Topic 6 Part 1 *[429 marks]*

1a. *[5 marks]*

Markscheme

$f'(x) = (\ln x)^2 + \frac{2x\ln x}{x}\left(= (\ln x)^2 + 2\ln x = \ln x(\ln x + 2)\right)$ ***M1A1***

$f'(x) = 0\ (\Rightarrow x = 1,\ x = e^{-2})$ ***M1***

Note: Award ***M1*** for an attempt to solve
$f'(x) = 0$.

$A(e^{-2}, 4e^{-2})$ **and** $B(1, 0)$ ***A1A1***

Note: The final ***A1*** is independent of prior working.

[5 marks]

Examiners report

This was answered very well. Candidates are very familiar with this type of question. Some lost a couple of marks by failing to find their final *y* coordinates, though only the weakest struggled with differentiation and so made little progress.

1b. *[3 marks]*

Markscheme

$f''(x) = \frac{2}{x}(\ln x + 1)$ ***A1***

$f''(x) = 0\ \left(\Rightarrow x = e^{-1}\right)$ ***(M1)***

inflexion point
(e^{-1}, e^{-1}) ***A1***

Note: ***M1*** for attempt to solve
$f''(x) = 0$.

[3 marks]

Examiners report

This was answered very well. Candidates are very familiar with this type of question. Some lost a couple of marks by failing to find their final *y* coordinates, though only the weakest struggled with differentiation and so made little progress.

2a. *[6 marks]*

Markscheme

attempt to differentiate implicitly ***M1***

$2x + \cos y\frac{\mathrm{d}y}{\mathrm{d}x} - y - x\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ ***A1A1***

Note: ***A1*** for differentiating
x^2 and sin *y* ; ***A1*** for differentiating *xy*.

substitute *x* and *y* by
π ***M1***

$2\pi - \frac{\mathrm{d}y}{\mathrm{d}x} - \pi - \pi\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\pi}{1+\pi}$ ***M1A1***

Note: ***M1*** for attempt to make d*y*/d*x* the subject. This could be seen earlier.

[6 marks]

Examiners report

Part a) proved an easy 6 marks for most candidates, while the majority failed to make any headway with part b), with some attempting to find the equation of their line in the form $y = mx + c$. Only the best candidates were able to see their way through to the given answer.

2b. *[3 marks]*

Markscheme

$\theta = \frac{\pi}{4} - \arctan\frac{\pi}{1+\pi}$ (or seen the other way) ***M1***

$\tan\theta = \tan\left(\frac{\pi}{4} - \arctan\frac{\pi}{1+\pi}\right) = \frac{1-\frac{\pi}{1+\pi}}{1+\frac{\pi}{1+\pi}}$ ***M1A1***

$\tan\theta = \frac{1}{1+2\pi}$ ***AG***

[3 marks]

Examiners report

Part a) proved an easy 6 marks for most candidates, while the majority failed to make any headway with part b), with some attempting to find the equation of their line in the form $y = mx + c$. Only the best candidates were able to see their way through to the given answer.

3. *[6 marks]*

Markscheme

$2s\frac{\mathrm{d}s}{\mathrm{d}t} + \frac{\mathrm{d}s}{\mathrm{d}t} - 2 = 0$ ***M1A1***

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = \frac{2}{2s+1}$ ***A1***

EITHER

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t}$ ***(M1)***

$\frac{\mathrm{d}v}{\mathrm{d}s} = \frac{-4}{(2s+1)^2}$ ***(A1)***

$a = \frac{-4}{(2s+1)^2}\frac{\mathrm{d}s}{\mathrm{d}t}$

OR

$2\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 + 2s\frac{\mathrm{d}^2s}{\mathrm{d}t^2} + \frac{\mathrm{d}^2s}{\mathrm{d}t^2} = 0$ ***(M1)***

$\underbrace{\frac{\mathrm{d}^2s}{\mathrm{d}t^2}}_{a} = \frac{-2\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2}{2s+1}$ ***(A1)***

THEN

$a = \frac{-8}{(2s+1)^3}$ ***A1***

[6 marks]

Examiners report

Despite the fact that many candidates were able to calculate the speed of the particle, many of them failed to calculate the acceleration. Implicit differentiation turned out to be challenging in this exercise showing in many cases a lack of understanding of independent/dependent variables. Very often candidates did not use the chain rule or implicit differentiation when attempting to find the acceleration. It was not uncommon to see candidates trying to differentiate implicitly with respect to t rather than s, but getting the variables muddled.

4.

[7 marks]

Markscheme

$x = \sin t,\ \mathrm{d}x = \cos t\,\mathrm{d}t$

$\int \frac{x^3}{\sqrt{1-x^2}}\mathrm{d}x = \int \frac{\sin^3 t}{\sqrt{1-\sin^2 t}}\cos t\,\mathrm{d}t$ ***M1***

$= \int \sin^3 t\,\mathrm{d}t$ ***(A1)***

$= \int \sin^2 t \sin t\,\mathrm{d}t$

$= \int (1-\cos^2 t)\sin t\,\mathrm{d}t$ ***M1A1***

$= \int \sin t\,\mathrm{d}t - \int \cos^2 t \sin t\,\mathrm{d}t$

$= -\cos t + \frac{\cos^3 t}{3} + C$ ***A1A1***

$= -\sqrt{1-x^2} + \frac{1}{3}\left(\sqrt{1-x^2}\right)^3 + C$ ***A1***

$\left(= -\sqrt{1-x^2}\,\left(1 - \frac{1}{3}(1-x^2)\right) + C\right)$

$\left(= -\frac{1}{3}\sqrt{1-x^2}\,(2+x^2) + C\right)$

[7 marks]

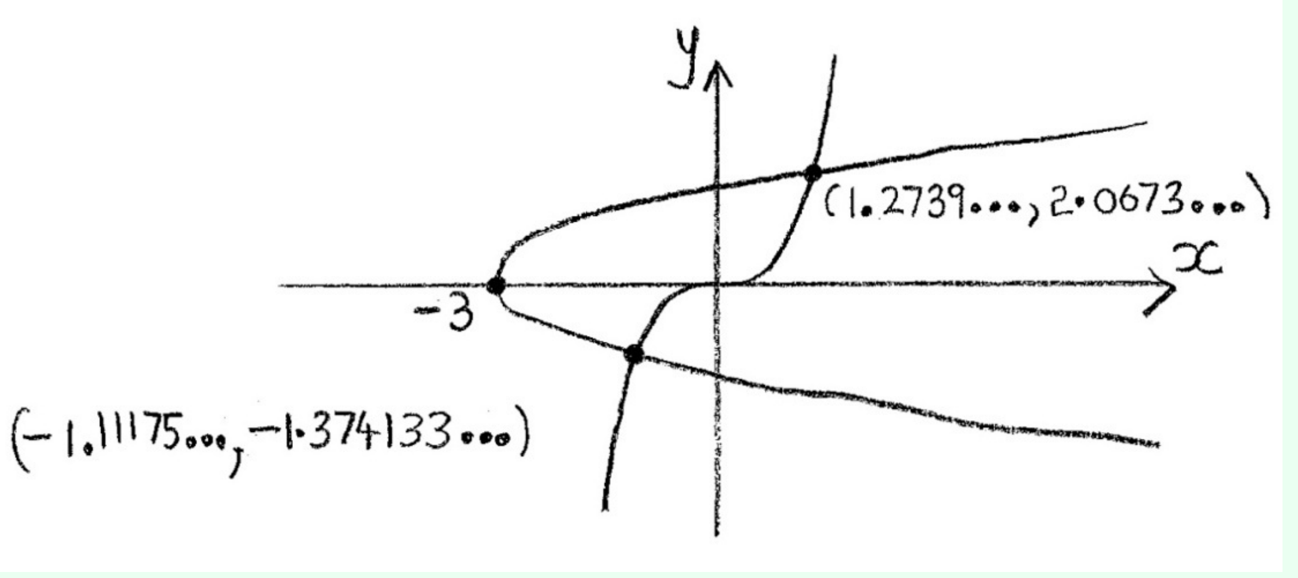
Examiners report

Just a few candidates got full marks in this question. Substitution was usually incorrectly done and lead to wrong results. A cosine term in the denominator was a popular error. Candidates often chose unhelpful trigonometric identities and attempted integration by parts. Results such as $\int \sin^3 t\,\mathrm{d}t = \frac{\sin^4 t}{4} + C$ were often seen along with other misconceptions concerning the manipulation/simplification of integrals were also noticed. Some candidates unsatisfactorily attempted to use $\arcsin x$. However, there were some good solutions involving an expression for the cube of $\sin t$ in terms of $\sin t$ and $\sin 3t$. Very few candidates re-expressed their final result in terms of x.

5.

[7 marks]

Markscheme



intersection points ***A1A1***

Note: Only either the *x*-coordinate or the *y*-coordinate is needed.

EITHER

$x = y^2 - 3 \Rightarrow y = \pm\sqrt{x+3}$ (accept $y = \sqrt{x+3}$) ***(M1)***

$A = \int_{-3}^{-1.111...} 2\sqrt{x+3}\,\mathrm{d}x + \int_{-1.111...}^{1.2739...} \sqrt{x+3} - x^3\,\mathrm{d}x$ ***(M1)A1A1***

= 3.4595... + 3.8841...

= 7.34 (3sf) ***A1***

OR

$y = x^3 \Rightarrow x = \sqrt[3]{y}$ ***(M1)***

$A = \int_{-1.374...}^{2.067...} \sqrt[3]{y} - (y^2 - 3)\mathrm{d}y$ ***(M1)A1***

= 7.34 (3sf) ***A2***

[7 marks]

Examiners report

This question proved challenging to most candidates. Just a few candidates were able to calculate the exact area between curves. Those candidates who tried to express the functions in terms *x* of instead of *y* showed better performances. Determining only $\sqrt{x+3}$ was a common error and forming appropriate definite integrals above and below the *x*-axis proved difficult. Although many candidates attempted to sketch the graphs, many found only one branch of the parabola and only one point of intersection; as the graph of the parabola was not complete, many candidates did not know which area they were trying to find. Not many split the integral correctly to find areas that would add up to the result. Premature rounding was usually seen and consequently final answers proved inaccurate.

6a.

[3 marks]

Markscheme

$L = \mathrm{CA} + \mathrm{AD}$ ***M1***

$\sin\alpha = \frac{a}{\mathrm{CA}} \Rightarrow \mathrm{CA} = \frac{a}{\sin\alpha}$ ***A1***

$\cos\alpha = \frac{b}{\mathrm{AD}} \Rightarrow \mathrm{AD} = \frac{b}{\cos\alpha}$ ***A1***

$L = \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha}$ ***AG***

[2 marks]

Examiners report

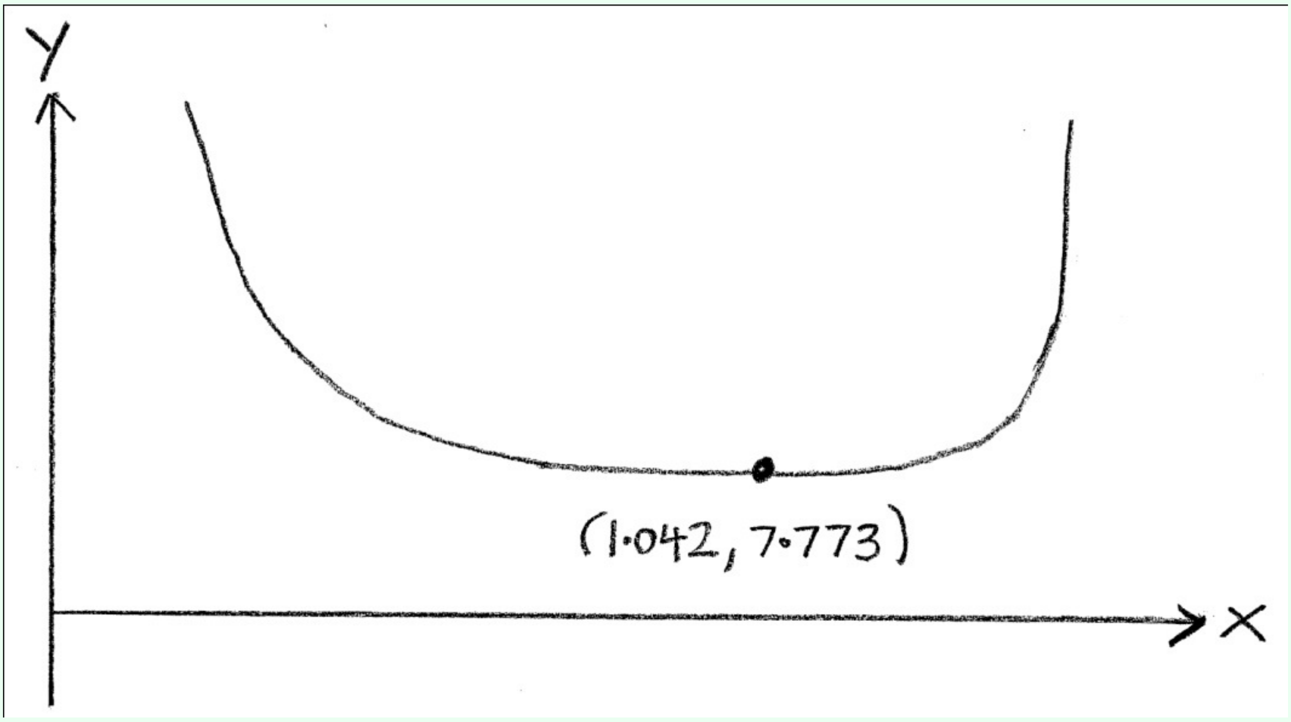
Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.
In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for
α and L. The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

6b. *[4 marks]*

Markscheme

$a = 5$ and $b = 1 \Rightarrow L = \frac{5}{\sin\alpha} + \frac{1}{\cos\alpha}$

METHOD 1



(M1)

minimum from graph

$\Rightarrow L = 7.77$ ***(M1)A1***

minimum of L gives the max length of the painting ***R1***

[4 marks]

METHOD 2

$\frac{dL}{d\alpha} = \frac{-5\cos\alpha}{\sin^2\alpha} + \frac{\sin\alpha}{\cos^2\alpha}$ ***(M1)***

(M1)

$\frac{dL}{d\alpha} = 0 \Rightarrow \frac{\sin^3\alpha}{\cos^3\alpha} = 5 \Rightarrow \tan\alpha = \sqrt[3]{5}\ (\alpha = 1.0416\ldots)$

minimum of L gives the max length of the painting ***R1***

maximum length = 7.77 ***A1***

[4 marks]

Examiners report

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.
In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for α and L. The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

6c. *[3 marks]*

Markscheme

$\frac{\mathrm{d}L}{\mathrm{d}\alpha} = \frac{-3k\cos\alpha}{\sin^2\alpha} + \frac{k\sin\alpha}{\cos^2\alpha}$ (or equivalent) ***M1A1A1***

[3 marks]

Examiners report

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.
In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for α and L. The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

6d. *[6 marks]*

Markscheme

$\frac{\mathrm{d}L}{\mathrm{d}\alpha} = \frac{-3k\cos^3\alpha + k\sin^3\alpha}{\sin^2\alpha\cos^2\alpha}$ ***(A1)***

$\frac{\mathrm{d}L}{\mathrm{d}\alpha} = 0 \Rightarrow \frac{\sin^3\alpha}{\cos^3\alpha} = \frac{3k}{k} \Rightarrow \tan\alpha = \sqrt[3]{3}$ $(\alpha = 0.96454...)$ ***M1A1***

$\tan\alpha = \sqrt[3]{3} \Rightarrow \frac{1}{\cos\alpha} = \sqrt{1+\sqrt[3]{9}}$ $(1.755...)$ ***(A1)***

and $\frac{1}{\sin\alpha} = \frac{\sqrt{1+\sqrt[3]{9}}}{\sqrt[3]{3}}$ $(1.216...)$ ***(A1)***

$L = 3k\left(\frac{\sqrt{1+\sqrt[3]{9}}}{\sqrt[3]{3}}\right) + k\sqrt{1+\sqrt[3]{9}}$ $(L = 5.405598...k)$ ***A1 N4***

[6 marks]

Examiners report

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.
In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for α and L. The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

6e. *[2 marks]*

Markscheme

$L < 8 \Rightarrow k > 1.48$ ***M1A1***

the minimum value is 1.48

[2 marks]

Examiners report

Part (a) was very well done by most candidates. Parts (b), (c) and (d) required a subtle balance between abstraction, differentiation skills and use of GDC.
In part (b), although candidates were asked to justify their reasoning, very few candidates offered an explanation for the maximum. Therefore most candidates did not earn the R1 mark in part (b). Also not as many candidates as anticipated used a graphical approach, preferring to use the calculus with varying degrees of success. In part (c), some candidates calculated the derivatives of inverse trigonometric functions. Some candidates had difficulty with parts (d) and (e). In part (d), some candidates erroneously used their alpha value from part (b). In part (d) many candidates used GDC to calculate decimal values for θ and L. The premature rounding of decimals led sometimes to inaccurate results. Nevertheless many candidates got excellent results in this question.

7a. *[2 marks]*

Markscheme

$4(x-0.5)^2+4$ ***A1A1***

Note: ***A1*** for two correct parameters, ***A2*** for all three correct.

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e).

7b. *[3 marks]*

Markscheme

translation

$\begin{pmatrix}0.5\\0\end{pmatrix}$ (allow "0.5 to the right") ***A1***

stretch parallel to *y*-axis, scale factor 4 (allow vertical stretch or similar) ***A1***

translation

$\begin{pmatrix}0\\4\end{pmatrix}$ (allow "4 up") ***A1***

Note: All transformations must state magnitude and direction.

Note: First two transformations can be in either order.

It could be a stretch followed by a single translation of

$\begin{pmatrix}0.5\\4\end{pmatrix}$. If the vertical translation is before the stretch it is

$\begin{pmatrix}0\\1\end{pmatrix}$.

[3 marks]

Examiners report

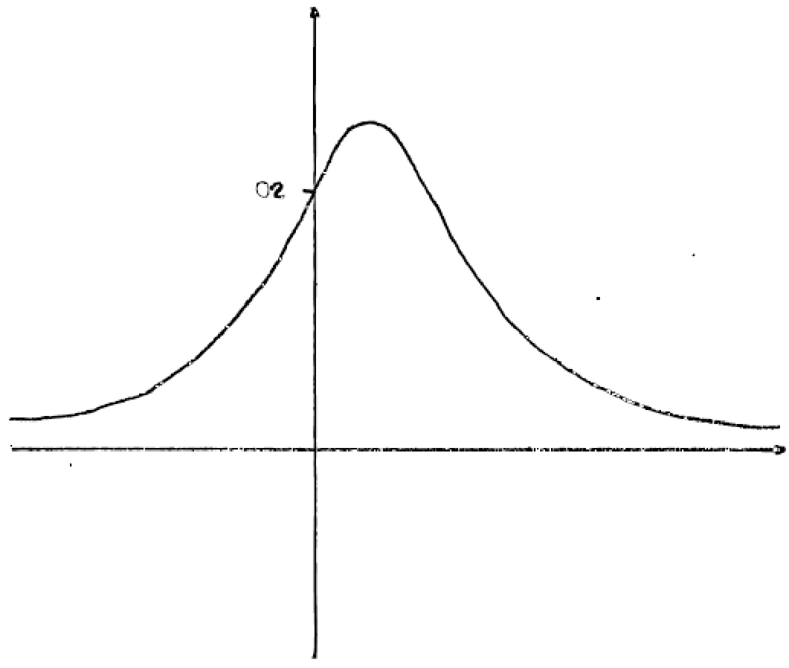
This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(b) Exam technique would have helped those candidates who could not get part (a) correct as any solution of the form given in the question could have led to full marks in part (b). Several candidates obtained expressions which were not of this form in (a) and so were unable to receive any marks in (b) Many missed the fact that if a vertical translation is performed before the vertical stretch it has a different magnitude to if it is done afterwards. Though on this occasion the markscheme was fairly flexible in the words it allowed to be used by candidates to describe the transformations it would be less risky to use the correct expressions.

7c.

[2 marks]

Markscheme



general shape (including asymptote and single maximum in first quadrant), ***A1***

intercept $\left(0, \frac{1}{5}\right)$ or maximum $\left(\frac{1}{2}, \frac{1}{4}\right)$ shown ***A1***

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(c) Generally the sketches were poor. The general rule for all sketch questions should be that any asymptotes or intercepts should be clearly labelled. Sketches do not need to be done on graph paper, but a ruler should be used, particularly when asymptotes are involved.

7d.

[2 marks]

Markscheme

$0 < f(x) \leqslant \frac{1}{4}$ ***A1A1***

Note: ***A1*** for $\leqslant \frac{1}{4}$, ***A1*** for $0 <$.

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e).

7e.

[3 marks]

Markscheme

let

$u = x - \frac{1}{2}$ ***A1***

$\frac{\mathrm{d}u}{\mathrm{d}x} = 1 \quad (\text{or } \mathrm{d}u = \mathrm{d}x)$ ***A1***

$\int \frac{1}{4x^2 - 4x + 5}\mathrm{d}x = \int \frac{1}{4\left(x - \frac{1}{2}\right)^2 + 4}\mathrm{d}x$ ***A1***

$\int \frac{1}{4u^2 + 4}\mathrm{d}u = \frac{1}{4}\int \frac{1}{u^2 + 1}\mathrm{d}u$ ***AG***

Note: If following through an incorrect answer to part (a), do not award final ***A1*** mark.

[3 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(e) and (f) were well done up to the final part of (f), in which candidates did not realise they needed to use the compound angle formula.

7f.

[7 marks]

Markscheme

$\int_1^{3.5} \frac{1}{4x^2 - 4x + 5}\mathrm{d}x = \frac{1}{4}\int_{0.5}^{3} \frac{1}{u^2 + 1}\mathrm{d}u$ ***A1***

Note: ***A1*** for correct change of limits. Award also if they do not change limits but go back to *x* values when substituting the limit

(even if there is an error in the integral).

$\frac{1}{4}[\arctan(u)]_{0.5}^{3}$ ***(M1)***

$\frac{1}{4}\left(\arctan(3) - \arctan\left(\frac{1}{2}\right)\right)$ ***A1***

let the integral $= I$

$\tan 4I = \tan\left(\arctan(3) - \arctan\left(\frac{1}{2}\right)\right)$ ***M1***

$\frac{3 - 0.5}{1 + 3 \times 0.5} = \frac{2.5}{2.5} = 1$ ***(M1)A1***

$4I = \frac{\pi}{4} \Rightarrow I = \frac{\pi}{16}$ ***A1AG***

[7 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(e) and (f) were well done up to the final part of (f), in which candidates did not realise they needed to use the compound angle formula.

8.

[6 marks]

Markscheme

volume

(M1)
$= \pi \int x^2 \mathrm{d}y$

(M1)(A1)
$x = \arcsin y + 1$

volume

A1
$= \pi \int_0^1 (\arcsin y + 1)^2 \mathrm{d}y$

Note: ***A1*** is for the limits, provided a correct integration of *y*.

A2 N5
$= 2.608993\ldots\pi = 8.20$

[6 marks]

Examiners report

Although it was recognised that the imprecise nature of the wording of the question caused some difficulties, these were overwhelmingly by candidates who were attempting to rotate around the
x
-axis. The majority of students who understood to rotate about the
y
-axis had no difficulties in writing the correct integral. Marks lost were for inability to find the correct value of the integral on the GDC (some clearly had the calculator in degrees) and also for poor rounding where the GDC had been used correctly. In the few instances where students seemed confused by the lack of precision in the question, benefit of the doubt was given and full points awarded.

9a.

[4 marks]

Markscheme

let the distance the cable is laid along the seabed be *y*

(M1)
$y^2 = x^2 + 200^2 - 2 \times x \times 200 \cos 60^\circ$
(or equivalent method)

(A1)
$y^2 = x^2 - 200x + 40000$

cost = *C* = 80*y* + 20*x* ***(M1)***

A1
$C = 80(x^2 - 200x + 40000)^{\frac{1}{2}} + 20x$

[4 marks]

Examiners report

Some surprising misconceptions were evident here, using right angled trigonometry in non right angled triangles etc. Those that used the cosine rule, usually managed to obtain the correct answer to part (a).

9b.

[2 marks]

Markscheme

(m to the nearest metre) ***(A1)A1***
$x = 55.2786\ldots = 55$

$\left(x = 100 - \sqrt{2000}\right)$

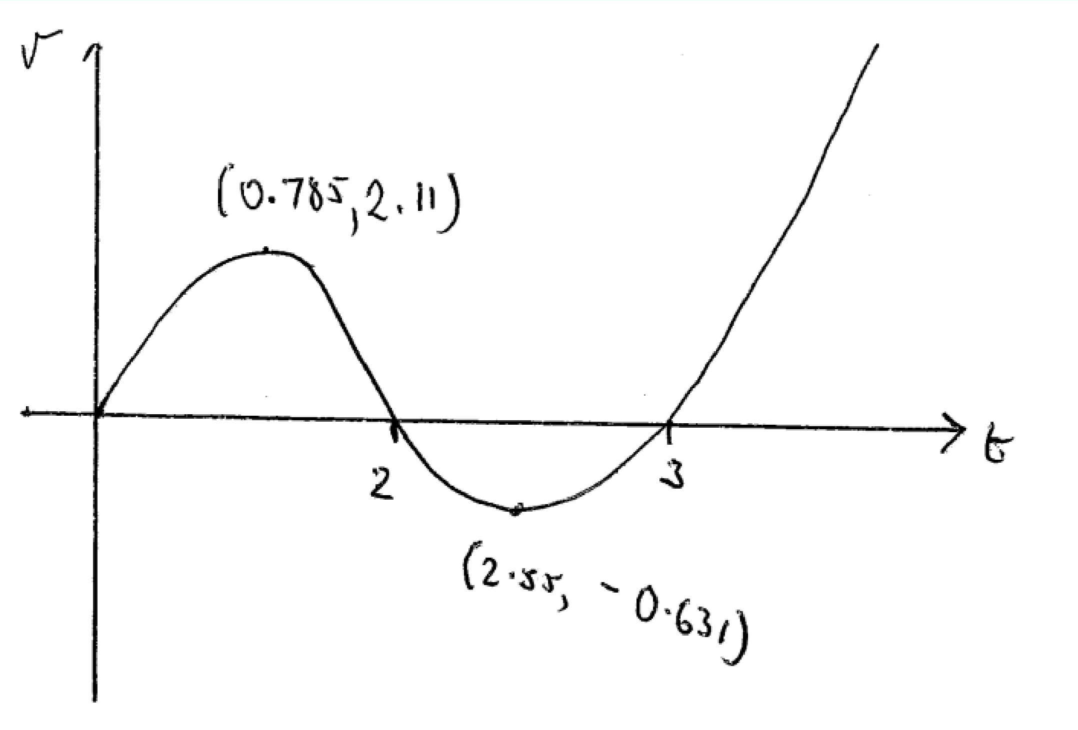
[2 marks]

Examiners report

Some surprising misconceptions were evident here, using right angled trigonometry in non right angled triangles etc. Many students attempted to find the value of the minimum algebraically instead of the simple calculator solution.

10a. *[3 marks]*

Markscheme



A1A1A1

Note: Award ***A1*** for general shape, ***A1*** for correct maximum and minimum, ***A1*** for intercepts.

Note: Follow through applies to (b) and (c).

[3 marks]

Examiners report

Part (a) was generally well done, although correct accuracy was often a problem.

10b. *[2 marks]*

Markscheme

$0 \leqslant t < 0.785$, $\left(\text{or } 0 \leqslant t < \frac{5-\sqrt{7}}{3}\right)$ (allow $t < 0.785$) ***A1***

and

$t > 2.55$ $\left(\text{or } t > \frac{5+\sqrt{7}}{3}\right)$ ***A1***

[2 marks]

Examiners report

Parts (b) and (c) were also generally quite well done.

10c. *[3 marks]*

Markscheme

$0 \leqslant t < 0.785, \left(\text{or } 0 \leqslant t < \frac{5-\sqrt{7}}{3}\right)$ ***A1***

(allow $t < 0.785$)

$2 < t < 2.55, \left(\text{or } 2 < t < \frac{5+\sqrt{7}}{3}\right)$ ***A1***

$t > 3$ ***A1***

[3 marks]

Examiners report

Parts (b) and (c) were also generally quite well done.

10d. *[3 marks]*

Markscheme

position of A:

$x_A = \int t^3 - 5t^2 + 6t \, dt$ ***(M1)***

$x_A = \frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2 \quad (+c)$ ***A1***

when $t = 0, \ x_A = 0$, so $c = 0$ ***R1***

[3 marks]

Examiners report

A variety of approaches were seen in part (d) and many candidates were able to obtain at least 2 out of 3. A number missed to consider the $+c$, thereby losing the last mark.

10e. *[4 marks]*

Markscheme

$\frac{dv_B}{dt} = -2v_B \Rightarrow \int \frac{1}{v_B} dv_B = \int -2dt$ ***(M1)***

$\ln|v_B| = -2t + c$ ***(A1)***

$v_B = Ae^{-2t}$ ***(M1)***

$v_B = -20$ when $t = 0$ so $v_B = -20e^{-2t}$ ***A1***

[4 marks]

Examiners report

Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

10f.

[6 marks]

Markscheme

$x_B = 10e^{-2t}(+c)$ ***(M1)(A1)***

$x_B = 20$ when $t = 0$ so $x_B = 10e^{-2t} + 10$ ***(M1)A1***

meet when

$\frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2 = 10e^{-2t} + 10$ ***(M1)***

$t = 4.41(290\ldots)$ ***A1***

[6 marks]

Examiners report

Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

11a.

[6 marks]

Markscheme

$f(2) = 9$ ***(A1)***

$f^{-1}(x) = (x-1)^{\frac{1}{3}}$ ***A1***

$(f^{-1})'(x) = \frac{1}{3}(x-1)^{-\frac{2}{3}}$ ***(M1)***

$(f^{-1})'(9) = \frac{1}{12}$ ***A1***

$f'(x) = 3x^2$ ***(M1)***

$\frac{1}{f'(2)} = \frac{1}{3\times 4} = \frac{1}{12}$ ***A1***

Note: The last ***M1*** and ***A1*** are independent of previous marks.

[6 marks]

Examiners report

There were many good attempts at parts (a) and (b), although in (b) many were unable to give a thorough justification.

11b.

[3 marks]

Markscheme

$g'(x) = \mathrm{e}^{x^2} + 2x^2\mathrm{e}^{x^2}$ ***M1A1***

as each part is positive ***R1***

$g'(x) > 0$

[3 marks]

Examiners report

There were many good attempts at parts (a) and (b), although in (b) many were unable to give a thorough justification.

11c. [4 marks]

Markscheme

to find the x-coordinate on

solve
$y = g(x)$
(M1)
$2 = xe^{x^2}$
(A1)
$x = 0.89605022078\ldots$
gradient

(M1)
$= (g^{-1})'(2) = \frac{1}{g'(0.896\ldots)}$
to 3sf ***A1***
$= \frac{1}{e^{(0.896\ldots)^2}\left(1+2\times(0.896\ldots)^2\right)} = 0.172$
(using the

function on gdc
$\frac{dy}{dx}$

$g'(0.896\ldots) = 5.7716028\ldots$

$\frac{1}{g'(0.896\ldots)} = 0.173$
[4 marks]

Examiners report

Few good solutions to parts (c) and (d)(ii) were seen although many were able to answer (d)(i) correctly.

11d. [6 marks]

Markscheme

(i)

A1
$(x^3+1)e^{(x^3+1)^2} = 2$
A1
$x = -0.470191\ldots$

(ii) **METHOD 1**

(M1)(A1)
$(g \circ f)'(x) = 3x^2 e^{(x^3+1)^2}\left(2(x^3+1)^2+1\right)$
(A1)
$(g \circ f)'(-0.470191\ldots) = 3.85755\ldots$
A1
$h'(2) = \frac{1}{3.85755\ldots} = 0.259\ (232\ldots)$
Note: The solution can be found without the student obtaining the explicit form of the composite function.

METHOD 2

A1
$h(x) = (f^{-1} \circ g^{-1})(x)$
M1
$h'(x) = (f^{-1})'\left(g^{-1}(x)\right) \times (g^{-1})'(x)$
M1
$= \frac{1}{3}\left(g^{-1}(x) - 1\right)^{-\frac{2}{3}} \times (g^{-1})'(x)$

$h'(2) = \frac{1}{3}\left(g^{-1}(2) - 1\right)^{-\frac{2}{3}} \times (g^{-1})'(2)$

$= \frac{1}{3}(0.89605\ldots - 1)^{-\frac{2}{3}} \times 0.171933\ldots$
A1 ***N4***
$= 0.259\ (232\ldots)$
[6 marks]

Examiners report

Few good solutions to parts (c) and (d)(ii) were seen although many were able to answer (d)(i) correctly.

12. *[6 marks]*

Markscheme

$\left[\frac{1}{3}(x-2)^3 + \ln x - \frac{1}{\pi}\cos \pi x\right]_1^2$ ***A1A1A1***

Note: Accept $\frac{1}{3}x^3 - 2x^2 + 4x$ in place of $\frac{1}{3}(x-2)^3$.

$= \left(0 + \ln 2 - \frac{1}{\pi}\cos 2\pi\right) - \left(-\frac{1}{3} + \ln 1 - \frac{1}{\pi}\cos \pi\right)$ ***(M1)***

$= \frac{1}{3} + \ln 2 - \frac{2}{\pi}$ ***A1A1***

Note: Award ***A1*** for any two terms correct, ***A1*** for the third correct.

[6 marks]

Examiners report

Generally well done, although quite a number of candidates were either unable to integrate the sine term or incorrectly evaluated the resulting cosine at the limits.

13a. *[4 marks]*

Markscheme

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+\cos x)(\cos x - x\sin x) - x\cos x(1-\sin x)}{(x+\cos x)^2}$ ***M1A1A1***

Note: Award ***M1*** for attempt at differentiation of a quotient and a product condoning sign errors in the quotient formula and the trig differentiations, ***A1*** for correct derivative of "*u*", ***A1*** for correct derivative of "*v*".

$= \frac{x\cos x + \cos^2 x - x^2\sin x - x\cos x\sin x - x\cos x + x\cos x\sin x}{(x+\cos x)^2}$ ***A1***

$= \frac{\cos^2 x - x^2\sin x}{(x+\cos x)^2}$ ***AG***

[4 marks]

Examiners report

The majority of candidates earned significant marks on this question. The product rule and the quotient rule were usually correctly applied, but a few candidates made an error in differentiating the denominator, obtaining $-\sin x$ rather than $1-\sin x$. A disappointing number of candidates failed to calculate the correct gradient at the specified point.

13b. *[3 marks]*

Markscheme

the derivative has value –1 ***(A1)***

the equation of the tangent line is

$(y-0) = (-1)\left(x - \frac{\pi}{2}\right)$ $\left(y = \frac{\pi}{2} - x\right)$ ***M1A1***

[3 marks]

Examiners report

The majority of candidates earned significant marks on this question. The product rule and the quotient rule were usually correctly applied, but a few candidates made an error in differentiating the denominator, obtaining $-\sin x$ rather than $1 - \sin x$. A disappointing number of candidates failed to calculate the correct gradient at the specified point.

14a.

[4 marks]

Markscheme

attempt at implicit differentiation ***M1***

EITHER

$\frac{2x}{y} - \frac{x^2}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x} - 2 = \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$ ***A1A1***

Note: Award ***A1*** for each side.

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{2x}{y} - 2}{\frac{1}{y} + \frac{x^2}{y^2}} \left(= \frac{2xy - 2y^2}{x^2 + y}\right)$ ***A1***

OR

after multiplication by y

$2x - 2y - 2x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}x}\ln y + y\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$ ***A1A1***

Note: Award ***A1*** for each side.

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2(x-y)}{1+2x+\ln y}$ ***A1***

[4 marks]

Examiners report

Most candidates were familiar with the concept of implicit differentiation and the majority found the correct derivative function. In part (b), a significant number of candidates didn't realise that the value of x was required.

14b.

[2 marks]

Markscheme

for $y = 1,\ x^2 - 2x = 0$

$x = (0 \text{ or})\ 2$ ***A1***

for $x = 2$,

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{5}$ ***A1***

[2 marks]

Examiners report

Most candidates were familiar with the concept of implicit differentiation and the majority found the correct derivative function. In part (b), a significant number of candidates didn't realise that the value of x was required.

15a. *[2 marks]*

Markscheme

by division or otherwise

$f(x) = 2 - \frac{5}{x+2}$ ***A1A1***

[2 marks]

Examiners report

Generally well done.

15b. *[2 marks]*

Markscheme

$f'(x) = \frac{5}{(x+2)^2}$ ***A1***

> 0 as $(x+2)^2 > 0$ (on D) ***R1AG***

Note: Do not penalise candidates who use the original form of the function to compute its derivative.

[2 marks]

Examiners report

In their answers to Part (b), most candidates found the derivative, but many assumed it was obviously positive.

15c. *[2 marks]*

Markscheme

$S = \left[-3, \frac{3}{2}\right]$ ***A2***

Note: Award ***A1A0*** for the correct endpoints and an open interval.

[2 marks]

Examiners report

[N/A]

15d.

[8 marks]

Markscheme

(i) **EITHER**

rearrange

$y = f(x)$ to make x the subject *M1*

obtain one-line equation, *e.g.*

$2x - 1 = xy + 2y$ ***A1***

$x = \frac{2y+1}{2-y}$ ***A1***

OR

interchange x and y ***M1***

obtain one-line equation, *e.g.*

$2y - 1 = xy + 2x$ ***A1***

$y = \frac{2x+1}{2-x}$ ***A1***

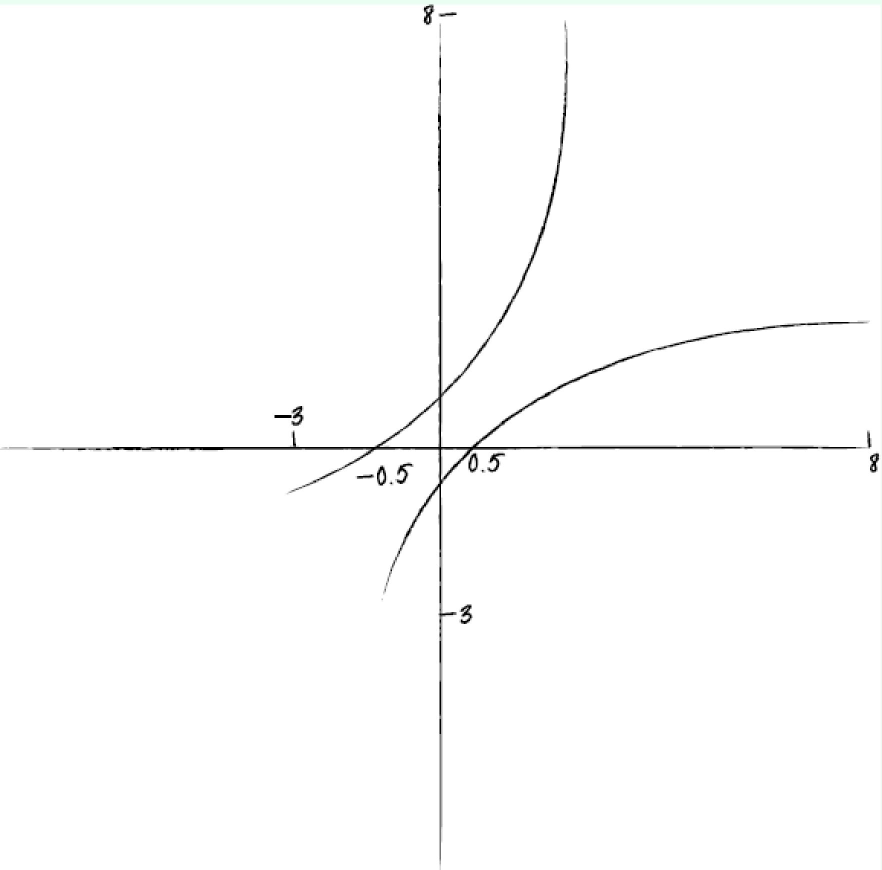
THEN

$f^{-1}(x) = \frac{2x+1}{2-x}$ ***A1***

Note: Accept

$\frac{5}{2-x} - 2$

(ii), (iii)



A1A1A1A1

[8 marks]

Note: Award ***A1*** for correct shape of

.
$y = f(x)$
Award ***A1*** for x intercept

seen. Award ***A1*** for y intercept
$\frac{1}{2}$
seen.
$-\frac{1}{2}$
Award ***A1*** for the graph of

being the reflection of
$y = f^{-1}(x)$
in the line
$y = f(x)$
. Candidates are not required to indicate the full domain, but
$y = x$
should not be shown approaching
$y = f(x)$
. Candidates, in answering (iii), can **FT** on their sketch in (ii).
$x = -2$

Examiners report

Part (d)(i) Generally well done, but some candidates failed to label their final expression as

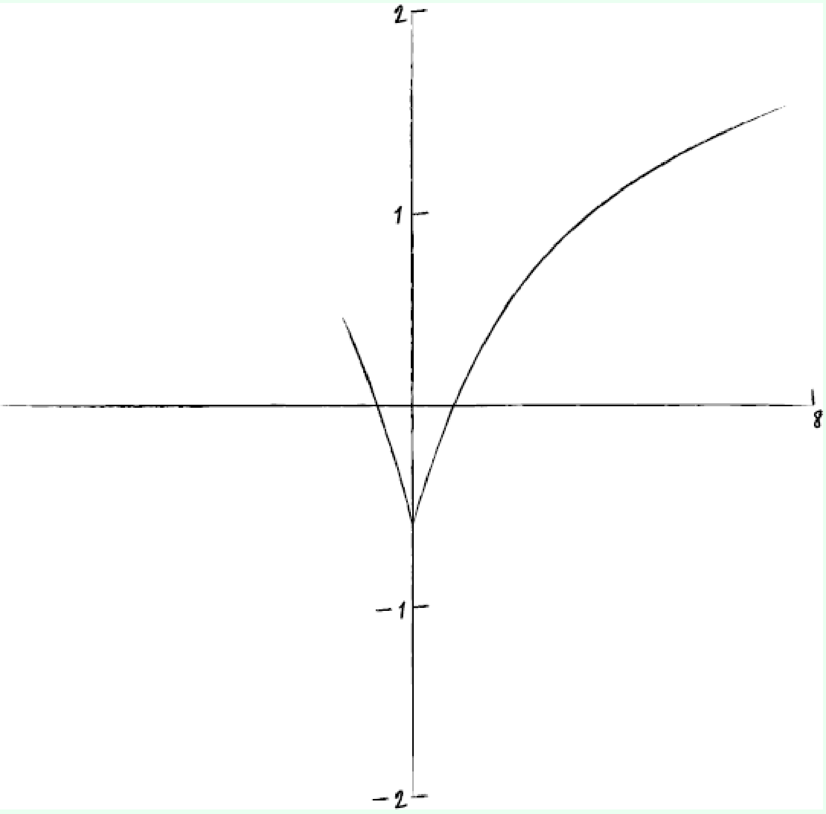
. Part (d)(ii) Marks were lost by candidates who failed to mark the intercepts with values.
$f^{-1}(x)$

15e.

[7 marks]

Markscheme

(i)



A1A1A1

Note: ***A1*** for correct sketch

, ***A1*** for symmetry, ***A1*** for correct domain (from –1 to +8).
$x > 0$

Note: Candidates can **FT** on their sketch in (d)(ii).

(ii) attempt to solve

(M1)
$f(x) = -\frac{1}{4}$
obtain

A1
$x = \frac{2}{9}$
use of symmetry or valid algebraic approach ***(M1)***

obtain

A1
$x = -\frac{2}{9}$

[7 marks]

Examiners report

Marks were also lost in this part and in part (e)(i) for graphs that went beyond the explicitly stated domain.

16a. *[4 marks]*

Markscheme

M1A1

$\int x\sec^2 x\mathrm{d}x = x\tan x - \int 1 \times \tan x\mathrm{d}x$

M1A1

$= x\tan x + \ln|\cos x|(+c)\ \ (= x\tan x - \ln|\sec x|(+c))$

[4 marks]

Examiners report

In part (a), a large number of candidates were able to use integration by parts correctly but were unable to use integration by substitution to then find the indefinite integral of tan x. In part (b), a large number of candidates attempted to solve the equation without direct use of a GDC's numerical solve command. Some candidates stated more than one solution for m and some specified m correct to two significant figures only.

16b. *[2 marks]*

Markscheme

attempting to solve an appropriate equation *eg*

(M1)

$m\tan m + \ln(\cos m) = 0.5$

$m = 0.822$ ***A1***

Note: Award ***A1*** if $m = 0.822$ is specified with other positive solutions.

[2 marks]

Examiners report

In part (a), a large number of candidates were able to use integration by parts correctly but were unable to use integration by substitution to then find the indefinite integral of tan x. In part (b), a large number of candidates attempted to solve the equation without direct use of a GDC's numerical solve command. Some candidates stated more than one solution for m and some specified m correct to two significant figures only.

17. *[6 marks]*

Markscheme

METHOD 1

$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{40}(60 - v)$ ***(M1)***

attempting to separate variables

$\int \frac{\mathrm{d}v}{60 - v} = \int \frac{\mathrm{d}t}{40}$ ***M1***

$-\ln(60 - v) = \frac{t}{40} + c$ ***A1***

(or equivalent) ***A1***

$c = -\ln 60$

attempting to solve for v when $t = 30$ ***(M1)***

$v = 60 - 60e^{-\frac{3}{4}}$

$v = 31.7\ (\mathrm{ms}^{-1})$ ***A1***

METHOD 2

$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{40}(60 - v)$ ***(M1)***

(or equivalent) ***M1***

$\frac{\mathrm{d}t}{\mathrm{d}v} = \frac{40}{60 - v}$

where

$\int_0^{v_f} \frac{40}{60 - v} \mathrm{d}v = 30$

is the velocity of the car after 30 seconds. ***A1A1***

v_f

attempting to solve

for

$\int_0^{v_f} \frac{40}{60 - v} \mathrm{d}v = 30$ ***(M1)***

v_f

$v = 31.7\ (\mathrm{ms}^{-1})$ ***A1***

[6 marks]

Examiners report

Most candidates experienced difficulties with this question. A large number of candidates did not attempt to separate the variables and instead either attempted to integrate with respect to v or employed constant acceleration formulae. Candidates that did separate the variables and attempted to integrate both sides either made a sign error, omitted the constant of integration or found an incorrect value for this constant. Almost all candidates were not aware that this question could be solved readily on a GDC.

18a. *[10 marks]*

Markscheme

(i) **METHOD 1**

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\sin x + \cos x$ ***A1***

$y\frac{\mathrm{d}y}{\mathrm{d}x} = (\cos x + \sin x)(-\sin x + \cos x)$ ***M1***

$= \cos^2 x - \sin^2 x$ ***A1***

$= \cos 2x$ ***AG***

METHOD 2

$y^2 = (\sin x + \cos x)^2$ ***A1***

$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2(\cos x + \sin x)(\cos x - \sin x)$ ***M1***

$y\frac{\mathrm{d}y}{\mathrm{d}x} = \cos^2 x - \sin^2 x$ ***A1***

$= \cos 2x$ ***AG***

(ii) attempting to separate variables

$\int y\ \mathrm{d}y = \int \cos 2x\ \mathrm{d}x$ ***M1***

$\frac{1}{2}y^2 = \frac{1}{2}\sin 2x + C$ ***A1A1***

Note: Award ***A1*** for a correct LHS and ***A1*** for a correct RHS.

$y = \pm(\sin 2x + A)^{\frac{1}{2}}$ ***A1***

(iii)

$\sin 2x + A \equiv (\cos x + \sin x)^2$ ***(M1)***

$(\cos x + \sin x)^2 = \cos^2 x + 2\sin x\cos x + \sin^2 x$

use of $\sin 2x \equiv 2\sin x \cos x$. ***(M1)***

A = 1 ***A1***

[10 marks]

Examiners report

Part (a) was not well done and was often difficult to mark. In part (a) (i), a large number of candidates did not know how to verify a solution, $y(x)$, to the given differential equation. Instead, many candidates attempted to solve the differential equation. In part (a) (ii), a large number of candidates began solving the differential equation by correctly separating the variables but then either neglected to add a constant of integration or added one as an afterthought. Many simple algebraic and basic integral calculus errors were seen. In part (a) (iii), many candidates did not realize that the solution given in part (a) (i) and the general solution found in part (a) (ii) were to be equated. Those that did know to equate these two solutions, were able to square both solution forms and correctly use the trigonometric identity $\sin 2x = 2\sin x\cos x$. Many of these candidates however started with incorrect solution(s).

18b. *[12 marks]*

Markscheme

(i) substituting

$x = \frac{\pi}{4}$ and $y = 2$ into

$y = (\sin 2x + A)^{\frac{1}{2}}$ ***M1***

so

$g(x) = (\sin 2x + 3)^{\frac{1}{2}}$. ***A1***

range g is

$\left[\sqrt{2},\ 2\right]$ ***A1A1A1***

Note: Accept [1.41, 2]. Award ***A1*** for each correct endpoint and ***A1*** for the correct closed interval.

(ii)

$\int_0^{\frac{\pi}{2}} (\sin 2x + 3)^{\frac{1}{2}}\, \mathrm{d}x$ ***(M1)(A1)***

= 2.99 ***A1***

(iii)

$\pi \int_0^{\frac{\pi}{2}} (\sin 2x + 3)\mathrm{d}x - \pi(1)\left(\frac{\pi}{2}\right)$ (or equivalent) ***(M1)(A1)(A1)***

Note: Award ***(M1)(A1)(A1)*** for

$\pi \int_0^{\frac{\pi}{2}} (\sin 2x + 2)\mathrm{d}x$

$= 17.946 - 4.935\ (= \frac{\pi}{2}(3\pi + 2) - \pi\left(\frac{\pi}{2}\right))$ ***A1***

Note: Award ***A1*** for

$\pi(\pi + 1)$.

[12 marks]

Examiners report

In part (b), a large number of candidates knew how to find a required area and a required volume of solid of revolution using integral calculus. Many candidates, however, used incorrect expressions obtained in part (a). In part (b) (ii), a number of candidates either neglected to state 'π' or attempted to calculate the volume of a solid of revolution of 'radius'

$f(x) - g(x)$.

19a. *[2 marks]*

Markscheme

EITHER

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent) ***M1A1***

Note: Accept $\theta = 180° - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent).

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$ (or equivalent) ***M1A1***

[2 marks]

Examiners report

Part (a) was reasonably well done. While many candidates exhibited sound trigonometric knowledge to correctly express θ in terms of x, many other candidates were not able to use elementary trigonometry to formulate the required expression for θ.

19b. *[2 marks]*

Markscheme

(i)

$\theta = 0.994 \left(= \arctan \frac{20}{13}\right)$ ***A1***

(ii)

$\theta = 1.19 \left(= \arctan \frac{5}{2}\right)$ ***A1***

[2 marks]

Examiners report

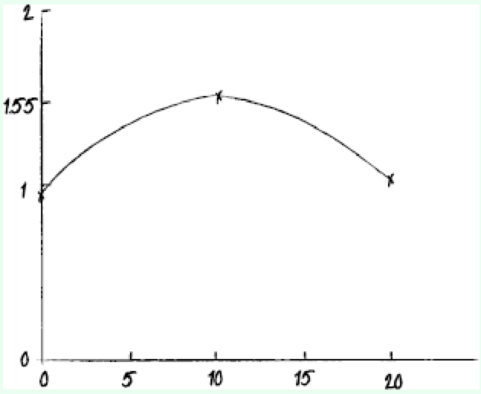
In part (b), a large number of candidates did not realize that θ could only be acute and gave obtuse angle values for θ. Many candidates also demonstrated a lack of insight when substituting endpoint x-values into θ.

19c. [2 marks]

Markscheme

correct shape. *A1*

correct domain indicated. *A1*



[2 marks]

Examiners report

In part (c), many candidates sketched either inaccurate or implausible graphs.

19d. [6 marks]

Markscheme

attempting to differentiate one $\arctan(f(x))$ term *M1*

EITHER

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ *A1A1*

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{8}{x^2} \times \frac{1}{1+\left(\frac{8}{x}\right)^2} - \frac{13}{(20-x)^2} \times \frac{1}{1+\left(\frac{13}{20-x}\right)^2}$

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$ *A1A1*

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{\frac{1}{8}}{1+\left(\frac{x}{8}\right)^2} + \frac{-\frac{1}{13}}{1+\left(\frac{20-x}{13}\right)^2}$

THEN

$= \frac{8}{x^2+64} - \frac{13}{569-40x+x^2}$ *A1*

$= \frac{8(569-40x+x^2)-13(x^2+64)}{(x^2+64)(x^2-40x+569)}$ *M1A1*

$= \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}$ *AG*

[6 marks]

Examiners report

In part (d), a large number of candidates started their differentiation incorrectly by failing to use the chain rule correctly.

19e. *[3 marks]*

Markscheme

Maximum light intensity at P occurs when

$\frac{\mathrm{d}\theta}{\mathrm{d}x}=0$ **(M1)**

either attempting to solve

$\frac{\mathrm{d}\theta}{\mathrm{d}x}=0$ for x or using the graph of either θ or $\frac{\mathrm{d}\theta}{\mathrm{d}x}$ **(M1)**

$x = 10.05$ (m) ***A1***

[3 marks]

Examiners report

For a question part situated at the end of the paper, part (e) was reasonably well done. A large number of candidates demonstrated a sound knowledge of finding where the maximum value of θ occurred and rejected solutions that were not physically feasible.

19f. *[4 marks]*

Markscheme

$\frac{\mathrm{d}x}{\mathrm{d}t}=0.5$ **(A1)**

At $x = 10$,

$\frac{\mathrm{d}\theta}{\mathrm{d}x}=0.000453\ \left(=\frac{5}{11029}\right)$ **(A1)**

use of

$\frac{\mathrm{d}\theta}{\mathrm{d}t}=\frac{\mathrm{d}\theta}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$ ***M1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t}=0.000227\ \left(=\frac{5}{22058}\right)\ (\mathrm{rad\ s^{-1}})$ ***A1***

Note: Award ***(A1)*** for $\frac{\mathrm{d}x}{\mathrm{d}t}=-0.5$ and ***A1*** for $\frac{\mathrm{d}\theta}{\mathrm{d}t}=-0.000227\ \left(=-\frac{5}{22058}\right)$.

Note: Implicit differentiation can be used to find $\frac{\mathrm{d}\theta}{\mathrm{d}t}$. Award as above.

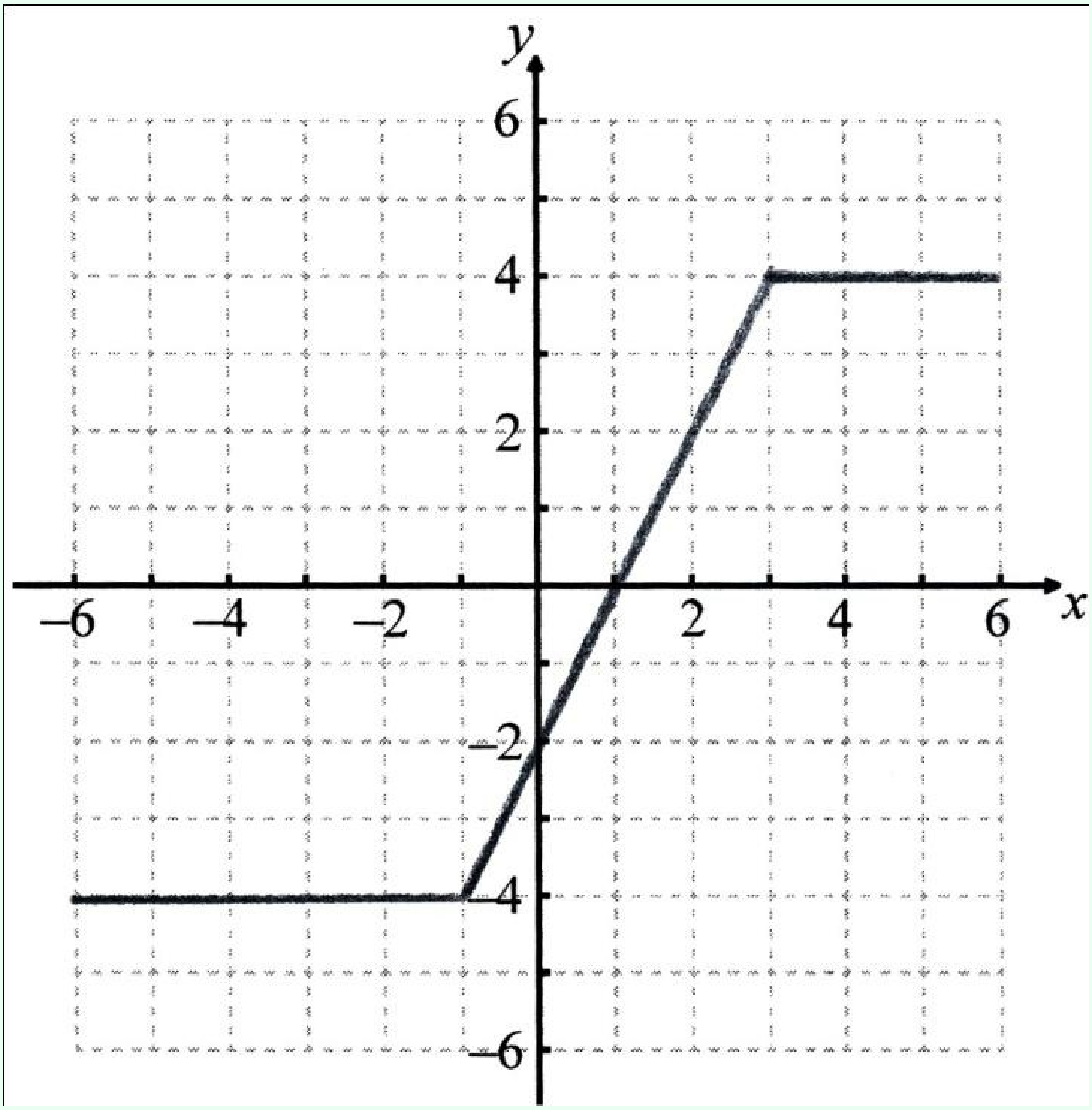
[4 marks]

Examiners report

In part (f), many candidates were able to link the required rates, however only a few candidates were able to successfully apply the chain rule in a related rates context.

20a. *[4 marks]*

Markscheme



M1A1A1A1

Note: Award ***M1*** for any of the three sections completely correct, ***A1*** for each correct segment of the graph. ***[4 marks]***

Examiners report

Most candidates were able to produce a good graph, and many were able to interpret that to get correct answers to part (b). The most common error was to give 4 as the answer to (b) (iii). Some candidates did not recognise that the "hence" in the question meant that they had to use their graph to obtain their answers to part (b).

20b. *[4 marks]*

Markscheme

(i) 0 ***A1***

(ii) 2 ***A1***

(iii) finding area of rectangle ***(M1)***

A1
-4

Note: Award ***M1A0*** for the answer 4.

[4 marks]

21a. *[3 marks]*

Markscheme

$\int_{\frac{1}{6}}^{1} \frac{k}{x} - \frac{1}{x}\mathrm{d}x = (k-1)[\ln x]_{\frac{1}{6}}^{1}$ ***M1A1***

Note: Award ***M1*** for $\int \frac{k}{x} - \frac{1}{x}\mathrm{d}x$ or $\int \frac{1}{x} - \frac{k}{x}\mathrm{d}x$ and ***A1*** for $(k-1)\ln x$ seen in part (a) or later in part (b).

$= (1-k)\ln\frac{1}{6}$ ***A1***

[3 marks]

Examiners report

Generally well answered by most candidates. Basic algebra sometimes let students down in the simplification of the ratio in part (c). It was not uncommon to see $\frac{\log A}{\log B}$ simplified to $\frac{A}{B}$

21b. *[2 marks]*

Markscheme

$\int_{1}^{\sqrt{6}} \frac{k}{x} - \frac{1}{x}\mathrm{d}x = (k-1)[\ln x]_{1}^{\sqrt{6}}$ ***(A1)***

Note: Award ***A1*** for correct change of limits.

$= (k-1)\ln\sqrt{6}$ ***A1***

[2 marks]

Examiners report

Generally well answered by most candidates. Basic algebra sometimes let students down in the simplification of the ratio in part (c). It was not uncommon to see $\frac{\log A}{\log B}$ simplified to $\frac{A}{B}$

21c. *[3 marks]*

Markscheme

$(1-k)\ln\frac{1}{6} = (k-1)\ln 6$ ***A1***

$(k-1)\ln\sqrt{6} = \frac{1}{2}(k-1)\ln 6$ ***A1***

Note: This simplification could have occurred earlier, and marks should still be awarded.

ratio is 2 (or 2:1) ***A1***

[3 marks]

Examiners report

Generally well answered by most candidates. Basic algebra sometimes let students down in the simplification of the ratio in part (c). It was not uncommon to see $\frac{\log A}{\log B}$ simplified to $\frac{A}{B}$

22.

[9 marks]

Markscheme

$4x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x}{y}$ ***M1A1***

Note: Allow follow through on incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$ from this point.

gradient of normal at $(a,\ b)$ is $\frac{b}{2a}$

Note: No further A marks are available if a general point is not used

equation of normal at $(a,\ b)$ is

$y - b = \frac{b}{2a}(x - a)\ \left(\Rightarrow y = \frac{b}{2a}x + \frac{b}{2}\right)$ ***M1A1***

substituting $(1,\ 0)$ ***M1***

$b = 0$ or $a = -1$ ***A1A1***

four points are

$(3,\ 0),\ (-3,\ 0),\ (-1,\ 4),\ (-1,\ -4)$ ***A1A1***

Note: Award ***A1A0*** for any two points correct.

[9 marks]

Examiners report

Many students were able to obtain the first marks in this question by implicit differentiation but few were able to complete the question successfully. There were a number of students obtaining the correct final answers, but could not be given the marks due to incorrect working. Most common was students giving the equation of the normal as $y - 0 = \frac{y}{2x}(x - 1)$, instead of taking a general point e.g. $(a,\ b)$

23a.

[5 marks]

Markscheme

EITHER

derivative of $\frac{x}{1-x}$ is $\frac{(1-x) - x(-1)}{(1-x)^2}$ ***M1A1***

$f'(x) = \frac{1}{2}\left(\frac{x}{1-x}\right)^{-\frac{1}{2}}\frac{1}{(1-x)^2}$ ***M1A1***

$= \frac{1}{2}x^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}$ ***AG***

$f'(x) > 0$ (for all $0 < x < 1$) so the function is increasing ***R1***

OR

$f(x) = \frac{x^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}$

$f'(x) = \frac{(1-x)^{\frac{1}{2}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - \frac{1}{2}x^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}(-1)}{1-x}$ ***M1A1***

$= \frac{1}{2}x^{-\frac{1}{2}}(1-x)^{-\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}}(1-x)^{-\frac{3}{2}}$ ***A1***

$= \frac{1}{2}x^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}[1 - x + x]$ ***M1***

$= \frac{1}{2}x^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}$ ***AG***

$f'(x) > 0$ (for all $0 < x < 1$) so the function is increasing ***R1***

[5 marks]

Examiners report

Part (a) was generally well done, although few candidates made the final deduction asked for. Those that lost other marks in this part were generally due to mistakes in algebraic manipulation. In part (b) whilst many students found the second derivative and set it equal to zero, few then confirmed that it was a point of inflexion. There were several good attempts for part (c), even though there were various points throughout the question that provided stopping points for other candidates.

23b. *[6 marks]*

Markscheme

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}$

M1A1

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1-x)^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{1}{2}}(1-x)^{-\frac{5}{2}}$

$= -\frac{1}{4}x^{-\frac{3}{2}}(1-x)^{-\frac{5}{2}}[1-4x]$

M1A1

$f''(x) = 0 \Rightarrow x = \frac{1}{4}$

$f''(x)$ changes sign at $x = \frac{1}{4}$ hence there is a point of inflexion ***R1***

A1

$x = \frac{1}{4} \Rightarrow y = \frac{1}{\sqrt{3}}$

the coordinates are $\left(\frac{1}{4}, \frac{1}{\sqrt{3}}\right)$

[6 marks]

Examiners report

Part (a) was generally well done, although few candidates made the final deduction asked for. Those that lost other marks in this part were generally due to mistakes in algebraic manipulation. In part (b) whilst many students found the second derivative and set it equal to zero, few then confirmed that it was a point of inflexion. There were several good attempts for part (c), even though there were various points throughout the question that provided stopping points for other candidates.

23c. *[11 marks]*

Markscheme

M1A1

$x = \sin^2\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sin\theta\cos\theta$

M1A1

$\int\sqrt{\frac{x}{1-x}}\mathrm{d}x = \int\sqrt{\frac{\sin^2\theta}{1-\sin^2\theta}}2\sin\theta\cos\theta\mathrm{d}\theta$

A1

$= \int 2\sin^2\theta\mathrm{d}\theta$

M1A1

$= \int 1-\cos 2\theta\mathrm{d}\theta$

A1

$= \theta - \frac{1}{2}\sin 2\theta + c$

A1

$\theta = \arcsin\sqrt{x}$

M1A1

$\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta = \sqrt{x}\sqrt{1-x} = \sqrt{x-x^2}$

hence

AG

$\int\sqrt{\frac{x}{1-x}}\mathrm{d}x = \arcsin\sqrt{x} - \sqrt{x-x^2} + c$

[11 marks]

Examiners report

Part (a) was generally well done, although few candidates made the final deduction asked for. Those that lost other marks in this part were generally due to mistakes in algebraic manipulation. In part (b) whilst many students found the second derivative and set it equal to zero, few then confirmed that it was a point of inflexion. There were several good attempts for part (c), even though there were various points throughout the question that provided stopping points for other candidates.

24.

[5 marks]

Markscheme

$\frac{dy}{dx} = 3x^2 - 12x + k$ ***M1A1***

For use of discriminant $b^2 - 4ac = 0$ or completing the square $3(x-2)^2 + k - 12$ ***(M1)***

$144 - 12k = 0$ ***(A1)***

Note: Accept trial and error, sketches of parabolas with vertex (2,0) or use of second derivative.

$k = 12$ ***A1***

[5 marks]

Examiners report

Generally candidates answer this question well using a diversity of methods. Surprisingly, a small number of candidates were successful in answering this question using the discriminant of the quadratic and in many cases reverted to trial and error to obtain the correct answer.

25.

[9 marks]

Markscheme

$x = r - \frac{r}{h}y$ or $x = \frac{r}{h}(h - y)$ (or equivalent) ***(A1)***

$\int \pi x^2 \mathrm{d}y$

$= \pi \int_0^h \left(r - \frac{r}{h}y\right)^2 \mathrm{d}y$ ***M1A1***

Note: Award ***M1*** for $\int x^2 \mathrm{d}y$ and ***A1*** for correct expression.

Accept

$\pi \int_0^h \left(\frac{r}{h}y - r\right)^2 \mathrm{d}y$ and $\pi \int_0^h \left(\pm\left(r - \frac{r}{h}x\right)\right)^2 \mathrm{d}x$

$= \pi \int_0^h \left(r^2 - \frac{2r^2}{h}y + \frac{r^2}{h^2}y^2\right) \mathrm{d}y$ ***A1***

Note: Accept substitution method and apply markscheme to corresponding steps.

$= \pi \left[r^2 y - \frac{r^2y^2}{h} + \frac{r^2y^3}{3h^2}\right]_0^h$ ***M1A1***

Note: Award ***M1*** for attempted integration of any quadratic trinomial.

$= \pi \left(r^2h - r^2h + \frac{1}{3}r^2h\right)$ ***M1A1***

Note: Award ***M1*** for attempted substitution of limits in a trinomial.

$= \frac{1}{3}\pi r^2 h$ ***A1***

Note: Throughout the question do not penalize missing d*x*/d*y* as long as the integrations are done with respect to correct variable.

[9 marks]

Examiners report

Most candidates attempted this question using either the formula given in the information booklet or the disk method. However, many were not successful, either because they started off with the incorrect expression or incorrect integration limits or even attempted to integrate the correct expression with respect to the incorrect variable.

26. *[8 marks]*

Markscheme

attempt to find intersections ***M1***

intersections are

A1A1

$\left(\frac{10}{m+2}, \frac{10m}{m+2}\right)$ and $\left(\frac{10m}{2m-1}, -\frac{10}{2m-1}\right)$

area of triangle

M1A1A1

$= \frac{1}{2} \times \frac{\sqrt{100+100m^2}}{(m+2)} \times \frac{\sqrt{100+100m^2}}{(2m-1)}$

$= \frac{50(1+m^2)}{(m+2)(2m-1)}$

minimum when

(M1)A1

$m = 3$

[8 marks]

Examiners report

Most candidates had difficulties with this question and did not go beyond the determination of the intersection points of the lines; in a few cases candidates set up the expression of the area, in some cases using unsimplified expressions of the coordinates.

27a. *[1 mark]*

Markscheme

A1

$(3.79, -5)$

[1 mark]

Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

27b. *[2 marks]*

Markscheme

A1A1

$p = 1.57$ or $\frac{\pi}{2}$, $q = 6.00$

[2 marks]

Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

27c. *[4 marks]*

Markscheme

(M1)(A1)

$f'(x) = 3\cos x - 4\sin x$

(A1)

$3\cos x - 4\sin x = 3 \Rightarrow x = 4.43...$

A1

$(y = -4)$

Coordinates are

$(4.43, -4)$

[4 marks]

Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

27d. *[7 marks]*

Markscheme

$m_{\text{normal}} = \frac{1}{m_{\text{tangent}}}$ ***(M1)***

gradient at P is $\frac{1}{4}$ so gradient of normal at P is -4 ***(A1)***

gradient at Q is 4 so gradient of normal at Q is $-\frac{1}{4}$ ***(A1)***

equation of normal at P is $y-3=\frac{1}{4}(x-1.570...)$ (or $y=0.25x+2.60...$) ***(M1)***

equation of normal at Q is $y-3=\frac{1}{4}(x-5.999...)$ (or $y=-0.25x+4.499...$) ***(M1)***

Note: Award the previous two ***M1*** even if the gradients are incorrect in $y-b=m(x-a)$ where (a,b) are coordinates of P and Q (or in $y=mx+c$ with c determined using coordinates of P and Q.

intersect at $(3.79,\ 3.55)$ ***A1A1***

Note: Award ***N2*** for 3.79 without other working.

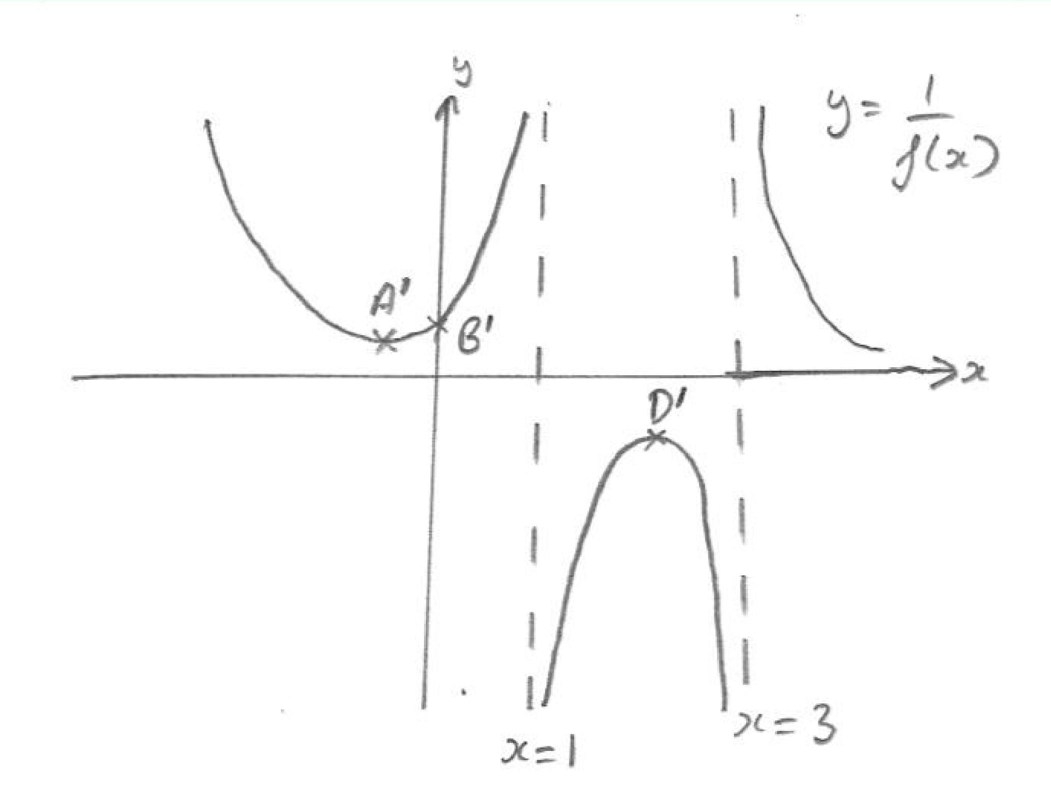
[7 marks]

Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

28a. *[3 marks]*

Markscheme



A1A1A1

Note: Award ***A1*** for correct shape.
Award ***A1*** for two correct asymptotes, and

and

$x = 1$

$x = 3$

Award ***A1*** for correct coordinates,

$\mathrm{A}'\left(-1, \frac{1}{4}\right)$, $\mathrm{B}'\left(0, \frac{1}{3}\right)$ and $\mathrm{D}'\left(2, -\frac{1}{3}\right)$

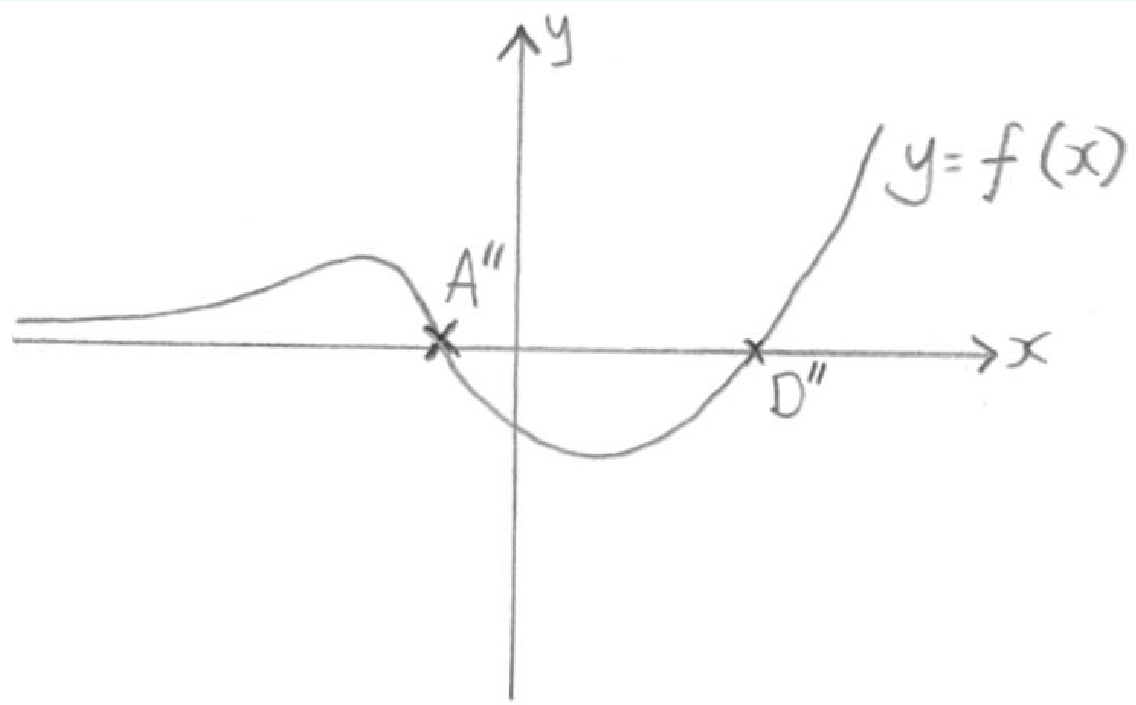
[3 marks]

Examiners report

Solutions to this question were generally disappointing. In (a), the shape of the graph was often incorrect and many candidates failed to give the equations of the asymptotes and the coordinates of the image points. In (b), many candidates produced incorrect graphs although the coordinates of the image points were often given correctly.

28b. [3 marks]

Markscheme



A1A1A1

Note: Award *A1* for correct general shape including the horizontal asymptote.
Award *A1* for recognition of 1 maximum point and 1 minimum point.
Award *A1* for correct coordinates, $A''(-1,0)$ and $D''(2,0)$.

[3 marks]

Examiners report

Solutions to this question were generally disappointing. In (a), the shape of the graph was often incorrect and many candidates failed to give the equations of the asymptotes and the coordinates of the image points. In (b), many candidates produced incorrect graphs although the coordinates of the image points were often given correctly.

29. [6 marks]

Markscheme

$x^3y = a\sin nx$

attempt to differentiate implicitly *M1*

$\Rightarrow 3x^2y + x^3\frac{\mathrm{d}y}{\mathrm{d}x} = an\cos nx$ *A2*

Note: Award *A1* for two out of three correct, *A0* otherwise.

$\Rightarrow 6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x} + x^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -an^2\sin nx$ *A2*

Note: Award *A1* for three or four out of five correct, *A0* otherwise.

$\Rightarrow 6xy + 6x^2\frac{\mathrm{d}y}{\mathrm{d}x} + x^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -an^2\sin nx$ *A1*

$\Rightarrow x^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 6x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 6xy + n^2x^3y = 0$

$\Rightarrow x^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 6x^2\frac{\mathrm{d}y}{\mathrm{d}x} + (n^2x^2+6)xy = 0$ *AG*

[6 marks]

Examiners report

Candidates who are comfortable using implicit differentiation found this to be a fairly straightforward question and were able to answer it in just a few lines. Many candidates, however, were unable to differentiate x^3y with respect to x and were therefore unable to proceed. Candidates whose first step was to write $y = \frac{a\sin nx}{x^3}$ were given no credit since the question required the use of implicit differentiation.

30a. *[1 mark]*

Markscheme

$e^{-x}\cos x = 0$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1***

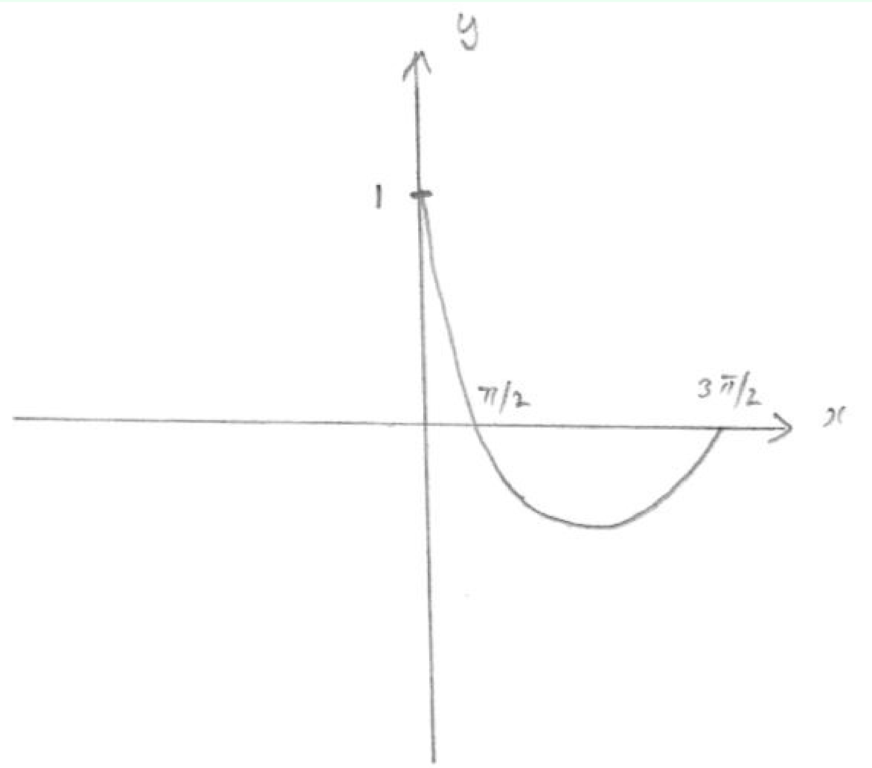
[1 mark]

Examiners report

Many candidates stated the two zeros of *f* correctly but the graph of *f* was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

30b. *[1 mark]*

Markscheme



A1

Note: Accept any form of concavity for

$x \in \left[0, \frac{\pi}{2}\right]$.

Note: Do not penalize unmarked zeros if given in part (a).

Note: Zeros written on diagram can be used to allow the mark in part (a) to be awarded retrospectively.

[1 mark]

Examiners report

Many candidates stated the two zeros of *f* correctly but the graph of *f* was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

30c. *[7 marks]*

Markscheme

attempt at integration by parts ***M1***

EITHER

$I = \int e^{-x}\cos x\,dx = -e^{-x}\cos x\,dx - \int e^{-x}\sin x\,dx$ ***A1***

$\Rightarrow I = -e^{-x}\cos x\,dx - \left[-e^{-x}\sin x + \int e^{-x}\cos x\,dx\right]$ ***A1***

$\Rightarrow I = \frac{e^{-x}}{2}(\sin x - \cos x) + C$ ***A1***

Note: Do not penalize absence of C.

OR

$I = \int e^{-x}\cos x\,dx = e^{-x}\sin x + \int e^{-x}\sin x\,dx$ ***A1***

$I = e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\,dx$ ***A1***

$\Rightarrow I = \frac{e^{-x}}{2}(\sin x - \cos x) + C$ ***A1***

Note: Do not penalize absence of C.

THEN

$\int_0^{\frac{\pi}{2}} e^{-x}\cos x\,dx = \left[\frac{e^{-x}}{2}(\sin x - \cos x)\right]_0^{\frac{\pi}{2}} = \frac{e^{-\frac{\pi}{2}}}{2} + \frac{1}{2}$ ***A1***

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} e^{-x}\cos x\,dx = \left[\frac{e^{-x}}{2}(\sin x - \cos x)\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = -\frac{e^{-\frac{3\pi}{2}}}{2} - \frac{e^{-\frac{\pi}{2}}}{2}$ ***A1***

ratio of $A:B$ is

$\frac{\frac{e^{-\frac{\pi}{2}}}{2}+\frac{1}{2}}{-\frac{e^{-\frac{3\pi}{2}}}{2}-\frac{e^{-\frac{\pi}{2}}}{2}}$ ***M1***

$= -\frac{e^{\frac{3\pi}{2}}\left(e^{-\frac{\pi}{2}}+1\right)}{e^{\frac{3\pi}{2}}\left(e^{-\frac{3\pi}{2}}+e^{-\frac{\pi}{2}}\right)}$ ***AG***

[7 marks]

Examiners report

Many candidates stated the two zeros of *f* correctly but the graph of *f* was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

31a. *[4 marks]*

Markscheme

let $f(x) = \frac{1}{2x+1}$ and using the result

$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ ***M1A1***

$f'(x) = \lim_{h\to 0}\left(\frac{\frac{1}{2(x+h)+1}-\frac{1}{2x+1}}{h}\right)$ ***A1***

$\Rightarrow f'(x) = \lim_{h\to 0}\left(\frac{[2x+1]-[2(x+h)+1]}{h[2(x+h)+1][2x+1]}\right)$ ***A1***

$\Rightarrow f'(x) = \lim_{h\to 0}\left(\frac{-2}{[2(x+h)+1][2x+1]}\right)$

$\Rightarrow f'(x) = \frac{-2}{(2x+1)^2}$ ***AG***

[4 marks]

Examiners report

Even though the definition of the derivative was given in the question, solutions to (a) were often disappointing with algebraic errors fairly common, usually due to brackets being omitted or manipulated incorrectly. Solutions to the proof by induction in (b) were often poor. Many candidates fail to understand that they have to assume that the result is true for $n = k$ and then show that this leads to it being true for $n = k+1$. Many candidates just write 'Let $n = k$' which is of course meaningless. The conclusion is often of the form 'True for $n = 1$, $n = k$ and $n = k+1$ therefore true by induction'. Credit is only given for a conclusion which includes a statement such as 'True for $n = k \Rightarrow$ true for $n = k+1$'.

31b. *[9 marks]*

Markscheme

let

$y = \frac{1}{2x+1}$

we want to prove that

$\frac{\mathrm{d}^n y}{\mathrm{d}x^n} = (-1)^n \frac{2^n n!}{(2x+1)^{n+1}}$

let $n = 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = (-1)^1 \frac{2^1 1!}{(2x+1)^{1+1}}$ ***M1***

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{(2x+1)^2}$ which is the same result as part (a)

hence the result is true for $n = 1$ ***R1***

assume the result is true for $n = k$: $\frac{\mathrm{d}^k y}{\mathrm{d}x^k} = (-1)^k \frac{2^k k!}{(2x+1)^{k+1}}$ ***M1***

$\frac{\mathrm{d}^{k+1} y}{\mathrm{d}x^{k+1}} = \frac{\mathrm{d}}{\mathrm{d}x}\left[(-1)^k \frac{2^k k!}{(2x+1)^{k+1}}\right]$ ***M1***

$\Rightarrow \frac{\mathrm{d}^{k+1} y}{\mathrm{d}x^{k+1}} = \frac{\mathrm{d}}{\mathrm{d}x}\left[(-1)^k 2^k k!(2x+1)^{-k-1}\right]$ ***(A1)***

$\Rightarrow \frac{\mathrm{d}^{k+1} y}{\mathrm{d}x^{k+1}} = (-1)^k 2^k k!(-k-1)(2x+1)^{-k-2} \times 2$ ***A1***

$\Rightarrow \frac{\mathrm{d}^{k+1} y}{\mathrm{d}x^{k+1}} = (-1)^{k+1} 2^{k+1}(k+1)!(2x+1)^{-k-2}$ ***(A1)***

$\Rightarrow \frac{\mathrm{d}^{k+1} y}{\mathrm{d}x^{k+1}} = (-1)^{k+1} \frac{2^{k+1}(k+1)!}{(2x+1)^{k+2}}$ ***A1***

hence if the result is true for $n = k$, it is true for $n = k+1$

since the result is true for $n = 1$, the result is proved by mathematical induction ***R1***

Note: Only award final ***R1*** if all the ***M*** marks have been gained.

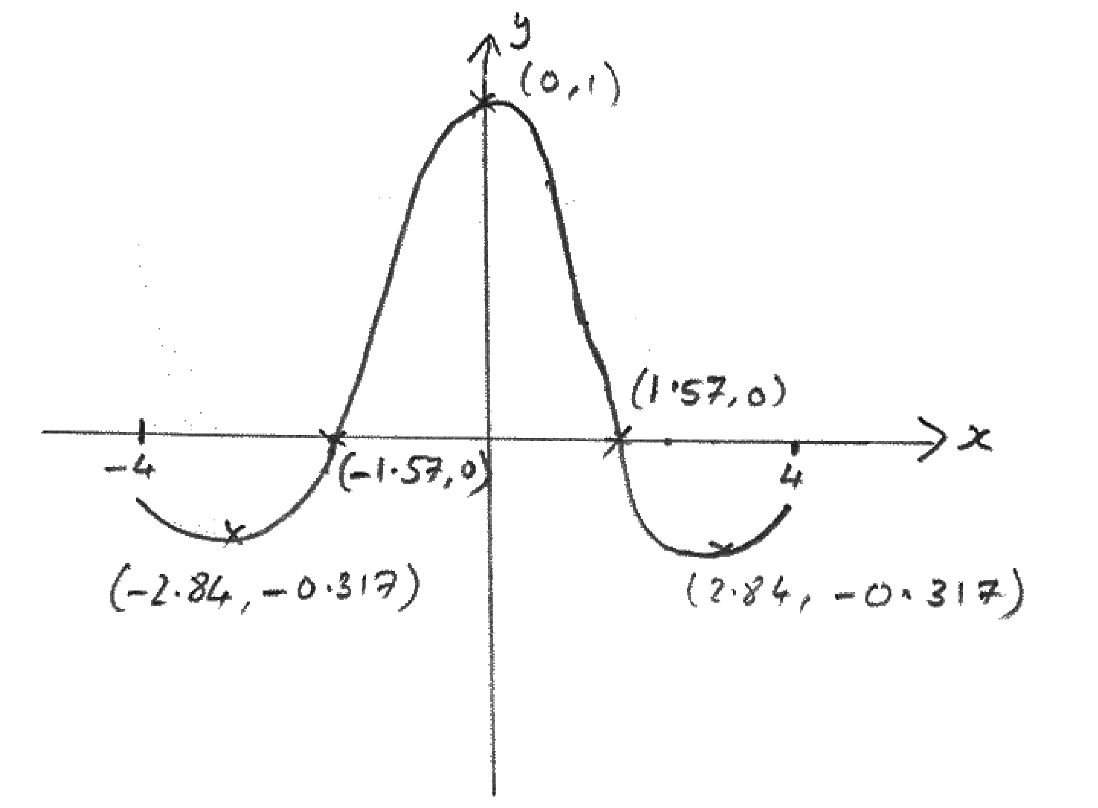
[9 marks]

Examiners report

Even though the definition of the derivative was given in the question, solutions to (a) were often disappointing with algebraic errors fairly common, usually due to brackets being omitted or manipulated incorrectly. Solutions to the proof by induction in (b) were often poor. Many candidates fail to understand that they have to assume that the result is true for $n = k$ and then show that this leads to it being true for $n = k+1$. Many candidates just write 'Let $n = k$' which is of course meaningless. The conclusion is often of the form 'True for $n = 1$, $n = k$ and $n = k+1$ therefore true by induction'. Credit is only given for a conclusion which includes a statement such as 'True for $n = k \Rightarrow$ true for $n = k+1$'.

32a. *[4 marks]*

Markscheme



A1A1A1A1

Note: Award ***A1*** for correct shape. Do not penalise if too large a domain is used,
A1 for correct x-intercepts,
A1 for correct coordinates of two minimum points,
A1 for correct coordinates of maximum point.
Accept answers which correctly indicate the position of the intercepts, maximum point and minimum points.

[4 marks]

Examiners report

Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

32b. *[1 mark]*

Markscheme

gradient at $x = 1$ is -0.786 ***A1***
[1 mark]

Examiners report

Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

32c. *[3 marks]*

Markscheme

gradient of normal is

$\frac{-1}{0.786}\ (= 1.272\ldots)$ ***(A1)***

when $x = 1$, $y = 0.3820\ldots$ ***(A1)***

Equation of normal is $y – 0.382 = 1.27(x – 1)$ ***A1***

$(\Rightarrow y = 1.27x - 0.890)$

[3 marks]

Examiners report

Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

33a. *[7 marks]*

Markscheme

$\frac{dv}{dt} = -v^2 - 1$

attempt to separate the variables ***M1***

$\int \frac{1}{1+v^2} dv = \int -1 dt$ ***A1***

$\arctan v = -t + k$ ***A1A1***

Note: Do not penalize the lack of constant at this stage.

when $t = 0$, $v = 1$ ***M1***

$\Rightarrow k = \arctan 1 = \left(\frac{\pi}{4}\right) = (45^\circ)$ ***A1***

$\Rightarrow v = \tan\left(\frac{\pi}{4} - t\right)$ ***A1***

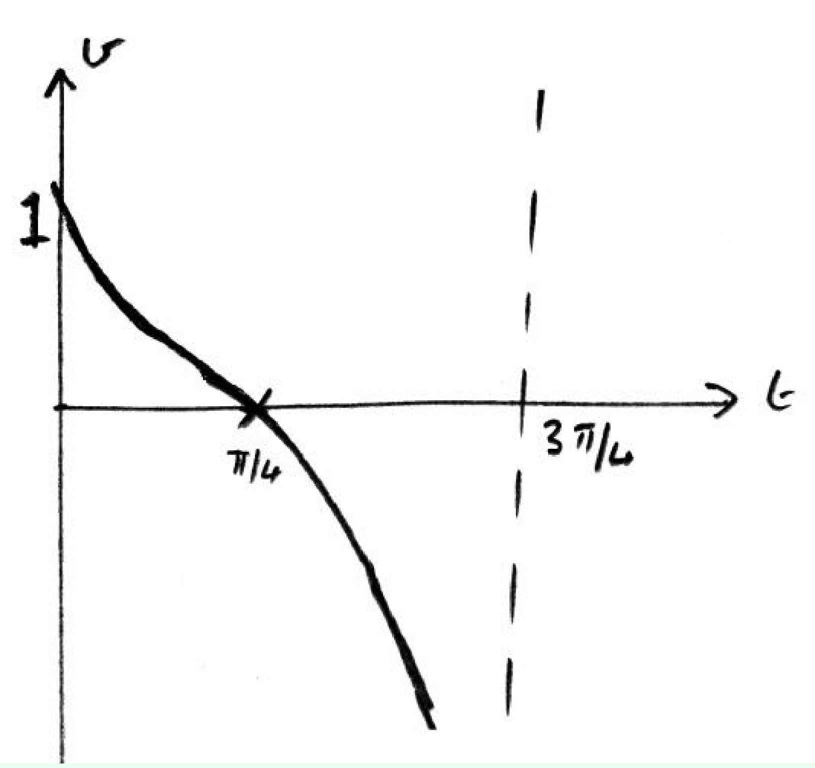
[7 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

33b. *[3 marks]*

Markscheme



A1A1A1

Note: Award ***A1*** for general shape,

A1 for asymptote,

A1 for correct t and v intercept.

Note: Do not penalise if a larger domain is used.

[3 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

33c. *[3 marks]*

Markscheme

(i)

$T = \frac{\pi}{4}$ ***A1***

(ii) area under curve

$= \int_0^{\frac{\pi}{4}} \tan\left(\frac{\pi}{4} - t\right)\mathrm{d}t$ ***(M1)***

$= 0.347 \left(= \frac{1}{2}\ln 2\right)$ ***A1***

[3 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

33d. *[5 marks]*

Markscheme

$v = \tan\left(\frac{\pi}{4} - t\right)$ ***M1***

$s = \int \tan\left(\frac{\pi}{4} - t\right)\mathrm{d}t$ ***(M1)***

$\int \frac{\sin\left(\frac{\pi}{4} - t\right)}{\cos\left(\frac{\pi}{4} - t\right)}\,\mathrm{d}t$ ***A1***

$= \ln\cos\left(\frac{\pi}{4} - t\right) + k$

when

$t = 0,\ s = 0$ ***A1***

$k = -\ln\cos\frac{\pi}{4}$ ***A1***

$s = \ln\cos\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4}\left(= \ln\left[\sqrt{2}\cos\left(\frac{\pi}{4} - t\right)\right]\right)$

[5 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

33e. *[4 marks]*

Markscheme

METHOD 1

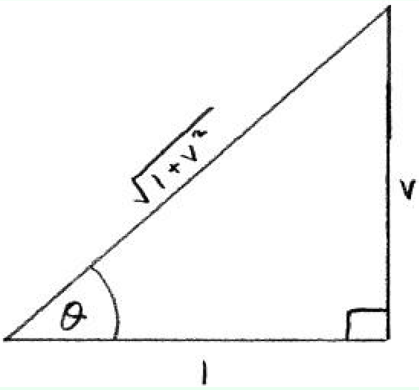
M1

$\frac{\pi}{4} - t = \arctan v$

$t = \frac{\pi}{4} - \arctan v$

$s = \ln\left[\sqrt{2}\cos\left(\frac{\pi}{4} - \frac{\pi}{4} + \arctan v\right)\right]$ *M1A1*

$s = \ln\left[\sqrt{2}\cos(\arctan v)\right]$



A1

$s = \ln\left[\sqrt{2}\cos\left(\arccos\frac{1}{\sqrt{1+v^2}}\right)\right]$

$= \ln\frac{\sqrt{2}}{\sqrt{1+v^2}}$ *AG*

$= \frac{1}{2}\ln\frac{2}{1+v^2}$

METHOD 2

$s = \ln\cos\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4}$ *M1*

$= -\ln\sec\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4}$ *M1*

$= -\ln\sqrt{1 + \tan^2\left(\frac{\pi}{4} - t\right)} - \ln\cos\frac{\pi}{4}$ *A1*

$= -\ln\sqrt{1+v^2} - \ln\cos\frac{\pi}{4}$ *A1*

$= \ln\frac{1}{\sqrt{1+v^2}} + \ln\sqrt{2}$ *AG*

$= \frac{1}{2}\ln\frac{2}{1+v^2}$

METHOD 3

M1

$v\frac{dv}{ds} = -v^2 - 1$ *M1*

$\int\frac{v}{v^2+1}dv = -\int 1 ds$ *A1*

$\frac{1}{2}\ln(v^2 + 1) = -s + k$

when

$s = 0,\ t = 0 \Rightarrow v = 1$ *A1*

$\Rightarrow k = \frac{1}{2}\ln 2$ *AG*

$\Rightarrow s = \frac{1}{2}\ln\frac{2}{1+v^2}$

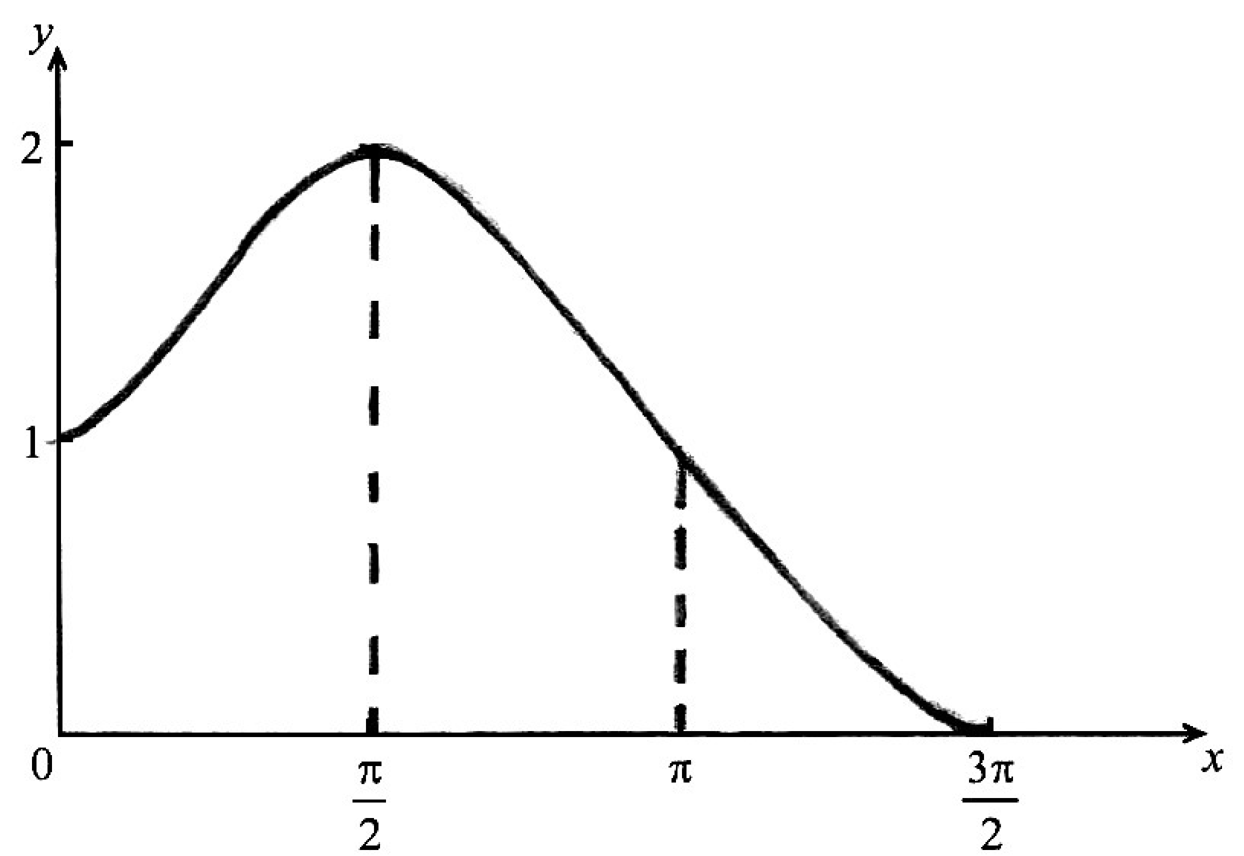
[4 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

34a. [1 mark]

Markscheme



A1

[1 mark]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of

and the insertion of the limits.
$\cos(2x)$

34b. [1 mark]

Markscheme

$(1+\sin x)^2 = 1 + 2\sin x + \sin^2 x$
A1
$= 1 + 2\sin x + \frac{1}{2}(1 - \cos 2x)$
AG
$= \frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x$
[1 mark]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of

and the insertion of the limits.
$\cos(2x)$

34c. *[4 marks]*

Markscheme

$V = \pi \int_0^{\frac{3\pi}{2}} (1 + \sin x)^2 \mathrm{d}x$ ***(M1)***

$= \pi \int_0^{\frac{3\pi}{2}} \left(\frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x\right) \mathrm{d}x$

$= \pi \left[\frac{3}{2}x - 2\cos x - \frac{\sin 2x}{4}\right]_0^{\frac{3\pi}{2}}$ ***A1***

$= \frac{9\pi^2}{4} + 2\pi$ ***A1A1***

[4 marks]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of $\cos(2x)$ and the insertion of the limits.

35. *[7 marks]*

Markscheme

proposition is true for *n* = 1 since

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{(1-x)^2}$ ***M1***

$= \frac{1!}{(1-x)^2}$ ***A1***

Note: Must see the 1! for the ***A1***.

assume true for *n* = *k*, $k \in \mathbb{Z}^+$, *i.e.* $\frac{\mathrm{d}^k y}{\mathrm{d}x^k} = \frac{k!}{(1-x)^{k+1}}$ ***M1***

consider

$\frac{\mathrm{d}^{k+1} y}{\mathrm{d}x^{k+1}} = \frac{\mathrm{d}\left(\frac{\mathrm{d}^k y}{\mathrm{d}x^k}\right)}{\mathrm{d}x}$ ***(M1)***

$= (k+1)k!(1-x)^{-(k+1)-1}$ ***A1***

$= \frac{(k+1)!}{(1-x)^{k+2}}$ ***A1***

hence, P_{k+1} is true whenever P_k is true, and P_1 is true, and therefore the proposition is true for all positive integers ***R1***

Note: The final ***R1*** is only available if at least 4 of the previous marks have been awarded.

[7 marks]

Examiners report

Most candidates were awarded good marks for this question. A disappointing minority thought that the $(k+1)$th derivative was the (k)th derivative multiplied by the first derivative. Providing an acceptable final statement remains a perennial issue.

36. *[7 marks]*

Markscheme

to find the points of intersection of the two curves

M1
$-x^2+2=x^3-x^2-bx+2$

$x^3-bx=x(x^2-b)=0$

A1A1
$\Rightarrow x=0;\ x=\pm\sqrt{b}$

M1
$A_1=\int_{-\sqrt{b}}^{0}\left[(x^3-x^2-bx+2)-(-x^2+2)\right]\mathrm{d}x\ \left(=\int_{-\sqrt{b}}^{0}(x^3-bx)\mathrm{d}x\right)$

$=\left[\frac{x^4}{4}-\frac{bx^2}{2}\right]_{-\sqrt{b}}^{0}$

A1
$=-\left(\frac{(-\sqrt{b})^4}{4}-\frac{b(-\sqrt{b})^2}{2}\right)=-\frac{b^2}{4}+\frac{b^2}{2}=\frac{b^2}{4}$

M1
$A_2=\int_0^{\sqrt{b}}\left[(-x^2+2)-(x^3-x^2-bx+2)\right]\mathrm{d}x$

$=\int_0^{\sqrt{b}}(-x^3+bx)\mathrm{d}x$

A1
$=\left[-\frac{x^4}{4}+\frac{bx^2}{2}\right]_0^{\sqrt{b}}=\frac{b^2}{4}$

therefore

AG
$A_1=A_2=\frac{b^2}{4}$

[7 marks]

Examiners report

Most candidates knew how to tackle this question. The most common error was in giving $+b$ and $-b$ as the x-coordinates of the point of intersection.

37a. *[3 marks]*

Markscheme

angle APB is a right angle

A1
$\Rightarrow \cos\theta=\frac{\mathrm{AP}}{4}\Rightarrow \mathrm{AP}=4\cos\theta$

Note: Allow correct use of cosine rule.

A1
$\text{arc PB}=2\times 2\theta=4\theta$

M1
$t=\frac{\mathrm{AP}}{3}+\frac{\mathrm{PB}}{6}$

Note: Allow use of their AP and their PB for the ***M1***.

AG
$\Rightarrow t=\frac{4\cos\theta}{3}+\frac{4\theta}{6}=\frac{4\cos\theta}{3}+\frac{2\theta}{3}=\frac{2}{3}(2\cos\theta+\theta)$

[3 marks]

Examiners report

The fairly easy trigonometry challenged a large number of candidates.

37b. *[2 marks]*

Markscheme

A1
$\frac{\mathrm{d}t}{\mathrm{d}\theta} = \frac{2}{3}(-2\sin\theta + 1)$
(or 30 degrees) ***A1***
$\frac{2}{3}(-2\sin\theta + 1) = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$
[2 marks]

Examiners report

Part (b) was very well done.

37c. *[3 marks]*

Markscheme

M1
$\frac{\mathrm{d}^2t}{\mathrm{d}\theta^2} = -\frac{4}{3}\cos\theta < 0 \ \left(\text{at } \theta = \frac{\pi}{6}\right)$
is maximized at
$\Rightarrow t$

R1
$\theta = \frac{\pi}{6}$
time needed to walk along arc AB is

$\frac{2\pi}{6}\ (\approx 1 \text{ hour})$
time needed to row from A to B is

$\frac{4}{3}\ (\approx 1.33 \text{ hour})$
hence, time is minimized in walking from A to B ***R1***

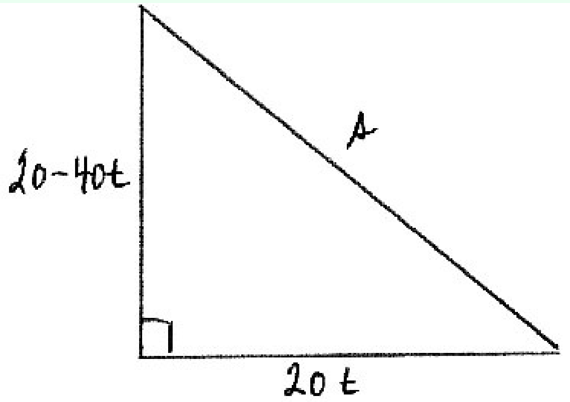
[3 marks]

Examiners report

Satisfactory answers were very rarely seen for (c). Very few candidates realised that a minimum can occur at the beginning or end of an interval.

38a. *[8 marks]*

Markscheme



(M1)

M1
$s^2 = (20t)^2 + (20 - 40t)^2$

A1
$s^2 = 2000t^2 - 1600t + 400$

to minimize s it is enough to minimize

s^2

A1
$f'(t) = 4000t - 1600$

setting

equal to 0 ***M1***
$f'(t)$

or 24 minutes ***A1***
$4000t - 1600 = 0 \Rightarrow t = \frac{2}{5}$

M1
$f''(t) = 4000 > 0$

at
$\Rightarrow$

is minimized
$t = \frac{2}{5},\ f(t)$

hence, the ships are closest at 12:24 ***A1***

Note: accept solution based on s.

[8 marks]

Examiners report

This was, disappointingly, a poorly answered question. Some tried to talk their way through the question without introducing the time variable. Even those who did use the distance as a function of time often did not check for a minimum.

38b. *[3 marks]*

Markscheme

M1A1
$f\left(\frac{2}{5}\right) = \sqrt{80}$

since

, the captains can see one another ***R1***
$\sqrt{80} < 9$

[3 marks]

Examiners report

This was, disappointingly, a poorly answered question. Some tried to talk their way through the question without introducing the time variable. Even those who did use the distance as a function of time often did not check for a minimum.

39a. *[3 marks]*

Markscheme

A1
$\frac{dy}{dx} = \frac{e}{\ln e}(2+2) = 4e$
at (2, e) the tangent line is

M1
$y - e = 4e(x-2)$
hence

A1
$y = 4ex - 7e$
[3 marks]

Examiners report

Nearly always correctly answered.

39b. *[11 marks]*

Markscheme

M1
$\frac{dy}{dx} = \frac{y}{\ln y}(x+2) \Rightarrow \frac{\ln y}{y}dy = (x+2)dx$

$\int \frac{\ln y}{y}dy = \int (x+2)dx$
using substitution

(M1)(A1)
$u = \ln y;\ du = \frac{1}{y}dy$
(A1)
$\Rightarrow \int \frac{\ln y}{y}dy = \int u du = \frac{1}{2}u^2$
A1A1
$\Rightarrow \frac{(\ln y)^2}{2} = \frac{x^2}{2} + 2x + c$
at (2, e),

M1
$\frac{(\ln e)^2}{2} = 6 + c$
A1
$\Rightarrow c = -\frac{11}{2}$

$\Rightarrow \frac{(\ln y)^2}{2} = \frac{x^2}{2} + 2x - \frac{11}{2} \Rightarrow (\ln y)^2 = x^2 + 4x - 11$
M1A1
$\ln y = \pm\sqrt{x^2+4x-11} \Rightarrow y = e^{\pm\sqrt{x^2+4x-11}}$
since y > 1,

R1
$f(x) = e^{\sqrt{x^2+4x-11}}$
Note: ***M1*** for attempt to make y the subject.

[11 marks]

Examiners report

Most candidates separated the variables and attempted the integrals. Very few candidates made use of the condition $y > 1$, so losing 2 marks.

39c. *[6 marks]*

Markscheme

EITHER

$x^2 + 4x - 11 > 0$ ***A1***

using the quadratic formula ***M1***

critical values are

$\frac{-4 \pm \sqrt{60}}{2}$ $\left(= -2 \pm \sqrt{15}\right)$ ***A1***

using a sign diagram or algebraic solution ***M1***

$x < -2 - \sqrt{15}$; $x > -2 + \sqrt{15}$ ***A1A1***

OR

$x^2 + 4x - 11 > 0$ ***A1***

by methods of completing the square ***M1***

$(x + 2)^2 > 15$ ***A1***

$\Rightarrow x + 2 < -\sqrt{15}$ or $x + 2 > \sqrt{15}$ ***(M1)***

$x < -2 - \sqrt{15}$; $x > -2 + \sqrt{15}$ ***A1A1***

[6 marks]

Examiners report

Part (c) was often well answered, sometimes with follow through.

39d. *[4 marks]*

Markscheme

$f(x) = f'(x) \Rightarrow f(x) = \frac{f(x)}{\ln f(x)}(x + 2)$ ***M1***

$\Rightarrow \ln(f(x)) = x + 2$ $\left(\Rightarrow x + 2 = \sqrt{x^2 + 4x - 11}\right)$ ***A1***

$\Rightarrow (x + 2)^2 = x^2 + 4x - 11 \Rightarrow x^2 + 4x + 4 = x^2 + 4x - 11$ ***A1***

$\Rightarrow 4 = -11$, hence $f(x) \neq f'(x)$ ***R1AG***

[4 marks]

Examiners report

Only the best candidates were successful on part (d).