]

(b) **METHOD 1**

$\left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{\left(-\frac{14}{3}\right)^2 + \left(-\frac{11}{3}\right)^2 + \left(\frac{4}{3}\right)^2}$ ***(M1)***

$= 6.08 \ (= \sqrt{37})$ ***A1***

METHOD 2

shortest distance $= \frac{\left|\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \times \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}\right|}$ ***(M1)***

$= \frac{|10i+8j+13k|}{\sqrt{1+4+4}}$

$= 6.08 \ (= \sqrt{37})$ ***A1***

[2 marks]

Total [7 marks]

Examiners report

Part (a) was not well done. Most candidates recognised the need to calculate a scalar product. Some candidates made careless sign or arithmetic errors when solving for λ. A few candidates neglected to express their final answer in the form '$\boldsymbol{r}$ ='.

Candidates who were successful in answering part (b) generally answered part (b) correctly. The large majority of successful candidates calculated $\left|\overrightarrow{\mathrm{OP}}\right|$.

7. *[5 marks]*

Markscheme

METHOD 1

$\left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{(1+s)^2 + (3+2s)^2 + (1-s)^2} \quad \left(= \sqrt{6s^2 + 12s + 11}\right)$ ***A1***

Note: Award ***A1*** if the square of the distance is found.

EITHER

attempt to differentiate: $\frac{\mathrm{d}}{\mathrm{d}s}\left|\overrightarrow{\mathrm{OP}}\right|^2 \quad (= 12s + 12)$ ***M1***

attempting to solve $\frac{\mathrm{d}}{\mathrm{d}s}\left|\overrightarrow{\mathrm{OP}}\right|^2 = 0$ for s ***(M1)***

$s = -1$ ***(A1)***

OR

attempt to differentiate: $\frac{\mathrm{d}}{\mathrm{d}s}\left|\overrightarrow{\mathrm{OP}}\right| \left(= \frac{6s+6}{\sqrt{6s^2+12s+11}}\right)$ ***M1***

attempting to solve $\frac{\mathrm{d}}{\mathrm{d}s}\left|\overrightarrow{\mathrm{OP}}\right| = 0$ for s ***(M1)***

$s = -1$ ***(A1)***

OR

attempt at completing the square: $\left(\left|\overrightarrow{\mathrm{OP}}\right|^2 = 6(s+1)^2 + 5\right)$ ***M1***

minimum value ***(M1)***

occurs at $s = -1$ ***(A1)***

THEN

the minimum length of $\overrightarrow{\mathrm{OP}}$ is $\sqrt{5}$ ***A1***

METHOD 2

the length of $\overrightarrow{\mathrm{OP}}$ is a minimum when $\overrightarrow{\mathrm{OP}}$ is perpendicular to $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ ***(R1)***

$\begin{pmatrix} 1+s \\ 3+2s \\ 1-s \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 0$ ***A1***

attempting to solve $1 + s + 6 + 4s - 1 + s = 0 \quad (6s + 6 = 0)$ for s ***(M1)***

$s = -1$ ***(A1)***

$\left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{5}$ ***A1***

[5 marks]

Examiners report

Generally well done. But there was a significant minority who didn't realise that they had to use calculus or completion of squares to minimise the length. Trying random values of s gained no marks. A number of candidates wasted time showing that their answer gave a minimum rather than a maximum value of the length.

8a. [4 marks]

Markscheme

(i) $\overrightarrow{\mathrm{AM}} = \frac{1}{2}\overrightarrow{\mathrm{AC}}$ ***(M1)***

$= \frac{1}{2}(c - a)$ ***A1***

(ii) $\overrightarrow{\mathrm{BM}} = \overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AM}}$ ***M1***

$= (a - b) + \frac{1}{2}(c - a)$ ***A1***

$\overrightarrow{\mathrm{BM}} = \frac{1}{2}a - b + \frac{1}{2}c$ ***AG***

[4 marks]

Examiners report

A fairly straightforward question for candidates confident in the use of and correct notation for relative position vectors. Sign errors were the most common, but the majority of candidates did not gain all the reasoning marks for part (c). In particular, it was necessary to observe that not only were two vectors parallel, but that they had a point in common.

8b. [5 marks]

Markscheme

(i) $\overrightarrow{\mathrm{RA}} = \frac{1}{3}\overrightarrow{\mathrm{BA}}$

$= \frac{1}{3}(\boldsymbol{a} - \boldsymbol{b})$ ***A1***

(ii) $\overrightarrow{\mathrm{RT}} = \frac{2}{3}\overrightarrow{\mathrm{RS}}$ ***(M1)***

$= \frac{2}{3}\left(\overrightarrow{\mathrm{RA}} + \overrightarrow{\mathrm{AS}}\right)$ or equivalent. ***A1A1***

$= \frac{2}{3}\left(\frac{1}{3}(a - b) + \frac{2}{3}(c - a)\right)$

$= \frac{2}{9}(a - b) + \frac{4}{9}(c - a)$ ***A1***

$\overrightarrow{\mathrm{RT}} = -\frac{2}{9}a - \frac{2}{9}b + \frac{4}{9}c$ ***AG***

[5 marks]

Examiners report

A fairly straightforward question for candidates confident in the use of and correct notation for relative position vectors. Sign errors were the most common, but the majority of candidates did not gain all the reasoning marks for part (c). In particular, it was necessary to observe that not only were two vectors parallel, but that they had a point in common.

8c. [5 marks]

Markscheme

$\overrightarrow{\mathrm{BT}} = \overrightarrow{\mathrm{BR}} + \overrightarrow{\mathrm{RT}}$

$= \frac{2}{3}\overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{RT}}$ ***(M1)***

$= \frac{2}{3}a - \frac{2}{3}b - \frac{2}{9}a - \frac{2}{9}b + \frac{4}{9}c$ ***A1***

$\overrightarrow{\mathrm{BT}} = \frac{8}{9}\left(\frac{1}{2}a - b + \frac{1}{2}c\right)$ ***A1***

point B is common to $\overrightarrow{\mathrm{BT}}$ and $\overrightarrow{\mathrm{BM}}$ and $\overrightarrow{\mathrm{BT}} = \frac{8}{9}\overrightarrow{\mathrm{BM}}$ ***R1R1***

so T lies on [BM] ***AG***

[5 marks]

Total [14 marks]

9.

[6 marks]

Markscheme

$n_1 = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ and $n_2 = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$ ***(A1)(A1)***

use of $\cos\theta = \frac{n_1 \bullet n_2}{|n_1||n_2|}$ ***(M1)***

$\cos\theta = \frac{7}{\sqrt{21}\sqrt{19}}$ $\left(= \frac{7}{\sqrt{399}}\right)$ ***(A1)(A1)***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$\theta = 69^\circ$ ***A1***

Note: Award ***A1*** for 111°.

[6 marks]

Examiners report

Reasonably well answered. A large number of candidates did not express their final answer correct to the nearest degree.

10a.

[4 marks]

Markscheme

attempting to find a normal to π eg $\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \times \begin{pmatrix} 8 \\ 11 \\ 6 \end{pmatrix}$ ***(M1)***

$\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \times \begin{pmatrix} 8 \\ 11 \\ 6 \end{pmatrix} = 17 \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ ***(A1)***

$r \bullet \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 12 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ ***M1***

$2x - 2y + z = 4$ (or equivalent) ***A1***

[4 marks]

Examiners report

Part (a) was reasonably well done. Some candidates made numerical errors when attempting to find a normal to π.

10b.

[4 marks]

Markscheme

$l_3 : r = \begin{pmatrix} 4 \\ 0 \\ 8 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}, t \in \mathbb{R}$ ***(A1)***

attempting to solve $\begin{pmatrix} 4 + 2t \\ -2t \\ 8 + t \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = 4$ for t ie $9t + 16 = 4$ for t ***M1***

$t = -\frac{4}{3}$ ***A1***

$\left(\frac{4}{3}, \frac{8}{3}, \frac{20}{3}\right)$ ***A1***

[4 marks]

Total [8 marks]

Examiners report

In part (b), a number of candidates were awarded follow through marks from numerical errors committed in part (a).

11a. *[3 marks]*

Markscheme

attempt at implicit differentiation ***M1***

$2x - 5x\frac{\mathrm{d}y}{\mathrm{d}x} - 5y + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***A1A1***

Note: ***A1*** for differentiation of $x^2 - 5xy$, ***A1*** for differentiation of y^2 and 7.

$2x - 5y + \frac{\mathrm{d}y}{\mathrm{d}x}(2y - 5x) = 0$

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5y-2x}{2y-5x}$ ***AG***

[3 marks]

Examiners report

[N/A]

11b. *[4 marks]*

Markscheme

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5\times1-2\times6}{2\times1-5\times6} = \frac{1}{4}$ ***A1***

gradient of normal $= -4$ ***A1***

equation of normal $y = -4x + c$ ***M1***

substitution of $(6,\ 1)$

$y = -4x + 25$ ***A1***

Note: Accept $y - 1 = -4(x - 6)$

[4 marks]

Examiners report

[N/A]

11c. *[8 marks]*

Markscheme

setting $\frac{5y-2x}{2y-5x} = 1$ ***M1***

$y = -x$ ***A1***

substituting into original equation ***M1***

$x^2 + 5x^2 + x^2 = 7$ ***(A1)***

$7x^2 = 7$

$x = \pm 1$ ***A1***

points $(1, -1)$ and $(-1, 1)$ ***(A1)***

distance $= \sqrt{8}$ $\left(= 2\sqrt{2}\right)$ ***(M1)A1***

[8 marks]

Total [15 marks]

Examiners report

[N/A]

12a. *[4 marks]*

Markscheme

L_1 and L_2 are not parallel, since $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \neq k \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$ ***R1***

if they meet, then $1 - \lambda = 1 + 2\mu$ and $2 + \lambda = 2 + \mu$ ***M1***

solving simultaneously $\Rightarrow \lambda = \mu = 0$ ***A1***

contradiction, $2 + 2\lambda = 4 + 6\mu \Rightarrow 2 \neq 4$ ***R1***

so lines are skew ***AG***

Note: Do not award the second ***R1*** if their values of parameters are incorrect.

[4 marks]

Examiners report

[N/A]

12b. *[4 marks]*

Markscheme

$\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix} (= 11) = \sqrt{6}\sqrt{41}\cos\theta$ ***M1A1***

$\cos\theta = \frac{11}{\sqrt{246}}$ ***(A1)***

$\theta = 45.5°$ (0.794 radians) ***A1***

[4 marks]

Examiners report

[N/A]

13. *[5 marks]*

Markscheme

point on line is

$x = \frac{-1-5\lambda}{5},\ y = \frac{9+5\lambda}{5},\ z = \lambda$ or similar ***M1A1***

Note: Accept use of point on the line or elimination of one of the variables using the equations of the planes

$\frac{-1-5\lambda}{5} - \frac{9+5\lambda}{5} + 2\lambda = k$ ***M1A1***

Note: Award ***M1A1*** if coordinates of point and equation of a plane is used to obtain linear equation in *k* or equations of the line are used in combination with equation obtained by elimination to get linear equation in *k*.

$k = -2$ ***A1***

[5 marks]

Examiners report

Many different attempts were seen, sometimes with success. Unfortunately many candidates wasted time with aimless substitutions showing little understanding of the problem.

14a. *[4 marks]*

Markscheme

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 0 \\ 6 \\ -6 \end{pmatrix} \Rightarrow \mathrm{AB} = \sqrt{72}$ ***A1***

$\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 6 \\ 0 \\ -6 \end{pmatrix} \Rightarrow \mathrm{AC} = \sqrt{72}$ ***A1***

so they are the same ***AG***

$\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{AC}} = 36 = \left(\sqrt{72}\right)\left(\sqrt{72}\right)\cos\theta$ ***(M1)***

$\cos\theta = \frac{36}{\left(\sqrt{72}\right)\left(\sqrt{72}\right)} = \frac{1}{2} \Rightarrow \theta = 60^\circ$ ***A1AG***

Note: Award ***M1A1*** if candidates find BC and claim that triangle ABC is equilateral.

[4 marks]

Examiners report

Parts (a) and (b) were well attempted with many candidates achieving full or nearly full marks in these parts. Surprisingly, many candidates were not able to find the coordinates of the midpoints in part (c) leading to incorrect equations of the planes and, in many cases this affected the performance in part (d). Very few answered part (d) correctly. In parts (e) and (f) very few candidates made good attempts of using the ‘show that’ procedure and used a parametric approach. In most attempts, candidates simply verified the condition using the answer given. In a few cases, candidates noticed that G is the barycentre of [ABCD] and found its coordinates successfully. Some candidates were also successful in determining the bounding angle using the information given. Unfortunately, many candidates instead of finding the angle simply quoted the result.

14b. *[4 marks]*

Markscheme

METHOD 1

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 0 & 6 & -6 \\ -6 & 0 & -6 \end{vmatrix} = -36i + 36j + 36k$ ***(M1)A1***

equation of plane is $x - y - z = k$ ***(M1)***

goes through A, B or C

$\Rightarrow x - y - z = 2$ ***A1***

[4 marks]

METHOD 2

$x + by + cz = d$ (or similar) ***M1***

$5 - 2b + 5c = d$

$5 + 4b - c = d$ ***A1***

$-1 - 2b - c = d$

solving simultaneously ***M1***

$b = -1,\ c = -1,\ d = 2$

so $x - y - z = 2$ ***A1***

[4 marks]

Examiners report

Parts (a) and (b) were well attempted with many candidates achieving full or nearly full marks in these parts. Surprisingly, many candidates were not able to find the coordinates of the midpoints in part (c) leading to incorrect equations of the planes and, in many cases this affected the performance in part (d). Very few answered part (d) correctly. In parts (e) and (f) very few candidates made good attempts of using the 'show that' procedure and used a parametric approach. In most attempts, candidates simply verified the condition using the answer given. In a few cases, candidates noticed that G is the barycentre of [ABCD] and found its coordinates successfully. Some candidates were also successful in determining the bounding angle using the information given. Unfortunately, many candidates instead of finding the angle simply quoted the result.

14c. *[4 marks]*

Markscheme

(i) midpoint is $(5, 1, 2)$, so equation of Π_1 is $y - z = -1$ ***A1A1***

(ii) midpoint is $(2, -2, 2)$, so equation of Π_2 is $x + z = 4$ ***A1A1***

Note: In each part, award ***A1*** for midpoint and ***A1*** for the equation of the plane.

[4 marks]

Examiners report

Parts (a) and (b) were well attempted with many candidates achieving full or nearly full marks in these parts. Surprisingly, many candidates were not able to find the coordinates of the midpoints in part (c) leading to incorrect equations of the planes and, in many cases this affected the performance in part (d). Very few answered part (d) correctly. In parts (e) and (f) very few candidates made good attempts of using the 'show that' procedure and used a parametric approach. In most attempts, candidates simply verified the condition using the answer given. In a few cases, candidates noticed that G is the barycentre of [ABCD] and found its coordinates successfully. Some candidates were also successful in determining the bounding angle using the information given. Unfortunately, many candidates instead of finding the angle simply quoted the result.

14d. *[3 marks]*

Markscheme

EITHER

solving the two equations above ***M1***

$$L: r = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$ ***A1***

OR

L has the direction of the vector product of the normal vectors to the planes Π_1 and Π_2 ***(M1)***

$$\begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix} = i - j - k$$ (or its opposite) ***A1***

THEN

direction is $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ as required ***R1***

[3 marks]

Examiners report

Parts (a) and (b) were well attempted with many candidates achieving full or nearly full marks in these parts. Surprisingly, many candidates were not able to find the coordinates of the midpoints in part (c) leading to incorrect equations of the planes and, in many cases this affected the performance in part (d). Very few answered part (d) correctly. In parts (e) and (f) very few candidates made good attempts of using the 'show that' procedure and used a parametric approach. In most attempts, candidates simply verified the condition using the answer given. In a few cases, candidates noticed that G is the barycentre of [ABCD] and found its coordinates successfully. Some candidates were also successful in determining the bounding angle using the information given. Unfortunately, many candidates instead of finding the angle simply quoted the result.

14e. *[3 marks]*

Markscheme

D is of the form $(4 - \lambda, -1 + \lambda, \lambda)$ ***M1***

$(1 + \lambda)^2 + (-1 - \lambda)^2 + (5 - \lambda)^2 = 72$ ***M1***

$3\lambda^2 - 6\lambda - 45 = 0$ ***A1***

$\lambda = 5$ or $\lambda = -3$

$D(-1, 4, 5)$ ***AG***

Note: Award ***M0M0A0*** if candidates just show that $D(-1, 4, 5)$ satisfies $AB = AD$.

Award ***M1M1A0*** if candidates also show that D is of the form $(4 - \lambda, -1 + \lambda, \lambda)$

[3 marks]

Examiners report

Parts (a) and (b) were well attempted with many candidates achieving full or nearly full marks in these parts. Surprisingly, many candidates were not able to find the coordinates of the midpoints in part (c) leading to incorrect equations of the planes and, in many cases this affected the performance in part (d). Very few answered part (d) correctly. In parts (e) and (f) very few candidates made good attempts of using the 'show that' procedure and used a parametric approach. In most attempts, candidates simply verified the condition using the answer given. In a few cases, candidates noticed that G is the barycentre of [ABCD] and found its coordinates successfully. Some candidates were also successful in determining the bounding angle using the information given. Unfortunately, many candidates instead of finding the angle simply quoted the result.

14f. *[6 marks]*

Markscheme

EITHER

G is of the form $(4-\lambda, -1+\lambda, \lambda)$ ***M1***

and $\mathrm{DG} = \mathrm{AG}, \mathrm{BG} \text{ or } \mathrm{CG}$ ***M1***

e.g. $(1+\lambda)^2 + (-1-\lambda)^2 + (5-\lambda)^2 = (5-\lambda)^2 + (5-\lambda)^2 + (5-\lambda)^2$

$(1+\lambda)^2 = (5-\lambda)^2$ ***A1***

$\lambda = 2$ ***AG***

$\mathrm{G}(2, 1, 2)$

OR

G is the centre of mass (barycentre) of the regular tetrahedron ABCD ***(M1)***

$\left(\frac{5+5+(-1)+(-1)}{4}, \frac{-2+4+(-2)+4}{4}, \frac{5+(-1)+(-1)+5}{4}\right)$ ***M1A1***

THEN

Note: the following part is independent of previous work and candidates may use ***AG*** to answer it (here it is possible to award ***M0M0A0A1M1A1***)

$\overrightarrow{GD} = \begin{pmatrix}-3\\3\\3\end{pmatrix}$ and ***A1***

$\cos\theta = \frac{-9}{(3\sqrt{3})(3\sqrt{3})} = -\frac{1}{3} \Rightarrow \theta = 109°$ (or 1.91 radians) ***M1A1***

[6 marks]

Examiners report

Parts (a) and (b) were well attempted with many candidates achieving full or nearly full marks in these parts. Surprisingly, many candidates were not able to find the coordinates of the midpoints in part (c) leading to incorrect equations of the planes and, in many cases this affected the performance in part (d). Very few answered part (d) correctly. In parts (e) and (f) very few candidates made good attempts of using the 'show that' procedure and used a parametric approach. In most attempts, candidates simply verified the condition using the answer given. In a few cases, candidates noticed that G is the barycentre of [ABCD] and found its coordinates successfully. Some candidates were also successful in determining the bounding angle using the information given. Unfortunately, many candidates instead of finding the angle simply quoted the result.

15. *[5 marks]*

Markscheme

perpendicular when

$\begin{pmatrix}1\\2\cos x\\0\end{pmatrix} \cdot \begin{pmatrix}-1\\2\sin x\\1\end{pmatrix} = 0$ ***(M1)***

$\Rightarrow -1 + 4\sin x\cos x = 0$ ***A1***

$\Rightarrow \sin 2x = \frac{1}{2}$ ***M1***

$\Rightarrow 2x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$ ***A1A1***

Note: Accept answers in degrees.

[5 marks]

Examiners report

Most candidates realised that the scalar product should be used to solve this problem and many obtained the equation $4\sin x\cos x = 1$. Candidates who failed to see that this could be written as $\sin 2x = 0.5$ usually made no further progress. The majority of those candidates who used this double angle formula carried on to obtain the solution $\frac{\pi}{12}$ but few candidates realised that $\frac{5\pi}{12}$ was also a solution.

16a. *[5 marks]*

Markscheme

in augmented matrix form

$$\left|\begin{array}{ccc|c} 1 & -3 & 1 & 3 \\ 1 & 5 & -2 & 1 \\ 0 & 16 & -6 & k \end{array}\right|$$

attempt to find a line of zeros ***(M1)***

$r_2 - r_1$

$$\left|\begin{array}{ccc|c} 1 & -3 & 1 & 3 \\ 0 & 8 & -3 & -2 \\ 0 & 16 & -6 & k \end{array}\right|$$ ***(A1)***

$r_3 - 2r_2$

$$\left|\begin{array}{ccc|c} 1 & -3 & 1 & 3 \\ 0 & 8 & -3 & -2 \\ 0 & 0 & 0 & k+4 \end{array}\right|$$ ***(A1)***

there is an infinite number of solutions when

$k = -4$ ***R1***

there is no solution when

$k \neq -4 \ (k \in \mathbb{R})$ ***R1***

Note: Approaches other than using the augmented matrix are acceptable.

[5 marks]

Examiners report

Many candidates were able to start this question, but only a few candidates gained full marks. Many candidates successfully used the augmented matrix in part (a) to find the correct answer. Part (b) was less successful with only a limited number of candidates using the calculator to its full effect here and with many candidates making arithmetic and algebraic errors. This was the hardest part of the question. Many candidates understood the vector techniques necessary to answer parts (c), (d) and (e) but a number made arithmetic and algebraic errors in the working.

16b. *[7 marks]*

Markscheme

using

$$\left|\begin{array}{ccc|c} 1 & -3 & 1 & 3 \\ 0 & 8 & -3 & -2 \\ 0 & 0 & 0 & k+4 \end{array}\right|$$

and letting

$z = \lambda$ ***(M1)***

$8y - 3\lambda = -2$ ***(A1)***

$\Rightarrow y = \frac{3\lambda - 2}{8}$

$x - 3y + z = 3$ ***(M1)***

$\Rightarrow x - \left(\frac{9\lambda - 6}{8}\right) + \lambda = 3$

$\Rightarrow 8x - 9\lambda + 6 + 8\lambda = 24$ ***(A1)***

$\Rightarrow x = \frac{18 + \lambda}{8}$ ***(M1)(A1)***

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{18}{8} \\ -\frac{2}{8} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{8} \\ \frac{3}{8} \\ 1 \end{pmatrix}$

$r = \begin{pmatrix} \frac{9}{4} \\ -\frac{1}{4} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}$ ***A1***

Note: Accept equivalent answers.

[7 marks]

Examiners report

Many candidates were able to start this question, but only a few candidates gained full marks. Many candidates successfully used the augmented matrix in part (a) to find the correct answer. Part (b) was less successful with only a limited number of candidates using the calculator to its full effect here and with many candidates making arithmetic and algebraic errors. This was the hardest part of the question. Many candidates understood the vector techniques necessary to answer parts (c), (d) and (e) but a number made arithmetic and algebraic errors in the working.

16c.

[5 marks]

Markscheme

recognition that $\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ is parallel to the plane ***(A1)***

direction normal of the plane is given by

$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 3 & 8 \\ 3 & -2 & 0 \end{vmatrix}$ ***(M1)***

$= 16\boldsymbol{i} + 24\boldsymbol{j} - 11\boldsymbol{k}$ ***A1***

Cartesian equation of the plane is given by $16x + 24y - 11z = d$ and a point which fits this equation is (1, 2, 0) ***(M1)***

$16 + 48 = d$

$d = 64$ ***A1***

hence Cartesian equation of plane is $16x + 24y - 11z = 64$ ***AG***

Note: Accept alternative methods using dot product.

[5 marks]

Examiners report

Many candidates were able to start this question, but only a few candidates gained full marks. Many candidates successfully used the augmented matrix in part (a) to find the correct answer. Part (b) was less successful with only a limited number of candidates using the calculator to its full effect here and with many candidates making arithmetic and algebraic errors. This was the hardest part of the question. Many candidates understood the vector techniques necessary to answer parts (c), (d) and (e) but a number made arithmetic and algebraic errors in the working.

16d.

[2 marks]

Markscheme

the plane crosses the z-axis when $x = y = 0$ ***(M1)***

coordinates of P are $\left(0, 0, -\frac{64}{11}\right)$ ***A1***

Note: Award ***A1*** for stating $z = -\frac{64}{11}$.

Note: Accept $\begin{pmatrix} 0 \\ 0 \\ -\frac{64}{11} \end{pmatrix}$.

[2 marks]

Examiners report

Many candidates were able to start this question, but only a few candidates gained full marks. Many candidates successfully used the augmented matrix in part (a) to find the correct answer. Part (b) was less successful with only a limited number of candidates using the calculator to its full effect here and with many candidates making arithmetic and algebraic errors. This was the hardest part of the question. Many candidates understood the vector techniques necessary to answer parts (c), (d) and (e) but a number made arithmetic and algebraic errors in the working.

16e.

[5 marks]

Markscheme

recognition that the angle between the line and the direction normal is given by: $\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 16 \\ 24 \\ -11 \end{pmatrix} = \sqrt{29}\sqrt{953}\cos\theta$ where θ is the angle between the line and the normal vector ***M1A1***

$\Rightarrow 122 = \sqrt{29}\sqrt{953}\cos\theta$ ***(A1)***

$\Rightarrow \theta = 42.8°$ (0.747 radians) ***(A1)***

hence the angle between the line and the plane is 90° – 42.8° = 47.2° (0.824 radians) ***A1***

[5 marks]

Note: Accept use of the formula $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\sin\theta$.

Examiners report

Many candidates were able to start this question, but only a few candidates gained full marks. Many candidates successfully used the augmented matrix in part (a) to find the correct answer. Part (b) was less successful with only a limited number of candidates using the calculator to its full effect here and with many candidates making arithmetic and algebraic errors. This was the hardest part of the question. Many candidates understood the vector techniques necessary to answer parts (c), (d) and (e) but a number made arithmetic and algebraic errors in the working.

17a. *[4 marks]*

Markscheme

METHOD 1

$9t_A = 7 - 4t_B$ and $3 + 6t_A = -6 + 7t_B$ ***M1A1***

solve simultaneously

$t_A = \frac{1}{3}$, $t_B = 1$ ***A1***

Note: Only need to see one time for the ***A1***.

therefore meet at (3, 1) ***A1***

[4 marks]

METHOD 2

path of A is a straight line: $y = -\frac{2}{3}x + 3$ ***M1A1***

Note: Award ***M1*** for an attempt at simultaneous equations.

path of B is a straight line: $y = -\frac{7}{4}x + \frac{25}{4}$ ***A1***

$-\frac{2}{3}x + 3 = -\frac{7}{4}x + \frac{25}{4}$ $(\Rightarrow x = 3)$

so the common point is (3, 1) ***A1***

[4 marks]

Examiners report

This was probably the least accessible question from section A. Most started by using the same value of *t* in attempting to find the common point, and so scored no marks. There were a number of very good candidates who set different parameters for *t* and correctly obtained (3,1) . There was slightly better understanding shown in part b), though some argued that the boats did not collide because their times were different, yet then provided incorrect times, or even no times at all.

17b. *[2 marks]*

Markscheme

METHOD 1

boats do not collide because the two times

(A1)
$\left(t_A = \frac{1}{3},\ t_B = 1\right)$
are different ***R1***

[2 marks]

METHOD 2

for boat A,
 and for boat B,
$9t = 3 \Rightarrow t = \frac{1}{3}$
$7 - 4t = 3 \Rightarrow t = 1$
times are different so boats do not collide ***R1AG***

[2 marks]

Examiners report

This was probably the least accessible question from section A. Most started by using the same value of *t* in attempting to find the common point, and so scored no marks. There were a number of very good candidates who set different parameters for *t* and correctly obtained (3,1) . There was slightly better understanding shown in part b), though some argued that the boats did not collide because their times were different, yet then provided incorrect times, or even no times at all.

18a. *[4 marks]*

Markscheme

Note: Accept alternative notation for vectors (*eg*
).
$\langle a, b, c\rangle$ or (a, b, c)
 and

(A1)
$n = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$
(M1)
$m = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ ***A1***
$\cos\theta = \frac{n \cdot m}{|n||m|}$
$\cos\theta = \frac{2+2+3}{\sqrt{1+4+9}\sqrt{4+1+1}} = \frac{7}{\sqrt{14}\sqrt{6}}$ ***A1***
$\theta = 40.2^\circ\ \ (0.702\ \text{rad})$

[4 marks]

Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of *k*. It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well.
Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

18b. [5 marks]

Markscheme

Note: Accept alternative notation for vectors (*eg* $\langle a, b, c\rangle$ or (a, b, c)).

METHOD 1

eliminate z from $x - 2y - 3z = 2$ and $2x - y - z = k$

$5x - y = 3k - 2 \Rightarrow x = \frac{y-(2-3k)}{5}$ ***M1A1***

eliminate y from $x - 2y - 3z = 2$ and $2x - y - z = k$

$3x + z = 2k - 2 \Rightarrow x = \frac{z-(2k-2)}{-3}$ ***A1***

$x = t$, $y = (2 - 3k) + 5t$ and $z = (2k - 2) - 3t$ ***A1A1***

$r = \begin{pmatrix} 0 \\ 2-3k \\ 2k-2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$ ***AG***

[5 marks]

METHOD 2

$\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix} \Rightarrow \text{direction is } \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$ ***M1A1***

Let $x = 0$ ***(M1)***

$0 - 2y - 3z = 2$ and $2 \times 0 - y - z = k$

solve simultaneously ***(M1)***

$y = 2 - 3k$ and $z = 2k - 2$ ***A1***

therefore $\mathbf{r} = \begin{pmatrix} 0 \\ 2-3k \\ 2k-2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$ ***AG***

[5 marks]

METHOD 3

substitute $x = t,\ y = (2 - 3k) + 5t$ and $z = (2k - 2) - 3t$ into π_1 and π_2 ***M1***

for $\pi_1 : t - 2(2 - 3k + 5t) - 3(2k - 2 - 3t) = 2$ ***A1***

for $\pi_2 : 2t - (2 - 3k + 5t) - (2k - 2 - 3t) = k$ ***A1***

the planes have a unique line of intersection ***R2***

therefore the line is $r = \begin{pmatrix} 0 \\ 2-3k \\ 2k-2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$ ***AG***

[5 marks]

Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of *k*. It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well.

Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

18c. *[5 marks]*

Markscheme

Note: Accept alternative notation for vectors (*eg* $\langle a, b, c\rangle$ or (a, b, c)).

$5 - t = (2 - 3k + 5t) + 3 = 2 - 2(2k - 2 - 3t)$ ***M1A1***

Note: Award ***M1A1*** if candidates use vector or parametric equations of L_2

eg $\begin{pmatrix} 0 \\ 2-3k \\ 2k-2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$ or $\begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix} + s\begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$

$t = 5 - 2s$

$2 - 3k + 5t = -3 + 2s$

$2k - 2 - 3t = 1 - s$

solve simultaneously ***M1***

$\Rightarrow k = 2, t = 1\ (s = 2)$ ***A1***

intersection point (1, 1, -1) ***A1***

[5 marks]

Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of k. It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well.
Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

18d. *[5 marks]*

Markscheme

Note: Accept alternative notation for vectors (*eg* $\langle a, b, c\rangle$ or (a, b, c)).

$\vec{l}_2 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ ***A1***

$\vec{l}_1 \times \vec{l}_2 = \begin{vmatrix} i & j & k \\ 1 & 5 & -3 \\ 2 & -2 & 1 \end{vmatrix} = \begin{pmatrix} -1 \\ -7 \\ -12 \end{pmatrix}$ ***(M1)A1***

$r \cdot \begin{pmatrix} 1 \\ 7 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 7 \\ 12 \end{pmatrix}$ ***(M1)***

$x + 7y + 12z = -4$ ***A1***

[5 marks]

$$\begin{pmatrix}\\12\end{pmatrix} \quad \begin{pmatrix}\\-1\end{pmatrix} \quad \begin{pmatrix}\\12\end{pmatrix}$$

Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of *k*. It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well.
Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

18e. *[5 marks]*

Markscheme

Note: Accept alternative notation for vectors (*eg* $\langle a, b, c\rangle$ or (a, b, c)).

Let θ be the angle between the lines $\vec{l}_1 = \begin{pmatrix}1\\2\\-3\end{pmatrix}$ and $\vec{l}_2 = $ [illegible]

$\cos\theta = \frac{|2-10-3|}{[illegible]} \Rightarrow \theta = 0.902334\ldots\ (51.699\ldots^\circ)$ ***(M1)***

as the triangle XYZ has a right angle at Y, ***(M1)***

$\mathrm{XZ} = a \Rightarrow \mathrm{YZ} = a\sin\theta$ and $\mathrm{XY} = a\cos\theta$ ***(M1)***

area $= 3 \Rightarrow \frac{a^2 \sin\theta\cos\theta}{2} = 3$ ***(M1)***

$a = 3.5122\ldots$ ***(A1)***

perimeter $= a + a\sin\theta + a\cos\theta = 8.44537\ldots = 8.45$ ***A1***

Note: If candidates attempt to find coordinates of Y and Z award ***M1*** for expression of vector YZ in terms of two parameters, ***M1*** for attempt to use perpendicular condition to determine relation between parameters, ***M1*** for attempt to use the area to find the parameters and ***A2*** for final answer.

[5 marks]

Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of *k*. It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well.
Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

19a. *[4 marks]*

Markscheme

,

$\overrightarrow{AB} = \begin{pmatrix}1\\0\\2\end{pmatrix} - \begin{pmatrix}1\\2\\0\end{pmatrix} = \begin{pmatrix}0\\-2\\2\end{pmatrix}$ ***A1A1***

Note: Award the above marks if the components are seen in the line below.

$\overrightarrow{AC} = \begin{pmatrix}2\\-1\\4\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}1\\-3\\1\end{pmatrix}$

(M1)A1

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k\\ 0 & -2 & 2\\ 1 & -3 & 1\end{vmatrix} = \begin{pmatrix}4\\2\\2\end{pmatrix}$

[4 marks]

Examiners report

Candidates showed a good understanding of the vector techniques required in this question.

19b. *[2 marks]*

Markscheme

area

(M1)

$= \frac{1}{2}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right|$

$= \frac{1}{2}\sqrt{4^2 + 2^2 + 2^2} = \frac{1}{2}\sqrt{24}\ \left(= \sqrt{6}\right)$ ***A1***

Note: Award ***M0A0*** for attempts that do not involve the answer to (a).

[2 marks]

Examiners report

Candidates showed a good understanding of the vector techniques required in this question.

20a. *[3 marks]*

Markscheme

(A1)

$\overrightarrow{PQ} = \begin{pmatrix}8\\6\\4\end{pmatrix}$

equation of line:

$r = \begin{pmatrix}-3\\-1\\2\end{pmatrix} + t\begin{pmatrix}8\\6\\4\end{pmatrix}$ (or equivalent) ***M1A1***

Note: Award ***M1A0*** if ***r*** **=** is omitted.

[3 marks]

Examiners report

There were many good answers to part (a) showing a clear understanding of finding the vector equation of a line. Unfortunately this understanding was marred by many students failing to write the equation properly resulting in just 2 marks out of the 3. The most common response was of the form

$L_1 = \begin{pmatrix}-3\\-1\\ \end{pmatrix} + \lambda\begin{pmatrix}4\\3\\ \end{pmatrix}$

which seemed a waste of a mark.

20b. *[4 marks]*

Markscheme

METHOD 1

$x: \; -4+5s=-3+8t$

$y: \; 2s=-1+6t$

$z: \; 4=2+4t$ ***M1***

solving any two simultaneously ***M1***

$t = 0.5$, $s = 1$ (or equivalent) ***A1***

verification that these values give R when substituted into **both** equations (or that the three equations are consistent and that one gives R) ***R1***

METHOD 2

(1, 2, 4) is given by $t = 0.5$ for L_1 and $s = 1$ for L_2 ***M1A1A1***

because (1, 2, 4) is on both lines it is the point of intersection of the two lines ***R1***

[4 marks]

Examiners report

In part (b) many students failed to verify that the lines do indeed intersect.

20c. *[3 marks]*

Markscheme

$\begin{pmatrix}5\\2\\0\end{pmatrix} \cdot \begin{pmatrix}4\\3\\2\end{pmatrix} = 26 = \sqrt{29} \times \sqrt{29}\cos\theta$ ***M1(A1)***

$\cos\theta = \frac{26}{29}$

$\theta = 0.459$ or $26.3°$ ***A1***

[3 marks]

Examiners report

Part (c) was very well done.

20d. *[6 marks]*

Markscheme

$\overrightarrow{RP} = \begin{pmatrix}-3\\-1\\2\end{pmatrix} - \begin{pmatrix}1\\2\\4\end{pmatrix} = \begin{pmatrix}-4\\-3\\-2\end{pmatrix}$, $\left|\overrightarrow{RP}\right| = \sqrt{29}$ ***(M1)A1***

Note: This could also be obtained from $\left|0.5\begin{pmatrix}8\\6\\4\end{pmatrix}\right|$

EITHER

$\overrightarrow{RS_1} = \begin{pmatrix}-4\\0\\4\end{pmatrix} - \begin{pmatrix}1\\2\\4\end{pmatrix} = \begin{pmatrix}-5\\-2\\0\end{pmatrix}$, $\left|\overrightarrow{RS_1}\right| = \sqrt{29}$ ***A1***

$\therefore \overrightarrow{OS_2} = \overrightarrow{OS_1} + 2\overrightarrow{S_1R} = \begin{pmatrix}-4\\0\\4\end{pmatrix} + 2\begin{pmatrix}5\\2\\0\end{pmatrix}$ ***M1A1***

$\left(\text{or } \overrightarrow{OS_2} = \overrightarrow{OR} + \overrightarrow{S_1R} = \begin{pmatrix}1\\2\\4\end{pmatrix} + \begin{pmatrix}5\\2\\0\end{pmatrix}\right)$

S_2 is $(6, 4, 4)$ $\left(\begin{pmatrix}6\\4\\4\end{pmatrix}\right)$ ***A1***

OR

$\begin{pmatrix}-4+5s\\2s\\4\end{pmatrix} - \begin{pmatrix}1\\2\\4\end{pmatrix} = \begin{pmatrix}5s-5\\2s-2\\0\end{pmatrix}$ ***M1***

$(5s-5)^2 + (2s-2)^2 = 29$ ***M1A1***

$29s^2 - 58s + 29 = 29$

$s(s-2) = 0,\ s = 0,\ 2$

$(6,\ 4,\ 4)$ (and $(-4,\ 0,\ 4)$) ***A1***

Note: There are several geometrical arguments possible using information obtained in previous parts, depending on what forms the previous answers had been given.

[6 marks]

Examiners report

In part (d) most candidates were able to obtain the first three marks, but few were able to find the second point.

20e. *[4 marks]*

Markscheme

EITHER

midpoint of $[\mathrm{PS}_1]$ is M(–3.5, –0.5, 3)   ***M1A1***

$\overrightarrow{\mathrm{RM}} = \begin{pmatrix}-4.5\\-2.5\\-1\end{pmatrix}$ ***A1***

OR

$\overrightarrow{\mathrm{RS}_1} = \begin{pmatrix}-5\\-2\\0\end{pmatrix}$ ***M1***

the direction of the line is $\overrightarrow{\mathrm{RS}_1} + \overrightarrow{\mathrm{RP}}$ ***M1A1***

$\begin{pmatrix}-5\\-2\\0\end{pmatrix} + \begin{pmatrix}-4\\-3\\-2\end{pmatrix} = \begin{pmatrix}-9\\-5\\-2\end{pmatrix}$

THEN

the equation of the line is:

$r = \begin{pmatrix}1\\2\\4\end{pmatrix} + t\begin{pmatrix}9\\5\\2\end{pmatrix}$ or equivalent   ***A1***

Note: Marks cannot be awarded for methods involving halving the angle, unless it is clear that the candidate considers also the equation of the plane of L_1 and L_2 to reduce the number of parameters involved to one (to obtain the vector equation of the required line).

[4 marks]

Examiners report

There were few correct answers to part (e).

21a. *[6 marks]*

Markscheme

(i)

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 5\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$ (or in column vector form) ***(A1)***

Note: Award ***A1*** if any one of the vectors, or its negative, representing the sides of the triangle is seen.

$\left|\overrightarrow{AB}\right| = \left|5\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}\right| = \sqrt{30}$

$\left|\overrightarrow{BC}\right| = \left|-\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}\right| = \sqrt{11}$

$\left|\overrightarrow{CA}\right| = \left|-4\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}\right| = \sqrt{33}$ ***A2***

Note: Award ***A1*** for two correct and ***A0*** for one correct.

(ii) **METHOD 1**

$\cos \mathrm{BAC} = \frac{20+4+2}{\sqrt{30}\sqrt{33}}$ ***M1A1***

Note: Award ***M1*** for an attempt at the use of the scalar product for two vectors representing the sides AB and AC, or their negatives,

A1 for the correct computation using their vectors.

$= \frac{26}{\sqrt{990}} \left(= \frac{26}{3\sqrt{110}}\right)$ ***A1***

Note: Candidates who use the modulus need to justify it – the angle is not stated in the question to be acute.

METHOD 2

using the cosine rule

$\cos \mathrm{BAC} = \frac{30+33-11}{2\sqrt{30}\sqrt{33}}$ ***M1A1***

$= \frac{26}{\sqrt{990}} \left(= \frac{26}{3\sqrt{110}}\right)$ ***A1***

[6 marks]

Examiners report

Many candidates confidently tackled most of this many-part question. Part (b)(i) As the answer was given, candidates were required to show they really knew how to work out a vector product in detail, not just by writing down a 3x3 determinant and then the final answer. Part (d) A few candidates failed to realise that the equation of a line is an equation not simply an expression. Part (e) A significant number of candidates did not realise that they could use their result for part (d).

21b. [5 marks]

Markscheme

A1

$\overrightarrow{\mathrm{BC}} \times \overrightarrow{\mathrm{CA}} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -1 & -3 & 1 \\ -4 & 4 & 1 \end{vmatrix}$

$\boldsymbol{i} + ((-3) \times 1 - 1 \times 4)$

$\boldsymbol{j} + (1 \times (-4) - (-1) \times 1)$

$\boldsymbol{k}$ ***M1A1***

$((-1) \times 4 - (-3) \times (-4))$

$= -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$ ***AG***

(ii) the area of

(M1)

$\Delta \mathrm{ABC} = \frac{1}{2}\left|\overrightarrow{\mathrm{BC}} \times \overrightarrow{\mathrm{CA}}\right|$

A1

$\frac{1}{2}\sqrt{(-7)^2 + (-3)^2 + (-16)^2}$

AG

$= \frac{1}{2}\sqrt{314}$

[5 marks]

Examiners report

Many candidates confidently tackled most of this many-part question. Part (b)(i) As the answer was given, candidates were required to show they really knew how to work out a vector product in detail, not just by writing down a 3x3 determinant and then the final answer. Part (d) A few candidates failed to realise that the equation of a line is an equation not simply an expression. Part (e) A significant number of candidates did not realise that they could use their result for part (d).

21c. [3 marks]

Markscheme

attempt at the use of "(***r*** – ***a***)

· ***n*** = 0" ***(M1)***

using ***r*** = *x**i*** + *y**j*** + *z**k***, ***a*** = OA and ***n*** = –7***i*** – 3***j*** – 16***k*** ***(A1)***

A1

$7x + 3y + 16z = 47$

Note: Candidates who adopt a 2-parameter approach should be awarded, ***A1*** for correct 2-parameter equations for *x*, *y* and *z*; ***M1*** for a serious attempt at elimination of the parameters; ***A1*** for the final Cartesian equation.

[3 marks]

Examiners report

Many candidates confidently tackled most of this many-part question. Part (b)(i) As the answer was given, candidates were required to show they really knew how to work out a vector product in detail, not just by writing down a 3x3 determinant and then the final answer. Part (d) A few candidates failed to realise that the equation of a line is an equation not simply an expression. Part (e) A significant number of candidates did not realise that they could use their result for part (d).

21d. *[2 marks]*

Markscheme

$\boldsymbol{r} = \overrightarrow{OA} + t\overrightarrow{AB}$ (or equivalent) ***M1***

$\boldsymbol{r} = (-\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + t(5\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k})$ ***A1***

Note: Award ***M1A0*** if "$\boldsymbol{r} =$" is missing.

Note: Accept forms of the equation starting with B or with the direction reversed.

[2 marks]

Examiners report

Many candidates confidently tackled most of this many-part question. Part (b)(i) As the answer was given, candidates were required to show they really knew how to work out a vector product in detail, not just by writing down a 3x3 determinant and then the final answer. Part (d) A few candidates failed to realise that the equation of a line is an equation not simply an expression. Part (e) A significant number of candidates did not realise that they could use their result for part (d).

21e. *[5 marks]*

Markscheme

(i)

$\overrightarrow{OD} = (-\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + t(5\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k})$

statement that $\overrightarrow{OD}\cdot\overrightarrow{BC} = 0$ ***(M1)***

A1

$\begin{pmatrix} -1+5t \\ 2-t \\ 3-2t \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} = 0$ ***A1***

$-2 - 4t = 0$ or $t = -\frac{1}{2}$ ***A1***

coordinates of D are $\left(-\frac{7}{2}, \frac{5}{2}, 4\right)$ ***A1***

Note: Different forms of $\overrightarrow{OD}$ give different values of t, but the same final answer.

(ii)

$t < 0 \Rightarrow$ D is not between A and B ***R1***

[5 marks]

Examiners report

Many candidates confidently tackled most of this many-part question. Part (b)(i) As the answer was given, candidates were required to show they really knew how to work out a vector product in detail, not just by writing down a 3x3 determinant and then the final answer. Part (d) A few candidates failed to realise that the equation of a line is an equation not simply an expression. Part (e) A significant number of candidates did not realise that they could use their result for part (d).

22a. *[2 marks]*

Markscheme

$\overrightarrow{\mathrm{CB}} = b - c$
$\overrightarrow{\mathrm{AC}} = b + c$ ***A1A1***

Note: Condone absence of vector notation in (a).

[2 marks]

Examiners report

Most candidates were able to find the expressions for the two vectors although a number were not able to do this. Most then tried to use Pythagoras' theorem and confused scalars and vectors. There were few correct responses to the second part. Candidates did not seem to be able to use the algebra of vectors comfortably.

22b. *[3 marks]*

Markscheme

$\overrightarrow{\mathrm{AC}} \cdot \overrightarrow{\mathrm{CB}} = (\boldsymbol{b} + \boldsymbol{c}) \cdot (\boldsymbol{b} - \boldsymbol{c})$ ***M1***

$= |\boldsymbol{b}|^2 - |\boldsymbol{c}|^2$ ***A1***

$= 0$ since $|\boldsymbol{b}| = |\boldsymbol{c}|$ ***R1***

Note: Only award the ***A1*** and ***R1*** if working indicates that they understand that they are working with vectors.

so $\overrightarrow{\mathrm{AC}}$ is perpendicular to $\overrightarrow{\mathrm{CB}}$ *i.e.* $\mathrm{A\hat{C}B}$ is a right angle ***AG***

[3 marks]

Examiners report

Most candidates were able to find the expressions for the two vectors although a number were not able to do this. Most then tried to use Pythagoras' theorem and confused scalars and vectors. There were few correct responses to the second part. Candidates did not seem to be able to use the algebra of vectors comfortably.

23a. *[2 marks]*

Markscheme

,

$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix}-1\\-1\\3\end{pmatrix}$ ***(M1)***

$\overrightarrow{\mathrm{SR}} = \begin{pmatrix}0-x\\5-y\\1-z\end{pmatrix}$

point S $= (1, 6, -2)$ ***A1***

[2 marks]

Examiners report

This was a multi-part question that was well answered by many candidates. Wrong answers to part (a) were mainly the result of failing to draw a diagram. Follow through benefitted many candidates. A high proportion of candidates lost the mark in (e) by not writing their answer as an equation in the form *r* = ...

23b. *[2 marks]*

Markscheme

A1

$\overrightarrow{\mathrm{PS}} = \begin{pmatrix}2-1\\4-1\\\ldots\end{pmatrix}$

$m = -2 \ldots 13$ ***A1***

$\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PS}} = \begin{pmatrix}-13\\7\\-2\end{pmatrix}$

[2 marks]

Examiners report

This was a multi-part question that was well answered by many candidates. Wrong answers to part (a) were mainly the result of failing to draw a diagram. Follow through benefitted many candidates. A high proportion of candidates lost the mark in (e) by not writing their answer as an equation in the form *r* = ...

23c. *[2 marks]*

Markscheme

area of parallelogram PQRS

M1

$= \left|\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PS}}\right| = \sqrt{(-13)^2 + 7^2 + (-2)^2}$ ***A1***

$= \sqrt{222} = 14.9$

[2 marks]

Examiners report

This was a multi-part question that was well answered by many candidates. Wrong answers to part (a) were mainly the result of failing to draw a diagram. Follow through benefitted many candidates. A high proportion of candidates lost the mark in (e) by not writing their answer as an equation in the form *r* = ...

23d.

[3 marks]

Markscheme

equation of plane is $-13x + 7y - 2z = d$ ***M1A1***

substituting any of the points given gives $d = 33$

$-13x + 7y - 2z = 33$ ***A1***

[3 marks]

Examiners report

This was a multi-part question that was well answered by many candidates. Wrong answers to part (a) were mainly the result of failing to draw a diagram. Follow through benefitted many candidates. A high proportion of candidates lost the mark in (e) by not writing their answer as an equation in the form $r = \ldots$

23e.

[1 mark]

Markscheme

equation of line is

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -13 \\ 7 \\ -2 \end{pmatrix}$$ ***A1***

Note: To get the ***A1*** must have $r =$ or equivalent.

[1 mark]

Examiners report

This was a multi-part question that was well answered by many candidates. Wrong answers to part (a) were mainly the result of failing to draw a diagram. Follow through benefitted many candidates. A high proportion of candidates lost the mark in (e) by not writing their answer as an equation in the form $r = \ldots$

23f.

[3 marks]

Markscheme

$169\lambda + 49\lambda + 4\lambda = 33$ ***M1***

$\lambda = \frac{33}{222}\ (= 0.149\ldots)$ ***A1***

closest point is

$\left(-\frac{143}{74}, \frac{77}{74}, -\frac{11}{37}\right)\ (= (-1.93,\ 1.04,\ -0.297))$ ***A1***

[3 marks]

Examiners report

This was a multi-part question that was well answered by many candidates. Wrong answers to part (a) were mainly the result of failing to draw a diagram. Follow through benefitted many candidates. A high proportion of candidates lost the mark in (e) by not writing their answer as an equation in the form $r = \ldots$

23g. *[4 marks]*

Markscheme

angle between planes is the same as the angle between the normals ***(R1)***

$\cos\theta = \frac{-13\times 1+7\times -2-2\times 1}{\sqrt{222}\times\sqrt{6}}$ ***M1A1***

(accept $\theta = 143^\circ$ or 2.49 radians or 0.652 radians) ***A1***

$\theta = 37.4^\circ$

[4 marks]

Examiners report

This was a multi-part question that was well answered by many candidates. Wrong answers to part (a) were mainly the result of failing to draw a diagram. Follow through benefitted many candidates. A high proportion of candidates lost the mark in (e) by not writing their answer as an equation in the form *r* = ...

24. *[7 marks]*

Markscheme

(a) for using normal vectors ***(M1)***

$$\begin{pmatrix}1\\2\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}=1-1=0$$ ***M1A1***

hence the two planes are perpendicular ***AG***

(b) **METHOD 1**

EITHER

$$\begin{vmatrix}i & j & k\\1 & 2 & -1\\1 & 0 & 1\end{vmatrix}=$$ $2\mathbf{i}-2\boldsymbol{j}-2\boldsymbol{k}$ ***M1A1***

OR

if $\begin{pmatrix}a\\b\\c\end{pmatrix}$ is normal to π_3, then

$a+2b-c=0$

and

$a+c=0$ ***M1***

a solution is $a=1$, $b=-1$, $c=-1$ ***A1***

THEN

π_3 has equation

$x-y-z=d$ ***(M1)***

as it goes through the origin, $d=0$ so

π_3 has equation

$x-y-z=0$ ***A1***

Note: The final ***(M1)A1*** are independent of previous working.

METHOD 2

$$r=\begin{pmatrix}0\\0\\0\end{pmatrix}+s\begin{pmatrix}1\\2\\-1\end{pmatrix}+t\begin{pmatrix}1\\0\\1\end{pmatrix}$$ ***A1(A1)A1A1***

[7 marks]

Examiners report

Although many candidates were successful in answering this question, a surprising number showed difficulties in working with normal vectors. In part (b) there were several candidates who found the cross product of the vectors but were unable to use it to write the equation of the plane.

25.

[4 marks]

Markscheme

$(\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{a} - \boldsymbol{b}) = \boldsymbol{a} \cdot \boldsymbol{a} + \boldsymbol{b} \cdot \boldsymbol{a} - \boldsymbol{a} \cdot \boldsymbol{b} - \boldsymbol{b} \cdot \boldsymbol{b}$ ***M1***

$= \boldsymbol{a} \cdot \boldsymbol{a} - \boldsymbol{b} \cdot \boldsymbol{b}$ ***A1***

$= |\boldsymbol{a}|^2 - |\boldsymbol{b}|^2 = 0$ since $|\boldsymbol{a}| = |\boldsymbol{b}|$ ***A1***

the **diagonals** are perpendicular ***R1***

Note: Accept geometric proof, awarding ***M1*** for recognizing OACB is a rhombus, ***R1*** for a clear indication that $(\boldsymbol{a} + \boldsymbol{b})$ and $(\boldsymbol{a} - \boldsymbol{b})$ are the diagonals, ***A1*** for stating that diagonals cross at right angles and ***A1*** for "hence dot product is zero".

Accept solutions using components in 2 or 3 dimensions.

[4 marks]

Examiners report

Many candidates found this more abstract question difficult. While there were some correct statements, they could not "show" the result that was asked. Some treated the vectors as scalars and notation was poor, making it difficult to follow what they were trying to do. Very few candidates realized that $a - b$ and $a + b$ were the diagonals of the parallelogram which prevented them from identifying the significance of the result proved. A number of candidates were clearly not aware of the difference between scalars and vectors.

26. *[20 marks]*

Markscheme

(a) **EITHER**

normal to plane given by

M1A1

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 3 & 2 \\ 6 & -3 & 2 \end{vmatrix}$$

$= 12\boldsymbol{i} + 8\boldsymbol{j} - 24\boldsymbol{k}$ ***A1***

equation of π is

(M1)

$3x + 2y - 6z = d$

as goes through (–2, 3, –2) so d = 12 ***M1A1***

AG

$\pi : 3x + 2y - 6z = 12$

OR

$x = -2 + 2\lambda + 6\mu$

$y = 3 + 3\lambda - 3\mu$

$z = -2 + 2\lambda + 2\mu$

eliminating μ

$x + 2y = 4 + 8\lambda$

M1A1A1

$2y + 3z = 12\lambda$

eliminating λ

M1A1A1

$3(x + 2y) - 2(2y + 3z) = 12$

AG

$\pi : 3x + 2y - 6z = 12$

[6 marks]

(b) therefore A(4, 0, 0), B(0, 6, 0) and C(0, 0, 2) ***A1A1A1***

Note: Award ***A1A1A0*** if position vectors given instead of coordinates.

[3 marks]

(c) area of base

M1

$\text{OAB} = \frac{1}{2} \times 4 \times 6 = 12$

M1A1

$V = \frac{1}{3} \times 12 \times 2 = 8$

[3 marks]

(d)

M1A1

$$\begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 3 = 7 \times 1 \times \cos\phi$$

so $\phi = \arccos\frac{3}{7}$

(accept 0.443 radians) ***M1A1***

$\theta = 90 - \arccos\frac{3}{7} = 25.4°$

[4 marks]

(e)

(M1)A1
$d = 4\sin\theta = \frac{12}{7} \quad (= 1.71)$
[2 marks]

(f)

M1A1
$8 = \frac{1}{3} \times \frac{12}{7} \times \text{area} \Rightarrow \text{area} = 14$
Note: If answer to part (f) is found in an earlier part, award ***M1A1***, regardless of the fact that it has not come from their answers to part (c) and part (e).

[2 marks]

Total [20 marks]

Examiners report

The question was generally well answered, although there were many students who failed to recognise that the volume was most logically found using a base as one of the coordinate planes.

27. *[5 marks]*

$2y + 8x = 4$

$-3x + 2y = -7$

$2x + 6 - 2x = 6$

$| = \left|\begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}\right|$

$\Rightarrow \left|\begin{pmatrix} 4 \\ -7 \\ 6 \end{pmatrix}\right|$

$= \sqrt{4^2 + (-7)^2 + 6^2}$

$= \sqrt{101}$

Examiners report

The majority of candidates understood what was required in part (a) of this question and gained the correct answer. Most candidates were able to do part (b) but few realised that they did not have to calculate $|\boldsymbol{a} + 2\boldsymbol{b}|$ as this is $|\boldsymbol{c}|$. Many candidates lost time on this question.

28a. [11 marks]

Markscheme

(i) use of ***a***

b =
·
a
|
b
|
(M1)
|$\cos\theta$

a

b = –1 ***(A1)***
·
a
|
= 7,
|
b
|
= 5 ***(A1)***
|
A1
$\cos\theta = -\frac{1}{35}$

(ii) the required cross product is

18***i*** - 24***j*** -18***k*** ***M1A1***

$\begin{vmatrix} i & j & k \\ 6 & 3 & 2 \\ 0 & -3 & 4 \end{vmatrix} =$

(iii) using ***r***

n = ***p***
·
n the equation of the plane is ***(M1)***
·
A1
$18x - 24y - 18z = 12 \ \ (3x - 4y - 3z = 2)$

(iv) recognizing that $z = 0$ ***(M1)***

x-intercept

, y-intercept
$= \frac{2}{3}$
(A1)
$= -\frac{1}{2}$
area

A1
$= \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{6}$

[11 marks]

Examiners report

This was the most accessible question in section B for the candidates. The majority of candidates produced partially correct answers to part (a), with nearly all candidates being able to use the scalar and vector product. Candidates found part (iv) harder and often did not appreciate the significance of letting $z = 0$. Candidates clearly found part (b) harder and again this was a point where candidates lost time. Many candidates attempted this using components, which was fine in part (i), fine, but time consuming in part (ii), and extremely complicated in part (iii). A number of candidates lost marks because they were careless in showing their working in part (ii) which required them to “show that”.

28b. *[8 marks]*

Markscheme

(i) $\boldsymbol{p} \cdot \boldsymbol{p} = |\boldsymbol{p}||\boldsymbol{p}|\cos 0$ ***M1A1***

$= |\boldsymbol{p}|^2$ ***AG***

(ii) consider the LHS, and use of result from part (i)

$|\boldsymbol{p}+\boldsymbol{q}|^2 = (\boldsymbol{p}+\boldsymbol{q}) \cdot (\boldsymbol{p}+\boldsymbol{q})$ ***M1***

$= \boldsymbol{p} \cdot \boldsymbol{p} + \boldsymbol{p} \cdot \boldsymbol{q} + \boldsymbol{q} \cdot \boldsymbol{p} + \boldsymbol{q} \cdot \boldsymbol{q}$ ***(A1)***

$= \boldsymbol{p} \cdot \boldsymbol{p} + 2\boldsymbol{p} \cdot \boldsymbol{q} + \boldsymbol{q} \cdot \boldsymbol{q}$ ***A1***

$= |\boldsymbol{p}|^2 + 2\boldsymbol{p} \cdot \boldsymbol{q} + |\boldsymbol{q}|^2$ ***AG***

(iii) **EITHER**

use of $\boldsymbol{p} \cdot \boldsymbol{q} \leqslant |\boldsymbol{p}||\boldsymbol{q}|$ ***M1***

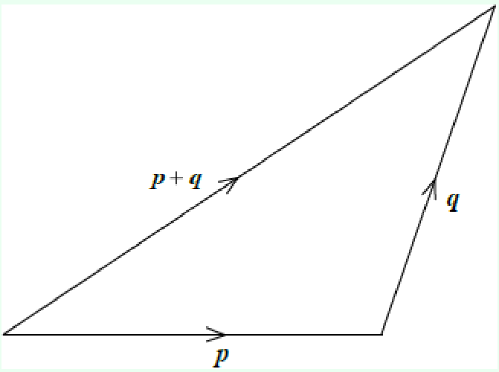
so $0 \leqslant |\boldsymbol{p}+\boldsymbol{q}|^2 = \boldsymbol{p} \cdot$

$|^2 + 2\boldsymbol{p}\cdot\boldsymbol{q} + |\boldsymbol{q}|^2 \leqslant |\boldsymbol{p}|^2 + 2|\boldsymbol{p}||\boldsymbol{q}| + |\boldsymbol{q}|^2$ ***A1***

take square root (of these positive quantities) to establish ***A1***

$|\boldsymbol{p}+\boldsymbol{q}| \leqslant |\boldsymbol{p}| + |\boldsymbol{q}|$ ***AG***

OR



M1M1

Note: Award ***M1*** for correct diagram and ***M1*** for correct labelling of vectors including arrows.

since the sum of any two sides of a triangle is greater than the third side,

$|\boldsymbol{p}| + |\boldsymbol{q}| > |\boldsymbol{p}+\boldsymbol{q}|$ ***A1***

when $\boldsymbol{p}$ and $\boldsymbol{q}$ are collinear

$|\boldsymbol{p}| + |\boldsymbol{q}| =$

|
$\boldsymbol{p}+\boldsymbol{q}$
|

|
$\boldsymbol{p}+\boldsymbol{q}$
$\Rightarrow$ |

|

$\leqslant$
$\boldsymbol{p}$
|
+
|
$\boldsymbol{q}$
|
AG
|

[8 marks]

Examiners report

This was the most accessible question in section B for the candidates. The majority of candidates produced partially correct answers to part (a), with nearly all candidates being able to use the scalar and vector product. Candidates found part (iv) harder and often did not appreciate the significance of letting $z = 0$. Candidates clearly found part (b) harder and again this was a point where candidates lost time. Many candidates attempted this using components, which was fine in part (i), fine, but time consuming in part (ii), and extremely complicated in part (iii). A number of candidates lost marks because they were careless in showing their working in part (ii) which required them to “show that”.

29. *[8 marks]*

Markscheme

(a)

$a = \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} \perp$ to the plane

$e = \begin{pmatrix} -2 \\ 1 \\ k \end{pmatrix}$ is parallel to the line ***(A1)(A1)***

Note: Award ***A1*** for each correct vector written down, even if not identified.

line $\perp$ plane $\Rightarrow$ e parallel to a

since $\begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} = t \begin{pmatrix} -2 \\ 1 \\ k \end{pmatrix} \Rightarrow k = \frac{1}{2}$ ***(M1)A1***

(b)

$4(3-2\lambda) - 2\lambda - \left(-1 + \frac{1}{2}\lambda\right) = 1$ ***(M1)(A1)***

Note: ***FT*** their value of k as far as possible.

$\lambda = \frac{8}{7}$ ***A1***

$\mathrm{P}\left(\frac{5}{7}, \frac{8}{7}, -\frac{3}{7}\right)$ ***A1***

[8 marks]

Examiners report

Solutions to this question were often disappointing. In (a), some candidates found the value of k, incorrectly, by taking the scalar product of the normal vector to the plane and the direction of the line. Such candidates benefitted partially from follow through in (b) but not fully because their line turned out to be parallel to the plane and did not intersect it.

30.

[5 marks]

Markscheme

METHOD 1

(a)

$|\boldsymbol{a}-\boldsymbol{b}| = \sqrt{|a|^2+|b|^2-2|a||b|\cos\alpha}$ ***M1***

$= \sqrt{2-2\cos\alpha}$ ***A1***

$|\boldsymbol{a}+\boldsymbol{b}| = \sqrt{|a|^2+|b|^2-2|a||b|\cos(\pi-\alpha)}$

$= \sqrt{2+2\cos\alpha}$ ***A1***

Note: Accept the use of a, b for $|\boldsymbol{a}|$, $|\boldsymbol{b}|$.

(b)

$= \sqrt{2+2\cos\alpha} = 3\sqrt{2-2\cos\alpha}$ ***M1***

$\cos\alpha = \frac{4}{5}$ ***A1***

METHOD 2

(a)

$|\boldsymbol{a}-\boldsymbol{b}| = 2\sin\frac{\alpha}{2}$ ***M1A1***

$|\boldsymbol{a}+\boldsymbol{b}| = 2\sin\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) = 2\cos\frac{\alpha}{2}$ ***A1***

Note: Accept the use of a, b for $|\boldsymbol{a}|$, $|\boldsymbol{b}|$.

(b)

$2\cos\frac{\alpha}{2} = 6\sin\frac{\alpha}{2}$ ***M1***

$\tan\frac{\alpha}{2} = \frac{1}{3} \Rightarrow \cos^2\frac{\alpha}{2} = \frac{9}{10}$ ***A1***

$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 = \frac{4}{5}$

[5 marks]

Examiners report

To solve this problem, candidates had to know either that $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{a}+\boldsymbol{b}) = |\boldsymbol{a}+\boldsymbol{b}|^2$ or that the diagonals of a parallelogram whose sides are $\boldsymbol{a}$ and $\boldsymbol{b}$ represent the vectors $\boldsymbol{a}+\boldsymbol{b}$ and $\boldsymbol{a}-\boldsymbol{b}$. It was clear from the scripts that many candidates were unaware of either result and were therefore unable to make any progress in this question.

31.

[8 marks]

Markscheme

(a)

$\cos\theta = \frac{ab}{|a||b|} = \frac{\sin 2\alpha\cos\alpha + \sin\alpha\cos 2\alpha - 1}{\sqrt{2}\times\sqrt{2}} \left(= \frac{\sin 3\alpha - 1}{2}\right)$ ***M1A1***

(b)

$a \perp b \Rightarrow \cos\theta = 0$ ***M1***

$\sin 2\alpha\cos\alpha + \sin\alpha\cos 2\alpha - 1 = 0$

$\alpha = 0.524 \left(= \frac{\pi}{6}\right)$ ***A1***

(c)

METHOD 1

$\begin{vmatrix} i & j & k \\ \sin 2\alpha & -\cos 2\alpha & 1 \\ \cos\alpha & -\sin\alpha & -1 \end{vmatrix}$ assuming $\alpha = \frac{7\pi}{6}$ ***(M1)***

Note: Allow substitution at any stage.

$\begin{vmatrix} i & j & k \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & -1 \end{vmatrix} = i\left(\frac{1}{2} - \frac{1}{2}\right) - j\left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) + k\left(\frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right)$ ***A1***

$= 0$ ***A1***

$\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel ***R1***

Note: Accept decimal equivalents.

METHOD 2

from (a)

$\cos\theta = -1 \ (\text{and } \sin\theta = 0)$ ***M1A1***

$a \times b = 0$ ***A1***

$\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel ***R1***

[8 marks]

Examiners report

This question was attempted by most candidates who in general were able to find the dot product of the vectors in part (a). However the simplification of the expression caused difficulties which affected the performance in part (b). Many candidates had difficulties in interpreting the meaning of $\boldsymbol{a} \times \boldsymbol{b} = 0$ in part (c).

32. [20 marks]

Markscheme

(a)

$\overrightarrow{OM} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $\overrightarrow{ON} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ ***A1A1A1***

[3 marks]

(b)

$\overrightarrow{MP} = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$ and $\overrightarrow{MN} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ ***A1A1***

$\overrightarrow{MP} \times \overrightarrow{MN} = \begin{pmatrix} i & j & k \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ ***(M1)A1***

[4 marks]

(c) (i) area of MNP $= \frac{1}{2}\left|\overrightarrow{MP} \times \overrightarrow{MN}\right|$ ***M1***

$= \frac{1}{2}\left|\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right|$

$= \frac{\sqrt{3}}{2}$ ***A1***

(ii) $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\overrightarrow{OG} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$

$\overrightarrow{AG} = \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$ ***A1***

since $\overrightarrow{AG} = 2(\overrightarrow{MP} \times \overrightarrow{MN})$

$\overrightarrow{AG}$ is perpendicular to MNP ***R1***

(iii) $r \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ ***M1A1***

$r \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 3$ (accept $-x + y + z = 3$) ***A1***

[7 marks]

(d)

$r = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$ ***A1***

$\begin{pmatrix} 2 - 2\lambda \\ 2\lambda \\ 2\lambda \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 3$

$-2 + 2\lambda + 2\lambda + 2\lambda = 3$ ***M1A1***

$\lambda = \frac{5}{6}$ ***A1***

coordinates of point

$r = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \frac{5}{6}\begin{pmatrix} -2 \\ 2 \\ \ldots \end{pmatrix}$ ***M1***

$r = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \frac{}{6}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

A1
$\left(\frac{1}{3}, \frac{5}{3}, \frac{5}{3}\right)$

[6 marks]

Total [20 marks]

Examiners report

This was the most successfully answered question in part B, with many candidates achieving full marks. There were a few candidates who misread the question and treated the cube as a unit cube. The most common errors were either algebraic or arithmetic mistakes. A variety of notation forms were seen but in general were used consistently. In a few cases, candidates failed to show all the work or set it properly.

33a. *[8 marks]*

Markscheme

(i)

$|a - b| = |a + b|$

(M1)
$\Rightarrow (a - b) \cdot (a - b) = (a + b) \cdot (a + b)$

A1
$\Rightarrow |a|^2 - 2a \cdot b + |b|^2 = |a|^2 + 2a \cdot b + |b|^2$

A1
$\Rightarrow 4a \cdot b = 0 \Rightarrow a \cdot b = 0$

therefore a and b are perpendicular ***R1***

Note: Allow use of 2-d components.

Note: Do not condone sloppy vector notation, so we must see something to the effect that $|c|^2 = c \cdot c$ is clearly being used for the ***M1***.

Note: Allow a correct geometric argument, for example that the diagonals of a parallelogram have the same length only if it is a rectangle.

(ii)

M1A1
$|a \times b|^2 = (|a||b|\sin\theta)^2 = |a|^2|b|^2\sin^2\theta$

M1
$|a|^2|b|^2 - (a \cdot b)^2 = |a|^2|b|^2 - |a|^2|b|^2\cos^2\theta$

A1
$= |a|^2|b|^2\left(1 - \cos^2\theta\right)$

$= |a|^2|b|^2\left(\sin^2\theta\right)$

AG
$\Rightarrow |a \times b|^2 = |a|^2|b|^2 - (a \cdot b)^2$

[8 marks]

Examiners report

(i) The majority of candidates were very sloppy in their use of vector notation. Some candidates used Cartesian coordinates, which was acceptable. Part (ii) was well done.

33b. *[7 marks]*

Markscheme

(i) area of triangle

(M1)

$= \frac{1}{2}\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}\right|$

A1

$= \frac{1}{2}|(b-a) \times (c-a)|$

A1

$= \frac{1}{2}|b \times c + b \times -a + -a \times c + -a \times -a|$

;

$b \times -a = a \times b$

;

$c \times a = -a \times c$

M1

$-a \times -a = 0$

hence, area of triangle is

AG

$\frac{1}{2}|a \times b + b \times c + c \times a|$

(ii) D is the foot of the perpendicular from B to AC

area of triangle

A1

ABC $= \frac{1}{2}\left|\overrightarrow{\mathrm{AC}}\right|\left|\overrightarrow{\mathrm{BD}}\right|$

therefore

M1

$\frac{1}{2}\left|\overrightarrow{\mathrm{AC}}\right|\left|\overrightarrow{\mathrm{BD}}\right| = \frac{1}{2}\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}\right|$

hence,

A1

$\left|\overrightarrow{\mathrm{BD}}\right| = \frac{\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}\right|}{\left|\overrightarrow{\mathrm{AC}}\right|}$

AG

$= \frac{|a \times b + b \times c + c \times a|}{|c-a|}$

[7 marks]

Examiners report

Part (i) was usually well started, but not completed satisfactorily. Many candidates understood the geometry involved in this part.

34a. *[6 marks]*

Markscheme

(a) **METHOD 1**

solving simultaneously (gdc) ***(M1)***

$x = 1 + 2z;\ y = -1 - 5z$ ***A1A1***

$L : r = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$ ***A1A1A1***

Note: 1^{st} ***A1*** is for ***r*** =.

[6 marks]

METHOD 2

direction of line

$= \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ 2 & 1 & 1 \end{vmatrix}$ (last two rows swapped) ***M1***

= 2***i*** – 5***j*** + ***k*** ***A1***

putting *z* = 0, a point on the line satisfies

$2x + y = 1,\ 3x + y = 2$ ***M1***

i.e. (1, –1, 0) ***A1***

the equation of the line is

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$ ***A1A1***

Note: Award ***A0A1*** if $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is missing.

[6 marks]

Examiners report

Candidates generally attempted this question but with varying degrees of success. Although (a) was answered best of all the parts, quite a few did not use correct notation to designate the vector equation of a line, i.e., ***r*** =, or its equivalent. In (b) some candidates incorrectly assumed the result and worked the question from there. In (c) some candidates did not understand the necessary relationships to make a meaningful attempt.

34b. *[4 marks]*

Markscheme

$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$ ***M1***

$= 6\boldsymbol{i} - 12\boldsymbol{k}$ ***A1***

hence, $\boldsymbol{n} = \boldsymbol{i} - 2\boldsymbol{k}$

therefore $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$

$\boldsymbol{n} \cdot \boldsymbol{a} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 1$ ***M1A1***

$\Rightarrow x - 2z = 1$ ***AG***

[4 marks]

Examiners report

Candidates generally attempted this question but with varying degrees of success. Although (a) was answered best of all the parts, quite a few did not use correct notation to designate the vector equation of a line, i.e., ***r*** =, or its equivalent. In (b) some candidates incorrectly assumed the result and worked the question from there. In (c) some candidates did not understand the necessary relationships to make a meaningful attempt.

34c. [6 marks]

Markscheme

METHOD 1

P = (–2, 4, 1), Q = $(x,\ y,\ z)$ ***A1***

$\overrightarrow{\text{PQ}} = \begin{pmatrix} x+2 \\ y-4 \\ z-1 \end{pmatrix}$ is perpendicular to $3x + y - z = 2$

$\Rightarrow \overrightarrow{\text{PQ}}$ is parallel to 3***i*** + ***j*** – ***k*** ***R1***

A1

$\Rightarrow x + 2 = 3t;\ y - 4 = t;\ z - 1 = -t$ ***A1***

$1 - z = t \Rightarrow x + 2 = 3 - 3z \Rightarrow x + 3z = 1$

solving simultaneously ***M1***

$x + 3z = 1;\ x - 2z = 1$ ***A1***

$5z = 0 \Rightarrow z = 0;\ x = 1,\ y = 5$

hence, Q = (1, 5, 0)

[6 marks]

METHOD 2

Line passing through PQ has equation

$\text{r} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$ ***M1A1***

Meets π_3 when: ***M1A1***

$-2 + 3t - 2(1 - t) = 1$

t = 1 ***A1***

Q has coordinates (1, 5, 0) ***A1***

[6 marks]

Examiners report

Candidates generally attempted this question but with varying degrees of success. Although (a) was answered best of all the parts, quite a few did not use correct notation to designate the vector equation of a line, i.e., ***r*** **=**, or its equivalent. In (b) some candidates incorrectly assumed the result and worked the question from there. In (c) some candidates did not understand the necessary relationships to make a meaningful attempt.

35a. [2 marks]

Markscheme

$\overrightarrow{\text{AB}} = b - a$ ***A1***

$\overrightarrow{\text{CB}} = a + b$ ***A1***

[2 marks]

Examiners report

This question was poorly done with most candidates having difficulties in using appropriate notation which made unclear the distinction between scalars and vectors. A few candidates scored at least one of the marks in (a) but most candidates had problems in setting up the proof required in (b) with many using a circular argument which resulted in a very poor performance in this part.

35b. *[3 marks]*

Markscheme

$\overrightarrow{AB} \cdot \overrightarrow{CB} = (b-a)\cdot(b+a)$ ***M1***

$= |b|^2 - |a|^2$ ***A1***

$= 0$ since $|b| = |a|$ ***R1***

Note: Only award the ***A1*** and ***R1*** if working indicates that they understand that they are working with vectors.

so $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CB}$ *i.e.* $A\hat{B}C$ is a right angle ***AG***

[3 marks]

Examiners report

This question was poorly done with most candidates having difficulties in using appropriate notation which made unclear the distinction between scalars and vectors. A few candidates scored at least one of the marks in (a) but most candidates had problems in setting up the proof required in (b) with many using a circular argument which resulted in a very poor performance in this part.

36a. *[2 marks]*

Markscheme

$\overrightarrow{AB} = \begin{pmatrix}-4\\4\\-3\end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix}-4\\3\\1\end{pmatrix}$ ***A1A1***

Note: Accept row vectors.

[2 marks]

Examiners report

Most candidates attempted this question and scored at least a few marks in (a) and (b). Part (c) was more challenging to many candidates who were unsure how to find the required distance. Part (d) was attempted by many candidates some of whom benefited from follow through marks due to errors in previous parts. However, many candidates failed to give the correct answer to this question due to the use of the simplified vector found in (b) showing little understanding of the role of the magnitude of this vector. Part (e) was poorly answered. Overall, this question was not answered to the expected level, showing that many candidates have difficulties with vectors and are unable to answer even standard questions on this topic.

36b. *[4 marks]*

Markscheme

normal $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}} = \begin{vmatrix} i & j & k \\ -4 & -1 & 3 \\ 4 & -3 & 1 \end{vmatrix} = \begin{pmatrix} 8 \\ 16 \\ 16 \end{pmatrix}$ ***M1A1***

so $n = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ ***A1***

$r \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ ***(M1)***

$x + 2y + 2z = 7$

Note: If attempt to solve by a system of equations:

Award ***A1*** for 3 correct equations, ***A1*** for eliminating a variable and ***A2*** for the correct answer.

[4 marks]

Examiners report

Most candidates attempted this question and scored at least a few marks in (a) and (b). Part (c) was more challenging to many candidates who were unsure how to find the required distance. Part (d) was attempted by many candidates some of whom benefited from follow through marks due to errors in previous parts. However, many candidates failed to give the correct answer to this question due to the use of the simplified vector found in (b) showing little understanding of the role of the magnitude of this vector. Part (e) was poorly answered. Overall, this question was not answered to the expected level, showing that many candidates have difficulties with vectors and are unable to answer even standard questions on this topic.

37. *[7 marks]*

Markscheme

METHOD 1

equation of journey of ship S_1

$r_1 = t\begin{pmatrix}10\\20\end{pmatrix}$

equation of journey of speedboat S_2 ,setting off k minutes later

$r_2 = \begin{pmatrix}70\\30\end{pmatrix} + (t-k)\begin{pmatrix}-60\\30\end{pmatrix}$ ***M1A1A1***

Note: Award ***M1*** for perpendicular direction, ***A1*** for speed, ***A1*** for change in parameter (*e.g.* by using $t-k$ or T, k being the time difference between the departure of the ships).

solve

$t\begin{pmatrix}10\\20\end{pmatrix} = \begin{pmatrix}70\\30\end{pmatrix} + (t-k)\begin{pmatrix}-60\\30\end{pmatrix}$ ***(M1)***

Note: ***M*** mark is for equating their two expressions.

$10t = 70 - 60t + 60k$

$20t = 30 + 30t - 30k$ ***M1***

Note: ***M*** mark is for obtaining two equations involving two different parameters.

$7t - 6k = 7$

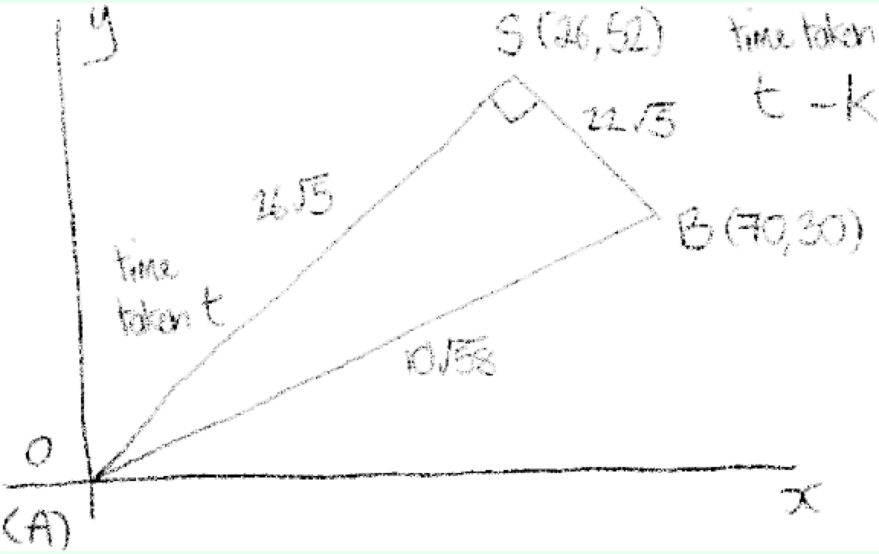
$-t + 3k = 3$

$k = \frac{28}{15}$ ***A1***

latest time is 11:52 ***A1***

[7 marks]

METHOD 2



$SB = 22\sqrt{5}$ (by perpendicular distance) ***M1A1***

$SA = 26\sqrt{5}$ (by Pythagoras or coordinates) ***M1A1***

$t = \frac{26\sqrt{5}}{10\sqrt{5}}$ ***A1***

$t - k = \frac{22\sqrt{5}}{30\sqrt{5}}$ ***A1***

$k = \frac{28}{15}$ leading to latest time 11:52 ***A1***

[7 marks]

Examiners report

Few candidates managed to make progress on this question. Many candidates did not attempt the problem and many that did make an attempt failed to draw a diagram that would have allowed them to make further progress. There were a variety of possible solution techniques but candidates seemed unable to interpret the equation of a straight line written in vector form or find a perpendicular direction. This meant that it was very difficult for meaningful progress to be made towards a solution.