

Topic 5 Part 1 [523 marks]

1a. [1 mark]

Markscheme

$$E(X) = np$$

$$\Rightarrow 10 = 30p$$

$$\Rightarrow p = \frac{1}{3} \quad \text{AI}$$

[1 mark]

Examiners report

Again this proved to be an accessible question for students with many students gaining full marks. Most candidates used the calculator to find the answers to parts (b) and (c) which is what was intended, but candidates should be aware that there are often marks for recognising what needs to be found, even if the candidate does not obtain the final correct answer. It is suggested that in this style of question, candidates should indicate what they are trying to find as well as giving the final answer.

1b. [2 marks]

Markscheme

$$P(X = 10) = \binom{30}{10} \left(\frac{1}{3}\right)^{10} \left(\frac{2}{3}\right)^{20} = 0.153 \quad \text{(MI)AI}$$

[2 marks]

Examiners report

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1c. [2 marks]

Markscheme

$$P(X \geq 15) = 1 - P(X \leq 14) \quad \text{(MI)}$$

$$= 1 - 0.9565\dots = 0.0435 \quad \text{AI}$$

[2 marks]

Examiners report

Again this proved to be an accessible question for students with many students gaining full marks. Most candidates used the calculator to find the answers to parts (b) and (c) which is what was intended, but candidates should be aware that there are often marks for recognising what needs to be found, even if the candidate does not obtain the final correct answer. It is suggested that in this style of question, candidates should indicate what they are trying to find as well as giving the final answer.

2a. [3 marks]

Markscheme

$$P(X = 5) = P(X = 3) + P(X = 4)$$

$$\frac{e^{-m}m^5}{5!} = \frac{e^{-m}m^3}{3!} + \frac{e^{-m}m^4}{4!} \quad \text{MI(AI)}$$

$$m^2 - 5m - 20 = 0$$

$$\Rightarrow m = \frac{5 + \sqrt{105}}{2} = (7.62) \quad \text{AI}$$

[3 marks]

Examiners report

Again this proved to be a successful question for many candidates with a good proportion of wholly correct answers seen. It was good to see students making good use of the calculator.

2b. [2 marks]

Markscheme

$$\begin{aligned} P(X > 2) &= 1 - P(X \leq 2) \quad (M1) \\ &= 1 - 0.018\dots \\ &= 0.982 \quad AI \\ [2 \text{ marks}] \end{aligned}$$

Examiners report

Again this proved to be a successful question for many candidates with a good proportion of wholly correct answers seen. It was good to see students making good use of the calculator.

3a. [3 marks]

Markscheme

$$\begin{aligned} \int_0^a \frac{1}{1+x^4} dx &= 1 \quad M2 \\ a &= 1.40 \quad AI \\ [3 \text{ marks}] \end{aligned}$$

Examiners report

Many candidates picked up some marks for this question, but only a few gained full marks. In part (a) many candidates did not appreciate the need for the calculator to find a value of a . Candidates had more success with part (b) with a number of candidates picking up follow through marks.

3b. [2 marks]

Markscheme

$$\begin{aligned} E(X) &= \int_0^a \frac{x}{1+x^4} dx \quad M1 \\ &= \frac{1}{2} \arctan(a^2) \\ &= 0.548 \quad AI \\ [2 \text{ marks}] \end{aligned}$$

Examiners report

Many candidates picked up some marks for this question, but only a few gained full marks. In part (a) many candidates did not appreciate the need for the calculator to find a value of a . Candidates had more success with part (b) with a number of candidates picking up follow through marks.

4a. [5 marks]

Markscheme

$$\begin{aligned} (i) \\ P(X > 225) &= 0.158\dots \quad (M1)(AI) \\ \text{expected number} \\ &= 450 \times 0.158\dots = 71.4 \quad AI \end{aligned}$$

$$\begin{aligned} (ii) \\ P(X < m) &= 0.7 \quad (M1) \\ \Rightarrow m &= 213 \text{ (grams)} \quad AI \\ [5 \text{ marks}] \end{aligned}$$

Examiners report

This was an accessible question for most students with many wholly correct answers seen. In part (b) a few candidates struggled to find the correct values from the calculator and in part (c) a small minority did not see the need to treat it as a binomial distribution.

4b. [6 marks]

Markscheme

$$\frac{270-\mu}{\sigma} = 1.40\dots \quad (M1)AI$$

$$\frac{250-\mu}{\sigma} = -1.03\dots \quad AI$$

Note: These could be seen in graphical form.

solving simultaneously (M1)

$$\mu = 258, \sigma = 8.19 \quad AIAI$$

[6 marks]

Examiners report

This was an accessible question for most students with many wholly correct answers seen. In part (b) a few candidates struggled to find the correct values from the calculator and in part (c) a small minority did not see the need to treat it as a binomial distribution.

4c. [3 marks]

Markscheme

$$X \sim N(80, 4^2)$$

$$P(X > 82) = 0.3085\dots \quad AI$$

recognition of the use of binomial distribution. (M1)

$$X \sim B(5, 0.3085\dots)$$

$$P(X = 3) = 0.140 \quad AI$$

[3 marks]

Examiners report

This was an accessible question for most students with many wholly correct answers seen. In part (b) a few candidates struggled to find the correct values from the calculator and in part (c) a small minority did not see the need to treat it as a binomial distribution.

5a. [2 marks]

Markscheme

$$\text{mean} = 2.06 \quad AI$$

$$\text{variance} = 1.94 \quad AI$$

[2 marks]

Examiners report

Many candidates picked up good marks for this question, but lost marks because of inattention to detail. The mean of the data was usually given correctly, but sometimes the variance was wrong. It may seem a small point, but the correct hypotheses should not mention the value of the estimated mean. Some candidates did not notice that some columns needed to be combined.

5b. [1 mark]

Markscheme

a Poisson distribution has the property that its mean and variance are the same **RI**

[1 mark]

Examiners report

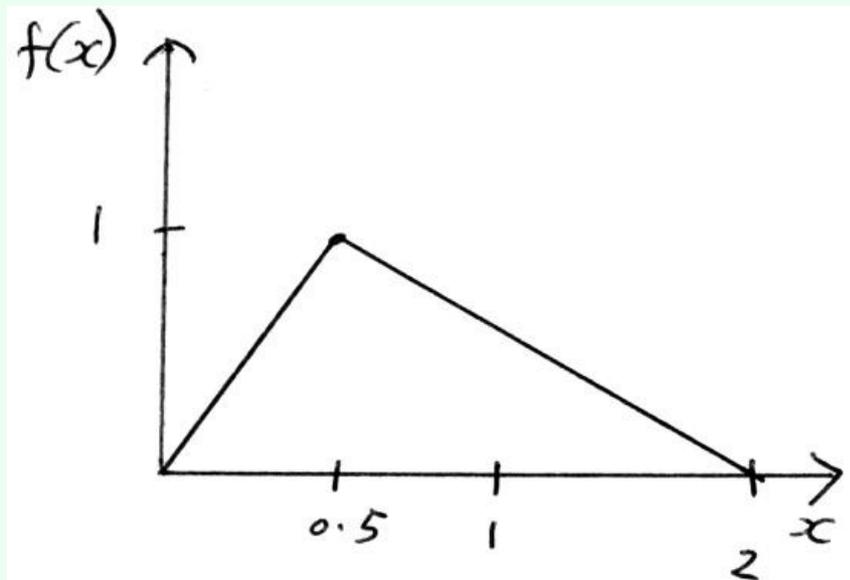
Many candidates picked up good marks for this question, but lost marks because of inattention to detail. The mean of the data was usually given correctly, but sometimes the variance was wrong. It may seem a small point, but the correct hypotheses should not mention the value of the estimated mean. Some candidates did not notice that some columns needed to be combined.

6a.

[3 marks]

Markscheme

piecewise linear graph



correct shape *AI*

with vertices (0, 0), (0.5, 1) and (2, 0) *AI*

LQ: $x = 0.5$, because the area of the triangle is 0.25 *RI*

[3 marks]

Examiners report

There was a curious issue about the lower quartile in part (a): The LQ coincides with a quarter of the range of the distribution $\frac{2}{4} = 0.5$. Sadly this is wrong reasoning – the correct reasoning involves a consideration of areas.

6b.

[4 marks]

Markscheme

(i)

$$E(X) = \int_0^{0.5} x \times 2x dx + \int_{0.5}^2 x \times \left(\frac{4}{3} - \frac{2}{3}x\right) dx = \frac{5}{6} (= 0.833\dots) \quad (MI)AI$$

(ii)

$$E(X^2) = \int_0^{0.5} x^2 \times 2x dx + \int_{0.5}^2 x^2 \times \left(\frac{4}{3} - \frac{2}{3}x\right) dx = \frac{7}{8} (= 0.875) \quad (MI)AI$$

[4 marks]

Examiners report

There was a curious issue about the lower quartile in part (a): The LQ coincides with a quarter of the range of the distribution $\frac{2}{4} = 0.5$. Sadly this is wrong reasoning – the correct reasoning involves a consideration of areas.

In part (b) many candidates used hand calculation rather than their GDC.

The random variable Y was not well understood, and that followed into incorrect calculations involving $Y - 2X$.

Markscheme

(i)

$$E(Y - 2X) = 2E(X) - 2E(X) = 0 \quad \mathbf{AI}$$

(ii)

$$\text{Var}(X) = \left(E(X^2) - E(X)^2 \right) = \frac{13}{72} \quad \mathbf{AI}$$

$$Y = X_1 + X_2 \Rightarrow \text{Var}(Y) = 2\text{Var}(X) \quad \mathbf{(MI)}$$

$$\text{Var}(Y - 2X) = 2\text{Var}(X) + 4\text{Var}(X) = \frac{13}{12} \quad \mathbf{MIAI}$$

[5 marks]

Examiners report

There was a curious issue about the lower quartile in part (a): The LQ coincides with a quarter of the range of the distribution $\frac{2}{4} = 0.5$. Sadly this is wrong reasoning – the correct reasoning involves a consideration of areas.

In part (b) many candidates used hand calculation rather than their GDC.

The random variable Y was not well understood, and that followed into incorrect calculations involving $Y - 2X$.

Markscheme

(i) attempt to use

$$cf(x) = \int f(u)du \quad \mathbf{MI}$$

obtain

$$cf(x) = \begin{cases} x^2, & 0 \leq x \leq 0.5, \\ \frac{4x}{3} - \frac{1}{3}x^2 - \frac{1}{3}, & 0.5 \leq x \leq 2, \end{cases}$$

A1

A2

(ii) attempt to solve

$$cf(x) = 0.5 \quad \mathbf{MI}$$

$$\frac{4x}{3} - \frac{1}{3}x^2 - \frac{1}{3} = 0.5 \quad \mathbf{(AI)}$$

obtain 0.775 **AI**

Note: Accept attempts in the form of an integral with upper limit the unknown median.

Note: Accept exact answer

$$2 - \sqrt{1.5}.$$

[7 marks]

Examiners report

There was a curious issue about the lower quartile in part (a): The LQ coincides with a quarter of the range of the distribution $\frac{2}{4} = 0.5$. Sadly this is wrong reasoning – the correct reasoning involves a consideration of areas.

In part (b) many candidates used hand calculation rather than their GDC.

The random variable Y was not well understood, and that followed into incorrect calculations involving $Y - 2X$.

Markscheme

$$\frac{m^{k-1}e^{-m}}{(k-1)!} = \frac{m^{k+1}e^{-m}}{(k+1)!} \quad \mathbf{MI}$$

$$\Rightarrow 1 = \frac{m^2}{(k+1)k} \quad \mathbf{AI}$$

Note: Award **AI** for any correct intermediate step.

$$\Rightarrow m^2 = (k+1)k \quad \mathbf{AG}$$

[2 marks]

Examiners report

Most candidates were able to complete part (a). The remainder of the question involved some understanding of the shape of the distribution and some facility with algebraic manipulation.

7b. [6 marks]

Markscheme

$$\begin{aligned}\frac{P(X=k)}{P(X=k-1)} &= \frac{e^{-m} \times \frac{m^k}{k!}}{e^{-m} \times \frac{m^{k-1}}{(k-1)!}} \quad \mathbf{MI} \\ &= \frac{m}{k} \quad \mathbf{AI} \\ &= \frac{\sqrt{k(k+1)}}{k} \quad \mathbf{MI} \\ &= \sqrt{\frac{k+1}{k}} > 1 \quad \mathbf{RI}\end{aligned}$$

so

$$P(X = k) > P(X = k - 1) \quad \mathbf{RI}$$

similarly

$$P(X = k) > P(X = k + 1) \quad \mathbf{RI}$$

hence k is the mode \mathbf{AG}

[6 marks]

Examiners report

Most candidates were able to complete part (a). The remainder of the question involved some understanding of the shape of the distribution and some facility with algebraic manipulation.

8. [6 marks]

Markscheme

tree diagram $\mathbf{(MI)}$

$$P(I|D) = \frac{P(D|I) \times P(I)}{P(D)} \quad \mathbf{(MI)}$$

$$= \frac{0.1 \times 0.2}{0.1 \times 0.2 + 0.8 \times 0.75} \quad \mathbf{AIAIAI}$$

$$\left(= \frac{0.02}{0.62} \right) = \frac{1}{31} \quad \mathbf{AI}$$

Note: Alternative presentation of results: \mathbf{MI} for labelled tree; \mathbf{AI} for initial branching probabilities, 0.2 and 0.8; \mathbf{AI} for at least the relevant second branching probabilities, 0.1 and 0.75; \mathbf{AI} for the ‘infected’ end-point probabilities, 0.02 and 0.6; \mathbf{MIAI} for the final conditional probability calculation.

[6 marks]

Examiners report

Candidates who drew a tree diagram, the majority, usually found the correct answer.

9.

[6 marks]

Markscheme

$$P(A) = \frac{\pi}{25\pi} \times \frac{1}{2} = \frac{1}{50} \quad (MI)AI$$

$$P(B) = \frac{8\pi}{25\pi} \times \frac{1}{2} = \frac{4}{25} \quad AI$$

$$P(C) = \frac{16\pi}{25\pi} \times \frac{1}{2} = \frac{8}{25} \quad AI$$

Note: The *MI* is for the use of 3 areas

$$E(X) = (0.5 \times 0) + \frac{1}{50} \times 10 + \frac{4}{25} \times 6 + \frac{8}{25} \times 3 = \frac{106}{50} (= 2.12) \quad MIAI$$

Note: The final *MI* is available if the probabilities are incorrect but sum to 1 or

[6 marks]

Examiners report

The key to this question was the recognition that the various probabilities were proportional to relevant areas. Most candidates realised this, but made mistakes translating this into correct calculations. Some candidates did not make use of the probability of $\frac{1}{2}$ of not hitting the target.

10a.

[2 marks]

Markscheme

$$k \int_0^{\frac{\pi}{2}} \sin x dx = 1 \quad MI$$

$$k[-\cos x]_0^{\frac{\pi}{2}} = 1$$

$$k = 1 \quad AI$$

[2 marks]

Examiners report

Most candidates scored maximum marks on this question. A few candidates found $k = -1$.

10b.

[5 marks]

Markscheme

$$E(X) = \int_0^{\frac{\pi}{2}} x \sin x dx \quad MI$$

integration by parts *MI*

$$[-x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x dx \quad AIAI$$

$$= 1 \quad AI$$

[5 marks]

Examiners report

Most candidates scored maximum marks on this question. A few candidates found $k = -1$.

10c. [3 marks]

Markscheme

$$\int_0^M \sin x dx = \frac{1}{2} \quad \text{MI}$$

$$[-\cos x]_0^M = \frac{1}{2} \quad \text{AI}$$

$$\cos M = \frac{1}{2}$$

$$M = \frac{\pi}{3} \quad \text{AI}$$

Note: accept

$$\arccos \frac{1}{2}$$

[3 marks]

Examiners report

Most candidates scored maximum marks on this question. A few candidates found $k = -1$.

11a. [2 marks]

Markscheme

$$m = \frac{300}{60} = 5 \quad \text{(AI)}$$

$$P(X = 0) = 0.00674 \quad \text{AI}$$

or

$$e^{-5}$$

[2 marks]

Examiners report

Parts (a) and (b) were answered successfully by many candidates. Some candidates had difficulty obtaining the correct inequality in (c).

11b. [1 mark]

Markscheme

$$E(X) = 5 \times 2 = 10 \quad \text{AI}$$

[1 mark]

Examiners report

Parts (a) and (b) were answered successfully by many candidates. Some candidates had difficulty obtaining the correct inequality in (c).

11c. [2 marks]

Markscheme

$$P(X > 10) = 1 - P(X \leq 10) \quad (M1)$$

$$= 0.417 \quad A1$$

[2 marks]

Examiners report

Parts (a) and (b) were answered successfully by many candidates. Some candidates had difficulty obtaining the correct inequality in (c).

12a. [2 marks]

Markscheme

$$X \sim B(5, 0.1) \quad (M1)$$

$$P(X = 2) = 0.0729 \quad A1$$

[2 marks]

Examiners report

This question was generally answered successfully. Many candidates used the tabular feature of their GDC for (b) thereby avoiding potential errors in the algebraic manipulation of logs and inequalities.

12b. [3 marks]

Markscheme

$$P(X \geq 1) = 1 - P(X = 0) \quad (M1)$$

$$0.9 < 1 - \left(\frac{9}{10}\right)^n \quad (M1)$$

$$n > \frac{\ln 0.1}{\ln 0.9}$$

$$n = 22 \text{ days} \quad A1$$

[3 marks]

Examiners report

This question was generally answered successfully. Many candidates used the tabular feature of their GDC for (b) thereby avoiding potential errors in the algebraic manipulation of logs and inequalities.

13a. [2 marks]

$$X \sim N(60.33, 1.95^2)$$

$$P(X < x) = 0.2 \Rightarrow x = 58.69 \text{ m}$$

Examiners report

Parts (a) and (b) were generally accessible to many candidates. In (c)(i) quite a few candidates missed the wording 'first throw', and consequently in (ii) used the incorrect probabilities.

13b.

[3 marks]

Markscheme

$$z = -0.8416\dots \quad (A1)$$

$$-0.8416 = \frac{56.52 - 59.39}{\sigma} \quad (M1)$$

$$\sigma \approx 3.41 \quad A1$$

[3 marks]

Examiners report

Parts (a) and (b) were generally accessible to many candidates. In (c)(i) quite a few candidates missed the wording 'first throw', and consequently in (ii) used the incorrect probabilities.

13c.

[10 marks]

Markscheme

Jan

$$X \sim N(60.33, 1.95^2); \text{ Sia}$$

$$X \sim N(59.50, 3.00^2)$$

(i) Jan:

$$P(X > 65) \approx 0.00831 \quad (MI)AI$$

Sia:

$$P(Y > 65) \approx 0.0334 \quad AI$$

Sia is more likely to qualify *RI***Note:** Only award *RI* if *(MI)* has been awarded.

(ii) Jan:

$$P(X \geq 1) = 1 - P(X = 0) \quad (MI)$$

$$= 1 - (1 - 0.00831 \dots)^3 \approx 0.0247 \quad (MI)AI$$

Sia:

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1 - 0.0334 \dots)^3 \approx 0.0968 \quad AI$$

Note: Accept 0.0240 and 0.0969.

hence,

$$P(X \geq 1 \text{ and } Y \geq 1) = 0.0247 \times 0.0968 = 0.00239 \quad (MI)AI$$

[10 marks]

Examiners report

Parts (a) and (b) were generally accessible to many candidates. In (c)(i) quite a few candidates missed the wording ‘first throw’, and consequently in (ii) used the incorrect probabilities.

14a.

[2 marks]

Markscheme

let S be the weight of tea in a random *Supermug* tea bag

$$S \sim N(4.2, 0.15^2)$$

$$P(S > 3.9) = 0.977 \quad (MI)AI$$

[2 marks]

Examiners report

For most candidates this was a reasonable start to the paper with many candidates gaining close to full marks. The most common error was in (b) where, surprisingly, many candidates did not realise the need to multiply the product of the two probabilities by 2 to gain the final answer. Weaker candidates often found problems in understanding how to correctly find the variance in both (c) and (d).

14b. [4 marks]

Markscheme

let M be the weight of tea in a random *Megamug* tea bag

$$M \sim N(5.6, 0.17^2)$$

$$P(M > 5.4) = 0.880\dots \quad (AI)$$

$$P(M < 5.4) = 1 - 0.880\dots = 0.119\dots \quad (AI)$$

required probability

$$= 2 \times 0.880\dots \times 0.119\dots = 0.211 \quad MIAI$$

[4 marks]

Examiners report

For most candidates this was a reasonable start to the paper with many candidates gaining close to full marks. The most common error was in (b) where, surprisingly, many candidates did not realise the need to multiply the product of the two probabilities by 2 to gain the final answer. Weaker candidates often found problems in understanding how to correctly find the variance in both (c) and (d).

14c. [4 marks]

Markscheme

$$P(S_1 + S_2 + S_3 + S_4 + S_5 < 20.5)$$

let

$$S_1 + S_2 + S_3 + S_4 + S_5 = A \quad (MI)$$

$$E(A) = 5E(S)$$

$$= 21 \quad AI$$

$$\text{Var}(A) = 5\text{Var}(S)$$

$$= 0.1125 \quad AI$$

$$A \sim N(21, 0.1125)$$

$$P(A < 20.5) = 0.0680 \quad AI$$

[4 marks]

Examiners report

For most candidates this was a reasonable start to the paper with many candidates gaining close to full marks. The most common error was in (b) where, surprisingly, many candidates did not realise the need to multiply the product of the two probabilities by 2 to gain the final answer. Weaker candidates often found problems in understanding how to correctly find the variance in both (c) and (d).

Markscheme

$$P(S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 - (M_1 + M_2 + M_3 + M_4 + M_5) > 0)$$

let

$$S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 - (M_1 + M_2 + M_3 + M_4 + M_5) = B \quad (MI)$$

$$E(B) = 7E(S) - 5E(M)$$

$$= 1.4 \quad AI$$

Note: Above *AI* is independent of first *MI*.

$$\text{Var}(B) = 7\text{Var}(S) + 5\text{Var}(M) \quad (MI)$$

$$= 0.302 \quad AI$$

$$P(B > 0) = 0.995 \quad AI$$

[5 marks]

Examiners report

For most candidates this was a reasonable start to the paper with many candidates gaining close to full marks. The most common error was in (b) where, surprisingly, many candidates did not realise the need to multiply the product of the two probabilities by 2 to gain the final answer. Weaker candidates often found problems in understanding how to correctly find the variance in both (c) and (d).

Markscheme

(i)

$$P(A \cup B) = P(A) + P(B) = 0.7 \quad AI$$

(ii)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (MI)$$

$$= P(A) + P(B) - P(A)P(B) \quad (MI)$$

$$= 0.3 + 0.4 - 0.12 = 0.58 \quad AI$$

[4 marks]

Examiners report

Most candidates attempted this question and answered it well. A few misconceptions were identified (eg $P(A \cup B) = P(A)P(B)$). Many candidates were unsure about the meaning of independent events.

15b.

[3 marks]

Markscheme

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.3 + 0.4 - 0.6 = 0.1 \quad \text{AI}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (\text{MI})$$

$$= \frac{0.1}{0.4} = 0.25 \quad \text{AI}$$

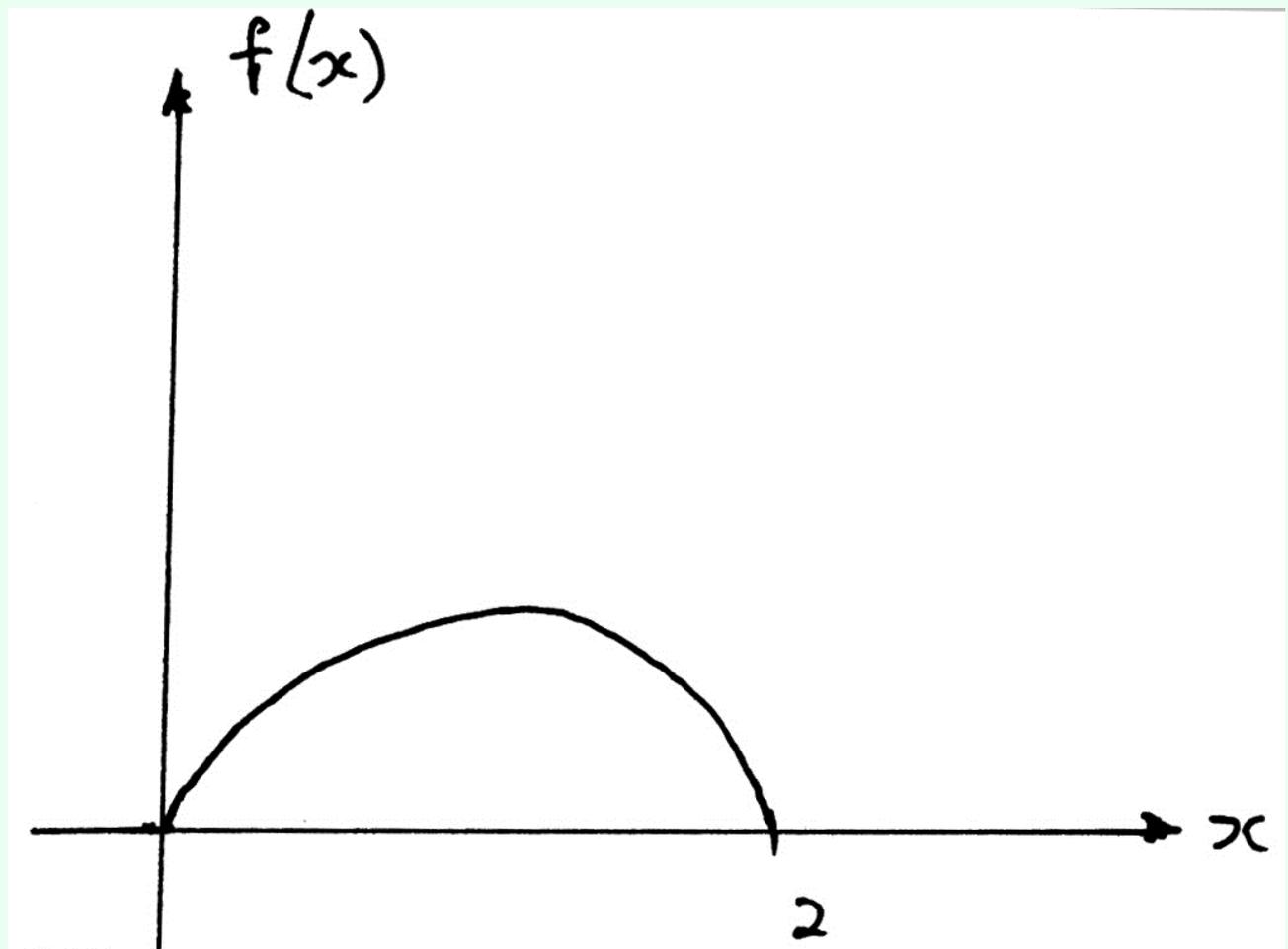
[3 marks]

Examiners report

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16a.

[1 mark]

Markscheme

AI

Note: Award AI for intercepts of 0 and 2 and a concave down curve in the given domain .

Note: Award A0 if the cubic graph is extended outside the domain $[0, 2]$.

[1 mark]

Examiners report

Most candidates completed this question well. A number extended the graph beyond the given domain.

16b. [5 marks]

Markscheme

$$\int_0^2 kx(x+1)(2-x)dx = 1 \quad (MI)$$

Note: The correct limits and =1 must be seen but may be seen later.

$$k \int_0^2 (-x^3 + x^2 + 2x)dx = 1 \quad AI$$

$$k \left[-\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2 \right]_0^2 = 1 \quad MI$$

$$k \left(-4 + \frac{8}{3} + 4 \right) = 1 \quad (AI)$$

$$k = \frac{3}{8} \quad AI$$

[5 marks]

Examiners report

Most candidates completed this question well. A number extended the graph beyond the given domain.

17a. [4 marks]

Markscheme

METHOD 1

P(3 defective in first 8)

$$= \binom{8}{3} \times \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} \times \frac{11}{12} \times \frac{10}{11} \times \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8} \quad MIAIAI$$

Note: Award *MI* for multiplication of probabilities with decreasing denominators.

Award *AI* for multiplication of correct eight probabilities.

Award *AI* for multiplying by

$$\binom{8}{3}.$$

$$= \frac{56}{195} \quad AI$$

METHOD 2

P(3 defective DVD players from 8)

$$= \frac{\binom{4}{3} \binom{11}{5}}{\binom{15}{8}} \quad MIAI$$

Note: Award *MI* for an expression of this form containing three combinations.

$$= \frac{\frac{4!}{3!1!} \times \frac{11!}{5!6!}}{\frac{15!}{8!7!}} \quad MI$$

$$= \frac{56}{195} \quad AI$$

[4 marks]

Examiners report

There were two main methods used to complete this question, the most common being a combinations approach. Those who did this coped well with the factorial simplification. Many who did not manage the first part were able to complete the second part successfully.

17b. [3 marks]

Markscheme

$$P(9^{\text{th}} \text{ selected is } 4^{\text{th}} \text{ defective player} | 3 \text{ defective in first } 8) = \frac{1}{7} \quad (AI)$$

$$P(9^{\text{th}} \text{ selected is } 4^{\text{th}} \text{ defective player}) = \frac{56}{195} \times \frac{1}{7} \quad MI$$

$$= \frac{8}{195} \quad AI$$

[3 marks]

Examiners report

There were two main methods used to complete this question, the most common being a combinations approach. Those who did this coped well with the factorial simplification. Many who did not manage the first part were able to complete the second part successfully.

18a. [1 mark]

Markscheme

$$P(x < 1.4) = 0.691 \quad (\text{accept } 0.692) \quad AI$$

[1 mark]

Examiners report

Part (a) was almost universally correctly answered, albeit with an accuracy penalty in some cases. In (b) it was generally recognised that the distribution was binomial, but with some wavering about the correct value of the parameter p . Part (c) was sometimes answered correctly, but not with much confidence.

18b. [3 marks]

Markscheme

METHOD 1

$$y \sim B(6, 0.3085\dots) \quad (MI)$$

$$P(Y \geq 4) = 1 - P(Y \leq 3) \quad (MI)$$

$$= 0.0775 \quad (\text{accept } 0.0778 \text{ if 3sf approximation from (a) used}) \quad AI$$

METHOD 2

$$X \sim B(6, 0.6914\dots) \quad (MI)$$

$$P(X \leq 2) \quad (MI)$$

$$= 0.0775 \quad (\text{accept } 0.0778 \text{ if 3sf approximation from (a) used}) \quad AI$$

[3 marks]

Examiners report

Part (a) was almost universally correctly answered, albeit with an accuracy penalty in some cases. In (b) it was generally recognised that the distribution was binomial, but with some wavering about the correct value of the parameter p . Part (c) was sometimes answered correctly, but not with much confidence.

18c. [2 marks]

Markscheme

$$P(x < 1 | x < 1.4) = \frac{P(x < 1)}{P(x < 1.4)} \quad \mathbf{M1}$$

$$= \frac{0.06680\dots}{0.6914\dots}$$

$$= 0.0966 \quad (\text{accept } 0.0967) \quad \mathbf{A1}$$

[2 marks]

Examiners report

Part (a) was almost universally correctly answered, albeit with an accuracy penalty in some cases. In (b) it was generally recognised that the distribution was binomial, but with some wavering about the correct value of the parameter p . Part (c) was sometimes answered correctly, but not with much confidence.

19a. [1 mark]

Markscheme

$$P(x = 0) = 0.607 \quad \mathbf{A1}$$

[1 mark]

Examiners report

Most candidates successfully answered (a) and (b). Although many found the correct answer to (c), communication of their reasoning was weak. This was also true for (d)(i). Answers to (d)(ii) were mostly scrappy and rarely worthy of credit.

19b. [2 marks]

Markscheme

EITHER

Using

$$X \sim \text{Po}(3) \quad (\mathbf{M1})$$

OR

Using

$$(0.6065\dots)^6 \quad (\mathbf{M1})$$

THEN

$$P(X = 0) = 0.0498 \quad \mathbf{A1}$$

[2 marks]

Examiners report

Most candidates successfully answered (a) and (b). Although many found the correct answer to (c), communication of their reasoning was weak. This was also true for (d)(i). Answers to (d)(ii) were mostly scrappy and rarely worthy of credit.

19c. [6 marks]

Markscheme

$$X \sim \text{Po}(0.5t) \quad (MI)$$

$$P(x \geq 1) = 1 - P(x = 0) \quad (MI)$$

$$P(x = 0) < 0.01 \quad AI$$

$$e^{-0.5t} < 0.01 \quad AI$$

$$-0.5t < \ln(0.01) \quad (MI)$$

$$t > 9.21 \text{ months}$$

therefore 10 months *AIN4*

Note: Full marks can be awarded for answers obtained directly from GDC if a systematic method is used and clearly shown.

[6 marks]

Examiners report

Most candidates successfully answered (a) and (b). Although many found the correct answer to (c), communication of their reasoning was weak. This was also true for (d)(i). Answers to (d)(ii) were mostly scrappy and rarely worthy of credit.

19d. [9 marks]

Markscheme

$$(i) \quad P(1 \text{ or } 2 \text{ accidents}) = 0.37908\dots \quad AI$$

$$E(B) = 1000 \times 0.60653\dots + 500 \times 0.37908\dots \quad MIAI$$

$$= \$796 \text{ (accept } \$797 \text{ or } \$796.07) \quad AI$$

$$(ii) \quad P(2000) = P(1000, 1000, 0) + P(1000, 0, 1000) + P(0, 1000, 1000) + \\ P(1000, 500, 500) + P(500, 1000, 500) + P(500, 500, 1000) \quad (MI)(AI)$$

Note: Award *MI* for noting that 2000 can be written both as

$$2 \times 1000 + 1 \times 0 \text{ and}$$

$$2 \times 500 + 1 \times 1000 .$$

$$= 3(0.6065\dots)^2(0.01437\dots) + 3(0.3790\dots)^2(0.6065\dots) \quad MIAI$$

$$= 0.277 \text{ (accept } 0.278) \quad AI$$

[9 marks]

Examiners report

Most candidates successfully answered (a) and (b). Although many found the correct answer to (c), communication of their reasoning was weak. This was also true for (d)(i). Answers to (d)(ii) were mostly scrappy and rarely worthy of credit.

20a. [2 marks]

Markscheme

$$z = \frac{200-205}{10} = -0.5 \quad (M1)$$

$$\text{probability} = 0.691 \text{ (accept } 0.692) \quad AI$$

Note: Award *MIA0* for 0.309 or 0.308

[2 marks]

Examiners report

As might be expected, (a) was well answered by many candidates, although those who gave 0.6915 straight from tables were given an arithmetic penalty. Parts (b) and (c), however, were not so well answered with errors in calculating the variances being the most common source of incorrect solutions. In particular, some candidates are still uncertain about the difference between nX and

$$\sum_{i=1}^n X_i .$$

20b. [4 marks]

Markscheme

let X be the total weight of the 5 oranges

then

$$E(X) = 5 \times 205 = 1025 \quad (A1)$$

$$\text{Var}(X) = 5 \times 100 = 500 \quad (M1)(A1)$$

$$P(X < 1000) = 0.132 \quad AI$$

[4 marks]

Examiners report

As might be expected, (a) was well answered by many candidates, although those who gave 0.6915 straight from tables were given an arithmetic penalty. Parts (b) and (c), however, were not so well answered with errors in calculating the variances being the most common source of incorrect solutions. In particular, some candidates are still uncertain about the difference between nX and

$$\sum_{i=1}^n X_i .$$

20c.

[5 marks]

Markscheme

let $Y = B - 3C$ where B is the weight of a random orange and C the weight of a random lemon (MI)

$$E(Y) = 205 - 3 \times 75 = -20 \quad (AI)$$

$$\text{Var}(Y) = 100 + 9 \times 9 = 181 \quad (MI)(AI)$$

$$P(Y > 0) = 0.0686 \quad AI$$

[5 marks]

Note: Award AI for 0.0681 obtained from tables

Examiners report

As might be expected, (a) was well answered by many candidates, although those who gave 0.6915 straight from tables were given an arithmetic penalty. Parts (b) and (c), however, were not so well answered with errors in calculating the variances being the most common source of incorrect solutions. In particular, some candidates are still uncertain about the difference between nX and

$$\sum_{i=1}^n X_i .$$

21.

[7 marks]

Markscheme

P(six in first throw)

$$= \frac{1}{6} \quad (AI)$$

P(six in third throw)

$$= \frac{25}{36} \times \frac{1}{6} \quad (MI)(AI)$$

P(six in fifth throw)

$$= \left(\frac{25}{36}\right)^2 \times \frac{1}{6}$$

P(A obtains first six)

$$= \frac{1}{6} + \frac{25}{36} \times \frac{1}{6} + \left(\frac{25}{36}\right)^2 \times \frac{1}{6} + \dots \quad (MI)$$

recognizing that the common ratio is

$$\frac{25}{36} \quad (AI)$$

P(A obtains first six)

$$= \frac{\frac{1}{6}}{1 - \frac{25}{36}} \quad (\text{by summing the infinite GP}) \quad MI$$

$$= \frac{6}{11} \quad AI$$

[7 marks]

Examiners report

This question proved difficult to the majority of the candidates although a few interesting approaches to this problem have been seen. Candidates who started the question by drawing a tree diagram were more successful, although a number of these failed to identify the geometric series.

Markscheme

EITHER

let

$$y_i = x_i - 12$$

$$\bar{x} = 10 \Rightarrow \bar{y} = -2 \quad \text{MIAI}$$

$$\sigma_x = \sigma_y = 3 \quad \text{AI}$$

$$\frac{\sum_{i=1}^{10} y_i^2}{10} - \bar{y}^2 = 9 \quad \text{MIAI}$$

$$\sum_{i=1}^{10} y_i^2 = 10(9 + 4) = 130 \quad \text{AI}$$

OR

$$\sum_{i=1}^{10} (x_i - 12)^2 = \sum_{i=1}^{10} x_i^2 - 24 \sum_{i=1}^{10} x_i + 144 \sum_{i=1}^{10} 1 \quad \text{MIAI}$$

$$\bar{x} = 10 \Rightarrow \sum_{i=1}^{10} x_i = 100 \quad \text{AI}$$

$$\sigma_x = 3, \frac{\sum_{i=1}^{10} x_i^2}{10} - \bar{x}^2 = 9 \quad \text{(MI)}$$

$$\Rightarrow \sum_{i=1}^{10} x_i^2 = 10(9 + 100) \quad \text{AI}$$

$$\sum_{i=1}^{10} (x_i - 12)^2 = 1090 - 2400 + 1440 = 130 \quad \text{AI}$$

[6 marks]

Examiners report

Very few candidates answered this question well, but among those a variety of nice approaches were seen. Most candidates though revealed an inability to deal with sigma expressions, especially

$\sum_{i=1}^{10} 144$. Some tried to use expectation algebra but could not then relate those results to sigma expressions (often the factor 10 was

forgotten). In a few cases candidates attempted to show the result using particular examples.

Markscheme

(a)

$$\int_0^1 ae^{-ax} dx = 1 - \frac{1}{\sqrt{2}} \quad \text{MIAI}$$

$$[-e^{-ax}]_0^1 = 1 - \frac{1}{\sqrt{2}} \quad \text{MIAI}$$

$$-e^{-a} + 1 = 1 - \frac{1}{\sqrt{2}} \quad \text{AI}$$

Note: Accept e^0 instead of 1.

$$e^{-a} = \frac{1}{\sqrt{2}}$$

$$e^a = \sqrt{2}$$

$$a = \ln 2^{\frac{1}{2}} \quad (\text{accept } -a = \ln 2^{-\frac{1}{2}}) \quad \text{AI}$$

$$a = \frac{1}{2} \ln 2 \quad \text{AG}$$

[6 marks]

(b)

$$\int_0^M ae^{-ax} dx = \frac{1}{2} \quad \text{MIAI}$$

$$[-e^{-ax}]_0^M = \frac{1}{2} \quad \text{AI}$$

$$-e^{-Ma} + 1 = \frac{1}{2}$$

$$e^{-Ma} = \frac{1}{2} \quad \text{AI}$$

$$Ma = \ln 2$$

$$M = \frac{\ln 2}{a} = 2 \quad \text{AI}$$

[5 marks]

(c)

$$P(1 < X < 3) = \int_1^3 ae^{-ax} dx \quad \text{MIAI}$$

$$= -e^{-3a} + e^{-a} \quad \text{AI}$$

$$P(X < 3 | X > 1) = \frac{P(1 < X < 3)}{P(X > 1)} \quad \text{MIAI}$$

$$= \frac{-e^{-3a} + e^{-a}}{1 - P(X < 1)} \quad \text{AI}$$

$$= \frac{-e^{-3a} + e^{-a}}{\frac{1}{\sqrt{2}}} \quad \text{AI}$$

$$= \sqrt{2}(-e^{-3a} + e^{-a})$$

$$= \sqrt{2} \left(-2^{-\frac{3}{2}} + 2^{-\frac{1}{2}} \right) \quad \text{AI}$$

$$= \frac{1}{2} \quad \text{AI}$$

Note: Award full marks for $P(X < 3 | X > 1) = P(X < 2) = \frac{1}{2}$ or quoting properties of exponential distribution.**[9 marks]****Total [20 marks]**

Examiners report

Many candidates did not attempt this question and many others were clearly not familiar with this topic. On the other hand, most of the candidates who were familiar with continuous random variables and knew how to start the questions were successful and scored well in parts (a) and (b). The most common errors were in the integral of e^{-at} , having the limits from $-\infty$ to 1, confusion over powers and signs ('-' sometimes just disappeared). Understanding of conditional probability was poor and marks were low in part (c). A small number of candidates from a small number of schools coped very competently with the algebra throughout the question.

24.

[5 marks]

Markscheme

(a) the total area under the graph of the pdf is unity (AI)

area

$$= c \int_0^1 x - x^2 dx$$

$$= c \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1 \quad AI$$

$$= \frac{c}{6}$$

$$\Rightarrow c = 6 \quad AI$$

(b)

$$E(X) = 6 \int_0^1 x^2 - x^3 dx \quad (M1)$$

$$= 6 \left(\frac{1}{3} - \frac{1}{4} \right) = \frac{1}{2} \quad AI$$

Note: Allow an answer obtained by a symmetry argument.

[5 marks]

Examiners report

Most candidates made a meaningful attempt at this question with many gaining the correct answers. One or two candidates did not attempt this question at all.

Markscheme

recognition of

$$X \sim B\left(6, \frac{4}{7}\right) \quad (M1)$$

$$P(X = 3) = \binom{6}{3} \left(\frac{4}{7}\right)^3 \left(\frac{3}{7}\right)^3 \left(= 20 \times \frac{4^3 \times 3^3}{7^6}\right) \quad A1$$

$$P(X = 2) = \binom{6}{3} \left(\frac{4}{7}\right)^2 \left(\frac{3}{7}\right)^4 \left(= 15 \times \frac{4^2 \times 3^4}{7^6}\right) \quad A1$$

$$\frac{P(X=3)}{P(X=2)} = \frac{80}{45} \left(= \frac{16}{9}\right) \quad A1$$

[4 marks]

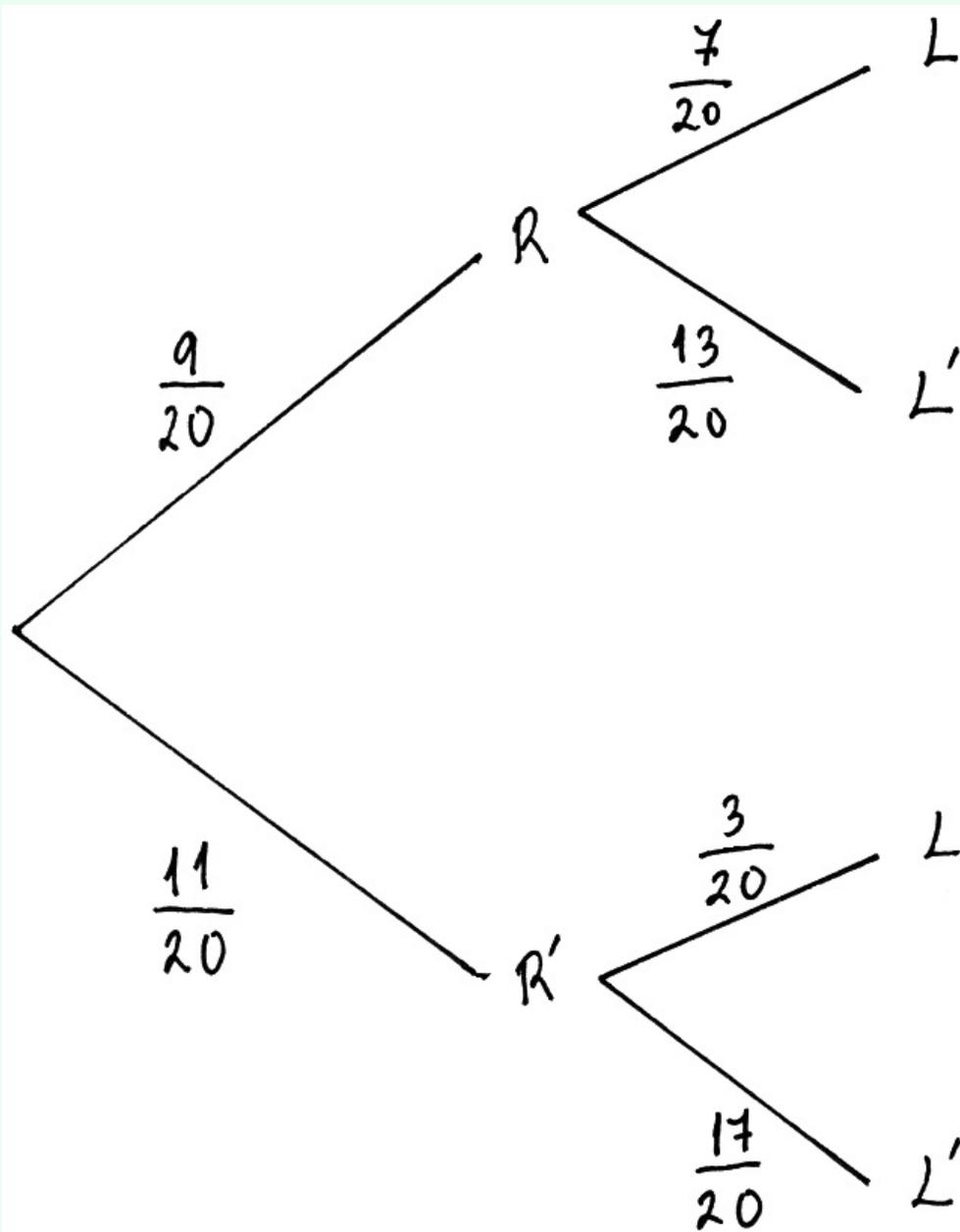
Examiners report

Many correct answers were seen to this and the majority of candidates recognised the need to use a Binomial distribution. A number of candidates, although finding the correct expressions for

$P(X = 3)$ and

$P(X = 4)$, were unable to perform the required simplification.

Markscheme



$$P(R' \cap L) = \frac{11}{20} \times \frac{3}{20} \quad A1$$

$$P(L) = \frac{9}{20} \times \frac{7}{20} + \frac{11}{20} \times \frac{3}{20} \quad A1$$

$$P(R'|L) = \frac{P(R' \cap L)}{P(L)} \quad (M1)$$

$$= \frac{33}{96} \left(= \frac{11}{32} \right) \quad A1$$

[5 marks]

Examiners report

This question was generally well answered with candidates who drew a tree diagram being the most successful.

27.

[5 marks]

Markscheme

$$X \sim N(\mu, \sigma^2)$$

$$P(X \leq 5) = 0.670 \Leftrightarrow \frac{5-\mu}{\sigma} = 0.4399\dots \quad \text{MIAI}$$

$$P(X > 7) = 0.124 \Leftrightarrow \frac{7-\mu}{\sigma} = 1.155\dots \quad \text{AI}$$

solve simultaneously

$$\mu + 0.4399\sigma = 5 \text{ and } \mu + 1.1552\sigma = 7 \quad \text{MI}$$

$$\mu = 3.77 \text{ (3 sf)} \quad \text{AI} \quad \text{N3}$$

the expected weight loss is 3.77 kg

Note: Award **A0** for

$\mu = 3.78$ (answer obtained due to early rounding).

[5 marks]

Examiners report

Although many candidates were successful in answering this question, a surprising number of candidates did not even attempt it. The main difficulty was in finding the correct z scores. A fairly common error was to misinterpret one of the conditions and obtain one of the equations as

$\frac{7-\mu}{\sigma} = -1.155\dots$. In some cases candidates failed to keep the accuracy throughout the question and obtained inaccurate answers.

28.

[6 marks]

Markscheme

(a)

$$P(X = 1) + P(X = 3) = P(X = 0) + P(X = 2)$$

$$me^{-m} + \frac{m^3e^{-m}}{6} = e^{-m} + \frac{m^2e^{-m}}{2} \quad \text{(MI)(AI)}$$

$$m^3 - 3m^2 + 6m - 6 = 0 \quad \text{(MI)}$$

$$m = 1.5961 \text{ (4 decimal places)} \quad \text{AI}$$

(b)

$$m = 1.5961 \Rightarrow P(1 \leq X \leq 2) = me^{-m} + \frac{m^2e^{-m}}{2} = 0.582 \quad \text{(MI)AI}$$

[6 marks]

Examiners report

Most candidates correctly stated the required equation for m . However, many algebraic errors in the simplification of this equation led to incorrect answers. Also, many candidates failed to find the value of m to the required accuracy, with many candidates giving answers correct to 4 sf instead of 4 dp. In part (b) many candidates did not realize that they needed to calculate $P(X = 1) + P(X = 2)$ and many attempts to calculate other combinations of probabilities were seen.

Markscheme

+	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

(a) let T be Tim's score

(i)

$$P(T = 6) = \frac{11}{9} (= 0.111 \text{ 3 sf}) \quad \mathbf{AI}$$

(ii)

$$P(T \geq 3) = 1 - P(T \leq 2) = 1 - \frac{1}{9} = \frac{8}{9} (= 0.889 \text{ 3 sf}) \quad \mathbf{(MI)AI}$$

[3 marks]

(b) let B be Bill's score

(i)

$$P(T = 6 \text{ and } B = 6) = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81} (= 0.012 \text{ 3 sf}) \quad \mathbf{(MI)AI}$$

(ii)

$$P(B = T) = P(2)P(2) + P(3)P(3) + \dots + P(6)P(6)$$

$$= \frac{1}{9} \times \frac{1}{9} + \frac{2}{9} \times \frac{2}{9} + \frac{3}{9} \times \frac{3}{9} + \frac{2}{9} \times \frac{2}{9} + \frac{1}{9} \times \frac{1}{9} \quad \mathbf{MI}$$

$$= \frac{19}{81} (= 0.235 \text{ 3 sf}) \quad \mathbf{AI}$$

[4 marks]

(c) (i) **EITHER**

$$P(X \leq 2) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \quad \mathbf{MIAI}$$

because

$$P(X \leq 2) = P((a, b, c, d) | a, b, c, d = 1, 2) \quad \mathbf{RI}$$

or equivalent

$$P(X \leq 2) = \frac{16}{81} \quad \mathbf{AG}$$

OR

there are sixteen possible permutations which are

Combinations	Number
1111	1
1112	4
1122	6
1222	4
2222	1

\mathbf{MIAI}

Note: This information may be presented in a variety of forms.

$$P(X \leq 2) = \frac{1+4+6+4+1}{81} \quad \mathbf{AI}$$

$$= \frac{16}{81} \quad \mathbf{AG}$$

(ii)

x	1	2	3	<i>AIAI</i>
$P(X = x)$	$\frac{1}{81}$	$\frac{15}{81}$	$\frac{65}{81}$	

(iii)

$$E(X) = \sum_{x=1}^3 xP(X = x) \quad (M1)$$

$$= \frac{1}{81} + \frac{30}{81} + \frac{195}{81}$$

$$= \frac{226}{81} \quad (2.79 \text{ to } 3 \text{ sf}) \quad AI$$

$$E(X^2) = \sum_{x=1}^3 x^2P(X = x)$$

$$= \frac{1}{81} + \frac{60}{81} + \frac{585}{81}$$

$$= \frac{646}{81} \quad (7.98 \text{ to } 3 \text{ sf}) \quad AI$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 \quad (M1)$$

$$= 0.191 \quad (3 \text{ sf}) \quad AI$$

Note: Award *MIA0* for answers obtained using rounded values

(e.g. $\text{Var}(X) = 0.196$).

[10 marks]

(d)

Combinations	Number
3311	6
3221	12

$$P(\text{total is } 8 \cap (X = 3)) = \frac{18}{81} \quad MIAI$$

since

$$P(X = 3) = \frac{65}{81}$$

$$P(\text{total is } 8 | (X = 3)) = \frac{P(\text{total is } 8 \cap (X = 3))}{P(X = 3)} \quad M1$$

$$= \frac{18}{65} (= 0.277) \quad AI$$

[4 marks]

Total [21 marks]

Examiners report

Most candidates with a reasonable understanding of probability managed to answer well parts (a), (b) and some of part (c). However some candidates did not realize that different scores were not equally likely which lead to incorrect answers in several parts.

Surprisingly, many candidates completed the table in part c) ii) with values that did not add up to 1. Very few candidates answered part (d) well. The enumeration of possible cases was sometimes attempted but with little success.

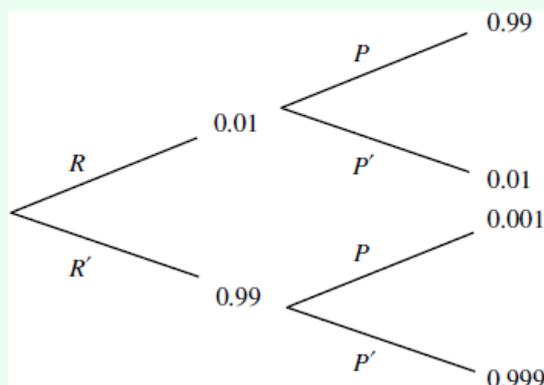
30a.

[2 marks]

Markscheme

R is 'rabbit with the disease'

P is 'rabbit testing positive for the disease'



$$P(P) = P(R \cap P) + P(R' \cap P)$$

$$= 0.01 \times 0.99 + 0.99 \times 0.001 \quad \text{MI}$$

$$= 0.01089 (= 0.0109) \quad \text{AI}$$

Note: Award **MI** for a correct tree diagram with correct probability values shown.

[2 marks]

Examiners report

There was a mixed performance in this question with some candidates showing good understanding of probability and scoring well and many others showing no understanding of conditional probability and difficulties in working with decimals. Very few candidates were able to provide a valid argument to justify their answer to part (b).

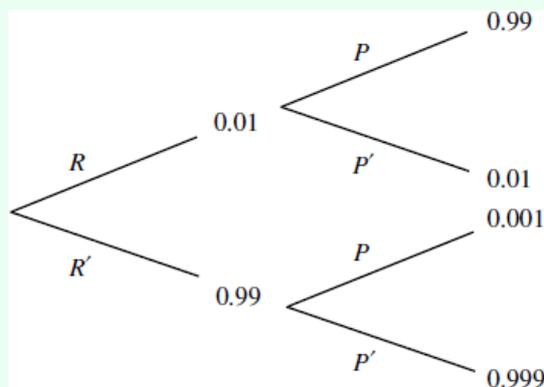
30b.

[3 marks]

Markscheme

R is 'rabbit with the disease'

P is 'rabbit testing positive for the disease'



$$P(R'|P) = \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.01 \times 0.99} (= \frac{0.00099}{0.01089}) \quad \text{MIAI}$$

$$\frac{0.00099}{0.01089} < \frac{0.001}{0.01} = 10\% \text{ (or other valid argument)} \quad \text{RI}$$

[3 marks]

Examiners report

There was a mixed performance in this question with some candidates showing good understanding of probability and scoring well and many others showing no understanding of conditional probability and difficulties in working with decimals. Very few candidates were able to provide a valid argument to justify their answer to part (b).

31a. [2 marks]

Markscheme

$$k \int_{-2}^0 (x+2)^2 dx + \int_0^{\frac{4}{3}} k dx = 1 \quad \text{MI}$$

$$\frac{8k}{3} + \frac{4k}{3} = 1$$

$$k = \frac{1}{4} \quad \text{AI}$$

Note: Only ft on positive values of k .

[2 marks]

Examiners report

Many candidates recognised that integration was the appropriate technique to solve this question but the fact that the function was piecewise proved problematic for many. Good use of technology by some candidates was seen but few sketches of the function were made. A sketch would have been helpful to many candidates when attempting to solve (b (ii)).

31b. [5 marks]

Markscheme

(i)

$$E(X) = \frac{1}{4} \int_{-2}^0 x(x+2)^2 dx + \frac{1}{4} \int_0^{\frac{4}{3}} x dx \quad \text{MI}$$

$$= \frac{1}{4} \times \frac{-4}{3} + \frac{2}{9}$$

$$= -\frac{1}{9} \quad (-0.111) \quad \text{AI}$$

(ii) median given by a such that

$$P(X < a) = 0.5$$

$$\frac{1}{4} \int_{-2}^a (x+2)^2 dx = 0.5 \quad \text{MI}$$

$$\left[\frac{(x+2)^3}{3} \right]_{-2}^a = 2 \quad (\text{AI})$$

$$(a+2)^3 - 0 = 6$$

$$a = \sqrt[3]{6} - 2 \quad (-0.183) \quad \text{AI}$$

[5 marks]

Examiners report

Many candidates recognised that integration was the appropriate technique to solve this question but the fact that the function was piecewise proved problematic for many. Good use of technology by some candidates was seen but few sketches of the function were made. A sketch would have been helpful to many candidates when attempting to solve (b (ii)).

32a.

[5 marks]

Markscheme

$$P(X < 30) = 0.4$$

$$P(X < 55) = 0.9$$

or relevant sketch (MI)

given

$$Z = \frac{X - \mu}{\sigma}$$

$$P(Z < z) = 0.4 \Rightarrow \frac{30 - \mu}{\sigma} = -0.253... \quad (AI)$$

$$P(Z < z) = 0.9 \Rightarrow \frac{55 - \mu}{\sigma} = 1.28... \quad (AI)$$

$$\mu = 30 + (0.253...) \times \sigma = 55 - (1.28...) \times \sigma \quad MI$$

$$\sigma = 16.3,$$

$$\mu = 34.1 \quad AI$$

Note: Accept 16 and 34.

Note: Working with 830 and 855 will only gain the two *M* marks.

[5 marks]

Examiners report

Candidates who had been prepared to solve questions from this part of the syllabus did well on the question. As a general point, candidates did not always write down clearly which distribution was being used. There were many candidates who seemed unfamiliar with the concept of Normal distributions as well as the Poisson and Binomial distributions and did not attempt the question. Parts (a) – (c) of the question were a variation on similar problems seen on previous examinations and there were a disappointing number of candidates who seemed unable to start the question. The use of 830 and 850 rather than minutes after 8am was seen and this caused students to lose marks despite knowing the method required. In general technology was used well and this was seen in (d) when solving a problem that involved a Poisson distribution. A number of candidates were unable to identify the Binomial distribution in (e).

32b.

[3 marks]

Markscheme

$$X \sim N(34.12..., 16.28...^2)$$

late to school
 $\Rightarrow X > 60$

$$P(X > 60) = 0.056... \quad (AI)$$

number of students late

$$= 0.0560... \times 1200 \quad (MI)$$

$$= 67 \text{ (to nearest integer) } AI$$

Note: Accept 62 for use of 34 and 16.

[3 marks]

Examiners report

Candidates who had been prepared to solve questions from this part of the syllabus did well on the question. As a general point, candidates did not always write down clearly which distribution was being used. There were many candidates who seemed unfamiliar with the concept of Normal distributions as well as the Poisson and Binomial distributions and did not attempt the question. Parts (a) – (c) of the question were a variation on similar problems seen on previous examinations and there were a disappointing number of candidates who seemed unable to start the question. The use of 830 and 850 rather than minutes after 8am was seen and this caused students to lose marks despite knowing the method required. In general technology was used well and this was seen in (d) when solving a problem that involved a Poisson distribution. A number of candidates were unable to identify the Binomial distribution in (e).

32c. [2 marks]

Markscheme

$$P(X > 60 | X > 30) = \frac{P(X > 60)}{P(X > 30)} \quad \mathbf{M1}$$

$$= 0.0935 \text{ (accept anything between } 0.093 \text{ and } 0.094) \quad \mathbf{A1}$$

Note: If 34 and 16 are used 0.0870 is obtained. This should be accepted.

[2 marks]

Examiners report

Candidates who had been prepared to solve questions from this part of the syllabus did well on the question. As a general point, candidates did not always write down clearly which distribution was being used. There were many candidates who seemed unfamiliar with the concept of Normal distributions as well as the Poisson and Binomial distributions and did not attempt the question. Parts (a) – (c) of the question were a variation on similar problems seen on previous examinations and there were a disappointing number of candidates who seemed unable to start the question. The use of 830 and 850 rather than minutes after 8am was seen and this caused students to lose marks despite knowing the method required. In general technology was used well and this was seen in (d) when solving a problem that involved a Poisson distribution. A number of candidates were unable to identify the Binomial distribution in (e).

32d. [3 marks]

Markscheme

let

L be the random variable of the number of students who leave school in a 30 minute interval

since

$$24 \times 30 = 720 \quad \mathbf{A1}$$

$$L \sim \text{Po}(720)$$

$$P(L \geq 700) = 1 - P(L \leq 699) \quad (\mathbf{M1})$$

$$= 0.777 \quad \mathbf{A1}$$

Note: Award $MIA0$ for

$P(L > 700) = 1 - P(L \leq 700)$ (this leads to 0.765).

[3 marks]

Examiners report

Candidates who had been prepared to solve questions from this part of the syllabus did well on the question. As a general point, candidates did not always write down clearly which distribution was being used. There were many candidates who seemed unfamiliar with the concept of Normal distributions as well as the Poisson and Binomial distributions and did not attempt the question. Parts (a) – (c) of the question were a variation on similar problems seen on previous examinations and there were a disappointing number of candidates who seemed unable to start the question. The use of 830 and 850 rather than minutes after 8am was seen and this caused students to lose marks despite knowing the method required. In general technology was used well and this was seen in (d) when solving a problem that involved a Poisson distribution. A number of candidates were unable to identify the Binomial distribution in (e).

32e.

[4 marks]

Markscheme

(i)

$Y \sim B($

200,

0.7767...) (MI)

$$E(Y) = 200 \times 0.7767... = 155 \quad AI$$

Note: On ft, use of 0.765 will lead to 153.

(ii)

$$P(Y > 150) = 1 - P(Y \leq 150) \quad (MI)$$

$$= 0.797 \quad AI$$

Note: Accept 0.799 from using rounded answer.

Note: On ft, use of 0.765 will lead to 0.666.

[4 marks]

Examiners report

Candidates who had been prepared to solve questions from this part of the syllabus did well on the question. As a general point, candidates did not always write down clearly which distribution was being used. There were many candidates who seemed unfamiliar with the concept of Normal distributions as well as the Poisson and Binomial distributions and did not attempt the question. Parts (a) – (c) of the question were a variation on similar problems seen on previous examinations and there were a disappointing number of candidates who seemed unable to start the question. The use of 830 and 850 rather than minutes after 8am was seen and this caused students to lose marks despite knowing the method required. In general technology was used well and this was seen in (d) when solving a problem that involved a Poisson distribution. A number of candidates were unable to identify the Binomial distribution in (e).

Markscheme

(a) Any consideration of

$$\int_0^1 f(x) dx \quad (M1)$$

$$0 \quad A1 \quad N2$$

(b) **METHOD 1**

Let the upper and lower quartiles be a and $-a$

$$\frac{\pi}{4} \int_a^1 \cos \frac{\pi t}{2} dt = 0.25 \quad M1$$

$$\Rightarrow \left[\frac{\pi}{4} \times \frac{2}{\pi} \sin \frac{\pi t}{2} \right]_a^1 = 0.25 \quad A1$$

$$\Rightarrow \left[\frac{1}{2} \sin \frac{\pi t}{2} \right]_a^1 = 0.25$$

$$\Rightarrow \left[\frac{1}{2} - \frac{1}{2} \sin \frac{\pi a}{2} \right] = 0.25 \quad A1$$

$$\Rightarrow \frac{1}{2} \sin \frac{\pi a}{2} = \frac{1}{4}$$

$$\Rightarrow \sin \frac{\pi a}{2} = \frac{1}{2}$$

$$\frac{\pi a}{2} = \frac{\pi}{6}$$

$$a = \frac{1}{3} \quad A1$$

Since the function is symmetrical about $t = 0$,

interquartile range is

$$\frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{2}{3} \quad R1$$

METHOD 2

$$\frac{\pi}{4} \int_{-a}^a \cos \frac{\pi t}{2} dt = 0.5 = \frac{\pi}{2} \int_0^a \cos \frac{\pi t}{2} dt \quad M1A1$$

$$\Rightarrow \left[\sin \frac{a\pi}{2} \right] = 0.5 \quad A1$$

$$\Rightarrow \frac{a\pi}{2} = \frac{\pi}{6}$$

$$\Rightarrow a = \frac{1}{3} \quad A1$$

The interquartile range is

$$\frac{2}{3} \quad R1$$

[7 marks]

Examiners report

All but the best candidates struggled with part (a). The vast majority either did not attempt it or let $t = 1$. There was no indication from any of the scripts that candidates wasted an undue amount of time in trying to solve part (a). Many candidates attempted part (b), but few had a full understanding of the situation and hence were unable to give wholly correct answers.

Markscheme

(a)

$$P(X \leq 84) = P(Z \leq -1.62\dots) = 0.0524 \quad (M1)A1 \quad N2$$

Note: Accept 0.0526.

(b)

$$P(Z \leq z) = 0.01 \Rightarrow z = -2.326\dots \quad (M1)$$

$$P(X \leq x) = P(Z \leq z) = 0.01 \Rightarrow z = -2.326\dots$$

$$x = 81.4 \quad (\text{accept } 81) \quad A1 \quad N2$$

(c)

$$P(X \leq 84) = 0.12 \Rightarrow z = -1.1749\dots \quad (M1)$$

mean is 88.3 (accept 88) $A1 \quad N2$

[6 marks]

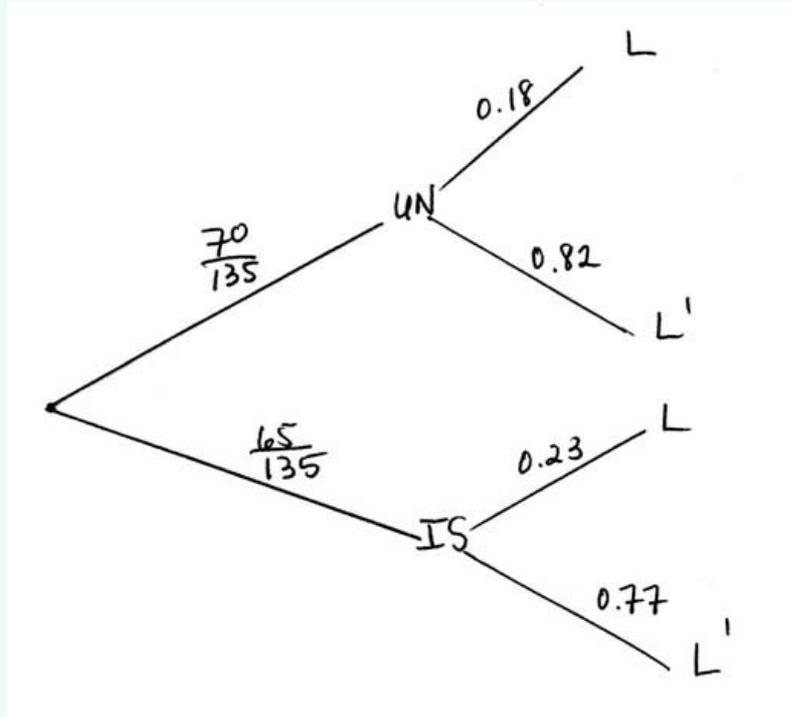
Examiners report

A fair amount of students did not use their GDC directly, but used tables and more traditional methods to answer this question. Part

(a) was answered correctly by most candidates using any method. A large number of candidates reversed the probabilities, i.e., failed to use a negative z value in parts (b) and (c), and hence did not obtain correct answers.

Markscheme

METHOD 1



(M1)

Let $P(I)$ be the probability of flying IS Air, $P(U)$ be the probability flying UN Air and $P(L)$ be the probability of luggage lost.

$$P(I|L) = \frac{P(I \cap L)}{P(L)} \quad \left(\text{or Bayes' formula, } P(I|L) = \frac{P(L|I)P(I)}{P(L|I)P(I) + P(L|U)P(U)} \right) \quad (M1)$$

$$= \frac{0.23 \times \frac{65}{135}}{0.18 \times \frac{70}{135} + 0.23 \times \frac{65}{135}} \quad A1A1A1$$

$$= \frac{299}{551} (= 0.543, \text{ accept } 0.542) \quad A1$$

[6 marks]

METHOD 2

Expected number of suitcases lost by UN Air is

$$0.18 \times 70 = 12.6 \quad M1A1$$

Expected number of suitcases lost by IS Air is

$$0.23 \times 65 = 14.95 \quad A1$$

$$P(I|L) = \frac{14.95}{12.6 + 14.95} \quad M1A1$$

$$= 0.543 \quad A1$$

[6 marks]

Examiners report

This question was well answered by the majority of candidates. Most candidates used either tree diagrams or expected value methods.

Markscheme

(a) mean for 30 days:

$$30 \times 0.2 = 6 \quad (A1)$$

$$P(X = 4) = \frac{6^4}{4!} e^{-6} = 0.134 \quad (M1)A1 \quad N3$$

[3 marks]

(b)

$$P(X > 3) = 1 - P(X \leq 3) = 1 - e^{-6}(1 + 6 + 18 + 36) = 0.849 \quad (M1)A1 \quad N2$$

[2 marks]

(c) **EITHER**

mean for five days:

$$5 \times 0.2 = 1 \quad (A1)$$

$$P(X = 0) = e^{-1} (= 0.368) \quad A1 \quad N2$$

OR

mean for one day: 0.2 (A1)

$$P(X = 0) = (e^{-0.2})^5 = e^{-1} (= 0.368) \quad A1 \quad N2$$

[2 marks]

(d) Required probability

$$= e^{-0.2} \times e^{-0.2} \times (1 - e^{-0.2}) \quad M1A1$$

$$= 0.122 \quad A1 \quad N3$$

[3 marks]

(e) Expected cost is

$$1850 \times 6 = 11\,100 \text{ Euros} \quad A1$$

[1 mark]

(f) On any one day

$$P(X = 0) = e^{-0.2}$$

Therefore,

$$\binom{5}{1} (e^{-0.2})^4 (1 - e^{-0.2}) = 0.407 \quad M1A1 \quad N2$$

[2 marks]

Total [13 marks]

Examiners report

Many candidates showed familiarity with the Poisson Distribution. Parts (a), (b), and (c) were straightforward, as long as candidates multiplied 0.2 by 30 to get the mean. Part (e) was answered successfully by most candidates. Parts (d) and (f) were done very poorly.

In part (d), most candidates calculated

$$P(X = 1) \text{ rather than}$$

$P(X \leq 1)$. Although some candidates realized the need for the Binomial in part (e), some incorrectly used 0.8 and 0.2.

Markscheme

(a) Using

$$\sum P(X = x) = 1 \quad (M1)$$

$$4c + 6c + 6c + 4c = 1 \quad (20c = 1) \quad AI$$

$$c = \frac{1}{20} \quad (= 0.05) \quad AI \quad NI$$

(b) Using

$$E(X) = \sum xP(X = x) \quad (M1)$$

$$= (1 \times 0.2) + (2 \times 0.3) + (3 \times 0.3) + (4 \times 0.2) \quad (A1)$$

$$= 2.5 \quad AI \quad NI$$

Notes: Only one of the first two marks can be implied.

Award *MIAIAI* if the x values are averaged only if symmetry is explicitly mentioned.

[6 marks]

Examiners report

This question was generally well done, but a few candidates tried integration for part (b).

Markscheme

EITHER

Using

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (M1)$$

$$0.6P(B) = P(A \cap B) \quad A1$$

Using

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to obtain

$$0.8 = 0.6 + P(B) - P(A \cap B) \quad A1$$

Substituting

$$0.6P(B) = P(A \cap B) \text{ into above equation} \quad M1$$

OR

As

$P(A|B) = P(A)$ then A and B are independent events $MIR1$

Using

$$P(A \cup B) = P(A) + P(B) - P(A) \times P(B) \quad A1$$

to obtain

$$0.8 = 0.6 + P(B) - 0.6 \times P(B) \quad A1$$

THEN

$$0.8 = 0.6 + 0.4P(B) \quad A1$$

$$P(B) = 0.5 \quad A1 \quad NI$$

[6 marks]

Examiners report

This question was generally well done, with a few candidates spotting an opportunity to use results for the *independent events* A and B .

39.

[5 marks]

Markscheme

(a) Use of

$$\bar{x} = \frac{\sum_{i=1}^4 x_i}{n} \quad (M1)$$

$$\bar{x} = \frac{(k-2)+k+(k+1)+(k+4)}{4} \quad (A1)$$

$$\bar{x} = \frac{4k+3}{4} \quad (= k + \frac{3}{4}) \quad A1 \quad N3$$

(b) Either attempting to find the new mean or subtracting 3 from **their** \bar{x} (M1)

$$\bar{x} = \frac{4k+3}{4} - 3 \quad (= \frac{4k-9}{4}, k - \frac{9}{4}) \quad A1 \quad N2$$

[5 marks]

Examiners report

This was an easy question that was well done by most candidates. Careless arithmetic errors caused some candidates not to earn full marks. Only a few candidates realised that part (b) could be answered correctly by directly subtracting 3 from their answer to part (a). Most successful responses were obtained by redoing the calculation from part (a).

40.

[6 marks]

Markscheme

Attempting to find the mode graphically or by using

$$f'(x) = 12x(2 - 3x) \quad (M1)$$

$$\text{Mode} = \frac{2}{3} \quad A1$$

Use of

$$E(X) = \int_0^1 xf(x)dx \quad (M1)$$

$$E(X) = \frac{3}{5} \quad A1$$

$$\int_{\frac{2}{3}}^{\frac{3}{5}} f(x)dx = 0.117 \quad (= \frac{1981}{16875}) \quad M1A1 \quad N4$$

[6 marks]

Examiners report

A significant number of candidates attempted to find the mode and the mean using calculus when it could be argued that these quantities could be found more efficiently with a GDC.

A significant proportion of candidates demonstrated a lack of understanding of the definitions governing the mean, mode and median of a continuous probability density function. A significant number of candidates attempted to calculate the median instead of either the mean or the mode. A number of candidates prematurely rounded their value for the mode i.e. subsequently using 0.7 for example rather than using the exact value of

$\frac{2}{3}$. A few candidates offered negative probability values or probabilities greater than one.

41.

[7 marks]

Markscheme

(a) 1, 2, 3, 4 *AI*

(b)

$$P(Y = 1) = \frac{2}{5} \quad \text{AI}$$

$$P(Y = 2) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10} \quad \text{AI}$$

$$P(Y = 3) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5} \quad \text{AI}$$

$$P(Y = 4) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{1}{10} \quad \text{AI}$$

(c)

$$E(Y) = 1 \times \frac{2}{5} + 2 \times \frac{3}{10} + 3 \times \frac{1}{5} + 4 \times \frac{1}{10} \quad \text{MI}$$

$$= 2 \quad \text{AI}$$

[7 marks]

Examiners report

Candidates found this question challenging with only better candidates gaining the correct answers. A number of students assumed incorrectly that the distribution was either Binomial or Geometric.

42.

[7 marks]

Markscheme

(a)

$$\int_{1.5}^{2.5} \frac{(x+1)^3}{60} dx = 0.4625 \quad (= 0.463) \quad \text{MIAI}$$

(b)

$$E(X) = \int_1^3 \frac{x(x+1)^3}{60} dx = 2.31 \quad \text{MIAI}$$

(c)

$$\int_1^m \frac{(x+1)^3}{60} dx = 0.5 \quad \text{MI}$$

$$\left[\frac{(x+1)^4}{240} \right]_1^m = 0.5 \quad (\text{AI})$$

$$m = 2.41 \quad \text{AI}$$

[7 marks]

Examiners report

Parts (a) and (b) were reasonably well done in general but (c) caused problems for many candidates where several misconceptions regarding the median were seen. The expectation was that candidates would use their GDCs to solve (a) and (b), and possibly even (c), although in the event most candidates did the integrations by hand. Those candidates using their GDCs made fewer mistakes in general than those doing the integrations analytically.

Markscheme

(a)

$$X \sim \text{Po}(3.2)$$

$$P(X = 4) = \frac{e^{-3.2} 3.2^4}{4!}$$

$$= 0.178 \quad \text{AI}$$

(b) (i)

$$\text{Var}(Y) = E(Y^2) - E^2(Y) \quad (M1)$$

$$m = 5.5 - m^2 \quad \text{AI}$$

$$m = 1.90 \text{ (or } m = -2.90 \text{ which is valid)} \quad \text{AI}$$

(ii)

$$Y \sim \text{Po}(1.90)$$

$$P(Y = 3) = \frac{e^{-1.90} 1.90^4}{3!} \quad (M1)$$

$$= 0.171 \quad \text{AI}$$

(c) Required probability

$$= 0.171 \times 0.178 = 0.0304 \text{ (accept 0.0305)} \quad (M1)AI$$

[8 marks]

Examiners report

Part (a) was correctly solved by most candidates, either using the formula or directly from their GDC. Solutions to (b), however, were extremely disappointing with the majority of candidates giving

$\sqrt{5}$, incorrectly, as their value of m . It was possible to apply follow through in (b) (ii) and (c) which were well done in general.

44a.

[11 marks]

Markscheme

(a)

$$X \sim N(231, 1.5^2)$$

$$P(X < 228) = 0.0228 \quad (M1)A1$$

Note: Accept 0.0227.

[2 marks]

(b) (i)

$$X \sim N(\mu, 1.5^2)$$

$$P(X < 228) = 0.002$$

$$\frac{228 - \mu}{1.5} = -2.878\dots \quad M1A1$$

$$\mu = 232 \text{ grams} \quad A1 \quad N3$$

(ii)

$$X \sim N(231, \sigma^2)$$

$$\frac{228 - 231}{\sigma} = -2.878\dots \quad M1A1$$

$$\sigma = 1.04 \text{ grams} \quad A1 \quad N3$$

[6 marks]

(c)

$$X \sim B(100, 0.002) \quad (M1)$$

$$P(X \leq 1) = 0.982\dots \quad (A1)$$

$$P(X \geq 2) = 1 - P(X \leq 1) = 0.0174 \quad A1$$

[3 marks]

Total [11 marks]

Examiners report

Part A was well done by many candidates although an arithmetic penalty was often awarded in (b)(i) for giving the new value of the mean to too many significant figures.

Markscheme

(a) Boys can be chosen in

$$\frac{6 \times 5}{2} = 15 \text{ ways } \quad (AI)$$

Girls can be chosen in

$$\frac{5 \times 4}{2} = 10 \text{ ways } \quad (AI)$$

Total

$$= 15 \times 10 = 150 \text{ ways } \quad AI$$

[3 marks]

(b) Number of ways

$$= 5 \times 4 = 20 \quad (M1)AI$$

[2 marks]

(c)

$$\frac{20}{150} \left(= \frac{2}{15} \right) \quad AI$$

[1 mark]

(d) **METHOD 1**

$$P(T) = \frac{1}{5}; P(A) = \frac{2}{5} \quad AI$$

$P(T \text{ or } A \text{ but not both})$

$$= P(T) \times P(A') + P(T') \times P(A) \quad MIAI$$

$$= \frac{1}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{2}{5} = \frac{11}{25} \quad AI$$

METHOD 2

Number of selections including Fred

$$= 5 \times \binom{5}{2} = 50 \quad AI$$

Number of selections including Tim but not Anna

$$= \binom{4}{2} = 6 \quad AI$$

Number of selections including Anna but not Tim

$$= 4 \times 4 = 16$$

Note: Both statements are needed to award *AI*.

$P(T \text{ or } A \text{ but not both})$

$$= \frac{6+16}{50} = \frac{11}{25} \quad MIAI$$

[4 marks]

Total [10 marks]

Examiners report

Candidates are known, however, to be generally uncomfortable with combinatorial mathematics and Part B caused problems for many candidates. Even some of those candidates who solved (a) and (b) correctly were then unable to deduce the answer to (c), sometimes going off on some long-winded solution which invariably gave the wrong answer. Very few correct solutions were seen to (d).

45.

[6 marks]

Markscheme

(a)

$$\int_0^2 kx(2-x)dx = 1 \quad \text{MIAI}$$

Note: Award *MI* for LHS and *AI* for setting = 1 at any stage.

$$\left[\frac{2k}{2}x^2 - \frac{k}{3}x^3 \right]_0^2 = 1 \quad \text{AI}$$

$$k\left(4 - \frac{8}{3}\right) = 1 \quad \text{AI}$$

$$k = \frac{3}{4} \quad \text{AG}$$

(b)

$$E(X) = \frac{3}{4} \int_0^2 x^2(2-x)dx \quad (\text{MI})$$

$$= 1 \quad \text{AI}$$

Note: Accept answers that indicate use of symmetry.

[6 marks]

Examiners report

The integration was particularly well done in this question. A number of students treated the distribution as discrete. On the whole a) was done well once the distribution was recognized although there was a certain amount of fudging to achieve the result. A significant number of students did not initially set the integral equal to 1. Very few noted the symmetry of the distribution in b).

46.

[5 marks]

Markscheme

$$P(M|G) = \frac{P(M \cap G)}{P(G)} \quad (\text{MI})$$

$$= \frac{0.2 \times 0.9}{0.2 \times 0.9 + 0.8 \times 0.7} \quad \text{MIAIAI}$$

$$= \frac{0.18}{0.74}$$

$$= \frac{9}{37} \quad \text{AI}$$

[5 marks]

Examiners report

Most candidates answered this question successfully. Some made arithmetic errors.

Markscheme

(a) (i)

$$\frac{P(X=x)}{P(X=x-1)} = \frac{\left(\frac{n!}{(n-x)!x!} \times p^x \times (1-p)^{n-x}\right)}{\left(\frac{n!}{(n-x+1)!(x-1)!} \times p^{x-1} \times (1-p)^{n-x+1}\right)} \quad \text{MIAI}$$

$$= \frac{(n-x+1)p}{x(1-p)} \quad \text{AG}$$

(ii) if

$P(X=x) > P(X=x-1)$ then

$$(n-x+1)p > x(1-p) \quad \text{(MIAI)}$$

$$np - xp + p > x - px \quad \text{AI}$$

$$x < (n+1)p \quad \text{AG}$$

(iii) to maximise the probability we also need

$$P(X=x) > P(X=x+1) \quad \text{(MI)}$$

$$\frac{(n-(x+1)+1)p}{(x+1)(1-p)} < 1$$

$$np - xp < x - xp + 1 - p$$

$$p(n+1) < x+1 \quad \text{AI}$$

hence

$$p(n+1) > x > p(n+1) - 1 \quad \text{(AI)}$$

so x is the integer part of

$(n+1)p$ i.e. the largest integer less than

$$(n+1)p \quad \text{AI}$$

[9 marks]

(b) the mode is the value which maximises the probability (RI)

$$20p > 13 > 20p - 1 \quad \text{MI}$$

$$\Rightarrow p > \frac{13}{20} = 0.65, \text{ and}$$

$$p < \frac{7}{10} = 0.70 \quad \text{AIAI}$$

(it follows that

$$0.65 < p < 0.7)$$

[4 marks]

Total [13 marks]

Examiners report

Many candidates made a reasonable attempt at (a)(i) and (ii) but few were able to show that the mode is the integer part of $(n+1)p$. Part (b) also proved difficult for most candidates with few correct solutions seen.

48.

[3 marks]

Markscheme

$$\begin{aligned} P(a \leq X \leq b) &= \int_a^b \frac{x}{18} dx \quad \text{MIAI} \\ &= \left[\frac{x^2}{36} \right]_a^b \quad \text{AI} \\ &= \frac{b^2 - a^2}{36} \quad \text{AG} \end{aligned}$$

[3 marks]

Examiners report

This was the best answered question on the paper, helped probably by the fact that rounding errors in finding the expected frequencies were not an issue. In (a), some candidates thought, incorrectly, that all they had to do was to show that

$$\int_0^6 f(x) dx = 1.$$

49.

[6 marks]

Markscheme

weight of glass = X

$$X \sim N(160, \sigma^2)$$

$$P(X < 160 + 14) = P(X < 174) = 0.75 \quad \text{(MI)(AI)}$$

Note:

$$P(X < 160 - 14) = P(X < 146) = 0.25 \text{ can also be used.}$$

$$P\left(Z < \frac{14}{\sigma}\right) = 0.75 \quad \text{(MI)}$$

$$\frac{14}{\sigma} = 0.6745 \dots \quad \text{(MI)(AI)}$$

$$\sigma = 20.8 \quad \text{AI}$$

[6 marks]

Examiners report

Of those students able to start the question, there were good solutions seen. Most students could have made better use of the GDC on this question.

50.

[15 marks]

Markscheme

Note: Accept exact answers in parts (a) to (c).

(a) number of patients in 30 minute period = X (AI)

$$X \sim \text{Po}(3) \quad (\text{MI})\text{AI}$$

[3 marks]

(b) number of patients in working period = Y (AI)

$$Y \sim \text{Po}(12) \quad (\text{MI})\text{AI}$$

[3 marks]

(c) number of working period with less than 10 patients = W (MI)(AI)

$$W \sim \text{B}(6, 0.2424\dots) \quad (\text{MI})\text{AI}$$

[4 marks]

(d) number of patients in t minute interval = X

$$X \sim \text{Po}(T)$$

$$P(X \geq 2) = 0.95$$

$$P(X = 0) + P(X = 1) = 0.05 \quad (\text{MI})(\text{AI})$$

$$e^{-T}(1 + T) = 0.05 \quad (\text{MI})$$

$$T = 4.74 \quad (\text{AI})$$

$$t = 47.4 \text{ minutes} \quad \text{AI}$$

[5 marks]

Total [15 marks]

Examiners report

Parts (a) and (b) were well answered, but many students were unable to recognise the Binomial distribution in part (c) and were unable to form the correct equation in part (d). There were many accuracy errors in this question.

51.

[6 marks]

Markscheme

(a)

$$p + q = 0.44 \quad \text{AI}$$

$$2.5p + 3.5q = 1.25 \quad (\text{MI})\text{AI}$$

$$p = 0.29, q = 0.15 \quad \text{AI}$$

(b) use of

$$\text{Var}(X) = E(X^2) - E(X)^2 \quad (\text{MI})$$

$$\text{Var}(X) = 2.10 \quad \text{AI}$$

[6 marks]

Examiners report

An easy question, well answered by most candidates. For the others it was disappointing that many did not use the fact that the probabilities add to unity.

52. [5 marks]

Markscheme

(a) required to solve

$$P\left(Z < \frac{21-15}{\sigma}\right) = 0.8 \quad (M1)$$

$$\frac{6}{\sigma} = 0.842\dots \quad (\text{or equivalent}) \quad (M1)$$

$$\Rightarrow \sigma = 7.13 \text{ (days)} \quad A1 \quad N1$$

(b) $P(\text{survival after 21 days}) = 0.337 \quad (M1)A1$

[5 marks]

Examiners report

A straightforward Normal distribution problem, but many candidates confused the z value with the probability.

53. [6 marks]

Markscheme

(a) required to solve

$$e^{-\lambda} + \lambda e^{-\lambda} = 0.123 \quad M1A1$$

solving to obtain

$$\lambda = 3.63 \quad A2 \quad N2$$

Note: Award **A2** if an additional negative solution is seen but **A0** if only a negative solution is seen.

(b)

$$P(0 < X < 9)$$

$$= P(X \leq 8) - P(X = 0) \quad (\text{or equivalent}) \quad (M1)$$

$$= 0.961 \quad A1$$

[6 marks]

Examiners report

Part (a) - Well done by most, although there were some answers that ignored the requirement of mathematical notation.

Part (b) - Not successfully answered by many. The main problem was not correctly interpreting the inequalities in the probability.

54.

[8 marks]

Markscheme

$$\int_{200}^M \frac{2.5(200)^{2.5}}{x^{3.5}} dx = 0.5 \quad \text{MIAIAI}$$

Note: Award M1 for the integral equal to 0.5

AIAI for the correct limits.

$$\frac{-200^{2.5}}{M^{2.5}} \left(\frac{-200^{2.5}}{200^{2.5}} \right) = 0.5 \quad \text{MIAIAI}$$

Note: Award *MI* for correct integration

AIAI for correct substitutions.

$$\frac{-200^{2.5}}{M^{2.5}} + 1 = 0.5 \Rightarrow M^{2.5} = 2(200)^{2.5} \quad \text{(AI)}$$

$$M = 264 \quad \text{AI}$$

[8 marks]

Examiners report

Many students used incorrect limits to the integral, although many did correctly let the integral equal to 0.5.

55.

[6 marks]

Markscheme

the waiting time,

$X \sim N(\mu, \sigma^2)$

18,

4²)

(a)

$$P(X > 25) = 0.0401 \quad \text{(MI)AI}$$

(b)

$$P(X < 20 | X > 15) \quad \text{(AI)}$$

$$= \frac{P(15 < X < 20)}{P(X > 15)} \quad \text{(AI)}$$

Note: Only one of the above *AI* marks can be implied.

$$= \frac{0.4648\dots}{0.7733\dots} = 0.601 \quad \text{(MI)AI}$$

[6 marks]

Examiners report

Part (a) was well answered, whilst few candidates managed to correctly use conditional probability for part (b).

Markscheme

(a) Ying:

Number of heads	0	1	2	3	<i>(M1)A1</i>
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

Mario:

Number of heads	0	1	2	<i>(M1)A1</i>
P	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	

$$\begin{aligned} P(\text{Ying wins}) &= \frac{1}{8} + \frac{3}{8}\left(\frac{2}{4} + \frac{1}{4}\right) + \frac{3}{8} \times \frac{1}{4} \\ &= \frac{16}{32} \quad \text{(M1)A1} \end{aligned}$$

$$\begin{aligned} P(\text{Mario wins}) &= \frac{1}{4}\left(\frac{3}{8} + \frac{1}{8}\right) + \frac{2}{4} \times \frac{1}{8} \\ &= \frac{6}{32} \quad \text{(M1)A1} \end{aligned}$$

$$\begin{aligned} P(\text{draw}) &= 1 - \frac{16}{32} - \frac{6}{32} \\ &= \frac{10}{32} \quad \text{A1} \end{aligned}$$

Ying's winnings:

X	5	-10	-2
P	$\frac{16}{32}$	$\frac{6}{32}$	$\frac{10}{32}$

$$\begin{aligned} \text{expected winnings} &= 5\left(\frac{16}{32}\right) - 10\left(\frac{6}{32}\right) - 2\left(\frac{10}{32}\right) \quad \text{M1A1} \\ &= 0 \quad \text{A1} \end{aligned}$$

[12 marks]

(b)

$$P(\text{Ying wins on 1st round}) = \frac{1}{2} \quad \text{(A1)}$$

$$P(\text{Ying wins on 2nd round}) = \frac{5}{16} \times \frac{1}{2} \quad \text{(M1)(A1)}$$

$$P(\text{Ying wins on 3rd round}) = \left(\frac{5}{16}\right)^2 \times \frac{1}{2} \text{ etc.} \quad \text{(A1)}$$

$$\begin{aligned} P(\text{Ying wins}) &= \frac{1}{2} + \frac{5}{16} \times \frac{1}{2} + \left(\frac{5}{16}\right)^2 \times \frac{1}{2} + \dots \quad \text{(M1)} \\ &= \frac{\frac{1}{2}}{1 - \frac{5}{16}} \quad \text{M1A1} \end{aligned}$$

$$\begin{aligned} &= \frac{8}{11} \quad \text{(} \\ &= 0.727) \quad \text{A1} \end{aligned}$$

[8 marks]

Total [20 marks]

Examiners report

There were some good attempts at this question, but there were also many candidates that were unable to maintain a clearly presented solution and consequently were unable to obtain marks that they should have been able to secure. Those that attempted part (b) usually made a good attempt.

57.

[6 marks]

Markscheme

(a)

 $H \sim N($

166.5,

 $5^2)$

$$P(H \geq 170) = 0.242\dots \quad (MI)(A1)$$

$$0.242\dots \times 63 = 15.2 \quad A1$$

so, approximately

15 students

(b) correct mean:

161.5 (cm) *A1*variance remains the same, *i.e.* 25 (cm²) *A2*

[6 marks]

Examiners report

A surprising number of students lacked the basic knowledge of the normal distribution and were unable to answer the first part of this question. Those students who showed a knowledge of the topic tended to answer the question well. In part (b) many students either had a misunderstanding of the difference between variance and standard deviation, or did not read the question properly.

58.

[7 marks]

Markscheme

(a)

 $X \sim \text{Po}(0.6)$

$$P(X \geq 1) = 1 - P(X = 0) \quad MI$$

$$= 0.451 \quad A1 \quad N1$$

(b)

 $Y \sim \text{Po}(2.4) \quad (MI)$

$$P(Y = 3) = 0.209 \quad A1$$

(c)

 $Z \sim \text{Po}(0.6n) \quad (MI)$

$$P(Z \geq 3) = 1 - P(Z \leq 2) > 0.8 \quad (MI)$$

Note: Only one of these *MI* marks may be implied.

$$n \geq 7.132\dots \text{ (hours)}$$

so, Mr Lee needs to fish for at least

8 complete hours *A1 N2*

Note: Accept a shown trial and error method that leads to a correct solution.

[7 marks]

Examiners report

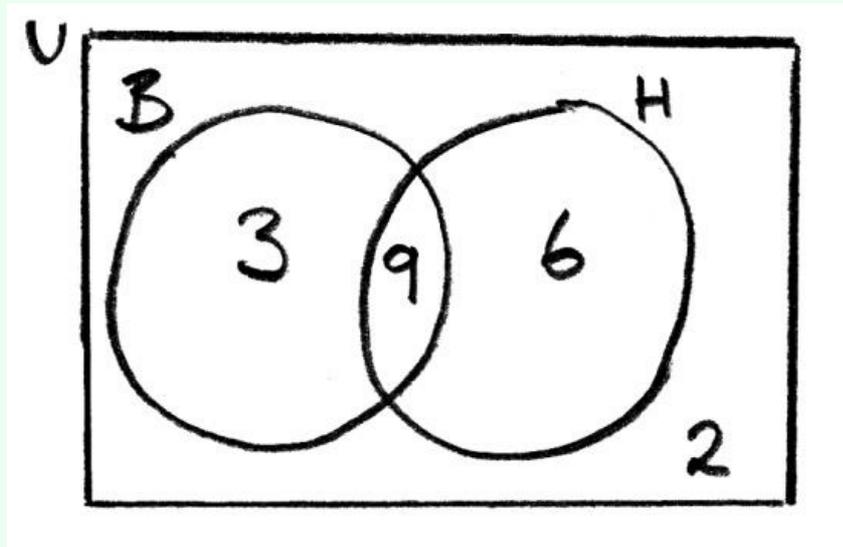
It was clear that many students had not been taught the topic and were consequently unable to make an attempt at the question. Of those students who were able to start, common errors were in a misunderstanding of the language. Many had difficulties in part (c) and “at least” in part (a) was sometimes misinterpreted.

59.

[4 marks]

Markscheme

(a)



A1A1

Note: Award *A1* for a diagram with two intersecting regions and at least the value of the intersection.

(b)
 $\frac{9}{20}$ *A1*(c)
 $\frac{9}{12} (= \frac{3}{4})$ *A1*

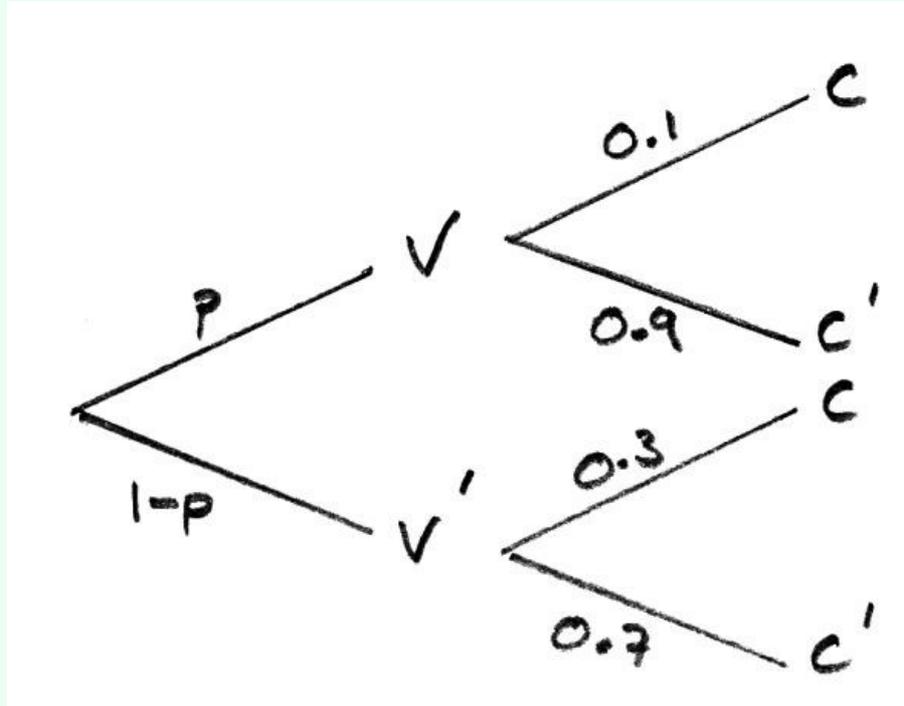
[4 marks]

Examiners report

Although this was the best done question on the paper, it was disappointing that a significant number of candidates produced Venn diagrams with key information missing.

Markscheme

(a)



using the law of total probabilities: (M1)

$$0.1p + 0.3(1 - p) = 0.22 \quad A1$$

$$0.1p + 0.3 - 0.3p = 0.22$$

$$0.2p = 0.88$$

$$p = \frac{0.88}{0.2} = 0.4$$

$$p = 40\% \text{ (accept}$$

$$0.4) \quad A1$$

(b) required probability

$$= \frac{0.4 \times 0.1}{0.22} \quad M1$$

$$= \frac{2}{11} \quad ($$

$$0.182) \quad A1$$

[5 marks]

Examiners report

Most candidates who successfully answered this question had first drawn a tree diagram, using a symbol to denote the probability that a randomly chosen person had received the influenza virus. For those who did not draw a tree diagram, there was poor understanding of how to apply the conditional probability formula.

Markscheme

(a)

$$X \sim N($$

998,

$$2.5^2) \quad \mathbf{MI}$$

$$P(X > 1000) = 0.212 \quad \mathbf{AG}$$

[1 mark]

(b)

$$X \sim B($$

5,

$$0.2119...)$$

evidence of binomial (MI)

$$P(X = 3) = \binom{5}{3} (0.2119...)^3 (0.7881...)^2 = 0.0591 \quad (\text{accept } 0.0592) \quad \mathbf{(MI)AI}$$

[3 marks]

(c)

$$P(X \geq 1) = 1 - P(X = 0) \quad \mathbf{(MI)}$$

$$1 - (0.7881...)^n > 0.99$$

$$(0.7881...)^n < 0.01 \quad \mathbf{AI}$$

Note: Award AI for line 2 or line 3 or equivalent.

$$n > 19.3 \quad \mathbf{(AI)}$$

minimum number of bottles required is

$$20 \quad \mathbf{AIN2}$$

[4 marks]

(d)

$$\frac{996 - \mu}{\sigma} = -1.1998 \quad (\text{accept}$$

$$1.2) \quad \mathbf{MIAI}$$

$$\frac{1000 - \mu}{\sigma} = 0.3999 \quad (\text{accept}$$

$$0.4) \quad \mathbf{MIAI}$$

$$\mu = 999(\text{ml}),$$

$$\sigma = 2.50(\text{ml}) \quad \mathbf{AIAI}$$

[6 marks]

(e) (i)

$$\frac{e^{-m}m^2}{2!} = \frac{e^{-m}m^3}{3!} + \frac{e^{-m}m^4}{4!} \quad \mathbf{MIAI}$$

$$\frac{m^2}{2} = \frac{m^3}{6} + \frac{m^4}{24}$$

$$12m^2 - 4m^3 - m^4 = 0 \quad \mathbf{(AI)}$$

$$m = -6,$$

$$0,$$

$$2$$

$$\Rightarrow m = 2 \quad \mathbf{AIN2}$$

(ii)

$$P(X > 2) = 1 - P(X \leq 2) \quad \mathbf{(MI)}$$

$$= 1 - P(X = 0) - P(X = 1) - P(X = 2)$$

$$= 1 - e^{-2} - 2e^{-2} - \frac{2^2 e^{-2}}{2!}$$

$$= 0.323 \quad AI$$

[6 marks]

Total [20 marks]

Examiners report

This was the best done of the section B questions, with the majority of candidates making the correct choice of probability distribution for each part. The main sources of errors: (b) missing out the binomial coefficient in the calculation; (c) failure to rearrange 'at least one bottle' in terms of the probability of obtaining no bottles; (d) using 1.2 rather than -1.2 in the inverse Normal or not performing an inverse Normal at all; (e)(ii) misinterpreting 'more than two'.