

## Topic 3 Part 2 [311 marks]

Let

$$f(x) = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}.$$

1a. [2 marks]

### Markscheme

$$\cos x = 0, \sin x = 0 \quad (M1)$$

$$x = \frac{n\pi}{2}, n \in \mathbb{Z} \quad AI$$

### Examiners report

Part (a) was well answered, although many candidates lost a mark through not giving sufficient solutions. It was rare for a student to receive no marks for part (b), but few solved the question by the easiest route, and as a consequence, there were frequently errors in the algebraic manipulation of the expression.

1b. [5 marks]

### Markscheme

**EITHER**

$$\begin{aligned} & \frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x} \quad M1 \quad AI \\ &= \frac{\sin(3x-x)}{\frac{1}{2}\sin 2x} \quad AI \quad AI \\ &= 2 \quad AI \end{aligned}$$

**OR**

$$\begin{aligned} & \frac{\sin 2x \cos x + \cos 2x \sin x}{\sin x} - \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x} \quad M1 \\ &= \frac{2 \sin x \cos^2 x + 2 \cos^2 x \sin x - \sin x}{\sin x} - \frac{2 \cos^3 x - \cos x - \sin^2 x \cos x}{\cos x} \quad AI \quad AI \\ &= 4 \cos^2 x - 1 - 2 \cos^2 x + 1 + 2 \sin^2 x \quad AI \\ &= 2 \cos^2 x + 2 \sin^2 x \\ &= 2 \quad AI \end{aligned}$$

[5 marks]

### Examiners report

Part (a) was well answered, although many candidates lost a mark through not giving sufficient solutions. It was rare for a student to receive no marks for part (b), but few solved the question by the easiest route, and as a consequence, there were frequently errors in the algebraic manipulation of the expression.

In the triangle ABC,  
 $\hat{A}BC = 90^\circ$ ,  
 $AC = \sqrt{2}$  and  $AB = BC + 1$ .

2a.

[3 marks]

## Markscheme

$$\cos \hat{A} = \frac{BA}{\sqrt{2}} \quad A1$$

$$\sin \hat{A} = \frac{BC}{\sqrt{2}} \quad A1$$

$$\cos \hat{A} - \sin \hat{A} = \frac{BA-BC}{\sqrt{2}} \quad R1$$

$$= \frac{1}{\sqrt{2}} \quad AG$$

[3 marks]

## Examiners report

Many good solutions to this question, although some students incorrectly stated the value of  $\arcsin\left(\frac{1}{2}\right)$ . A surprising number of students had greater difficulties with part (d).

2b.

[8 marks]

## Markscheme

$$\cos^2 \hat{A} - 2 \cos \hat{A} \sin \hat{A} + \sin^2 \hat{A} = \frac{1}{2} \quad M1A1$$

$$1 - 2 \sin \hat{A} \cos \hat{A} = \frac{1}{2} \quad M1A1$$

$$\sin 2\hat{A} = \frac{1}{2} \quad M1$$

$$2\hat{A} = 30^\circ \quad A1$$

$$\text{angles in the triangle are } 15^\circ \text{ and } 75^\circ \quad A1A1$$

**Note:** Accept answers in radians.

[8 marks]

## Examiners report

Many good solutions to this question, although some students incorrectly stated the value of  $\arcsin\left(\frac{1}{2}\right)$ . A surprising number of students had greater difficulties with part (d).

2c.

[6 marks]

## Markscheme

$$BC^2 + (BC + 1)^2 = 2 \quad M1A1$$

$$2BC^2 + 2BC - 1 = 0 \quad A1$$

$$BC = \frac{-2+\sqrt{12}}{4} \left( = \frac{\sqrt{3}-1}{2} \right) \quad M1A1$$

$$\sin \hat{A} = \frac{BC}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} \quad A1$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4} \quad AG$$

[6 marks]

## Examiners report

Many good solutions to this question, although some students incorrectly stated the value of  $\arcsin\left(\frac{1}{2}\right)$ . A surprising number of students had greater difficulties with part (d).

2d.

[4 marks]

### Markscheme

EITHER

$$\begin{aligned} h &= AB\sin\hat{A} \quad MI \\ &= (BC + 1)\sin\hat{A} \quad AI \\ &= \frac{\sqrt{3}+1}{2} \times \frac{\sqrt{6}-\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \quad MIAI \end{aligned}$$

OR

$$\begin{aligned} \frac{1}{2}AB.BC &= \frac{1}{2}AC.h \quad MI \\ \frac{\sqrt{3}-1}{2} \cdot \frac{\sqrt{3}+1}{2} &= \sqrt{2}h \quad AI \\ \frac{2}{4} &= \sqrt{2}h \quad MI \\ h &= \frac{1}{2\sqrt{2}} \quad AI \end{aligned}$$

[4 marks]

### Examiners report

Many good solutions to this question, although some students incorrectly stated the value of  $\arcsin(\frac{1}{2})$ . A surprising number of students had greater difficulties with part (d).

The function  
 $f(x) = 3\sin x + 4\cos x$  is defined for  
 $0 < x < 2\pi$ .

3a.

[1 mark]

### Markscheme

$$(3.79, -5) \quad AI$$

[1 mark]

### Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

3b.

[2 marks]

### Markscheme

$$p = 1.57 \text{ or } \frac{\pi}{2}, q = 6.00 \quad AIAI$$

[2 marks]

### Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

3c.

[4 marks]

## Markscheme

$$f'(x) = 3 \cos x - 4 \sin x \quad (M1)(A1)$$

$$3 \cos x - 4 \sin x = 3 \Rightarrow x = 4.43... \quad (A1)$$

$$(y = -4) \quad A1$$

Coordinates are

$$(4.43, -4)$$

[4 marks]

## Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

3d.

[7 marks]

## Markscheme

$$m_{\text{normal}} = \frac{1}{m_{\text{tangent}}} \quad (M1)$$

gradient at P is

−4 so gradient of normal at P is

$$\frac{1}{4} \quad (A1)$$

gradient at Q is 4 so gradient of normal at Q is

$$-\frac{1}{4} \quad (A1)$$

equation of normal at P is

$$y - 3 = \frac{1}{4}(x - 1.570...) \text{ (or } y = 0.25x + 2.60...) \quad (M1)$$

equation of normal at Q is

$$y - 3 = \frac{1}{4}(x - 5.999...) \text{ (or } y = -0.25x + \underbrace{4.499...}) \quad (M1)$$

**Note:** Award the previous two **M1** even if the gradients are incorrect in

$y - b = m(x - a)$  where

$(a, b)$  are coordinates of P and Q (or in

$y = mx + c$  with  $c$  determined using coordinates of P and Q.

intersect at

$$(3.79, 3.55) \quad A1A1$$

**Note:** Award **N2** for 3.79 without other working.

[7 marks]

## Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

4.

[5 marks]

## Markscheme

perpendicular when

$$\begin{pmatrix} 1 \\ 2 \cos x \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \sin x \\ 1 \end{pmatrix} = 0 \quad (M1)$$

$$\Rightarrow -1 + 4 \sin x \cos x = 0 \quad AI$$

$$\Rightarrow \sin 2x = \frac{1}{2} \quad M1$$

$$\Rightarrow 2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12} \quad AIAI$$

**Note:** Accept answers in degrees.

[5 marks]

## Examiners report

Most candidates realised that the scalar product should be used to solve this problem and many obtained the equation

$4 \sin x \cos x = 1$ . Candidates who failed to see that this could be written as

$\sin 2x = 0.5$  usually made no further progress. The majority of those candidates who used this double angle formula carried on to obtain the solution

$\frac{\pi}{12}$  but few candidates realised that  $\frac{5\pi}{12}$  was also a solution.

5.

[6 marks]

## Markscheme

### METHOD 1

$$\frac{\cos A + \sin A}{\cos A - \sin A} = \sec 2A + \tan 2A$$

consider right hand side

$$\sec 2A + \tan 2A = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A} \quad MIAI$$

$$= \frac{\cos^2 A + 2 \sin A \cos A + \sin^2 A}{\cos^2 A - \sin^2 A} \quad AIAI$$

**Note:** Award *AI* for recognizing the need for single angles and *AI* for recognizing  $\cos^2 A + \sin^2 A = 1$ .

$$= \frac{(\cos A + \sin A)^2}{(\cos A + \sin A)(\cos A - \sin A)} \quad MIAI$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A} \quad AG$$

### METHOD 2

$$\frac{\cos A + \sin A}{\cos A - \sin A} = \frac{(\cos A + \sin A)^2}{(\cos A + \sin A)(\cos A - \sin A)} \quad MIAI$$

$$= \frac{\cos^2 A + 2 \sin A \cos A + \sin^2 A}{\cos^2 A - \sin^2 A} \quad AIAI$$

**Note:** Award *AI* for correct numerator and *AI* for correct denominator.

$$= \frac{1 + \sin 2A}{\cos 2A} \quad MIAI$$

$$= \sec 2A + \tan 2A \quad AG$$

[6 marks]

## Examiners report

Solutions to this question were good in general with many candidates realising that multiplying the numerator and denominator by  $(\cos A + \sin A)$  might be helpful.

The function  $f$  is defined on the domain

$\left[0, \frac{3\pi}{2}\right]$  by

$$f(x) = e^{-x} \cos x.$$

6a.

[1 mark]

## Markscheme

$$e^{-x} \cos x = 0$$

$$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{AI}$$

[1 mark]

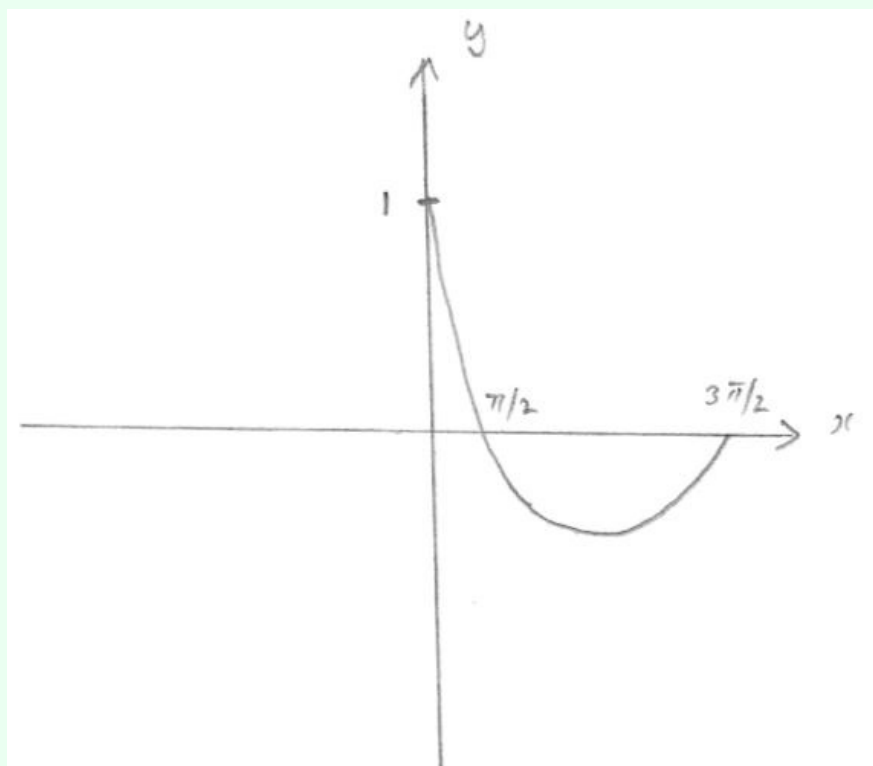
## Examiners report

Many candidates stated the two zeros of  $f$  correctly but the graph of  $f$  was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

6b.

[1 mark]

## Markscheme



AI

**Note:** Accept any form of concavity for  $x \in [0, \frac{\pi}{2}]$ .

**Note:** Do not penalize unmarked zeros if given in part (a).

**Note:** Zeros written on diagram can be used to allow the mark in part (a) to be awarded retrospectively.

[1 mark]

## Examiners report

Many candidates stated the two zeros of  $f$  correctly but the graph of  $f$  was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

## Markscheme

attempt at integration by parts *MI*

**EITHER**

$$I = \int e^{-x} \cos x dx = -e^{-x} \cos x - \int e^{-x} \sin x dx \quad \text{AI}$$

$$\Rightarrow I = -e^{-x} \cos x - [-e^{-x} \sin x + \int e^{-x} \cos x dx] \quad \text{AI}$$

$$\Rightarrow I = \frac{e^{-x}}{2} (\sin x - \cos x) + C \quad \text{AI}$$

**Note:** Do not penalize absence of  $C$ .

**OR**

$$I = \int e^{-x} \cos x dx = e^{-x} \sin x + \int e^{-x} \sin x dx \quad \text{AI}$$

$$I = e^{-x} \sin x - e^{-x} \cos x - \int e^{-x} \cos x dx \quad \text{AI}$$

$$\Rightarrow I = \frac{e^{-x}}{2} (\sin x - \cos x) + C \quad \text{AI}$$

**Note:** Do not penalize absence of  $C$ .

**THEN**

$$\int_0^{\frac{\pi}{2}} e^{-x} \cos x dx = \left[ \frac{e^{-x}}{2} (\sin x - \cos x) \right]_{\frac{\pi}{2}}^0 = \frac{e^{-\frac{\pi}{2}}}{2} + \frac{1}{2} \quad \text{AI}$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} e^{-x} \cos x dx = \left[ \frac{e^{-x}}{2} (\sin x - \cos x) \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = -\frac{e^{-\frac{3\pi}{2}}}{2} - \frac{e^{-\frac{\pi}{2}}}{2} \quad \text{AI}$$

ratio of A:B is

$$\begin{aligned} & \frac{\frac{e^{-\frac{\pi}{2}}}{2} + \frac{1}{2}}{\frac{e^{-\frac{3\pi}{2}}}{2} + \frac{e^{-\frac{\pi}{2}}}{2}} \\ &= \frac{\frac{3\pi}{2} \left( e^{-\frac{\pi}{2}} + 1 \right)}{\frac{3\pi}{2} \left( e^{-\frac{3\pi}{2}} + e^{-\frac{\pi}{2}} \right)} \quad \text{MI} \\ &= \frac{e^{\pi} \left( e^{\frac{\pi}{2}} + 1 \right)}{e^{\pi} + 1} \quad \text{AG} \end{aligned}$$

[7 marks]

## Examiners report

Many candidates stated the two zeros of  $f$  correctly but the graph of  $f$  was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

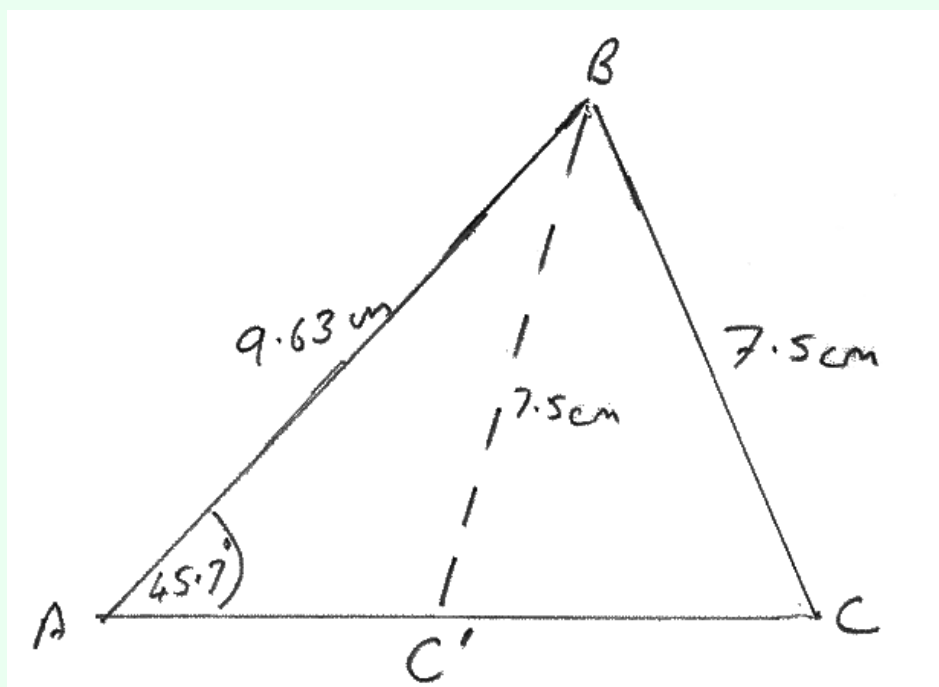
Consider a triangle ABC with

$\hat{A}C = 45.7^\circ$ ,  $AB = 9.63$  cm and  $BC = 7.5$  cm .

7a.

[2 marks]

## Markscheme



A2

**Note:** Accept 2 separate triangles. The diagram(s) should show that one triangle has an acute angle and the other triangle has an obtuse angle. The values 9.63, 7.5 and 45.7 and/or the letters, A, B, C' and C should be correctly marked on the diagram(s).

[2 marks]

## Examiners report

Surprisingly few candidates were able to demonstrate diagrammatically the situation for the ambiguous case of the sine rule. More were successful in trying to apply it or to use the cosine rule. However, there were still a surprisingly large number of candidates who were only able to find one possible answer for AC.

7b.

[6 marks]

## Markscheme

### METHOD 1

$$\frac{\sin 45.7}{7.5} = \frac{\sin C}{9.63} \quad M1$$

$$\Rightarrow \hat{C} = 66.77\dots^\circ, 113.2\dots^\circ \quad (A1)(A1)$$

$$\Rightarrow \hat{B} = 67.52\dots^\circ, 21.07\dots^\circ \quad (A1)$$

$$\frac{b}{\sin B} = \frac{7.5}{\sin 45.7} \Rightarrow b = 9.68(\text{cm}), b = 3.77(\text{cm}) \quad A1A1$$

**Note:** If only the acute value of

$\hat{C}$  is found, award *M1(A1)(A0)(A0)A1A0*.

### METHOD 2

$$7.5^2 = 9.63^2 + b^2 - 2 \times 9.63 \times b \cos 45.7^\circ \quad M1A1$$

$$b^2 - 13.45\dots b + 36.48\dots = 0$$

$$b = \frac{13.45\dots \pm \sqrt{13.45\dots^2 - 4 \times 36.48\dots}}{2} \quad (M1)(A1)$$

$$AC = 9.68(\text{cm}), AC = 3.77(\text{cm}) \quad A1A1$$

[6 marks]

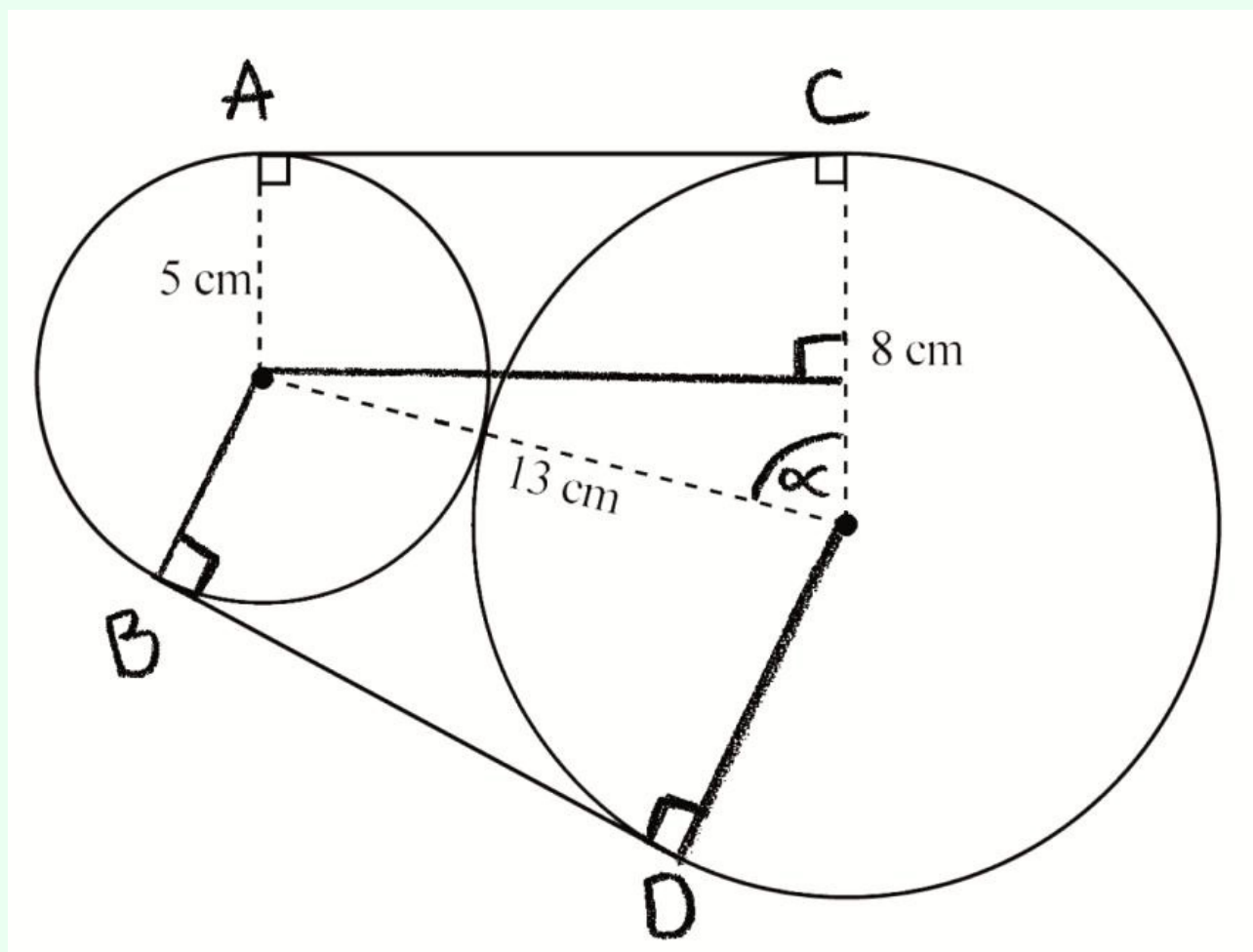
## Examiners report

Surprisingly few candidates were able to demonstrate diagrammatically the situation for the ambiguous case of the sine rule. More were successful in trying to apply it or to use the cosine rule. However, there were still a surprisingly large number of candidates who were only able to find one possible answer for AC.

8.

[8 marks]

## Markscheme



$$AC = BD = \sqrt{13^2 - 3^2} = 12.64... \quad (A1)$$

$$\cos \alpha = \frac{3}{13} \Rightarrow \alpha = 1.337... (76.65...^\circ) \quad (M1)(A1)$$

attempt to find either arc length AB or arc length CD  $(M1)$

$$\text{arc length AB} = 5(\pi - 2 \times 0.232...) (= 13.37...) \quad (A1)$$

$$\text{arc length CD} = 8(\pi + 2 \times 0.232...) (= 28.85...) \quad (A1)$$

$$\text{length of string} = 13.37... + 28.85... + 2(12.64...) \quad (M1)$$

$$= 67.5 \text{ (cm)} \quad A1$$

[8 marks]

## Examiners report

Given that this was the last question in section A it was pleasing to see a good number of candidates make a start on the question. As would be expected from a question at this stage of the paper, more limited numbers of candidates gained full marks. A number of candidates made the question very difficult by unnecessarily splitting the angles required to find the final answer into combinations of smaller angles, all of which required a lot of work and time.

9.

[4 marks]

## Markscheme

$$\sin \alpha = \sqrt{1 - \left(-\frac{3}{4}\right)^2} = \frac{\sqrt{7}}{4} \quad (M1)A1$$

attempt to use double angle formula  $M1$

$$\sin 2\alpha = 2\frac{\sqrt{7}}{4}\left(-\frac{3}{4}\right) = -\frac{3\sqrt{7}}{8} \quad A1$$

**Note:**

$\frac{\sqrt{7}}{4}$  seen would normally be awarded  $M1A1$ .

[4 marks]

## Examiners report

Many candidates scored full marks on this question, though their explanations for part a) often lacked clarity. Most preferred to use some kind of right-angled triangle rather than (perhaps in this case) the more sensible identity  $\sin^2 \alpha + \cos^2 \alpha = 1$ .

In the triangle PQR,  $PQ = 6$ ,  $PR = k$  and  $\hat{PQR} = 30^\circ$ .

10a.

[4 marks]

## Markscheme

attempt to apply cosine rule *MI*

$$4^2 = 6^2 + QR^2 - 2 \cdot QR \cdot 6 \cos 30^\circ \text{ ( or$$

$$QR^2 - 6\sqrt{3} QR + 20 = 0 ) \quad \textit{AI}$$

$$QR = 3\sqrt{3} + \sqrt{7} \text{ or } QR = 3\sqrt{3} - \sqrt{7} \quad \textit{AIAI}$$

[4 marks]

## Examiners report

Candidates using the sine rule here made little or no progress. With the cosine rule, the two values are obtained quite quickly, which was the case for a majority of candidates. A small number were able to write down the correct quadratic equation to be solved, but then made arithmetical errors en route to their final solution(s). Part b) was often left blank. The better candidates were able to deduce  $k = 3$ , though the solution  $k \geq 6$  was rarely, if at all, seen by examiners.

10b.

[3 marks]

## Markscheme

**METHOD 1**

$$k \geq 6 \quad \textit{AI}$$

$$k = 6 \sin 30^\circ = 3 \quad \textit{MIAI}$$

**Note:** The *MI* in (b) is for recognizing the right-angled triangle case.

**METHOD 2**

$$k \geq 6 \quad \textit{AI}$$

use of discriminant:

$$108 - 4(36 - k^2) = 0 \quad \textit{MI}$$

$$k = 3 \quad \textit{AI}$$

**Note:**  $k = \pm 3$  is *MIA0*.

[3 marks]

## Examiners report

Candidates using the sine rule here made little or no progress. With the cosine rule, the two values are obtained quite quickly, which was the case for a majority of candidates. A small number were able to write down the correct quadratic equation to be solved, but then made arithmetical errors en route to their final solution(s). Part b) was often left blank. The better candidates were able to deduce  $k = 3$ , though the solution  $k \geq 6$  was rarely, if at all, seen by examiners.

Consider the curve defined by the equation  $x^2 + \sin y - xy = 0$ .

## Markscheme

attempt to differentiate implicitly **MI**

$$2x + \cos y \frac{dy}{dx} - y - x \frac{dy}{dx} = 0 \quad \text{AIAI}$$

**Note:** **AI** for differentiating

$x^2$  and  $\sin y$ ; **AI** for differentiating  $xy$ .

substitute  $x$  and  $y$  by

$$\pi \quad \text{MI}$$

$$2\pi - \frac{dy}{dx} - \pi - \pi \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{\pi}{1+\pi} \quad \text{MIAI}$$

**Note:** **MI** for attempt to make  $dy/dx$  the subject. This could be seen earlier.

[6 marks]

## Examiners report

Part a) proved an easy 6 marks for most candidates, while the majority failed to make any headway with part b), with some attempting to find the equation of their line in the form  $y = mx + c$ . Only the best candidates were able to see their way through to the given answer.

## Markscheme

$$\theta = \frac{\pi}{4} - \arctan \frac{\pi}{1+\pi} \text{ (or seen the other way)} \quad \text{MI}$$

$$\tan \theta = \tan \left( \frac{\pi}{4} - \arctan \frac{\pi}{1+\pi} \right) = \frac{1 - \frac{\pi}{1+\pi}}{1 + \frac{\pi}{1+\pi}} \quad \text{MIAI}$$

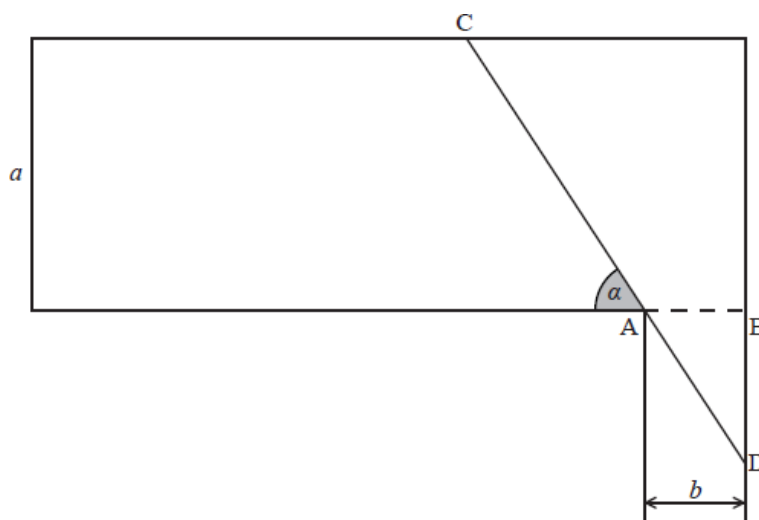
$$\tan \theta = \frac{1}{1+2\pi} \quad \text{AG}$$

[3 marks]

## Examiners report

Part a) proved an easy 6 marks for most candidates, while the majority failed to make any headway with part b), with some attempting to find the equation of their line in the form  $y = mx + c$ . Only the best candidates were able to see their way through to the given answer.

The diagram shows the plan of an art gallery  $a$  metres wide. [AB] represents a doorway, leading to an exit corridor  $b$  metres wide. In order to remove a painting from the art gallery, CD (denoted by  $L$ ) is measured for various values of  $\alpha$ , as represented in the diagram.



## Markscheme

$$L = CA + AD \quad MI$$

$$\sin \alpha = \frac{a}{CA} \Rightarrow CA = \frac{a}{\sin \alpha} \quad AI$$

$$\cos \alpha = \frac{b}{AD} \Rightarrow AD = \frac{b}{\cos \alpha} \quad AI$$

$$L = \frac{a}{\sin \alpha} + \frac{b}{\cos \alpha} \quad AG$$

[2 marks]

## Examiners report

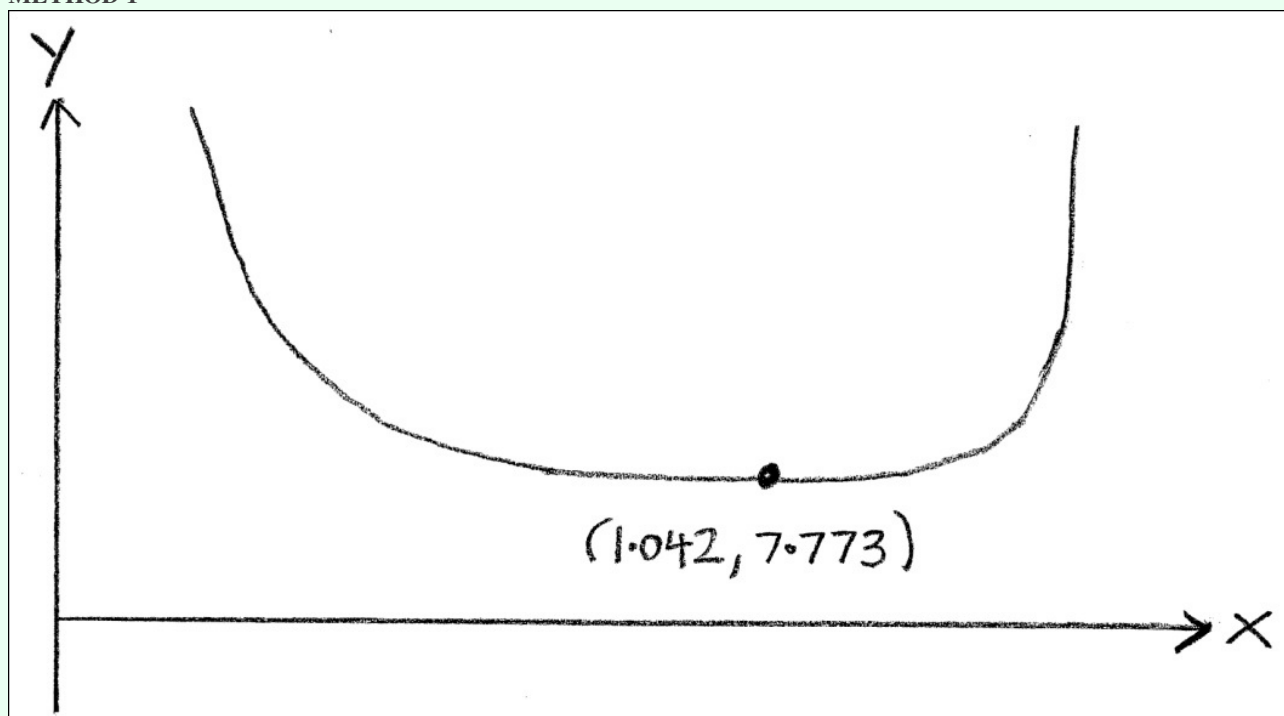
Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.

In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for  $\alpha$  and  $L$ . The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

## Markscheme

$$a = 5 \text{ and } b = 1 \Rightarrow L = \frac{5}{\sin \alpha} + \frac{1}{\cos \alpha}$$

### METHOD 1



(M1)

minimum from graph

$$\Rightarrow L = 7.77 \quad (M1)AI$$

minimum of  $L$  gives the max length of the painting  $R1$

[4 marks]

### METHOD 2

$$\frac{dL}{d\alpha} = \frac{-5 \cos \alpha}{\sin^2 \alpha} + \frac{\sin \alpha}{\cos^2 \alpha} \quad (M1)$$

$$\frac{dL}{d\alpha} = 0 \Rightarrow \frac{\sin^3 \alpha}{\cos^3 \alpha} = 5 \Rightarrow \tan \alpha = \sqrt[3]{5} \quad (\alpha = 1.0416...) \quad (M1)$$

minimum of  $L$  gives the max length of the painting  $R1$

$$\text{maximum length} = 7.77 \quad AI$$

[4 marks]

## Examiners report

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.

In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for  $\alpha$  and  $L$ . The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

12c. [3 marks]

## Markscheme

$$\frac{dL}{d\alpha} = \frac{-3k \cos \alpha}{\sin^2 \alpha} + \frac{k \sin \alpha}{\cos^2 \alpha} \quad (\text{or equivalent}) \quad \text{MIAIAI}$$

[3 marks]

## Examiners report

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.

In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for  $\alpha$  and  $L$ . The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

12d. [6 marks]

## Markscheme

$$\frac{dL}{d\alpha} = \frac{-3k \cos^3 \alpha + k \sin^3 \alpha}{\sin^2 \alpha \cos^2 \alpha} \quad (A1)$$

$$\frac{dL}{d\alpha} = 0 \Rightarrow \frac{\sin^3 \alpha}{\cos^3 \alpha} = \frac{3k}{k} \Rightarrow \tan \alpha = \sqrt[3]{3} \quad (\alpha = 0.96454\dots) \quad \text{MIAI}$$

$$\tan \alpha = \sqrt[3]{3} \Rightarrow \frac{1}{\cos \alpha} = \sqrt{1 + \sqrt[3]{9}} \quad (1.755\dots) \quad (A1)$$

$$\text{and } \frac{1}{\sin \alpha} = \frac{\sqrt{1 + \sqrt[3]{9}}}{\sqrt[3]{3}} \quad (1.216\dots) \quad (A1)$$

$$L = 3k \left( \frac{\sqrt{1 + \sqrt[3]{9}}}{\sqrt[3]{3}} \right) + k \sqrt{1 + \sqrt[3]{9}} \quad (L = 5.405598\dots k) \quad \text{AI} \quad \text{N4}$$

[6 marks]

## Examiners report

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.

In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for  $\alpha$  and  $L$ . The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

## Markscheme

$$L \leq 8 \Rightarrow k \geq 1.48 \quad \text{MIAI}$$

the minimum value is 1.48

[2 marks]

## Examiners report

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.

In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for  $\alpha$  and  $L$ . The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

Consider the planes

$$\pi_1 : x - 2y - 3z = 2 \text{ and } \pi_2 : 2x - y - z = k.$$

## Markscheme

**Note:** Accept alternative notation for vectors (eg

$\langle a, b, c \rangle$  or  $(a, b, c)$ ).

$$n = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \text{ and}$$

$$m = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \quad (AI)$$

$$\cos \theta = \frac{n \cdot m}{|n||m|} \quad (MI)$$

$$\cos \theta = \frac{2+2+3}{\sqrt{1+4+9}\sqrt{4+1+1}} = \frac{7}{\sqrt{14}\sqrt{6}} \quad AI$$

$$\theta = 40.2^\circ \quad (0.702 \text{ rad}) \quad AI$$

[4 marks]

## Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved.

Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of  $k$ . It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well.

Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

## Markscheme

**Note:** Accept alternative notation for vectors (eg  $\langle a, b, c \rangle$  or  $(a, b, c)$ ).

### METHOD 1

eliminate  $z$  from  $x - 2y - 3z = 2$  and  $2x - y - z = k$

$$5x - y = 3k - 2 \Rightarrow x = \frac{y - (2 - 3k)}{5} \quad \text{MIAI}$$

eliminate  $y$  from  $x - 2y - 3z = 2$  and  $2x - y - z = k$

$$3x + z = 2k - 2 \Rightarrow x = \frac{z - (2k - 2)}{-3} \quad \text{AI}$$

$$x = t, y = (2 - 3k) + 5t \text{ and } z = (2k - 2) - 3t \quad \text{AIAI}$$

$$r = \begin{pmatrix} 0 \\ 2 - 3k \\ 2k - 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} \quad \text{AG}$$

[5 marks]

### METHOD 2

$$\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix} \Rightarrow \text{direction is } \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} \quad \text{MIAI}$$

Let  $x = 0$

$$0 - 2y - 3z = 2 \text{ and } 2 \times 0 - y - z = k \quad \text{(MI)}$$

solve simultaneously (MI)

$$y = 2 - 3k \text{ and } z = 2k - 2 \quad \text{AI}$$

therefore  $\mathbf{r}$

$$= \begin{pmatrix} 0 \\ 2 - 3k \\ 2k - 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} \quad \text{AG}$$

[5 marks]

### METHOD 3

substitute

$$x = t, y = (2 - 3k) + 5t \text{ and } z = (2k - 2) - 3t \text{ into } \pi_1 \text{ and } \pi_2 \quad \text{MI}$$

for

$$\pi_1 : t - 2(2 - 3k + 5t) - 3(2k - 2 - 3t) = 2 \quad \text{AI}$$

for

$$\pi_2 : 2t - (2 - 3k + 5t) - (2k - 2 - 3t) = k \quad \text{AI}$$

the planes have a unique line of intersection **R2**

therefore the line is

$$r = \begin{pmatrix} 0 \\ 2 - 3k \\ 2k - 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} \quad \text{AG}$$

[5 marks]

## Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of  $k$ . It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well. Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

13c.

[5 marks]

## Markscheme

**Note:** Accept alternative notation for vectors (eg  $\langle a, b, c \rangle$  or  $(a, b, c)$ ).

$$5 - t = (2 - 3k + 5t) + 3 = 2 - 2(2k - 2 - 3t) \quad \text{MIAI}$$

**Note:** Award **MIAI** if candidates use vector or parametric equations of  $L_2$

eg

$$\begin{pmatrix} 0 \\ 2 - 3k \\ 2k - 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix} + s \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix} \text{ or}$$

$$\Rightarrow \begin{cases} t = 5 - 2s \\ 2 - 3k + 5t = -3 + 2s \\ 2k - 2 - 3t = 1 + s \end{cases}$$

solve simultaneously **MI**

$$k = 2, t = 1 \ (s = 2) \quad \text{AI}$$

intersection point (

1,

1,

-1) **AI**

[5 marks]

## Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of  $k$ . It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well. Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

13d.

[5 marks]

## Markscheme

**Note:** Accept alternative notation for vectors (eg  $\langle a, b, c \rangle$  or  $(a, b, c)$ ).

$$\vec{l}_2 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \quad \text{AI}$$

$$\vec{l}_1 \times \vec{l}_2 = \begin{vmatrix} i & j & k \\ 1 & 5 & -3 \\ 2 & -2 & 1 \end{vmatrix} = \begin{pmatrix} -1 \\ -7 \\ -12 \end{pmatrix} \quad (\text{MI})\text{AI}$$

$$r \cdot \begin{pmatrix} 1 \\ 7 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 7 \\ 12 \end{pmatrix} \quad (\text{MI})$$

$$x + 7y + 12z = -4 \quad \text{AI}$$

[5 marks]

## Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of  $k$ . It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well. Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

13e.

[5 marks]

## Markscheme

**Note:** Accept alternative notation for vectors (eg  $\langle a, b, c \rangle$  or  $(a, b, c)$ ).

Let

$\theta$  be the angle between the lines

$$\vec{l}_1 = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} \quad \text{and}$$

$$\vec{l}_2 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

$$\cos \theta = \frac{|2-10-3|}{\sqrt{35}\sqrt{9}} \Rightarrow \theta = 0.902334... \quad 51.699...^\circ \quad (\text{MI})$$

as the triangle XYZ has a right angle at Y,

$$XZ = a \Rightarrow YZ = a \sin \theta \quad \text{and} \quad XY = a \cos \theta \quad (\text{MI})$$

$$\text{area} = 3 \Rightarrow \frac{a^2 \sin \theta \cos \theta}{2} = 3 \quad (\text{MI})$$

$$a = 3.5122... \quad (\text{AI})$$

perimeter

$$= a + a \sin \theta + a \cos \theta = 8.44537... = 8.45 \quad \text{AI}$$

**Note:** If candidates attempt to find coordinates of Y and Z award **MI** for expression of vector YZ in terms of two parameters, **MI** for attempt to use perpendicular condition to determine relation between parameters, **MI** for attempt to use the area to find the parameters and **A2** for final answer.

[5 marks]

## Examiners report

Although this was the last question in part B, it was answered surprisingly well by many candidates, except for part (e). Even those who had not done so well elsewhere often gained a number of marks in some parts of the question. Nevertheless the presence of parameters seemed to have blocked the abilities of weaker candidates to solve situations in which vectors were involved. Mathematical skills for this particular question were sometimes remarkable, however, calculations proved incomplete due to the way that planes were presented. Most candidates found a correct angle in part (a). Occasional arithmetic errors in calculating the magnitude of a vector and dot product occurred. In part (b) the vector product approach was popular. In some case candidates simply verified the result by substitution. There was a lot of simultaneous equation solving, much of which was not very pretty. In part (c), a number of candidates made errors when attempting to solve a system of equations involving parameters. Many of the results for the point were found in terms of  $k$ . It was notorious that candidates did not use their GDC to try to find the coordinates of the intersection point between lines. In part (d), a number of candidates used an incorrect point but this part was often done well. Very few excellent answers to part (e) were seen using an efficient method. Most candidates attempted methods involving heavy algebraic manipulation and had little success in this part of the question.

14.

[6 marks]

## Markscheme

$$\frac{1}{2}r^2 \times 1 = 7 \quad \text{MI}$$

$$r = 3.7 \dots (= \sqrt{14}) \text{ (or } 37 \dots \text{ mm)} \quad \text{AI}$$

$$\text{height} = 2r \cos\left(\frac{\pi-1}{2}\right) \text{ (or } 2r \sin \frac{1}{2}) \quad \text{(MI)(AI)}$$

$$3.59 \text{ or anything that rounds to } 3.6 \quad \text{AI}$$

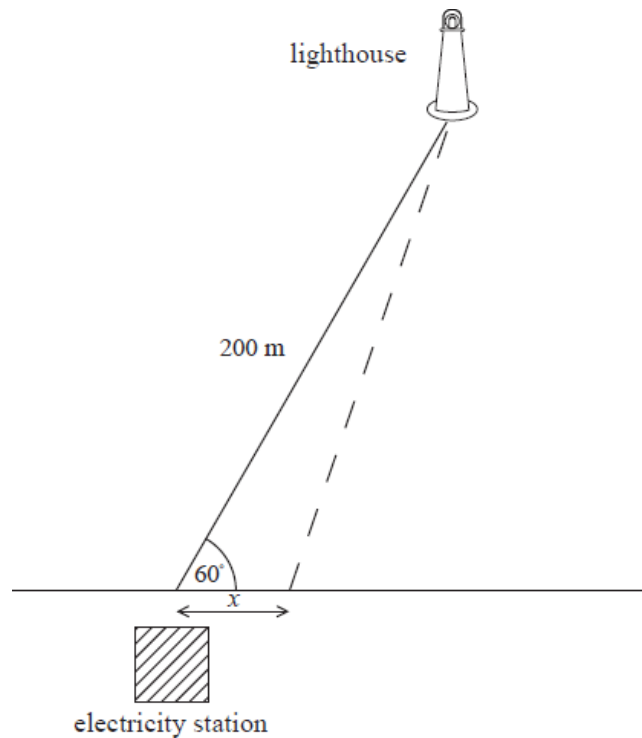
$$\text{so the dimensions are } 3.7 \text{ by } 3.6 \text{ (cm or } 37 \text{ by } 36 \text{ mm)} \quad \text{AI}$$

[6 marks]

## Examiners report

Most students found the value of  $r$ , but a surprising number had difficulties finding the height of the rectangle by any one of the many methods possible. Those that did, frequently failed to round their final answer to the required accuracy leading to few students obtaining full marks on this question. A surprising number of students found the area – clearly misinterpreting the meaning of “dimensions”.

An electricity station is on the edge of a straight coastline. A lighthouse is located in the sea 200 m from the electricity station. The angle between the coastline and the line joining the lighthouse with the electricity station is  $60^\circ$ . A cable needs to be laid connecting the lighthouse to the electricity station. It is decided to lay the cable in a straight line to the coast and then along the coast to the electricity station. The length of cable laid along the coastline is  $x$  metres. This information is illustrated in the diagram below.



The cost of laying the cable along the sea bed is US\$80 per metre, and the cost of laying it on land is US\$20 per metre.

15a.

[4 marks]

## Markscheme

let the distance the cable is laid along the seabed be  $y$

$$y^2 = x^2 + 200^2 - 2 \times x \times 200 \cos 60^\circ \quad (M1)$$

(or equivalent method)

$$y^2 = x^2 - 200x + 40000 \quad (A1)$$

$$\text{cost} = C = 80y + 20x \quad (M1)$$

$$C = 80(x^2 - 200x + 40000)^{\frac{1}{2}} + 20x \quad A1$$

[4 marks]

## Examiners report

Some surprising misconceptions were evident here, using right angled trigonometry in non right angled triangles etc. Those that used the cosine rule, usually managed to obtain the correct answer to part (a).

15b.

[2 marks]

## Markscheme

$$x = 55.2786 \dots = 55 \text{ (m to the nearest metre)} \quad (A1)A1$$

$$(x = 100 - \sqrt{2000})$$

[2 marks]

## Examiners report

Some surprising misconceptions were evident here, using right angled trigonometry in non right angled triangles etc. Many students attempted to find the value of the minimum algebraically instead of the simple calculator solution.

16a.

[3 marks]

### Markscheme

attempt at use of

$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)} \quad M1$$

$$\frac{1}{p} = \frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{5} \times \frac{1}{8}} \quad (= \frac{1}{3}) \quad A1$$

$$p = 3 \quad A1$$

**Note:** the value of  $p$  needs to be stated for the final mark.

[3 marks]

## Examiners report

Those candidates who used the addition formula for the tangent were usually successful.

16b.

[3 marks]

### Markscheme

$$\tan\left(\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)\right) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = 1 \quad M1A1$$

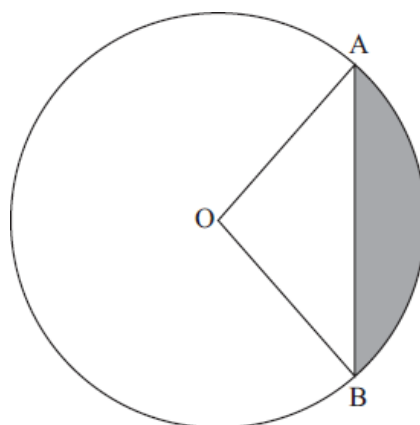
$$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right) = \frac{\pi}{4} \quad A1$$

[3 marks]

## Examiners report

Some candidates left their answer as the tangent of an angle, rather than the angle itself.

A circle of radius 4 cm, centre O, is cut by a chord [AB] of length 6 cm.



*diagram not to scale*

17a.

[2 marks]

## Markscheme

**EITHER**

$\hat{A}OB = 2 \arcsin\left(\frac{3}{4}\right)$  or equivalent (*eg*

$\hat{A}OB = 2 \arctan\left(\frac{3}{\sqrt{7}}\right), \hat{A}OB = 2 \arccos\left(\frac{\sqrt{7}}{4}\right)$  **(M1)**

**OR**

$\cos \hat{A}OB = \frac{4^2 + 4^2 - 6^2}{2 \times 4 \times 4} \left(= -\frac{1}{8}\right)$  **(M1)**

**THEN**

$= 1.696$  (correct to 4sf) **AI**

[2 marks]

## Examiners report

This was generally well done. In part (a), a number of candidates expressed the required angle either in degrees or in radians stated to an incorrect number of significant figures.

17b.

[3 marks]

## Markscheme

use of area of segment = area of sector – area of triangle **(M1)**

$= \frac{1}{2} \times 4^2 \times 1.696 - \frac{1}{2} \times 4^2 \times \sin 1.696$  **(A1)**

$= 5.63 \text{ (cm}^2\text{)}$  **AI**

[3 marks]

## Examiners report

This was generally well done. In part (b), some candidates demonstrated a correct method to calculate the shaded area using an incorrect formula for the area of a sector.

## Markscheme

attempting to solve for

$\cos x$  or for  $u$  where

$u = \cos x$  or for  $x$  graphically.    *(M1)*

**EITHER**

$\cos x = \frac{2}{3}$  (and 2)    *(A1)*

**OR**

$x = 48.1897 \dots^\circ$     *(A1)*

**THEN**

$x = 48^\circ$     *A1*

**Note:** Award *(M1)(A1)A0* for

$x = 48^\circ, 132^\circ$ .

**Note:** Award *(M1)(A1)A0* for 0.841 radians.

[3 marks]

## Examiners report

Part (a) was generally well done. Some candidates did not follow instructions and express their final answer correct to the nearest degree. A large number of candidates successfully employed a graphical approach.

## Markscheme

attempting to solve for

$\sec x$  or for

$v$  where

$v = \sec x$ .    *(M1)*

$\sec x = \pm\sqrt{2}$  (and  $\pm\sqrt{\frac{2}{3}}$ )    *(A1)*

$\sec x = \pm\sqrt{2}$     *A1*

[3 marks]

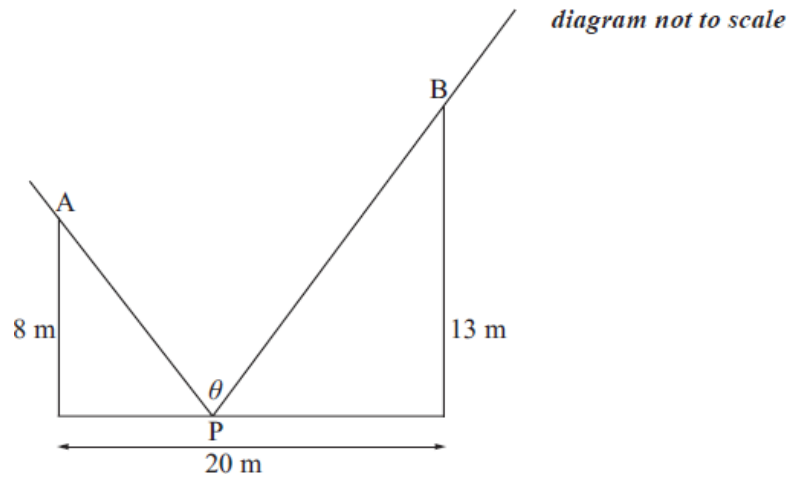
## Examiners report

Part (b) was not well done. Common errors included attempting to solve for  $x$  rather than for  $\sec x$ , either omitting or not considering  $\sec x = -\sqrt{2}$ , not rejecting  $\sec x = \pm\sqrt{\frac{2}{3}}$  and not working with exact values.

A straight street of width 20 metres is bounded on its parallel sides by two vertical walls, one of height 13 metres, the other of height 8 metres. The intensity of light at point P at ground level on the street is proportional to the angle

$\theta$  where

$\theta = \angle APB$ , as shown in the diagram.



19a. [2 marks]

## Markscheme

**EITHER**

$$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right) \text{ (or equivalent)} \quad MIAI$$

**Note:** Accept

$$\theta = 180^\circ - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right) \text{ (or equivalent)}.$$

**OR**

$$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right) \text{ (or equivalent)} \quad MIAI$$

[2 marks]

## Examiners report

Part (a) was reasonably well done. While many candidates exhibited sound trigonometric knowledge to correctly express  $\theta$  in terms of  $x$ , many other candidates were not able to use elementary trigonometry to formulate the required expression for  $\theta$ .

19b. [2 marks]

## Markscheme

(i)

$$\theta = 0.994 \left( = \arctan \frac{20}{13} \right) \quad AI$$

(ii)

$$\theta = 1.19 \left( = \arctan \frac{5}{2} \right) \quad AI$$

[2 marks]

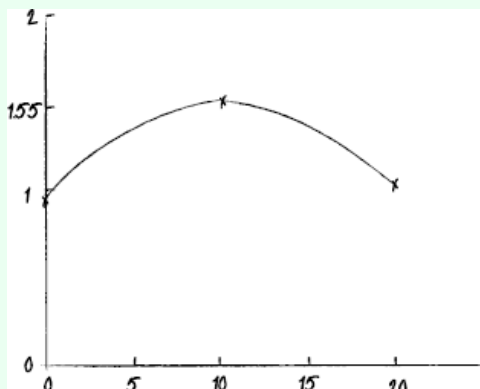
## Examiners report

In part (b), a large number of candidates did not realize that  $\theta$  could only be acute and gave obtuse angle values for  $\theta$ . Many candidates also demonstrated a lack of insight when substituting endpoint  $x$ -values into  $\theta$ .

19c.

[2 marks]

## Markscheme

correct shape. *AI*correct domain indicated. *AI*

[2 marks]

## Examiners report

In part (c), many candidates sketched either inaccurate or implausible graphs.

19d.

[6 marks]

## Markscheme

attempting to differentiate one

 $\arctan(f(x))$  term *MI***EITHER**

$$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$$

$$\frac{d\theta}{dx} = \frac{8}{x^2} \times \frac{1}{1 + \left(\frac{8}{x}\right)^2} - \frac{13}{(20-x)^2} \times \frac{1}{1 + \left(\frac{13}{20-x}\right)^2} \quad \text{AIAI}$$

**OR**

$$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$$

$$\frac{d\theta}{dx} = \frac{\frac{1}{8}}{1 + \left(\frac{x}{8}\right)^2} + \frac{-\frac{1}{13}}{1 + \left(\frac{20-x}{13}\right)^2} \quad \text{AIAI}$$

**THEN**

$$= \frac{8}{x^2 + 64} - \frac{13}{569 - 40x + x^2} \quad \text{AI}$$

$$= \frac{8(569 - 40x + x^2) - 13(x^2 + 64)}{(x^2 + 64)(x^2 - 40x + 569)} \quad \text{MIAI}$$

$$= \frac{5(744 - 64x - x^2)}{(x^2 + 64)(x^2 - 40x + 569)} \quad \text{AG}$$

[6 marks]

## Examiners report

In part (d), a large number of candidates started their differentiation incorrectly by failing to use the chain rule correctly.

19e. [3 marks]

## Markscheme

Maximum light intensity at P occurs when

$$\frac{d\theta}{dx} = 0. \quad (M1)$$

either attempting to solve

$$\frac{d\theta}{dx} = 0 \text{ for } x \text{ or using the graph of either}$$

$\theta$  or

$$\frac{d\theta}{dx} \quad (M1)$$

$$x = 10.05 \text{ (m)} \quad AI$$

[3 marks]

## Examiners report

For a question part situated at the end of the paper, part (e) was reasonably well done. A large number of candidates demonstrated a sound knowledge of finding where the maximum value of  $\theta$  occurred and rejected solutions that were not physically feasible.

19f. [4 marks]

## Markscheme

$$\frac{dx}{dt} = 0.5 \quad (A1)$$

At  $x = 10$ ,

$$\frac{d\theta}{dx} = 0.000453 \left( = \frac{5}{11029} \right). \quad (A1)$$

use of

$$\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt} \quad M1$$

$$\frac{d\theta}{dt} = 0.000227 \left( = \frac{5}{22058} \right) \text{ (rad s}^{-1}\text{)} \quad AI$$

**Note:** Award (AI) for

$$\frac{dx}{dt} = -0.5 \text{ and } AI \text{ for}$$

$$\frac{d\theta}{dt} = -0.000227 \left( = -\frac{5}{22058} \right).$$

**Note:** Implicit differentiation can be used to find

$$\frac{d\theta}{dt}. \text{ Award as above.}$$

[4 marks]

## Examiners report

In part (f), many candidates were able to link the required rates, however only a few candidates were able to successfully apply the chain rule in a related rates context.

## Markscheme

area of triangle

$$= \frac{1}{2}(2x)^2 \sin \frac{\pi}{3} \quad (M1)$$

$$= x^2 \sqrt{3} \quad A1$$

**Note:** A

$0.5 \times \text{base} \times \text{height}$  calculation is acceptable.

area of sector

$$= \frac{\theta}{2} r^2 = \frac{\pi}{6} r^2 \quad (M1)A1$$

area of triangle is twice the area of the sector

$$\Rightarrow 2 \left( \frac{\pi}{6} r^2 \right) = x^2 \sqrt{3} \quad M1$$

$$\Rightarrow r = x \sqrt{\frac{3\sqrt{3}}{\pi}} \quad \text{or equivalent} \quad A1$$

[6 marks]

## Examiners report

The majority of candidates obtained the correct answer. A small minority of candidates used degree measure rather than radian measure, or failed to notice that the triangle was equilateral.

## Markscheme

### METHOD 1

If the areas are in arithmetic sequence, then so are the angles. *(M1)*

$$\Rightarrow S_n = \frac{n}{2}(a + l) \Rightarrow \frac{12}{2}(\theta + 2\theta) = 18\theta \quad \text{M1A1}$$

$$\Rightarrow 18\theta = 2\pi \quad \text{(A1)}$$

$$\theta = \frac{\pi}{9} \quad \text{(accept}$$

$$20^\circ) \quad \text{A1}$$

[5 marks]

### METHOD 2

$$a_{12} = 2a_1 \quad \text{(M1)}$$

$$\frac{12}{2}(a_1 + 2a_1) = \pi r^2 \quad \text{M1A1}$$

$$3a_1 = \frac{\pi r^2}{6}$$

$$\frac{3}{2}r^2\theta = \frac{\pi r^2}{6} \quad \text{(A1)}$$

$$\theta = \frac{2\pi}{18} = \frac{\pi}{9} \quad \text{(accept}$$

$$20^\circ) \quad \text{A1}$$

[5 marks]

### METHOD 3

Let smallest angle =  $a$ , common difference =  $d$

$$a + 11d = 2a \quad \text{(M1)}$$

$$a = 11d \quad \text{A1}$$

$$S_n = \frac{12}{2}(2a + 11d) = 2\pi \quad \text{M1}$$

$$6(2a + a) = 2\pi \quad \text{(A1)}$$

$$18a = 2\pi$$

$$a = \frac{\pi}{9} \quad \text{(accept}$$

$$20^\circ) \quad \text{A1}$$

[5 marks]

## Examiners report

Stronger candidates had little problem with this question, but a significant minority of weaker candidates were unable to access the question or worked with area and very quickly became confused. Candidates who realised that the area of each sector was proportional to the angle usually gained the correct answer.

22.

[5 marks]

## Markscheme

$$\frac{9}{\sin C} = \frac{12}{\sin B} \quad (M1)$$

$$\frac{9}{\sin C} = \frac{12}{\sin 2C} \quad A1$$

Using double angle formula

$$\frac{9}{\sin C} = \frac{12}{2 \sin C \cos C} \quad M1$$

$$\Rightarrow 9(2 \sin C \cos C) = 12 \sin C$$

$$\Rightarrow 6 \sin C(3 \cos C - 2) = 0 \quad \text{or equivalent} \quad (A1)$$

$$(\sin C \neq 0)$$

$$\Rightarrow \cos C = \frac{2}{3} \quad A1$$

[5 marks]

## Examiners report

There were many totally correct solutions to this question, but again a significant minority did not make much progress. The most common reasons for this were that candidates immediately assumed that because the question asked for the cosine of  $\hat{C}$  that they should use the cosine rule, or they did not draw a diagram and then confused which angles were opposite which sides.

23.

[6 marks]

## Markscheme

(a)

$$y = \arccos(1.2 - \cos x) \quad A1$$

$$y = \arcsin(1.4 - \sin x) \quad A1$$

(b) The solutions are

$$x = 1.26, y = 0.464 \quad A1A1$$

$$x = 0.464, y = 1.26 \quad A1A1$$

[6 marks]

## Examiners report

The majority of candidates obtained the first two marks. Candidates who used their GDC to solve this question did so successfully, although few candidates provided a sketch as the rubric requires. Attempts to use “solver” only gave one solution. Some candidates did not give the solutions as coordinate pairs, but simply stated the  $x$  and  $y$  values.

## Markscheme

(a)

$$CD = AC - AD = b - c \cos A \quad \text{RIAG}$$

[1 mark]

(b) METHOD 1

$$BC^2 = BD^2 + CD^2 \quad (M1)$$

$$a^2 = (c \sin A)^2 + (b - c \cos A)^2 \quad (A1)$$

$$= c^2 \sin^2 A + b^2 - 2bc \cos A + c^2 \cos^2 A \quad A1$$

$$= b^2 + c^2 - 2bc \cos A \quad A1$$

[4 marks]

METHOD 2

$$BD^2 = AB^2 - AD^2 = BC^2 - CD^2 \quad (M1)(A1)$$

$$\Rightarrow c^2 - c^2 \cos^2 A = a^2 - b^2 + 2bc \cos A - c^2 \cos^2 A \quad A1$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc \cos A \quad A1$$

[4 marks]

(c) METHOD 1

$$b^2 = a^2 + c^2 - 2ac \cos 60^\circ \Rightarrow b^2 = a^2 + c^2 - ac \quad (M1)A1$$

$$\Rightarrow c^2 - ac + a^2 - b^2 = 0 \quad M1$$

$$\Rightarrow c = \frac{a \pm \sqrt{(-a)^2 - 4(a^2 - b^2)}}{2} \quad (M1)A1$$

$$= \frac{a \pm \sqrt{4b^2 - 3a^2}}{2} = \frac{a}{2} \pm \sqrt{\frac{4b^2 - 3a^2}{4}} \quad (M1)A1$$

$$= \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2} \quad AG$$

**Note:** Candidates can only obtain a maximum of the first three marks if they **verify** that the answer given in the question satisfies the equation.

[7 marks]

METHOD 2

$$b^2 = a^2 + c^2 - 2ac \cos 60^\circ \Rightarrow b^2 = a^2 + c^2 - ac \quad (M1)A1$$

$$c^2 - ac = b^2 - a^2 \quad (M1)$$

$$c^2 - ac + \left(\frac{a}{2}\right)^2 = b^2 - a^2 + \left(\frac{a}{2}\right)^2 \quad M1A1$$

$$\left(c - \frac{a}{2}\right)^2 = b^2 - \frac{3}{4}a^2 \quad (A1)$$

$$c - \frac{a}{2} = \pm \sqrt{b^2 - \frac{3}{4}a^2} \quad A1$$

$$\Rightarrow c = \frac{1}{2}a \pm \sqrt{b^2 - \frac{3}{4}a^2} \quad AG$$

[7 marks]

## Examiners report

The majority of the candidates attempted part A of this question. Parts (a) and (b) were answered reasonably well. In part (c), many candidates scored the first two marks, but failed to recognize that the result was a quadratic equation, and hence did not progress further.

## Markscheme

### METHOD 1

AC = 5 and

AB =  $\sqrt{13}$  (may be seen on diagram) (AI)

$$\cos \alpha = \frac{3}{5} \text{ and}$$

$$\sin \alpha = \frac{4}{5} \quad (AI)$$

$$\cos \beta = \frac{3}{\sqrt{13}} \text{ and}$$

$$\sin \beta = \frac{2}{\sqrt{13}} \quad (AI)$$

**Note:** If only the two cosines are correctly given award (AI)(AI)(A0).

Use of

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad (MI)$$

$$= \frac{3}{5} \times \frac{3}{\sqrt{13}} + \frac{4}{5} \times \frac{2}{\sqrt{13}} \quad (\text{substituting}) \quad MI$$

$$= \frac{17}{5\sqrt{13}}$$

$$\left( = \frac{17\sqrt{13}}{65} \right) \quad AI \quad NI$$

[6 marks]

### METHOD 2

AC = 5 and

AB =  $\sqrt{13}$  (may be seen on diagram) (AI)

Use of

$$\cos(\alpha + \beta) = \frac{AC^2 + AB^2 - BC^2}{2(AC)(AB)} \quad (MI)$$

$$= \frac{25 + 13 - 36}{2 \times 5 \times \sqrt{13}} \quad \left( = \frac{1}{5\sqrt{13}} \right) \quad AI$$

Use of

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta \quad (MI)$$

$$\cos \alpha = \frac{3}{5} \text{ and}$$

$$\cos \beta = \frac{3}{\sqrt{13}} \quad (AI)$$

$$\cos(\alpha - \beta) = \frac{17}{5\sqrt{13}} \quad \left( = 2 \times \frac{3}{5} \times \frac{3}{\sqrt{13}} - \frac{1}{5\sqrt{13}} \right) \quad \left( = \frac{17\sqrt{13}}{65} \right) \quad AI \quad NI$$

[6 marks]

## Examiners report

Many candidates used a lot of space answering this question, but were generally successful. A few candidates incorrectly used the formula for the cosine of the difference of angles. An interesting alternative solution was noted, in which the side AB is reflected in AD and the required result follows from the use of the cosine rule.

26.

[6 marks]

## Markscheme

10 cm water depth corresponds to

$$16 \sec\left(\frac{\pi x}{36}\right) - 32 = -6 \quad (AI)$$

Rearranging to obtain an equation of the form

$$\sec\left(\frac{\pi x}{36}\right) = k \text{ or equivalent}$$

*i.e.* making a trigonometrical function the subject of the equation. **MI**

$$\cos\left(\frac{\pi x}{36}\right) = \frac{8}{13} \quad (AI)$$

$$\frac{\pi x}{36} = \pm \arccos \frac{8}{13} \quad MI$$

$$x = \pm \frac{36}{\pi} \arccos \frac{8}{13} \quad AI$$

**Note:** Do not penalise the omission of  $\pm$ .

Width of water surface is

$$\frac{72}{\pi} \arccos \frac{8}{13} \text{ (cm)} \quad RI \quad NI$$

**Note:** Candidate who starts with 10 instead of  $-6$  has the potential to gain the two **MI** marks and the **RI** mark.

[6 marks]

## Examiners report

This was a question in context which proved difficult for many candidates. Many appeared not to have fully comprehended the implications of the details of the diagram. A few candidates attempted integration, for no apparent reason.

27.

[6 marks]

## Markscheme

(a) Either finding depths graphically, using

$$\sin \frac{\pi t}{6} = \pm 1 \text{ or solving}$$

$$h'(t) = 0 \text{ for } t \quad (MI)$$

$$h(t)_{\max} = 12 \text{ (m)}, h(t)_{\min} = 4 \text{ (m)} \quad AIAI \quad N3$$

(b) Attempting to solve

$$8 + 4 \sin \frac{\pi t}{6} = 8 \text{ algebraically or graphically} \quad (MI)$$

$$t \in [0, 6] \cup [12, 18] \cup \{24\} \quad AIAI \quad N3$$

[6 marks]

## Examiners report

Not as well done as expected with most successful candidates using a graphical approach. Some candidates confused  $t$  and  $h$  and subsequently stated the values of  $t$  for which the water depth was either at a maximum and a minimum. Some candidates simply gave the maximum and minimum coordinates without stating the maximum and minimum depths.

In part (b), a large number of candidates left out  $t = 24$  from their final answer. A number of candidates experienced difficulties solving the inequality via algebraic means. A number of candidates specified incorrect intervals or only one correct interval.

28.

[7 marks]

## Markscheme

### METHOD 1

Attempting to use the cosine rule *i.e.*

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \hat{BAC} \quad (M1)$$

$$6^2 = 8.75^2 + AC^2 - 2 \times 8.75 \times AC \times \cos 37.8^\circ \text{ (or equivalent)} \quad A1$$

Attempting to solve the quadratic in AC *e.g.* graphically, numerically or with quadratic formula **MIA1**

Evidence from a sketch graph or their quadratic formula ( $AC = \dots$ ) that there are two values of AC to determine. **(A1)**

$$AC = 9.60 \text{ or } AC = 4.22 \quad A1A1 \quad N4$$

**Note:** Award **(M1)A1MIA1(A0)A1A0** for one correct value of AC.

[7 marks]

### METHOD 2

Attempting to use the sine rule *i.e.*

$$\frac{BC}{\sin \hat{BAC}} = \frac{AB}{\sin \hat{ACB}} \quad (M1)$$

$$\sin C = \frac{8.75 \sin 37.8^\circ}{6} \quad (= 0.8938\dots) \quad (A1)$$

$$C = 63.3576\dots^\circ \quad A1$$

$$C = 116.6423\dots^\circ \text{ and } B = 78.842\dots^\circ \text{ or } B = 25.5576\dots^\circ \quad A1$$

### EITHER

Attempting to solve

$$\frac{AC}{\sin 78.842\dots^\circ} = \frac{6}{\sin 37.8^\circ} \text{ OR } \frac{AC}{\sin 25.5576\dots^\circ} = \frac{6}{\sin 37.8^\circ} \quad M1$$

### OR

Attempting to solve

$$AC^2 = 8.75^2 + 6^2 - 2 \times 8.75 \times 6 \times \cos 25.5576\dots^\circ \text{ or}$$

$$AC^2 = 8.75^2 + 6^2 - 2 \times 8.75^2 \times 6 \times \cos 78.842\dots^\circ \quad M1$$

$$AC = 9.60 \text{ or } AC = 4.22 \quad A1A1 \quad N4$$

**Note:** Award **(M1)(A1)A1A0MIA1A0** for one correct value of AC.

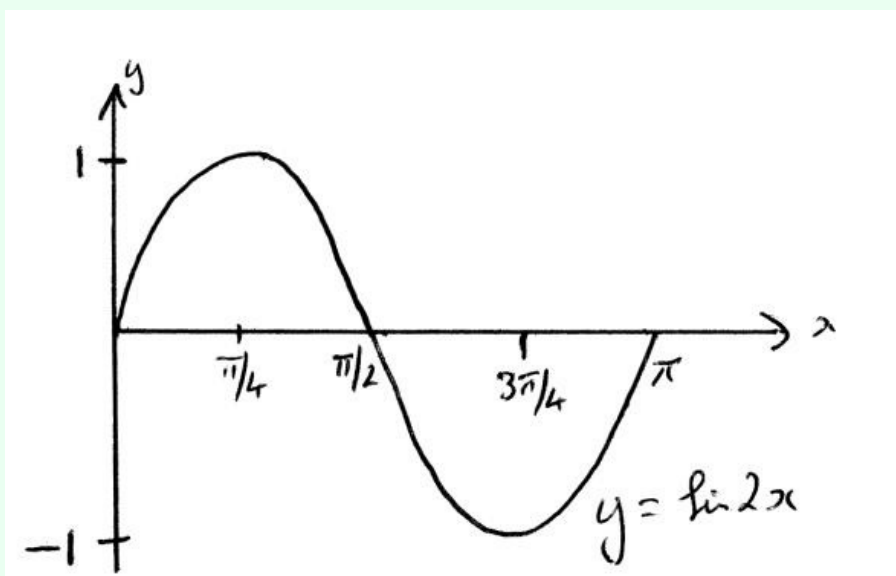
[7 marks]

## Examiners report

A large proportion of candidates did not identify the ambiguous case and hence they only obtained one correct value of AC. A number of candidates prematurely rounded intermediate results (angles) causing inaccurate final answers.

## Markscheme

(a)



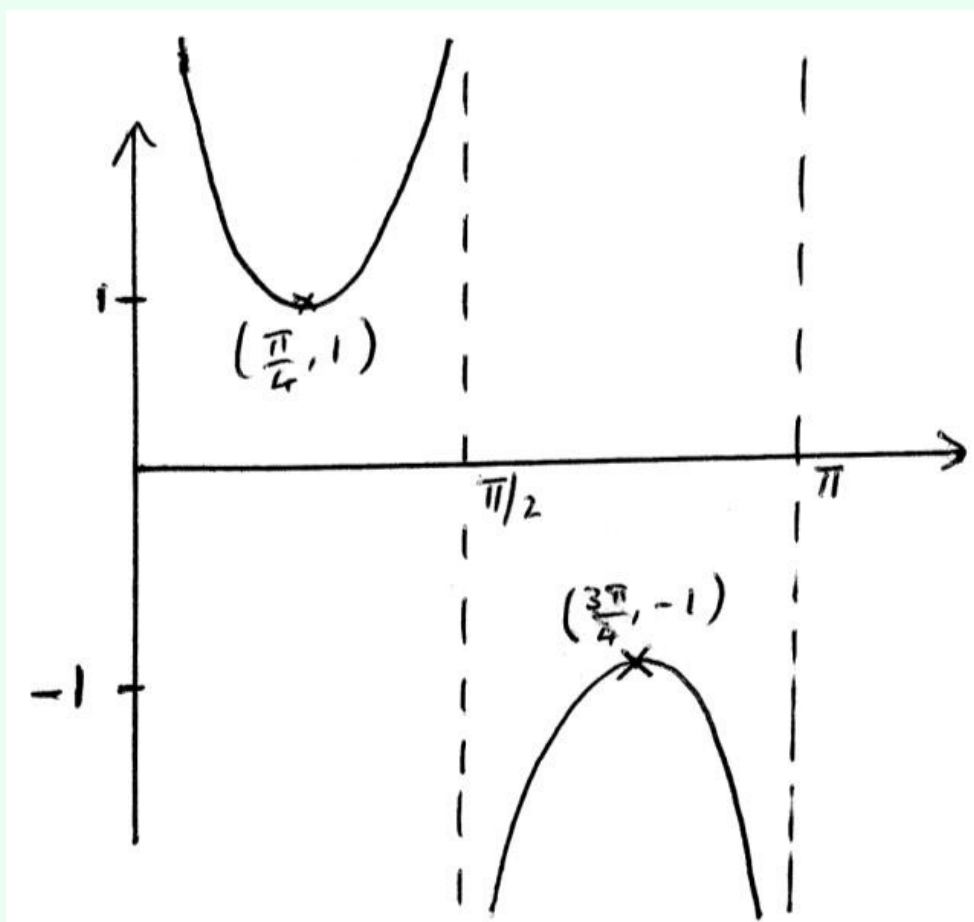
A2

**Note:** Award *A1* for shape.

*A1* for scales given on each axis.

[2 marks]

(b)



A5

Asymptotes

$$x = 0, x = \frac{\pi}{2}, x = \pi$$

$$\text{Max } \left(\frac{3\pi}{4}, -1\right),$$

$$\text{Min } \left(\frac{\pi}{4}, 1\right)$$

**Note:** Award *A1* for shape

**A2** for asymptotes, **A1** for one error, **A0** otherwise.

**A1** for max.

**A1** for min.

**[5 marks]**

(c)

$$\tan x + \cot x \equiv \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \quad \textbf{M1}$$

$$\equiv \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \quad \textbf{A1}$$

$$\equiv \frac{1}{\frac{1}{2} \sin 2x} \quad \textbf{A1}$$

$$\equiv 2 \csc 2x \quad \textbf{AG}$$

**[3 marks]**

(d)

$$\tan 2x + \cot 2x \equiv 2 \csc 4x \quad (\textbf{M1})$$

Max is at

$$\left(\frac{3\pi}{8}, -2\right) \quad \textbf{A1A1}$$

Min is at

$$\left(\frac{\pi}{8}, 2\right) \quad \textbf{A1A1}$$

**[5 marks]**

(e)

$$\csc 2x = 1.5 \tan x - 0.5$$

$$\frac{1}{2} \tan x + \frac{1}{2} \cot x = \frac{3}{2} \tan x - \frac{1}{2} \quad \textbf{M1}$$

$$\tan x + \cot x = 3 \tan x - 1$$

$$2 \tan x - \frac{1}{\tan x} - 1 = 0 \quad \textbf{M1}$$

$$2 \tan^2 x - \tan x - 1 = 0 \quad \textbf{A1}$$

$$(2 \tan x + 1)(\tan x - 1) = 0 \quad \textbf{M1}$$

$$\tan x = -\frac{1}{2} \text{ or } 1 \quad \textbf{A1}$$

$$x = \frac{\pi}{4} \quad \textbf{A1}$$

**Note:** Award **A0** for answer in degrees or if more than one value given for  $x$ .

**[6 marks]**

**Total [21 marks]**

## Examiners report

Although the better candidates scored well on this question, it was disappointing to see that a number of candidates did not appear to be well prepared and made little progress. It was disappointing that a small minority of candidates were unable to sketch  $y = \sin 2x$ . Most candidates who completed part (a) attempted part (b), although not always successfully. In many cases the coordinates of the local maximum and minimum points and the equations of the asymptotes were not clearly stated. Part (c) was attempted by the vast majority of candidates. The responses to part (d) were disappointing with a significant number of candidates ignoring the hence and attempting differentiation which more often than not resulted in either arithmetic or algebraic errors. A reasonable number of candidates gained the correct answer to part (e), but a number tried to solve the equation in terms of  $\sin x$  and  $\cos x$  and made little progress.

30.

[7 marks]

### Markscheme

$$\frac{\sin B}{6.5} = \frac{\sin 35^\circ}{4} \quad M1$$

$$\hat{B} = 68.8^\circ \text{ or } 111^\circ \quad A1A1$$

$$\hat{C} = 76.2^\circ \text{ or }$$

$$33.8^\circ \quad (\text{accept}$$

$$34^\circ) \quad A1$$

$$\frac{AB}{\sin C} = \frac{BC}{\sin A}$$

$$\frac{AB}{\sin 76.2^\circ} = \frac{4}{\sin 35^\circ} \quad (M1)$$

$$AB = 6.77 \text{ cm} \quad A1$$

$$\frac{AB}{\sin 33.8^\circ} = \frac{4}{\sin 35^\circ}$$

$$AB = 3.88 \text{ cm}$$

$$(\text{accept } 3.90) \quad A1$$

[7 marks]

### Examiners report

Most candidates realised that the sine rule was the best option although some used the more difficult cosine rule which was an alternative method. Many candidates failed to realise that there were two solutions even though the question suggested this. Many candidates were given an arithmetic penalty for giving one of the possible of values  $\hat{B}$  as  $112.2^\circ$  instead of  $111^\circ$ .

The angle

$\theta$  lies in the first quadrant and

$\cos \theta = \frac{1}{3}$ .

31a.

[1 mark]

### Markscheme

$$\sin \theta = \frac{\sqrt{8}}{3} \quad A1$$

[1 mark]

### Examiners report

[N/A]

31b.

[2 marks]

### Markscheme

$$\tan 2\theta = \frac{2 \times \sqrt{8}}{1-8} = -\frac{2\sqrt{8}}{7} \quad \left(-\frac{4\sqrt{2}}{7}\right) \quad M1A1$$

[2 marks]

### Examiners report

[N/A]

31c. [3 marks]

## Markscheme

$$\cos^2\left(\frac{\theta}{2}\right) = \frac{1+\frac{1}{3}}{2} = \frac{2}{3} \quad M1A1$$

$$\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{6}}{3} \quad A1$$

[3 marks]

## Examiners report

[N/A]

In the triangle ABC,

$AB = 2\sqrt{3}$  ,  $AC = 9$  and

$\hat{BAC} = 150^\circ$  .

32a. [3 marks]

## Markscheme

$$BC^2 = 12 + 81 + 2 \times 2\sqrt{3} \times 9 \times \frac{\sqrt{3}}{2} = 147 \quad M1A1$$

$$BC = 7\sqrt{3} \quad A1$$

[3 marks]

## Examiners report

[N/A]

32b. [4 marks]

## Markscheme

area of triangle

$$ABC = \frac{1}{2} \times 9 \times 2\sqrt{3} \times \frac{1}{2} \left( = \frac{9\sqrt{3}}{2} \right) \quad M1A1$$

therefore

$$\frac{1}{2} \times AD \times 7\sqrt{3} = \frac{9\sqrt{3}}{2} \quad M1$$

$$AD = \frac{9}{7} \quad A1$$

[4 marks]

## Examiners report

[N/A]

Consider the following system of equations:

$$x + y + z = 1$$

$$2x + 3y + z = 3$$

$$x + 3y - z = \lambda$$

where

$$\lambda \in \mathbb{R} .$$

33a.

[4 marks]

## Markscheme

using row operations, *MI*

to obtain 2 equations in the same 2 variables *AIAI*

for example

$$y - z = 1$$

$$2y - 2z = \lambda - 1$$

the fact that one of the left hand sides is a multiple of the other left hand side indicates that the equations do not have a unique solution, or equivalent *RIAG*

[4 marks]

## Examiners report

[N/A]

33b.

[4 marks]

## Markscheme

(i)

$$\lambda = 3 \quad \textit{AI}$$

(ii) put

$$z = \mu \quad \textit{MI}$$

then

$$y = 1 + \mu \quad \textit{AI}$$

and

$$x = -2\mu \text{ or equivalent} \quad \textit{AI}$$

[4 marks]

## Examiners report

[N/A]

## Markscheme

taking cross products with  $\mathbf{a}$ , **M1**

$\mathbf{a}$

$$\times (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{a}$$

$$\times \mathbf{0} = \mathbf{0} \quad \mathbf{A1}$$

using the algebraic properties of vectors and the fact that  $\mathbf{a}$

$$\times \mathbf{a} = \mathbf{0}, \quad \mathbf{M1}$$

$\mathbf{a}$

$$\times \mathbf{b} + \mathbf{a}$$

$$\times \mathbf{c} = \mathbf{0} \quad \mathbf{A1}$$

$\mathbf{a}$

$$\times \mathbf{b} = \mathbf{c}$$

$$\times \mathbf{a} \quad \mathbf{AG}$$

taking cross products with  $\mathbf{b}$ , **M1**

$\mathbf{b}$

$$\times (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{0}$$

$\mathbf{b}$

$$\times \mathbf{a} + \mathbf{b}$$

$$\times \mathbf{c} = \mathbf{0} \quad \mathbf{A1}$$

$\mathbf{a}$

$$\times \mathbf{b} = \mathbf{b}$$

$$\times \mathbf{c} \quad \mathbf{AG}$$

this completes the proof

[6 marks]

## Examiners report

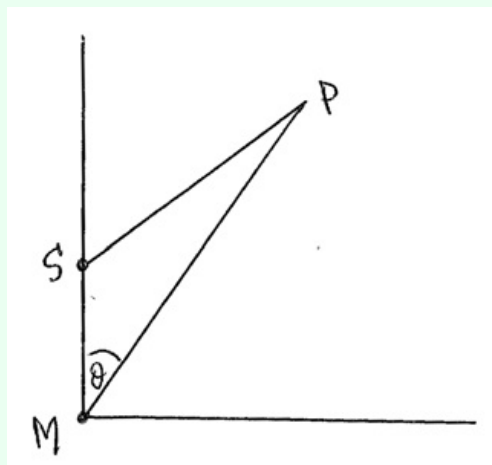
[N/A]

A ship, S, is 10 km north of a motorboat, M, at 12.00pm. The ship is travelling northeast with a constant velocity of  $20 \text{ km hr}^{-1}$ . The motorboat wishes to intercept the ship and it moves with a constant velocity of  $30 \text{ km hr}^{-1}$  in a direction  $\theta$  degrees east of north. In order for the interception to take place, determine

35a.

[4 marks]

## Markscheme



let the interception occur at the point P,  $t$  hrs after 12:00

then,  $SP = 20t$  and  $MP = 30t$  *AI*

using the sine rule,

$$\frac{SP}{MP} = \frac{2}{3} = \frac{\sin \theta}{\sin 135} \quad \text{MIAI}$$

whence

$$\theta = 28.1 \quad \text{AI}$$

[4 marks]

## Examiners report

[N/A]

35b.

[5 marks]

## Markscheme

using the sine rule again,

$$\frac{MP}{MS} = \frac{\sin 135}{\sin(45 - 28.1255\dots)} \quad \text{MIAI}$$

$$30t = 10 \times \frac{\sin 135}{\sin 16.8745\dots} \quad \text{MI}$$

$$t = 0.81199\dots \quad \text{AI}$$

the interception occurs at 12:49 *AI*

[5 marks]

## Examiners report

[N/A]