

## Topic 2 Part 4 [47 marks]

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Consider the equation

$$9x^3 - 45x^2 + 74x - 40 = 0.$$

1a. Write down the numerical value of the sum and of the product of the roots of this equation. [1 mark]

1b. The roots of this equation are three consecutive terms of an arithmetic sequence. [6 marks]  
Taking the roots to be  $\alpha, \alpha \pm \beta$ , solve the equation.

The function  $f$  is defined, for

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, \text{ by}$$

$$f(x) = 2 \cos x + x \sin x.$$

2a. Determine whether  $f$  is even, odd or neither even nor odd. [3 marks]

2b. Show that [2 marks]  
 $f''(0) = 0$ .

2c. John states that, because [2 marks]  
 $f''(0) = 0$ , the graph of  $f$  has a point of inflexion at the point  $(0, 2)$ . Explain briefly whether John's statement is correct or not.

The function  $f$  is defined on the domain

$$x \geq 0 \text{ by}$$

$$f(x) = e^x - x^e.$$

3a. (i) Find an expression for [3 marks]

$$f'(x).$$

(ii) Given that the equation

$$f'(x) = 0 \text{ has two roots, state their values.}$$

3b. Sketch the graph of  $f$ , showing clearly the coordinates of the maximum and minimum. [3 marks]

3c. Hence show that [1 mark]  
 $e^\pi > \pi^e$ .

The function  $f$  is defined by

$$f(x) = \begin{cases} 2x - 1, & x \leq 2 \\ ax^2 + bx - 5, & 2 < x < 3 \end{cases}$$

where  $a$ ,

$b \in \mathbb{R}$ .

4a. Given that  $f$  and its derivative, [6 marks]

$f'$ , are continuous for all values in the domain of  $f$ , find the values of  $a$  and  $b$ .

4b. Show that  $f$  is a one-to-one function. [3 marks]

4c. Obtain expressions for the inverse function [5 marks]

$f^{-1}$  and state their domains.

5. Given that  $(x - 2)$  is a factor of [6 marks]

$f(x) = x^3 + ax^2 + bx - 4$  and that division

$f(x)$  by  $(x - 1)$  leaves a remainder of  $-6$ , find the value of  $a$  and the value of  $b$ .

6. The function  $f$  is of the form [6 marks]

$$f(x) = \frac{x+a}{bx+c},$$

$x \neq -\frac{c}{b}$ . Given that the graph of  $f$  has asymptotes  $x = -4$  and  $y = -2$ , and that the point

$(\frac{2}{3}, 1)$  lies on the graph, find the values of  $a$ ,  $b$  and  $c$ .