

Topic 3 Part 1 [449 marks]

1a. [2 marks]

Markscheme

$$\sin(\pi x^{-1}) = 0 \quad \frac{\pi}{x} = \pi, 2\pi(\dots) \quad (AI)$$

$$x = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10} \quad AI$$

[2 marks]

Examiners report

There were a pleasing number of candidates who answered part (a) correctly. Fewer were successful with part (b). It was expected by this stage of the paper that candidates would be able to just write down the value of the integral rather than use substitution to evaluate it.

1b. [3 marks]

Markscheme

$$\left[\cos(\pi x^{-1}) \right]_{\frac{1}{n+1}}^{\frac{1}{n}} \quad MI$$

$$= \cos(\pi n) - \cos(\pi(n+1)) \quad AI$$

$$= 2 \text{ when } n \text{ is even and } = -2 \text{ when } n \text{ is odd} \quad AI$$

[3 marks]

Examiners report

There were a pleasing number of candidates who answered part (a) correctly. Fewer were successful with part (b). It was expected by this stage of the paper that candidates would be able to just write down the value of the integral rather than use substitution to evaluate it.

1c. [2 marks]

Markscheme

$$\int_{0.1}^1 |\pi x^{-2} \sin(\pi x^{-1})| dx = 2 + 2 + \dots + 2 = 18 \quad (MI)AI$$

[2 marks]

Examiners report

There were disappointingly few correct answers to part (c) with candidates not realising that it was necessary to combine the previous two parts in order to write down the answer.

2a. [2 marks]

Markscheme

$$4(x - 0.5)^2 + 4 \quad AIAI$$

Note: *AI* for two correct parameters, *A2* for all three correct.

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e).

2b.

[3 marks]

Markscheme

translation

$$\begin{pmatrix} 0.5 \\ 0 \end{pmatrix} \text{ (allow “0.5 to the right”) } \quad \text{AI}$$

stretch parallel to y-axis, scale factor 4 (allow vertical stretch or similar) **AI**

translation

$$\begin{pmatrix} 0 \\ 4 \end{pmatrix} \text{ (allow “4 up”) } \quad \text{AI}$$

Note: All transformations must state magnitude and direction.

Note: First two transformations can be in either order.

It could be a stretch followed by a single translation of

$$\begin{pmatrix} 0.5 \\ 4 \end{pmatrix}. \text{ If the vertical translation is before the stretch it is } \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

[3 marks]

Examiners report

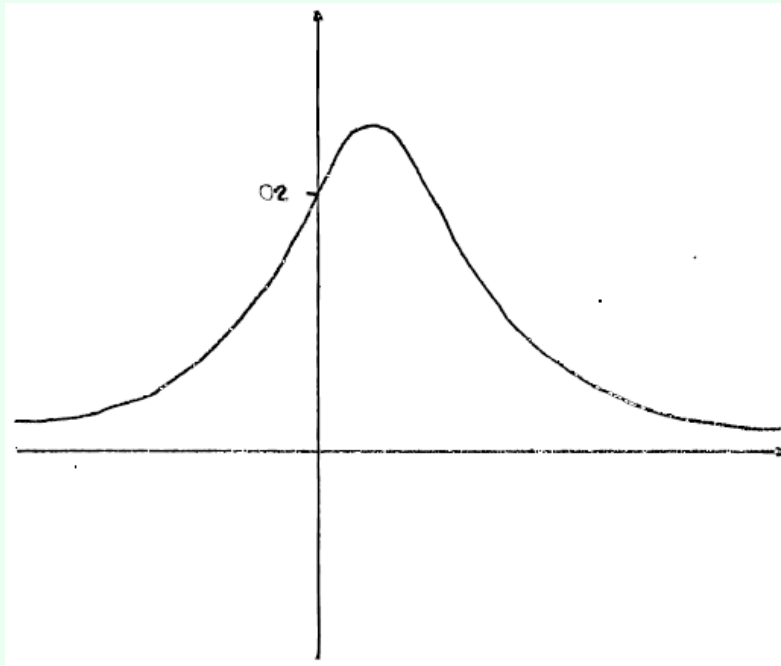
This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(b) Exam technique would have helped those candidates who could not get part (a) correct as any solution of the form given in the question could have led to full marks in part (b). Several candidates obtained expressions which were not of this form in (a) and so were unable to receive any marks in (b). Many missed the fact that if a vertical translation is performed before the vertical stretch it has a different magnitude to if it is done afterwards. Though on this occasion the markscheme was fairly flexible in the words it allowed to be used by candidates to describe the transformations it would be less risky to use the correct expressions.

2c.

[2 marks]

Markscheme



general shape (including asymptote and single maximum in first quadrant), *AI*

intercept

$(0, \frac{1}{5})$ or maximum

$(\frac{1}{2}, \frac{1}{4})$ shown *AI*

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(c) Generally the sketches were poor. The general rule for all sketch questions should be that any asymptotes or intercepts should be clearly labelled. Sketches do not need to be done on graph paper, but a ruler should be used, particularly when asymptotes are involved.

2d.

[2 marks]

Markscheme

$$0 < f(x) \leq \frac{1}{4}$$

Note: *AI* for

$$, AI \text{ for } \leq \frac{1}{4}$$

$$0 <$$

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e).

2e.

[3 marks]

Markscheme

let

$$u = x - \frac{1}{2}$$

$$\frac{du}{dx} = 1 \quad (\text{or } du = dx)$$

$$\int \frac{1}{4x^2 - 4x + 5} dx = \int \frac{1}{4\left(x - \frac{1}{2}\right)^2 + 4} dx$$

$$\int \frac{1}{4u^2 + 4} du = \frac{1}{4} \int \frac{1}{u^2 + 1} du$$

Note: If following through an incorrect answer to part (a), do not award final **AI** mark.

[3 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(e) and (f) were well done up to the final part of (f), in which candidates did not realise they needed to use the compound angle formula.

2f.

[7 marks]

Markscheme

$$\int_1^{3.5} \frac{1}{4x^2 - 4x + 5} dx = \frac{1}{4} \int_{0.5}^3 \frac{1}{u^2 + 1} du$$

Note: **AI** for correct change of limits. Award also if they do not change limits but go back to x values when substituting the limit (even if there is an error in the integral).

$$\frac{1}{4} [\arctan(u)]_{0.5}^3$$

$$\frac{1}{4} \left(\arctan(3) - \arctan\left(\frac{1}{2}\right) \right)$$

let the integral = I

$$\tan 4I = \tan \left(\arctan(3) - \arctan\left(\frac{1}{2}\right) \right)$$

$$\frac{3 - 0.5}{1 + 3 \times 0.5} = \frac{2.5}{2.5} = 1$$

$$4I = \frac{\pi}{4} \Rightarrow I = \frac{\pi}{16}$$

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(e) and (f) were well done up to the final part of (f), in which candidates did not realise they needed to use the compound angle formula.

Markscheme

attempt at use of

$$\stackrel{MI}{\tan(A+B)} = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$$

$$\stackrel{AI}{\frac{1}{p}} = \frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}} \left(= \frac{1}{3} \right)$$

$$\stackrel{AI}{p} = 3$$

Note: the value of p needs to be stated for the final mark.

[3 marks]

Examiners report

Those candidates who used the addition formula for the tangent were usually successful.

Markscheme

$$\stackrel{MIAI}{\tan\left(\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)\right)} = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = 1$$

$$\stackrel{AI}{\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)} = \frac{\pi}{4}$$

[3 marks]

Examiners report

Some candidates left their answer as the tangent of an angle, rather than the angle itself.

Markscheme

$$\stackrel{MI}{\cos x = 2\cos^2\frac{1}{2}x - 1}$$

$$\stackrel{AG}{\cos\frac{1}{2}x = \pm\sqrt{\frac{1+\cos x}{2}}}$$

positive as

$$\stackrel{RI}{0 \leq x \leq \pi}$$

$$\stackrel{AG}{\cos\frac{1}{2}x = \sqrt{\frac{1+\cos x}{2}}}$$

[2 marks]

Examiners report

[N/A]

Markscheme

$$\stackrel{(MI)}{\cos 2\theta = 1 - 2\sin^2\theta}$$

$$\stackrel{AI}{\sin\frac{1}{2}x = \sqrt{\frac{1-\cos x}{2}}}$$

[2 marks]

Examiners report

[N/A]

4c.

[4 marks]

Markscheme

$$\begin{aligned} & \sqrt{2} \int_0^{\frac{\pi}{2}} \cos \frac{1}{2}x + \sin \frac{1}{2}x dx \\ &= \sqrt{2} \left[2 \sin \frac{1}{2}x - 2 \cos \frac{1}{2}x \right]_0^{\frac{\pi}{2}} \\ &= \sqrt{2}(0) - \sqrt{2}(0 - 2) \\ &= 2\sqrt{2} \end{aligned}$$

[4 marks]

Examiners report

[N/A]

5.

[5 marks]

Markscheme

METHOD 1

$$AD^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos 60^\circ$$

(or

$$AD^2 = 1^2 + 3^2 - 2 \times 1 \times 3 \times \cos 60^\circ$$

Note: *MI* for use of cosine rule with 60° angle.

$$\begin{aligned} AD^2 &= 7 \\ \cos DAC &= \frac{9+7-4}{2 \times 3 \times \sqrt{7}} \end{aligned}$$

Note: *M1* for use of cosine rule involving

\hat{D}

$$\cos DAC = \frac{2}{\sqrt{7}}$$

METHOD 2

let point E be the foot of the perpendicular from D to AC

EC = 1 (by similar triangles, or triangle properties) *MIAI*

(or AE = 2)

and

$$DE = \sqrt{3} \quad (\text{by Pythagoras}) \quad (MI)AI$$

$$\cos DAC = \frac{2}{\sqrt{7}}$$

Note: If first *MI* not awarded but remainder of the question is correct award *M0A0MIAIA1*.

[5 marks]

Examiners report

[N/A]

6.

[6 marks]

Markscheme

$$\sin^2 x + \cos^2 x + 2 \sin x \cos x = \frac{4}{9}$$

using

$$\sin^2 x + \cos^2 x = 1$$

$$2 \sin x \cos x = -\frac{5}{9}$$

using

$$2 \sin x \cos x = \sin 2x$$

$$\sin 2x = -\frac{5}{9}$$

$$\cos 4x = 1 - 2 \sin^2 2x$$

Note: Award this **MI** for decomposition of $\cos 4x$ using double angle formula anywhere in the solution.

$$= 1 - 2 \times \frac{25}{81}$$

[6 marks]

Examiners report

[N/A]

7a.

[3 marks]

Markscheme

(a)

$$\frac{\pi}{2}(1.57), \frac{3\pi}{2}(4.71)$$

hence the coordinates are

$$\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$$

Examiners report

[N/A]

7b.

[5 marks]

Markscheme

(i)

$$\pi \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (x^2 - (x + 2 \cos x)^2) dx$$

Note: Award **AI** for

, **AI** for correct limits and **AI** for

$x^2 - (x + 2 \cos x)^2$

π

(ii)

$$6\pi^2 (= 59.2)$$

Notes: Do not award **ft** from (b)(i).

[5 marks]

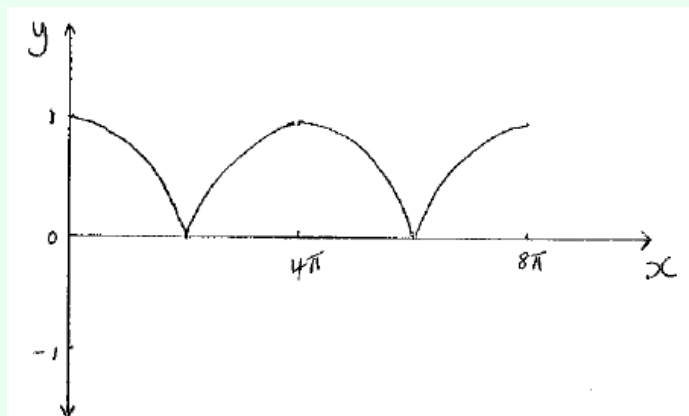
Examiners report

[N/A]

8a.

[2 marks]

Markscheme

*A1A1*

Note: Award *A1* for correct shape and *A1* for correct domain and range.

[2 marks]

Examiners report

[N/A]

8b.

[3 marks]

Markscheme

$\left| \cos\left(\frac{x}{4}\right) \right| = \frac{1}{2}$
 $x = \frac{4\pi}{3}$
 attempting to find any other solutions *MI*

Note: Award (*MI*) if at least one of the other solutions is correct (in radians or degrees) or clear use of symmetry is seen.

$$x = 8\pi - \frac{4\pi}{3} = \frac{20\pi}{3}$$

$$x = 4\pi - \frac{4\pi}{3} = \frac{8\pi}{3}$$

$$x = 4\pi + \frac{4\pi}{3} = \frac{16\pi}{3}$$

Note: Award *A1* for all other three solutions correct and no extra solutions.

Note: If working in degrees, then max *A0M1A0*.

[3 marks]

Examiners report

[N/A]

Markscheme

(a)

$\sin x$, $\sin 2x$ and $4\sin x \cos^2 x$

$$r = \frac{2\sin x \cos x}{\sin x} = 2\cos x$$

Note: Accept

$$\frac{\sin 2x}{\sin x}$$

[1 mark]

(b) **EITHER**

$$|r| < 1 \Rightarrow |2\cos x| < 1$$

OR

$$-1 < r < 1 \Rightarrow -1 < 2\cos x < 1$$

THEN

$$0 < \cos x < \frac{1}{2} \text{ for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$-\frac{\pi}{2} < x < -\frac{\pi}{3} \text{ or } \frac{\pi}{3} < x < \frac{\pi}{2}$$

[3 marks]

(c)

$$S_{\infty} = \frac{\sin x}{1 - 2\cos x}$$

$$S_{\infty} = \frac{\sin(\arccos(\frac{1}{4}))}{2\cos(\arccos(\frac{1}{4}))}$$

$$= \frac{\frac{4}{5}}{\frac{1}{2}}$$

Note: Award **A1** for correct numerator and **A1** for correct denominator.

$$\frac{\sqrt{15}}{5}$$

[3 marks]

Total [7 marks]

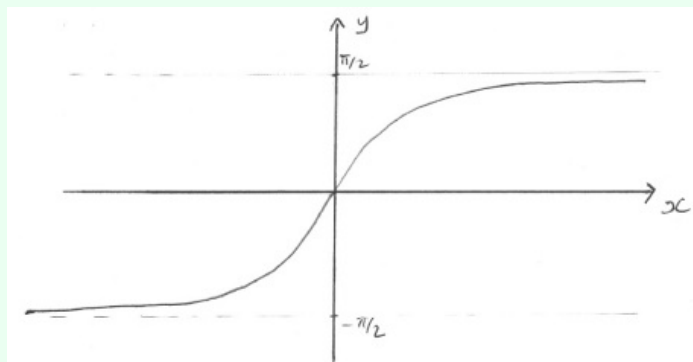
Examiners report

[N/A]

10a.

[2 marks]

Markscheme

*A1A1*

Note: *A1* for correct shape, *A1* for asymptotic behaviour at

$$y = \pm \frac{\pi}{2}$$

[2 marks]

Examiners report

[N/A]

10b.

[2 marks]

Markscheme

$$h \circ g(x) = \arctan\left(\frac{1}{x}\right)$$

domain of

is equal to the domain of

h

$$g: x \in \mathbb{R}, x \neq 0$$

[2 marks]

Examiners report

[N/A]

10c.

[7 marks]

Markscheme

(i)

$$f(x) = \arctan(x) + \arctan\left(\frac{1}{x}\right)$$

$$f'(x) = \frac{1}{1+x^2} + \frac{1}{1+\frac{1}{x^2}} \times -\frac{1}{x^2}$$

$$f'(x) = \frac{1}{1+x^2} + \frac{x^2}{x^2+1}$$

$$= \frac{1}{1+x^2} - \frac{1}{1+x^2}$$

$$= 0$$

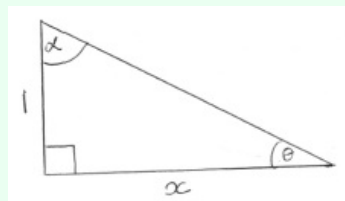
(ii) **METHOD 1** f is a constant **RI**

when

$$x > 0$$

$$f(1) = \frac{\pi}{4} + \frac{\pi}{4}$$

$$= \frac{\pi}{2}$$

METHOD 2

from diagram

$$\theta = \arctan \frac{1}{x}$$

$$\alpha = \arctan x$$

$$\theta + \alpha = \frac{\pi}{2}$$

$$f(x) = \frac{\pi}{2}$$

METHOD 3

$$\tan(f(x)) = \tan\left(\arctan(x) + \arctan\left(\frac{1}{x}\right)\right)$$

$$\frac{x + \frac{1}{x}}{1 - \frac{1}{x^2}} = 0, \text{ so}$$

$$f(x) = \frac{\pi}{2} \text{ (for } x > 0)$$

Examiners report

[N/A]

10d.

[3 marks]

Markscheme

(i) Nigel is correct. **AI****METHOD 1**

is an odd function and

 $\arctan(x)$

is an odd function

composition of two odd functions is an odd function and sum of two odd functions is an odd function **RI****METHOD 2**

$$f(-x) = \arctan(-x) + \arctan\left(-\frac{1}{x}\right) = -\arctan(x) - \arctan\left(\frac{1}{x}\right) = -f(x)$$

therefore f is an odd function. **RF**

(ii)

$$f(x) = -\frac{\pi}{2}$$

[3 marks]

Examiners report

[N/A]

11.

[6 marks]

Markscheme

(a) METHOD 1

$$\frac{MI}{AI} = \frac{1.5}{4}$$
$$\alpha = 0.769^{\circ} \text{ (44.0}^{\circ}\text{)}$$

METHOD 2

using the cosine rule:

$$\frac{MI}{AI} = \frac{3^2}{4^2 + 4^2 - 2(4)(4)\cos\alpha}$$
$$\alpha = 0.769^{\circ} \text{ (44.0}^{\circ}\text{)}$$

[2 marks]

(b) one segment

$$\frac{MIAI}{AI} = \frac{1}{2} \times 4^2 \times 0.76879 - \frac{1}{2} \times 4^2 \sin(0.76879)$$
$$= 0.58819\text{K}$$
$$\frac{AI}{AI} = 1.18 \text{ (cm}^2\text{)}$$

Note: Award **MI** only if both sector and triangle are considered.

[4 marks]

Total [6 marks]

Examiners report

[N/A]

12.

[8 marks]

Markscheme

(a)

$$\begin{aligned}
 & \sin(x+y)\sin(x-y) \\
 &= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\
 &= \sin^2 x \cos^2 y + \sin x \sin y \cos x \cos y - \sin x \sin y \cos x \cos y - \cos^2 x \sin^2 y \\
 &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\
 &= \sin^2 x (1 - \sin^2 y) - \sin^2 y (1 - \sin^2 x) \\
 &= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y \\
 &= \sin^2 x - \sin^2 y
 \end{aligned}$$

AG
[4 marks]

(b)

$$\begin{aligned}
 & f(x) = \sin^2 x - \frac{1}{4} \\
 & \text{range is} \\
 & f \in \left[-\frac{1}{4}, \frac{3}{4}\right]
 \end{aligned}$$

AIAI

Note: Award **AI** for each end point. Condone incorrect brackets.

[2 marks]

(c)

$$\begin{aligned}
 & g(x) = \frac{1}{\sin^2 x - \frac{1}{4}} \\
 & \text{range is} \\
 & g \in]-\infty, -4] \cup \left[\frac{4}{3}, \infty\right[
 \end{aligned}$$

AIAI

Note: Award **AI** for each part of range. Condone incorrect brackets.

[2 marks]

Total [8 marks]

Examiners report

Part a) often proved to be an easy 4 marks for candidates. A number were surprisingly content to gain the first 3 marks but were unable to make the final step by substituting

$$1 - \sin^2 y$$

for $\cos^2 y$. Parts b) and c) were more often than not, problematic. Some puzzling ‘working’ was often seen, with candidates making little headway. Otherwise good candidates were able to answer part b), though correct solutions for c) were a rarity. The range was sometimes seen, but gained no marks.

$$g \in \left[-4, \frac{4}{3}\right]$$

13a.

[2 marks]

Markscheme

$$A = \frac{1}{2} \times 10^2 \times \theta - \frac{1}{2} \times 10^2 \times \sin \theta$$

MIAI

Note: Award **MI** for use of area of segment = area of sector – area of triangle.

$$= 50\theta - 50 \sin \theta$$

AG
[2 marks]

Examiners report

Part (a) was very well done. Most candidates knew how to calculate the area of a segment. A few candidates used

$$r = 20$$

13b.

[3 marks]

Markscheme

METHOD 1

unshaded area

$$= \frac{\pi \times 10^2}{2} - 50(\theta - \sin \theta)$$

(or equivalent eg

$$\begin{matrix} (M1) \\ (A1) \end{matrix} 50\pi - 50\theta + 50\sin \theta$$

$$50\theta - 50\sin \theta = \frac{1}{2}(50\pi - 50\theta + 50\sin \theta)$$

$$3\theta - 3\sin \theta - \pi = 0$$

$$\begin{matrix} AI \\ \Rightarrow \end{matrix} \theta = 1.969 \text{ (rad)}$$

METHOD 2

$$\begin{matrix} (M1)(A1) \\ 50\theta - 50\sin \theta = \frac{1}{3} \left(\frac{\pi \times 10^2}{2} \right) \end{matrix}$$

$$3\theta - 3\sin \theta - \pi = 0$$

$$\begin{matrix} AI \\ \Rightarrow \end{matrix} \theta = 1.969 \text{ (rad)}$$

Examiners report

Part (b) challenged a large proportion of candidates. A common error was to equate the unshaded area and the shaded area. Some candidates expressed their final answer correct to three significant figures rather than to the four significant figures specified.

Markscheme

(i) **METHOD 1**

$$\begin{aligned} & \text{by de Moivre's theorem} \quad \text{A1} \\ (1 + i \tan \theta)^n + (1 - i \tan \theta)^n &= \left(1 + i \frac{\sin \theta}{\cos \theta}\right)^n + \left(1 - i \frac{\sin \theta}{\cos \theta}\right)^n \\ &= \left(\frac{\cos \theta + i \sin \theta}{\cos \theta}\right)^n + \left(\frac{\cos \theta - i \sin \theta}{\cos \theta}\right)^n \quad \text{M1} \end{aligned}$$

recognition that $\frac{\cos \theta - i \sin \theta}{\cos \theta}$ is the complex conjugate of $\frac{\cos \theta + i \sin \theta}{\cos \theta}$ (R1)
use of the fact that the operation of complex conjugation commutes with the operation of raising to an integer power:

$$\begin{aligned} & \text{A1} \\ \left(\frac{\cos \theta - i \sin \theta}{\cos \theta}\right)^n &= \frac{\cos n\theta - i \sin n\theta}{\cos^n \theta} \\ (1 + i \tan \theta)^n + (1 - i \tan \theta)^n &= \frac{2 \cos n\theta}{\cos^n \theta} \quad \text{METHOD 2} \end{aligned}$$

$$\begin{aligned} & \text{M1} \\ (1 + i \tan \theta)^n + (1 - i \tan \theta)^n &= (1 + i \tan \theta)^n + (1 + i \tan(-\theta))^n \\ &= \frac{(\cos \theta + i \sin \theta)^n}{\cos^n \theta} + \frac{(\cos(-\theta) + i \sin(-\theta))^n}{\cos^n \theta} \quad \text{M1A1} \end{aligned}$$

Note: Award **M1** for converting to cosine and sine terms.

use of de Moivre's theorem (M1)

$$\begin{aligned} & \text{A1} \\ &= \frac{1}{\cos^n \theta} (\cos n\theta + i \sin n\theta + \cos(-n\theta) + i \sin(-n\theta)) \\ &= \frac{2 \cos n\theta}{\cos^n \theta} \quad \text{as } \cos(-n\theta) = \cos n\theta \quad \text{and} \quad \sin(-n\theta) = -\sin n\theta \quad \text{R1AG} \\ \text{(ii)} \quad & \frac{(1 + i \tan \frac{3\pi}{8})^4 + (1 - i \tan \frac{3\pi}{8})^4}{\cos^4 \frac{3\pi}{8}} = \frac{2 \cos(4 \times \frac{3\pi}{8})}{\cos^4 \frac{3\pi}{8}} \quad \text{A1} \\ &= \frac{2 \cos \frac{3\pi}{2}}{\cos^4 \frac{3\pi}{8}} \quad \text{R1AG} \\ &= 0 \quad \text{as } \cos \frac{3\pi}{2} = 0 \end{aligned}$$

Note: The above working could involve theta and the solution of $\cos(4\theta) = 0$

so is a root of the equation **AG**

$$\begin{aligned} \text{(iii)} \quad & \text{either } \frac{i \tan \frac{3\pi}{8}}{\cos \frac{3\pi}{8}} \quad \text{A1} \\ & -i \tan \frac{3\pi}{8} \quad \text{or} \quad -i \tan \frac{\pi}{8} \quad \text{or} \quad i \tan \frac{\pi}{8} \end{aligned}$$

Note: Accept.

$$\begin{aligned} & \text{Accept.} \quad i \tan \frac{5\pi}{8} \quad \text{or} \quad i \tan \frac{7\pi}{8} \\ & -(1 + \sqrt{2})i \quad \text{or} \quad (1 - \sqrt{2})i \quad \text{or} \quad (-1 + \sqrt{2})i \end{aligned}$$

[10 marks]

Examiners report

Fairly successful.

14b.

[13 marks]

Markscheme

(i) **(M1)**
 $\tan \frac{\pi}{4} = \frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$
A1
 $\tan^2 \frac{\pi}{8} + 2 \tan \frac{\pi}{8} - 1 = 0$
 let $t = \tan \frac{\pi}{8}$
 attempting to solve **M1**
 $t^2 + 2t - 1 = 0$ for t
A1
 $t = -1 \pm \sqrt{2}$
 is a first quadrant angle and \tan is positive in this quadrant, so
 $\frac{\pi}{8}$ **R1**
 $\tan \frac{\pi}{8} > 0$
AG
 $\tan \frac{\pi}{8} = \sqrt{2} - 1$
 (ii) **A1**
 $\cos 4x = 2\cos^2 2x - 1$
M1
 $= 2(2\cos^2 x - 1)^2 - 1$
A1
 $= 2(4\cos^4 x - 4\cos^2 x + 1) - 1$
AG
 $= 8\cos^4 x - 8\cos^2 x + 1$

Note: Accept equivalent complex number derivation.

(iii)
M1
 $\int_0^{\frac{\pi}{8}} \frac{2 \cos 4x}{\cos^2 x} dx = 2 \int_0^{\frac{\pi}{8}} \frac{8\cos^4 x - 8\cos^2 x + 1}{\cos^2 x} dx$
 $= 2 \int_0^{\frac{\pi}{8}} 8\cos^2 x - 8 + \sec^2 x dx$

Note: The **M1** is for an integrand involving no fractions.

use of **M1**
 $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$
A1
 $= 2 \int_0^{\frac{\pi}{8}} 4 \cos 2x - 4 + \sec^2 x dx$
A1
 $= [4 \sin 2x - 8x + 2 \tan x]_0^{\frac{\pi}{8}}$
 (or equivalent) **A1**
 $= 4\sqrt{2} - \pi - 2$
[13 marks]

Total [23 marks]

Examiners report

(i) Most candidates attempted to use the hint. Those who doubled the angle were usually successful – but many lost the final mark by not giving a convincing reason to reject the negative solution to the intermediate quadratic equation. Those who halved the angle got nowhere.

(ii) The majority of candidates obtained full marks.

(iii) This was poorly answered, few candidates realising that part of the integrand could be re-expressed using $\frac{1}{\cos^2 x} = \sec^2$ which can be immediately integrated.

15a.

[3 marks]

Markscheme

each triangle has area **(M1)**
 $\frac{1}{8}x^2 \sin \frac{2\pi}{n}$ (use of $\frac{1}{2}ab \sin C$)
 there are
 triangles so **A1**
 $n A = \frac{1}{8}nx^2 \sin \frac{2\pi}{n}$
A1
 $C = \frac{(\frac{1}{8}nx^2 \sin \frac{2\pi}{n})}{8}$
 so **AG**
 $C = \frac{n}{8}x^2 \sin \frac{2\pi}{n}$
[3 marks]

Examiners report

Most candidates found this a difficult question with a large number of candidates either not attempting it or making little to no progress. In part (a), a number of candidates attempted to show the desired result using specific regular polygons. Some candidates attempted to fudge the result.

15b. [4 marks]

Markscheme

attempting to find the least value of

such that **(M1)**
 $n \frac{n}{2\pi} \sin \frac{2\pi}{n} > 0.99$
A1

$n = 26$

attempting to find the least value of

such that **(M1)**
 $n \frac{n}{2\pi} \sin \frac{2\pi}{n} > 0.99$
 (and so a regular polygon with 21 sides) **A1**
 $n = 21$

Note: Award **(M0)A0(M1)A1** if is not considered and is correctly considered.

Award **(M1)A1(M0)A0** for . $\frac{n}{2\pi} \sin \frac{2\pi}{n} > 0.99$ $\frac{n \sin \frac{2\pi}{n}}{\pi(1 + \cos \frac{\pi}{n})} > 0.99$

[4 marks] $n = 26$

Examiners report

In part (b), the overwhelming majority of candidates that obtained either or or both used either a GDC numerical solve feature or a graphical approach rather than a tabular approach which is more appropriate for a discrete variable such as the number of sides of a regular polygon. Some candidates wasted valuable time by showing that (a given result).

$$C = \frac{n \sin \frac{2\pi}{n}}{\pi(1 + \cos \frac{\pi}{n})}$$

16a. [2 marks]

Markscheme

use of to obtain **M1**

$$A = \frac{1}{2}qr \sin \theta = \frac{1}{2}(x+2)(5-x)^2 \sin 30^\circ$$

$$= \frac{1}{4}(x+2)(25-10x+x^2)$$

$$A = \frac{1}{4}(x^3 - 8x^2 + 5x + 50)$$

[2 marks]

Examiners report

This question was generally well done. Parts (a) and (b) were straightforward and well answered.

16b.

[3 marks]

Markscheme

- (i) $\frac{dA}{dx} = \frac{1}{4}(3x^2 - 16x + 5) = \frac{1}{4}(3x - 1)(x - 5)$ **A1**
 (ii) **METHOD 1**

EITHER

M1A1
~~OR~~ $\frac{dA}{dx} = \frac{1}{4}\left(3\left(\frac{1}{3}\right)^2 - 16\left(\frac{1}{3}\right) + 5\right) = 0$

M1A1
~~THEN~~ $\frac{dA}{dx} = \frac{1}{4}\left(3\left(\frac{1}{3}\right) - 1\right)\left(\left(\frac{1}{3}\right) - 5\right) = 0$

so when $\frac{dA}{dx} = 0 = \frac{1}{3}$ **AG**
METHOD 2

solving for $\frac{dA}{dx} = 0$ **M1**
A1

$-2 < x < 5 \Rightarrow x = \frac{1}{3}$

so when $\frac{dA}{dx} = 0 = \frac{1}{3}$ **AG**
METHOD 3

a correct graph of $\frac{dA}{dx}$ versus x **M1**

the graph clearly showing that when $\frac{dA}{dx} = 0 = \frac{1}{3}$ **A1**

so when $\frac{dA}{dx} = 0 = \frac{1}{3}$ **AG**
[3 marks]

Examiners report

This question was generally well done. Parts (a) and (b) were straightforward and well answered.

16c.

[7 marks]

Markscheme

- (i) $\frac{d^2A}{dx^2} = \frac{1}{2}(3x - 8)$ **A1**
 for $x = \frac{1}{3}$, $\frac{d^2A}{dx^2} = -3.5 (< 0)$ **A1**
 so gives the maximum area of triangle **AG**
 $x = \frac{1}{3}$ **PQR**

(ii) $A_{\max} = \frac{343}{8} (= 12.7) \text{ (cm}^2\text{)}$ **A1**

(iii) $PQ = PR = \left(\frac{14}{3}\right)^2 \text{ (cm)}$ **(A1)**

(M1)(A1) $QR^2 = \left(\frac{7}{3}\right)^2 + \left(\frac{14}{3}\right)^4 - 2\left(\frac{7}{3}\right)\left(\frac{14}{3}\right)^2 \cos 30^\circ$

$= 391.702 \dots$

A1
 $QR = 19.8 \text{ (cm)}$
[7 marks]

Total [12 marks]

Examiners report

This question was generally well done. Parts (c) (i) and (ii) were also well answered with most candidates correctly applying the second derivative test and displaying sound reasoning skills.

Part (c) (iii) required the use of the cosine rule and was reasonably well done. The most common error committed by candidates in attempting to find the value of QR was to use $PR = \frac{14}{3} \text{ (cm)}$ rather than $\left(\frac{14}{3}\right)^2 \text{ (cm)}$. The occasional candidate used $\cos 30^\circ = \frac{1}{2}$.

$PR = \frac{14}{3} \text{ (cm)}$ **OR** $PR = \left(\frac{14}{3}\right)^2 \text{ (cm)}$ $\cos 30^\circ = \frac{1}{2}$

Markscheme

METHOD 1

squaring both equations **M1**

$$9\sin^2 B + 24\sin B \cos C + 16\cos^2 C = 36$$

$$9\cos^2 B + 24\cos B \sin C + 16\sin^2 C = 1$$

adding the equations and using **M1** to obtain $\cos^2 \theta + \sin^2 \theta = 1$

$$24(\sin B \cos C + \cos B \sin C) = 12$$

$$24\sin(B + C) = 12$$

$$\sin(B + C) = \frac{1}{2}$$

METHOD 2

substituting for $\sin B$ and to obtain $\sin B \cos B$

$$\sin(B + C) = \left(\frac{6-4\cos C}{3} \right) \cos C + \left(\frac{1-4\sin C}{3} \right) \sin C$$

substituting for $\sin C$ and to obtain $\sin C \cos C$

$$\sin(B + C) = \sin B \left(\frac{6-3\sin B}{4} \right) + \cos B \left(\frac{1-3\cos B}{4} \right)$$

Adding the two equations for :

$$2\sin(B + C) = \frac{(18\sin B + 24\cos C) + (4\sin C + 3\cos B) - 25}{12}$$

$$\sin(B + C) = \frac{36+1-25}{24}$$

$$\sin(B + C) = \frac{1}{2}$$

METHOD 3

substituting and to obtain $\sin B \sin C$

$$\sin(B + C) = \left(\frac{6-4\cos C}{3} \right) \cos C + \cos B \left(\frac{1-3\cos B}{4} \right)$$

$$\sin(B + C) = \sin B \left(\frac{6-3\sin B}{4} \right) + \left(\frac{1-4\sin C}{3} \right) \sin C$$

$$(or\ equivalent) \quad 2\sin(B + C) = \frac{6\cos C + \sin C - 4}{3} + \frac{6\sin B + \cos B - 3}{4}$$

$$2\sin(B + C) = \frac{(18\sin B + 24\cos C) + (4\sin C + 3\cos B) - 25}{12}$$

$$\sin(B + C) = \frac{36+1-25}{24}$$

$$\sin(B + C) = \frac{1}{2}$$

[6 marks]

Examiners report

Most candidates found this a difficult question with a large number of candidates either not attempting it or making little to no progress. In part (a), most successful candidates squared both equations, added them together, used $\cos^2 \theta + \sin^2 \theta = 1$ and then simplified their result to show that $\sin(B + C) = \frac{1}{2}$. A number of candidates started with a correct alternative method (see the markscheme for alternative approaches) but were unable to follow them through fully.

17b.

[5 marks]

Markscheme

so **R1**
 $\sin A = \sin(180^\circ - (B + C))$
A1
 $\sin(B + C) = \frac{1}{2} \Rightarrow \sin A = \frac{1}{2}$
or **A1**
 $\Rightarrow A = 30^\circ$
if, then **R1**
 $A = 150^\circ < 30^\circ$
for example, , ie a contradiction **R1**
 $3 \sin B + 4 \cos C < \frac{3}{2} + 4 < 6$
only one possible value **AG**
(A = 30°)

[5 marks]

Total [11 marks]

Examiners report

In part (b), a small percentage of candidates were able to obtain or but were then unable to demonstrate or explain why is the only possible value for triangle ABC.
 $B = 30^\circ$ (50°) (150°) $(A = 30^\circ)$
 $A = 30^\circ$

18a.

[3 marks]

Markscheme

METHOD 1

M1A1
area = $\pi 2^2 - \frac{1}{2} 2^2 \theta$ (= 3π)

Note: Award **M1** for using area formula.

A1
 $\Rightarrow 2\theta = \pi \Rightarrow \theta = \frac{\pi}{2}$

Note: Degrees loses final A1

METHOD 2

let
 $x = 2\pi - \theta$
M1
area = $\frac{1}{2} 2^2 x$ (= 3π)
A1
 $\Rightarrow x = \frac{3}{2} \pi$
A1
 $\Rightarrow \theta = \frac{\pi}{2}$

METHOD 3

Area of circle is **A1**
Shaded area is $\frac{4\pi}{4}$ of the circle **(R1)**
A1
 $\Rightarrow \theta = \frac{\pi}{2}$
[3 marks]

Examiners report

Good methods. Some candidates found the larger angle.

18b. [2 marks]

Markscheme

A1
 arc length $= 2\frac{3\pi}{2}$
 perimeter $= 2\frac{3\pi}{2} + 2 \times 2$
A1
 $= 3\pi + 4$
[2 marks]

Total [5 marks]

Examiners report

Generally good, some forgot the radii.

19. [6 marks]

Markscheme

$\tan x + \tan 2x = 0$
M1
 $\tan x + \frac{2 \tan x}{1 - \tan^2 x} = 0$
A1
 $\tan x - \tan^3 x + 2 \tan x = 0$
(M1)
 $\tan x(3 - \tan^2 x) = 0$
A1
 $\tan x = 0 \Rightarrow x = 0, x = 180^\circ$

Note: If seen anywhere award **A0**
 $x = 360^\circ$

A1
 $\tan x = \sqrt{3} \Rightarrow x = 60^\circ, 240^\circ$
A1
 $\tan x = -\sqrt{3} \Rightarrow x = 120^\circ, 300^\circ$
[6 marks]

Examiners report

[N/A]

20a. [4 marks]

Markscheme

any attempt to use sine rule **M1**

A1
 $\frac{AB}{\sin \frac{2\pi}{3}} = \frac{\sqrt{3}}{\sin(\frac{2\pi}{3} - \theta)}$
A1
 $= \frac{\sqrt{3}}{\sin \frac{2\pi}{3} \cos \theta - \cos \frac{2\pi}{3} \sin \theta}$

Note: Condone use of degrees.

A1
 $= \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta}$
 $\frac{AB}{2} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta}$
AG
 $\therefore AB = \frac{2\sqrt{3}}{\sqrt{3} \cos \theta + \sin \theta}$
[4 marks]

Examiners report

[N/A]

20b. [4 marks]

Markscheme

METHOD 1

$$\begin{aligned} \text{M1A1} \quad (AB)' &= \frac{-3(-\sqrt{3} \sin \theta + \cos \theta)}{\text{setting } \text{M1} \sqrt{3} \cos \theta + \sin \theta)^2} \\ (AB)' &= 0 \end{aligned}$$

$$\tan \theta = \frac{1}{\sqrt{3}} \quad \text{A1}$$

$$\theta = \frac{\pi}{6}$$

METHOD 2

$$\begin{aligned} AB &= \frac{\sqrt{3} \sin \frac{\pi}{3}}{\sin(\frac{2\pi}{3} - \theta)} \quad \text{minimum (when)} \quad \text{is maximum} \quad \text{M1} \\ AB &= \frac{\sqrt{3}}{\sin(\frac{2\pi}{3} - \theta)} \quad \text{A1} \end{aligned}$$

$$\sin(\frac{2\pi}{3} - \theta) = 1 \quad \text{M1}$$

$$\frac{2\pi}{3} - \theta = \frac{\pi}{2} \quad \text{A1}$$

$$\theta = \frac{\pi}{6}$$

METHOD 3

shortest distance from B to AC is perpendicular to AC **R1**

$$\theta = \frac{\pi}{6} \quad \text{M1A2}$$

[4 marks]

Total [8 marks]

Examiners report

[N/A]

21a. [2 marks]

Markscheme

$$\begin{aligned} \text{A1} \quad g \circ f(x) &= \frac{\tan x + 1}{\tan x - 1} \\ \text{A1} \end{aligned}$$

$$x \neq \frac{\pi}{4}, 0 \leq x < \frac{\pi}{2}$$

[2 marks]

Examiners report

[N/A]

21b. [2 marks]

Markscheme

$$\begin{aligned} \text{M1A1} \quad \frac{\tan x + 1}{\tan x - 1} &= \frac{\frac{\sin x}{\cos x} + 1}{\frac{\sin x}{\cos x} - 1} \\ \text{AG} \quad &= \frac{\sin x + \cos x}{\sin x - \cos x} \\ \text{[2 marks]} \end{aligned}$$

Examiners report

[N/A]

Markscheme

METHOD 1

$$\begin{aligned} \frac{dy}{dx} &= \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(2 \sin x \cos x - \cos^2 x - \sin^2 x) - (2 \sin x \cos x + \cos^2 x + \sin^2 x)} \\ &= \frac{-2}{\cos^2 x + \sin^2 x - 2 \sin x \cos x} \\ &= \frac{-2}{1 - 2 \sin 2x} \end{aligned}$$

Substitute $\frac{\pi}{6}$ into any formula for $\frac{dy}{dx}$

$$\begin{aligned} &= \frac{-2}{1 - 2 \sin \frac{\pi}{3}} \\ &= \frac{-2}{1 - \sqrt{3}} \\ &= \frac{1 - \sqrt{3}}{-2} \\ &= \frac{1}{2} - \frac{\sqrt{3}}{2} \\ &= \frac{-4}{2 \times \sqrt{4} \sqrt{3^2 + 1}} \\ &= -8 - 4\sqrt{3} \end{aligned}$$

METHOD 2

$$\begin{aligned} \frac{dy}{dx} &= \frac{(\tan x - 1)\sec^2 x - (\tan x + 1)\sec^2 x}{(\tan x - 1)^2} \\ &= \frac{-2\sec^2 x}{(\tan x - 1)^2} \\ &= \frac{-2 \left(\frac{4}{3} \right)}{\left(\tan \frac{\pi}{6} - 1 \right)^2} = \frac{-2 \left(\frac{4}{3} \right)}{\left(\frac{1}{\sqrt{3}} - 1 \right)^2} = \frac{-8}{(1 - \sqrt{3})^2} \end{aligned}$$

Note: Award **M1** for substitution .

$$\frac{-8}{(1 - \sqrt{3})^2} = \frac{-8}{(4 - 2\sqrt{3})} \frac{(4 + 2\sqrt{3})}{(4 + 2\sqrt{3})} = -8 - 4\sqrt{3}$$

Examiners report

[N/A]

Markscheme

Area **M1**

$$\begin{aligned} &= \left| \int_0^{\frac{\pi}{6}} \frac{\sin x + \cos x}{\sin x - \cos x} dx \right| \\ &= \left| \left[\ln |\sin x - \cos x| \right]_0^{\frac{\pi}{6}} \right| \end{aligned}$$

Note: Condone absence of limits and absence of modulus signs at this stage.

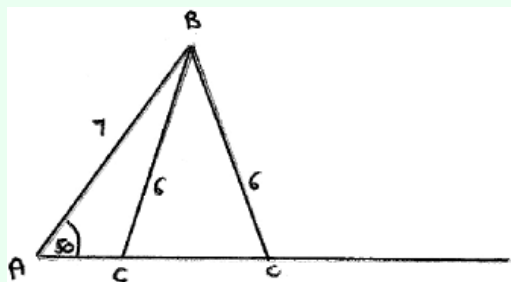
$$\begin{aligned} &= \left| \ln \left| \sin \frac{\pi}{6} - \cos \frac{\pi}{6} \right| - \ln |\sin 0 - \cos 0| \right| \\ &= \left| \ln \left| \frac{1}{2} - \frac{\sqrt{3}}{2} \right| - 0 \right| \\ &= \left| \ln \left(\frac{\sqrt{3} - 1}{2} \right) \right| \\ &= \ln \left(\frac{2}{\sqrt{3} - 1} \right) \\ &= \ln \left(\frac{2}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \right) \\ &= \ln (\sqrt{3} + 1) \end{aligned}$$

Total [16 marks]

Examiners report

[N/A]

Markscheme



METHOD 1

$$\frac{6}{\sin 50} = \frac{7}{\sin C} \Rightarrow \sin C = \frac{7 \sin 50}{6}$$

$$C = 63.344 \dots$$

$$\text{or } C = 116.655 \dots$$

$$B = 13.344 \dots \quad (\text{or } B = 66.656 \dots)$$

$$\text{area} = \frac{1}{2} \times 6 \times 7 \times \sin 13.344 \dots \quad (\text{or } \frac{1}{2} \times 6 \times 7 \times \sin 66.656 \dots)$$

$$4.846 \dots \quad (\text{or } = 19.281 \dots)$$

$$\text{so answer is } 4.85 \text{ (cm}^2\text{)}$$

METHOD 2

$$6^2 = 7^2 + b^2 - 2 \times 7b \cos 50$$

or equivalent method to solve the above equation (M1)

$$b^2 - 14b \cos 50 + 13 = 0$$

$$b = 7.1912821 \dots \quad \text{or } b = 1.807744 \dots$$

$$\text{area} = \frac{1}{2} \times 7 \times 1.8077 \dots \sin 50 = 4.846 \dots$$

$$(\text{or } \frac{1}{2} \times 7 \times 7.1912821 \dots \sin 50 = 19.281 \dots)$$

$$\text{so answer is } 4.85 \text{ (cm}^2\text{)}$$

METHOD 3

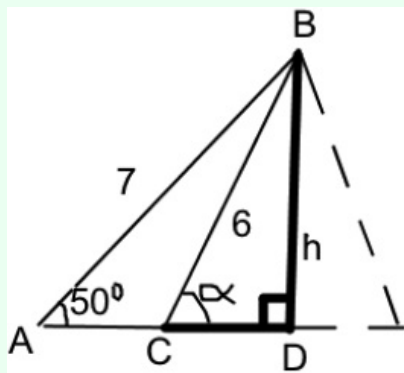


Diagram showing triangles and (M1)
 ACB and ADB

$$h = 7 \sin(50) = 5.3623 \dots \text{ (cm)}$$

$$\alpha = \arcsin \frac{h}{6} = 63.3442 \dots$$

$$AC = AD - CD = 7 \cos 50 - 6 \cos \alpha = 1.8077 \dots \text{ (cm)}$$

$$\text{area} = \frac{1}{2} \times 1.8077 \dots \times 5.3623 \dots$$

$$= 4.85 \text{ (cm}^2\text{)}$$

Total [6 marks]

Examiners report

Most candidates scored 4/6 showing that candidates do not have enough experience with the ambiguous case. Very few candidates drew a suitable diagram that would have illustrated this fact which could have helped them to understand the requirement that the answer should be less than 10. In fact many candidates ignored this requirement or used it incorrectly to solve an inequality.

23a.

[6 marks]

Markscheme

(i)

$$(\cos \theta + i \sin \theta)^5$$

$$= \cos^5 \theta + 5i \cos^4 \theta \sin \theta + 10i^2 \cos^3 \theta \sin^2 \theta +$$

A1A1

$$10i^3 \cos^2 \theta \sin^3 \theta + 5i^4 \cos \theta \sin^4 \theta + i^5 \sin^5 \theta$$

$$(= \cos^5 \theta + 5i \cos^4 \theta \sin \theta - 10 \cos^3 \theta \sin^2 \theta -$$

$$10i \cos^2 \theta \sin^3 \theta + 5 \cos \theta \sin^4 \theta + i \sin^5 \theta)$$

Note: Award first **A1** for correct binomial coefficients.

(ii)

M1

$$(\operatorname{cis} \theta)^5 = \operatorname{cis} 5\theta = \cos 5\theta + i \sin 5\theta$$

$$= \cos^5 \theta + 5i \cos^4 \theta \sin \theta - 10 \cos^3 \theta \sin^2 \theta - 10i \cos^2 \theta \sin^3 \theta +$$

A1

$$5 \cos \theta \sin^4 \theta + i \sin^5 \theta$$

Note: Previous line may be seen in (i)

equating imaginary terms **M1**

AG

$$\sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta$$

(iii) equating real terms

A1

$$\cos 5\theta = \cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta$$

[6 marks]

Examiners report

In part (i) many candidates tried to multiply it out the binomials rather than using the binomial theorem. In parts (ii) and (iii) many candidates showed poor understanding of complex numbers and made no attempt to equate real and imaginary parts. In a some cases the correct answer to part (iii) was seen although it was unclear how it was obtained.

23b.

[4 marks]

Markscheme

M1

$$(r \operatorname{cis} \alpha)^5 = 1 \Rightarrow r^5 \operatorname{cis} 5\alpha = 1 \operatorname{cis} 0$$

A1

$$r^5 = 1 \Rightarrow r = 1$$

(M1)

$$5\alpha = 0 \pm 360k, k \in \mathbb{Z} \Rightarrow \alpha = 72k$$

A1

$$\alpha = 72^\circ$$

Note: Award **M1A0** if final answer is given in radians.

[4 marks]

Examiners report

This question was poorly done. Very few candidates made a good attempt to apply De Moivre's theorem and most of them could not even equate the moduli to obtain .

r

23c.

[4 marks]

Markscheme

use of **OR** the imaginary part of is **(M1)**

$$\sin(5 \times 72) = 0 \quad 1 \quad 0$$

A1

$$0 = 5\cos^4\alpha \sin\alpha - 10\cos^2\alpha \sin^3\alpha + \sin^5\alpha$$

M1

$$\sin\alpha \neq 0 \Rightarrow 0 = 5(1 - \sin^2\alpha)^2 - 10(1 - \sin^2\alpha)\sin^2\alpha + \sin^4\alpha$$

Note: Award **M1** for replacing $\cos^2\alpha$

A1

$$0 = 5(1 - 2\sin^2\alpha + \sin^4\alpha) - 10\sin^2\alpha + 10\sin^4\alpha + \sin^4\alpha$$

Note: Award **A1** for any correct simplification.

so **AG**

$$16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$$

[4 marks]

Examiners report

This question was poorly done. From the few candidates that attempted it, many candidates started by writing down what they were trying to prove and made no progress.

23d.

[5 marks]

Markscheme

M1A1

$$\sin^2\alpha = \frac{20 \pm \sqrt{400 - 320}}{32}$$

$$\sin\alpha = \pm \sqrt{\frac{20 \pm \sqrt{80}}{32}}$$

A1

$$\sin\alpha = \frac{\pm \sqrt{10 \pm 2\sqrt{5}}}{4}$$

Note: Award **A1** regardless of signs. Accept equivalent forms with integral denominator, simplification may be seen later.

as , we have to take both positive signs (or equivalent argument) **R1**

$$\sin 72^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4} = 0.866 \dots$$

Note: Allow verification of correct signs with calculator if clearly stated

A1

$$\sin 72^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

[5 marks]**Total [19 marks]**

Examiners report

Very few made a serious attempt to answer this question. Also very few realised that they could use the answers given in part (c) to attempt this part.

24a. [2 marks]

Markscheme

$$\begin{aligned} A &= \frac{1}{2} \times 5 \times 12 \times \sin 100^\circ \\ &= 29.5 \text{ (cm}^2\text{)} \end{aligned}$$

(M1)
A1
[2 marks]

Examiners report

[N/A]

24b. [2 marks]

Markscheme

$$\begin{aligned} AC^2 &= 5^2 + 12^2 - 2 \times 5 \times 12 \times \cos 100^\circ \\ \text{therefore } AC &= 13.8 \text{ (cm)} \end{aligned}$$

(M1)
A1
[2 marks]

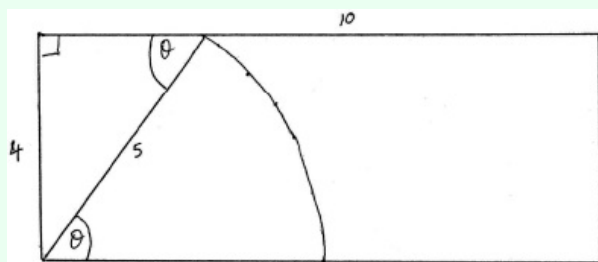
Total [4 marks]

Examiners report

[N/A]

25a. [4 marks]

Markscheme



EITHER

$$\begin{aligned} \text{area of triangle} &= \frac{1}{2} \times 3 \times 4 \quad (= 6) \\ \text{area of sector} &= \frac{1}{2} \arcsin\left(\frac{4}{5}\right) \times 5^2 \quad (= 11.5911 \dots) \end{aligned}$$

A1
A1

OR

$$\int_0^4 \sqrt{25 - x^2} \, dx$$

M1A1
THEN

$$\begin{aligned} \text{total area} &= 17.5911 \dots \text{ m}^2 \\ \text{percentage} &= \frac{17.5911 \dots}{40} \times 100 = 44\% \end{aligned}$$

(A1)
A1
[4 marks]

Examiners report

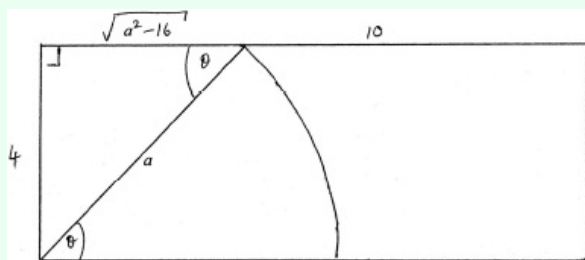
[N/A]

25b.

[4 marks]

Markscheme

METHOD 1



area of triangle = $\frac{1}{2} \times 4 \times \sqrt{a^2 - 16}$

(A1)

$$\theta = \arcsin\left(\frac{4}{a}\right)$$

area of sector

$$= \frac{1}{2} r^2 \theta = \frac{1}{2} a^2 \arcsin\left(\frac{4}{a}\right)$$

therefore total area

$$= 2\sqrt{a^2 - 16} + \frac{1}{2} a^2 \arcsin\left(\frac{4}{a}\right) = 20$$

rearrange to give:

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40$$

METHOD 2

$$\int_0^4 \sqrt{a^2 - x^2} dx = 20$$

use substitution

$$x = a \sin \theta, \quad \frac{dx}{d\theta} = a \cos \theta$$

$$\int_0^{\arcsin\left(\frac{4}{a}\right)} a^2 \cos^2 \theta d\theta = 20$$

$$\frac{a^2}{2} \int_0^{\arcsin\left(\frac{4}{a}\right)} (\cos 2\theta + 1) d\theta = 20$$

$$a^2 \left[\left(\frac{\sin 2\theta}{2} + \theta \right) \right]_0^{\arcsin\left(\frac{4}{a}\right)} = 40$$

$$a^2 \left[(\sin \theta \cos \theta + \theta) \right]_0^{\arcsin\left(\frac{4}{a}\right)} = 40$$

$$a^2 \arcsin\left(\frac{4}{a}\right) + a^2 \left(\frac{4}{a}\right) \sqrt{1 - \left(\frac{4}{a}\right)^2} = 40$$

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40$$

[4 marks]

Examiners report

[N/A]

25c.

[2 marks]

Markscheme

solving using

(A2)

$$\text{GDC} \Rightarrow a = 5.53 \text{ cm}$$

[2 marks]

Total [10 marks]

Examiners report

[N/A]

26a.

[1 mark]

Markscheme

area of

$$\text{AOP} = \frac{1}{2} r^2 \sin \theta$$

[1 mark]

Examiners report

The majority of candidates were able to find the area of Triangle AOP correctly. Most were then able to get an expression for the other triangle. In the final section, few saw the connection between the area of the sector and the relationship.

26b. [2 marks]

Markscheme

(MI)
 $TP = r \tan \theta$
 area of POT

$$= \frac{1}{2} r (r \tan \theta)$$

$$= \frac{1}{2} r^2 \tan \theta$$

[2 marks]

Examiners report

The majority of candidates were able to find the area of Triangle AOP correctly. Most were then able to get an expression for the other triangle. In the final section, few saw the connection between the area of the sector and the relationship.

26c. [2 marks]

Markscheme

area of sector OAP

$$= \frac{1}{2} r^2 \theta$$

area of triangle OAP < area of sector OAP < area of triangle POT **RI**

$$\frac{1}{2} r^2 \sin \theta < \frac{1}{2} r^2 \theta < \frac{1}{2} r^2 \tan \theta$$

$$\sin \theta < \theta < \tan \theta$$

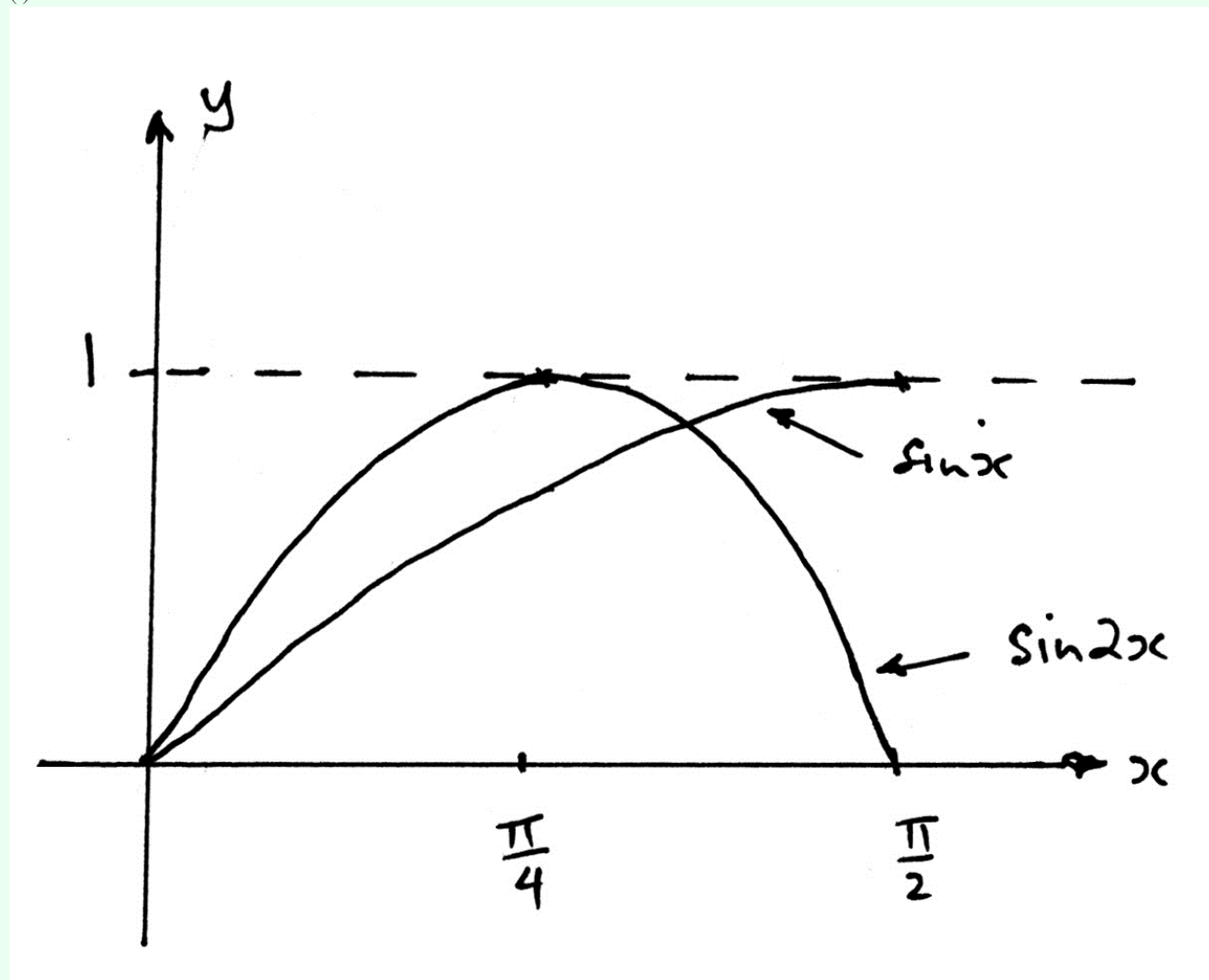
[2 marks]

Examiners report

The majority of candidates were able to find the area of Triangle AOP correctly. Most were then able to get an expression for the other triangle. In the final section, few saw the connection between the area of the sector and the relationship.

Markscheme

(i)



A2

Note: Award **A1** for correct

, **A1** for correct

$\sin x$

$\sin 2x$

Note: Award **A1A0** for two correct shapes with

and/or 1 missing.

$\frac{\pi}{2}$

Note: Condone graph outside the domain.

(ii)

$$\sin 2x = \sin x$$

$$0 \leq x \leq \frac{\pi}{2}$$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x (2 \cos x - 1) = 0$$

$$x = 0, \frac{\pi}{3}$$

(iii) area

$$\int_0^{\frac{\pi}{3}} (\sin 2x - \sin x) dx$$

Note: Award **M1** for an integral that contains limits, not necessarily correct, with

and

$\sin x$ subtracted in either order.

$\sin 2x$

$$\begin{aligned}
 &= \left[-\frac{1}{2} \cos 2x + \cos x \right]_0^{\frac{\pi}{3}} \\
 &= \left(-\frac{1}{2} \cos \frac{2\pi}{3} + \cos \frac{\pi}{3} \right) - \left(-\frac{1}{2} \cos 0 + \cos 0 \right) \\
 &= \frac{3}{4} - \frac{1}{2} \\
 &= \frac{1}{4}
 \end{aligned}$$

[9 marks]

Examiners report

A significant number of candidates did not seem to have the time required to attempt this question satisfactorily.

Part (a) was done quite well by most but a number found sketching the functions difficult, the most common error being poor labelling of the axes.

Part (ii) was done well by most the most common error being to divide the equation by

and so omit the $x = 0$ value. Many recognised the value from the graph and corrected this in their final solution.

The final part was done well by many candidates.

Many candidates found (b) challenging. Few were able to substitute the dx expression correctly and many did not even seem to recognise the need for this term. Those that did tended to be able to find the integral correctly. Most saw the need for the double angle expression although many did not change the limits successfully.

Few candidates attempted part c). Those who did get this far managed the sketch well and were able to explain the relationship required. Among those who gave a response to this many were able to get the result although a number made errors in giving the inverse function. On the whole those who got this far did it well.

27b.

[8 marks]

Markscheme

$\int_0^1 \frac{1}{\sqrt{x}} dx = \int_0^{\frac{\pi}{6}} \sqrt{\frac{4\sin^2\theta}{3}} \times 8\sin\theta \cos\theta d\theta$
 Note: Award $M1$ for substitution and reasonable attempt at finding expression for dx in terms of $d\theta$, first $A1$ for correct limits, second $A1$ for correct substitution for dx .

$$\begin{aligned}
 &\int_0^{\frac{\pi}{6}} 8\sin^2\theta d\theta \\
 &\int_0^{\frac{\pi}{6}} 4 - 4\cos 2\theta d\theta \\
 &= \left[4\theta - 2\sin 2\theta \right]_0^{\frac{\pi}{6}} \\
 &= \left(\frac{2\pi}{3} - 2\sin \frac{\pi}{3} \right) - 0 \\
 &= \frac{2\pi}{3} - \sqrt{3}
 \end{aligned}$$

[8 marks]

Examiners report

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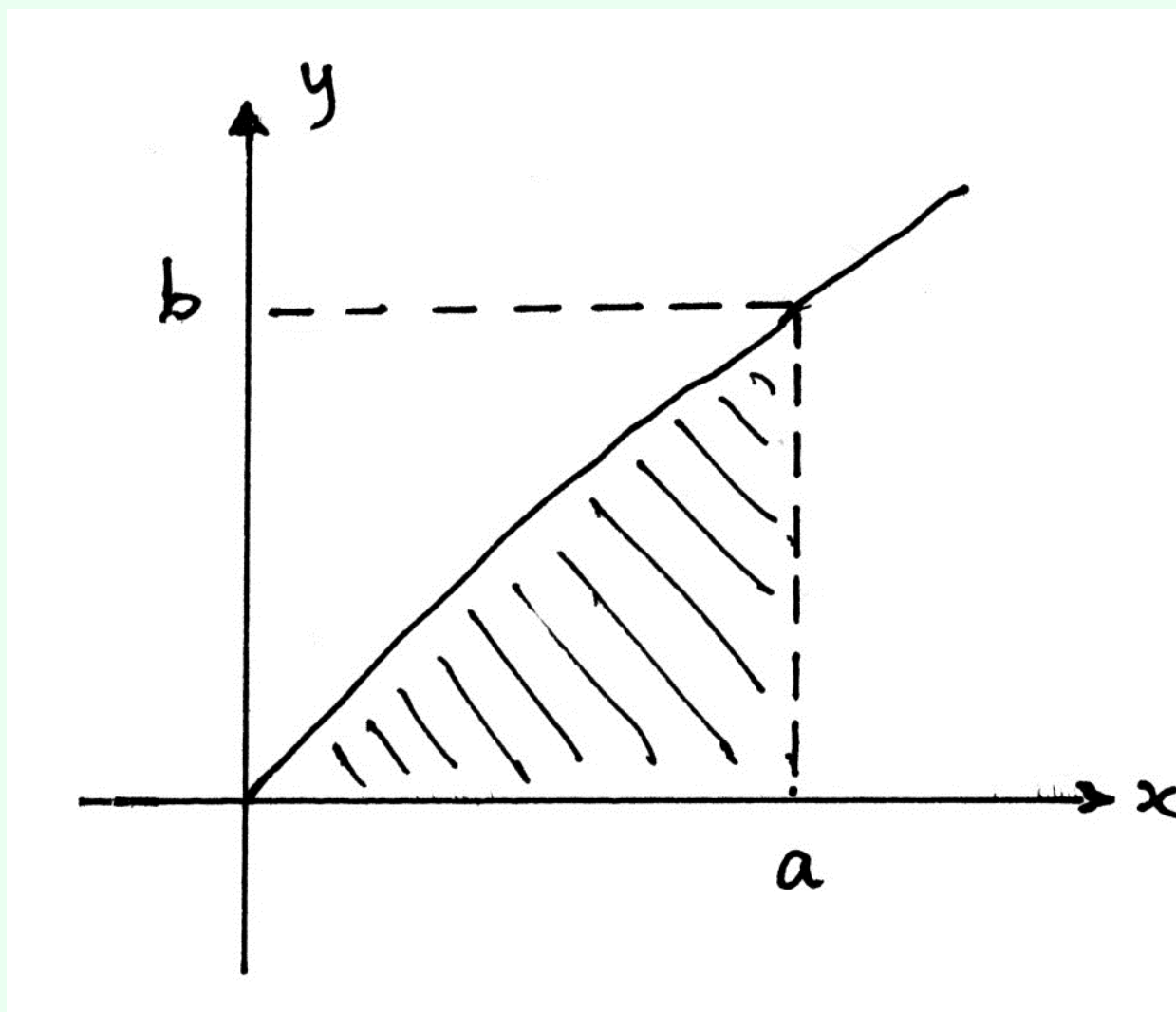
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Markscheme

(i)

**MI**

from the diagram above

the shaded area

$$\begin{aligned}
 &\text{RI} \\
 &= \int_0^a f(x) dx = ab - \int_0^b f^{-1}(y) dy \\
 &\text{AG} \\
 &= ab - \int_0^b f^{-1}(x) dx
 \end{aligned}$$

(ii)

$$\begin{aligned}
 &\text{AI} \\
 f(x) &= \arcsin \frac{x}{4} \Rightarrow f^{-1}(x) = 4 \sin x \\
 &\text{MIAIAI} \\
 \int_0^2 \arcsin\left(\frac{x}{4}\right) dx &= \frac{\pi}{3} - \int_0^{\frac{\pi}{6}} 4 \sin x dx
 \end{aligned}$$

Note: Award **AI** for the limitseen anywhere, **AI** for all else correct.

$$\frac{\pi}{6}$$

$$\begin{aligned}
 &\text{AI} \\
 &= \frac{\pi}{3} - [-4 \cos x]_0^{\frac{\pi}{6}} \\
 &\text{AI} \\
 &= \frac{\pi}{3} - 4 + 2\sqrt{3}
 \end{aligned}$$

Note: Award no marks for methods using integration by parts.

[8 marks]

Examiners report

A significant number of candidates did not seem to have the time required to attempt this question satisfactorily.

Part (a) was done quite well by most but a number found sketching the functions difficult, the most common error being poor labelling of the axes.

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and so omit the $x = 0$ value. Many recognised the value from the graph and corrected this in their final solution.

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28.

[5 marks]

Markscheme

area of triangle

$$\begin{aligned} \text{POQ} &= \frac{1}{2} 8^2 \sin 59^\circ \\ &= 27.43 \quad (A1) \end{aligned}$$

area of sector

$$\begin{aligned} &= \pi 8^2 \frac{59}{360} \\ &= 32.95 \quad (A1) \end{aligned}$$

area between arc and chord

$$= 32.95 - 27.43$$

$$\begin{aligned} &= 5.52 \quad (A1) \end{aligned}$$

[5 marks]

Examiners report

This was an easy starter question, with most candidates gaining full marks. Others lost marks through premature rounding or the incorrect use of radian measure.

Markscheme

let the length of one side of the triangle be x

consider the triangle consisting of a side of the triangle and two radii

EITHER

$$x^2 = r^2 + r^2 - 2r^2 \cos 120^\circ$$

$$= 3r^2$$

OR

$$x = 2r \cos 30^\circ$$

THEN

$$x = r\sqrt{3}$$

so perimeter

$$= 3\sqrt{3}r$$

now consider the area of the triangle

area

$$= 3 \times \frac{1}{2} r^2 \sin 120^\circ$$

$$= 3 \times \frac{\sqrt{3}}{4} r^2$$

$$\frac{P}{A} = \frac{3\sqrt{3}r}{\frac{3\sqrt{3}}{4}r^2}$$

$$= \frac{4}{r}$$

Note: Accept alternative methods

[6 marks]

Examiners report

It was pleasing to see some very slick solutions to this question. There were various reasons for the less successful attempts: not drawing a diagram; drawing a diagram, but putting one vertex of the triangle at the centre of the circle; drawing the circle inside the triangle; the side of the triangle being denoted by r the symbol used in the question for the radius of the circle.

Markscheme

(a)

$$\begin{array}{l} \text{M1} \\ -2 = 1 + k \sin\left(\frac{\pi}{6}\right) \end{array}$$

$$\begin{array}{l} \text{A1} \\ -3 = \frac{1}{2}k \end{array}$$

$$\begin{array}{l} \text{AG} \quad \text{N0} \\ k = -6 \end{array}$$

(b) **METHOD 1**

maximum

$$\begin{array}{l} \text{M1} \\ \Rightarrow \sin x = -1 \end{array}$$

$$\begin{array}{l} \text{A1} \\ a = \frac{3\pi}{2} \end{array}$$

$$b = 1 - 6(-1)$$

$$\begin{array}{l} \text{A1} \quad \text{N2} \\ = 7 \end{array}$$

METHOD 2

$$\begin{array}{l} \text{M1} \\ y' = 0 \end{array}$$

$$k \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$\begin{array}{l} \text{A1} \\ a = \frac{3\pi}{2} \end{array}$$

$$b = 1 - 6(-1)$$

$$\begin{array}{l} \text{A1} \quad \text{N2} \\ = 7 \end{array}$$

Note: Award *A1A1* for

$$\left(\frac{3\pi}{2}, 7\right)$$

[5 marks]

Examiners report

This was the most successfully answered question in the paper. Part (a) was done well by most candidates. In part (b), a small number of candidates used knowledge about transformations of functions to identify the coordinates of B. Most candidates used differentiation.

Markscheme

(a) METHOD 1

let

and

$$x = \arctan \frac{1}{2} \Rightarrow \tan x = \frac{1}{2}$$

$$y = \arctan \frac{1}{3} \Rightarrow \tan y = \frac{1}{3}$$

$$\text{MI} \quad \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = 1$$

so,

AIAG

$$x+y = \arctan 1 = \frac{\pi}{4}$$

METHOD 2

for

,
if $x, y > 0$

$$\text{MI} \quad \arctan x + \arctan y = \arctan \left(\frac{x+y}{1-xy} \right)$$

$$xy < 1$$

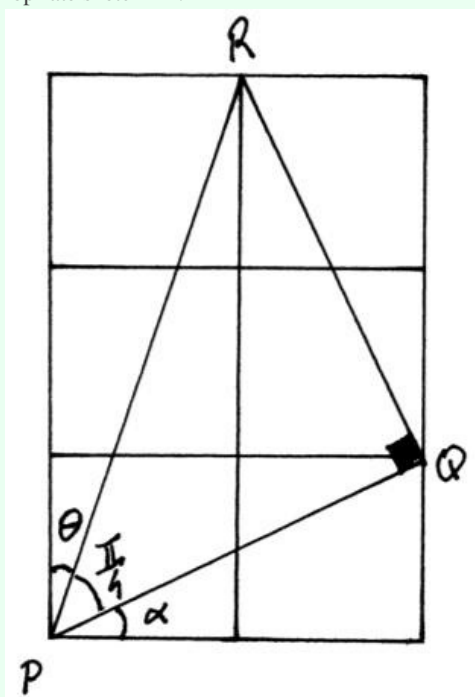
so,

AIAG

$$\text{METHOD 3} \quad \arctan \frac{1}{2} + \arctan \frac{1}{3} = \arctan \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} \right) = \frac{\pi}{4}$$

an appropriate sketch MI

e.g.



correct reasoning leading to

$$\text{RIAG} \quad \frac{\pi}{4}$$

(b) METHOD 1

$$\text{(MI)} \quad \arctan(2) + \arctan(3) = \frac{\pi}{2} - \arctan\left(\frac{1}{2}\right) + \frac{\pi}{2} - \arctan\left(\frac{1}{3}\right)$$

$$\text{(AI)} \quad = \pi - \left(\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) \right)$$

Note: Only one of the previous two marks may be implied.

$$\text{AI} \quad \text{NI} \quad = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

METHOD 2

let

and

$$x = \arctan 2 \Rightarrow \tan x = 2$$

$$y = \arctan 3 \Rightarrow \tan y = 3$$

$$\text{(MI)} \quad \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{2+3}{1-2 \times 3} = -1$$

as

$$\frac{\pi}{4} < x < \frac{\pi}{2} \quad (\text{accept } 0 < x < \frac{\pi}{2})$$

and

$$\frac{\pi}{4} < y < \frac{\pi}{2} \quad (\text{accept } 0 < y < \frac{\pi}{2})$$

(RI)

$$\frac{\pi}{4} < x + y < \pi \quad (\text{accept } 0 < x + y < \pi)$$

Note: Only one of the previous two marks may be implied.

so,

$$x + y = \frac{3\pi}{4}$$

METHOD 3

for

$$x, y > 0$$

(MI)

$$\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right) + \pi \text{ if } xy > 1$$

so,

(AI)

$$\arctan 2 + \arctan 3 = \arctan\left(\frac{2+3}{1-6}\right) + \pi$$

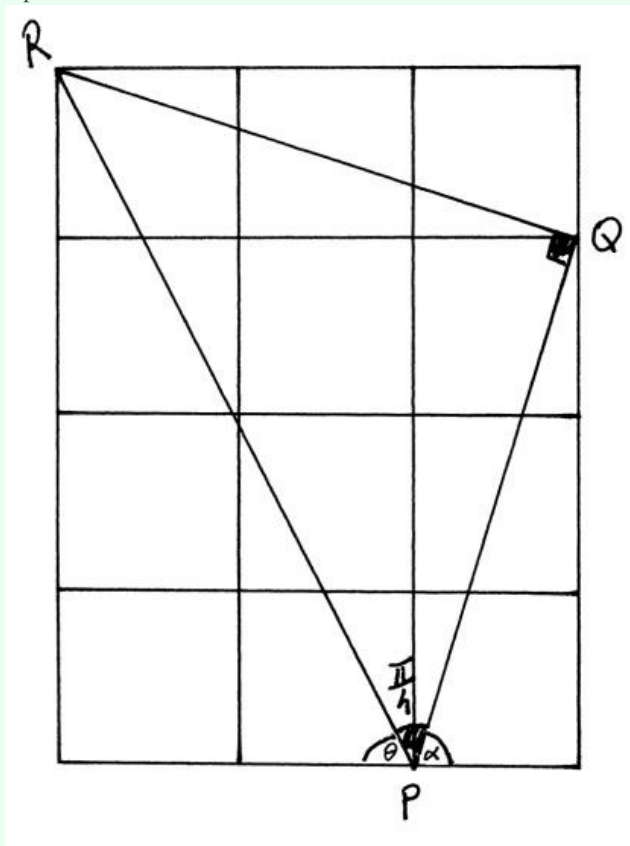
Note: Only one of the previous two marks may be implied.

$$= \frac{3\pi}{4}$$

METHOD 4

an appropriate sketch **MI**

e.g.



correct reasoning leading to

$$\frac{3\pi}{4}$$

[5 marks]

Examiners report

Most candidates had difficulties with this question due to a number of misconceptions, including

and
 $\arctan x = \tan^{-1} x = \frac{\cos x}{\sin x}$
 , showing that, although candidates were familiar with the notation, they did not understand its meaning. Part (a) was done well
 $\arctan x = \frac{\arcsin x}{\arccos x}$
 among candidates who recognized arctan as the inverse of the tangent function but just a few were able to identify the relationship between parts (a) and (b). Very few candidates attempted a geometrical approach to this question.

32.

[5 marks]

Markscheme

$$A = \frac{AI}{2}(R^2 - r^2)$$

$$B = \frac{AI}{2}r^2$$

from

, we have

$$A : B = 2 : 1$$

$$\frac{MI}{R^2 - r^2} = 2r^2$$

$$R = \sqrt{3}r$$

hence exact value of the ratio

$$R : r \text{ is } \sqrt{3} : 1$$

[5 marks]

Examiners report

This question was successfully answered by most candidates using a variety of correct approaches. A few candidates, however, did not use a parameter for the angle, but instead substituted an angle directly, e.g.,

$$\text{or} \\ \frac{\pi}{2} \\ \frac{\pi}{4}$$

33.

[8 marks]

$$n^2 + n + 1 > 2n + 1$$

$$n^2 + n + 1 > n^2 - 1$$

$$\cos \theta = \frac{(2n+1)^2 + (n^2-1)^2 - (n^2+n+1)^2}{2(2n+1)(n^2-1)}$$

$$= \frac{-2n^3 - n^2 + 2n + 1}{2(2n+1)(n^2-1)}$$

$$= -\frac{(n-1)(n+1)(2n+1)}{2(2n+1)(n^2-1)}$$

$$= -\frac{1}{2}$$

$$\theta = 120^\circ$$

Examiners report

There were very few complete and accurate answers to part a). The most common incorrect response was to state the triangle inequality and feel that this was sufficient.

Many substituted a particular value for n and illustrated the result. Most students recognised the need for the Cosine rule and applied it correctly. Many then expanded and simplified to the correct answer. There was significant fudging in the middle on some papers.

There were many good responses to this question.

Markscheme

(a)

$$\begin{aligned} & \text{MIAI} \\ \sin(2n+1)x \cos x - \cos(2n+1)x \sin x &= \sin(2n+1)x - x \\ & \text{AG} \\ &= \sin 2nx \end{aligned}$$

[2 marks]

(b) if $n = 1$ **MI**

$$\text{LHS} = \cos x$$

$$\begin{aligned} & \text{MI} \\ \text{RHS} &= \frac{\sin 2x}{2 \sin x} = \frac{2 \sin x \cos x}{2 \sin x} = \cos x \\ \text{so LHS} &= \text{RHS and the statement is true for } n = 1 \quad \text{RI} \end{aligned}$$

assume true for $n = k$ **MI****Note:** Only award **MI** if the word **true** appears.Do **not** award **MI** for 'let $n = k$ ' only.Subsequent marks are independent of this **MI**.

so

$$\cos x + \cos 3x + \cos 5x + \dots + \cos(2k-1)x = \frac{\sin 2kx}{2 \sin x}$$

if $n = k+1$ then

$$\begin{aligned} & \text{MI} \\ \cos x + \cos 3x + \cos 5x + \dots + \cos(2k-1)x + \cos(2k+1)x \\ &= \frac{\text{AI}}{\sin 2kx} + \cos(2k+1)x \\ &= \frac{\text{MI}}{\sin 2kx + 2 \cos(2k+1)x \sin x} \\ &= \frac{\text{MI}}{\sin(2k+1)x \cos x - \cos(2k+1)x \sin x + 2 \cos(2k+1)x \sin x} \\ &= \frac{\text{AI}}{\sin(2k+1)x \cos x + \cos(2k+1)x \sin x} \\ &= \frac{\text{MI}}{\sin(2k+2)x} \\ &= \frac{\text{AI}}{\sin 2(k+1)x} \end{aligned}$$

so if true for $n = k$, then also true for $n = k+1$

as true for $n = 1$ then true for all

$$\text{RI}$$

$$n \in \mathbb{Z}^+$$

Note: Final **RI** is independent of previous work.

[12 marks]

(c)

$$\frac{\text{MIAI}}{\sin 4x} = \frac{1}{2}$$

$$\begin{aligned} \sin 4x &= \sin x \\ \text{but this is impossible} \\ 4x = x &\Rightarrow x = 0 \end{aligned}$$

$$\text{AI}$$

$$4x = \pi - x \Rightarrow x = \frac{\pi}{5}$$

$$\text{AI}$$

$$4x = 2\pi + x \Rightarrow x = \frac{2\pi}{3}$$

$$\text{AI}$$

$$4x = 3\pi - x \Rightarrow x = \frac{3\pi}{5}$$

for not including any answers outside the domain **RI****Note:** Award the first **MIAI** for correctly obtaining

or equivalent and subsequent marks as appropriate including the answers

$$8\cos^2 x - 4\cos x - 1 = 0$$

$$\left(-\frac{1}{6} \frac{1 \pm \sqrt{5}}{\text{marks}} \right)$$

Total [20 marks]

Examiners report

This question showed the weaknesses of many candidates in dealing with formal proofs and showing their reasoning in a logical manner. In part (a) just a few candidates clearly showed the result and part (b) showed that most candidates struggle with the formality of a proof by induction. The logic of many solutions was poor, though sometimes contained correct trigonometric work. Very few candidates were successful in answering part (c) using the unit circle. Most candidates attempted to manipulate the equation to obtain a cubic equation but made little progress. A few candidates guessed $\frac{2\pi}{3}$ as a solution but were not able to determine the other solutions.

35. [4 marks]

Markscheme

$$\begin{aligned} & AI \\ a &= 3 \\ & AI \\ c &= 2 \\ & \text{period} \\ & (MI) \\ & = \frac{2\pi}{b} = 3 \\ & AI \\ b &= \frac{2\pi}{3} (= 2.09) \\ & [4 \text{ marks}] \end{aligned}$$

Examiners report

Most candidates were able to find a and c , but many had difficulties with finding b .

36. [6 marks]

Markscheme

$$\begin{aligned} AC &= AB = 10 \text{ (cm)} & AI \\ \text{triangle OBC is equilateral} & & (MI) \\ BC &= 6 \text{ (cm)} & AI \\ \text{EITHER} \\ \hat{BAC} &= 2 \arcsin \frac{3}{10} & MIAI \\ (\text{accept } 0.609 \text{ radians}) & & AI \\ \hat{BAC} &= 34.9^\circ \\ \text{OR} \\ \cos \hat{BAC} &= \frac{10^2 + 10^2 - 6^2}{2 \times 10 \times 10} = \frac{164}{200} & MIAI \\ (\text{accept } 0.609 \text{ radians}) & & AI \\ \hat{BAC} &= 34.9^\circ \\ \text{Note: Other valid methods may be seen.} \end{aligned}$$

[6 marks]

Examiners report

The question was generally well answered, but some students attempted to find the length of arc BC.

Markscheme

(a)

(allied) *AI*

$$\text{OAB} = \pi - \theta$$

recognizing OAB as an isosceles triangle *MI*

so

$$\hat{\text{A}}\hat{\text{B}}\hat{\text{O}} = \pi - \theta$$

(alternate) *AG*

$$\text{BOC} = \pi - \theta$$

Note: This can be done in many ways, including a clear diagram.*[3 marks]*

(b) area of trapezium is

(MI)

$$T = \text{area}_{\triangle \text{BOC}} + \text{area}_{\triangle \text{AOB}}$$

MIAI

$$= \frac{1}{2}r^2 \sin(\pi - \theta) + \frac{1}{2}r^2 \sin(2\theta - \pi)$$

AG

$$= \frac{1}{2}r^2 \sin \theta - \frac{1}{2}r^2 \sin 2\theta$$

[3 marks]

(c) (i)

$$\frac{dT}{d\theta} = \frac{1}{2}r^2 \cos \theta - r^2 \cos 2\theta$$

for maximum area

MI

$$\frac{1}{2}r^2 \cos \theta - r^2 \cos 2\theta = 0$$

AG

$$\cos \theta = 2 \cos 2\theta$$

(ii)

(AI)

$$\theta_{\max} = 2.205 \dots$$

AI

$$\frac{1}{2} \sin \theta_{\max} - \frac{1}{2} \sin 2\theta_{\max} = 0.880$$

*[5 marks]***Total [11 marks]**

Examiners report

In part (a) students had difficulties supporting their statements and were consequently unable to gain all the marks here. There were some good attempts at parts (b) and (c) although many students failed to recognise r as a constant and hence differentiated it, often incorrectly.

38.

[6 marks]

Markscheme

$$\sin\left(x + \frac{\pi}{3}\right) = \sin x \cos\left(\frac{\pi}{3}\right) + \cos x \sin\left(\frac{\pi}{3}\right)$$

$$\sin x \cos\left(\frac{\pi}{3}\right) + \cos x \sin\left(\frac{\pi}{3}\right) = 2 \sin x \sin\left(\frac{\pi}{3}\right)$$

$$\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x = 2 \times \frac{\sqrt{3}}{2} \sin x$$

dividing by

and rearranging MI
 $\cos x$

$$\tan x = \frac{\sqrt{3}}{2\sqrt{3}-1}$$

rationalizing the denominator MI

$$11 \tan x = 6 + \sqrt{3}$$

[6 marks]

Examiners report

Most candidates were able to make a meaningful start to this question, but a significant number were unable to find an appropriate expression for

or to rationalise the denominator.
 $\tan x$

39.

[6 marks]

Markscheme

use of cosine rule:

$$BC = \sqrt{(8^2 + 7^2 - 2 \times 7 \times 8 \cos 70)} = 8.6426 \dots$$

Note: Accept an expression for

$$BC^2$$

$$BD = 5.7617 \dots \quad (CD = 2.88085 \dots)$$

use of sine rule:

$$\hat{B} = \arcsin\left(\frac{7 \sin 70}{8.6426}\right) = 49.561 \dots^\circ \quad (\hat{C} = 60.4387 \dots^\circ)$$

use of cosine rule:

$$AD = \sqrt{8^2 + BD^2 - 2 \times BD \times 8 \cos B} = 6.12 \text{ (cm)}$$

Note: Scale drawing method not acceptable.

[6 marks]

Examiners report

Well done.

Markscheme

(a) the area of the first sector is

$$\frac{1}{2}2^2\theta$$

the sequence of areas is

$$(AI)$$

$$2\theta, 2k\theta, 2k^2\theta \dots$$

the sum of these areas is

$$(M1)$$

$$2\theta(1 + k + k^2 + \dots)$$

$$(M1A1)$$

$$= \frac{2\theta}{1-k} = 4\pi$$

hence

$$(AG)$$

$$\theta = 2\pi(1 - k)$$

Note: Accept solutions where candidates deal with angles instead of area.

[5 marks]

(b) the perimeter of the first sector is

$$(A1)$$

$$4 + 2\theta$$

the perimeter of the third sector is

$$(A1)$$

$$4 + 2k^2\theta$$

the given condition is

$$(M1)$$

$$4 + 2k^2\theta = 2 + \theta$$

which simplifies to

$$(A1)$$

$$2 = \theta(1 - 2k^2)$$

eliminating

, obtain cubic in k :

$$\theta$$

$$(A1)$$

$$\pi(1 - k)(1 - 2k^2) - 1 = 0$$

or equivalent

solve for $k = 0.456$ and then

$$(A1A1)$$

$$\theta = 3.42$$

[7 marks]

Total [12 marks]

Examiners report

This was a disappointingly answered question.

Part(a) - Many candidates correctly assumed that the areas of the sectors were proportional to their angles, but did not actually state that fact.

Part(b) - Few candidates seem to know what the term ‘perimeter’ means.

Markscheme

(a) area

$$= \frac{1}{2} \times BC \times AB \times \sin B \quad (M1)$$

$$(10 = \frac{1}{2} \times 5 \times 6 \times \sin B)$$

$$\sin B = \frac{2}{3} \quad (A1)$$

(b)

$$\cos B = \pm \frac{\sqrt{5}}{3} (= \pm 0.7453\dots) \text{ or } B = 41.8\dots \text{ and } 138.1\dots \quad (A1)$$

$$AC^2 = BC^2 + AB^2 - 2 \times BC \times AB \times \cos B \quad (M1)$$

$$AC = \sqrt{5^2 + 6^2 - 2 \times 5 \times 6 \times 0.7453\dots} \text{ or } \sqrt{5^2 + 6^2 + 2 \times 5 \times 6 \times 0.7453\dots}$$

$$AC = 4.03 \text{ or } 10.28 \quad (A1A1)$$

[6 marks]

Examiners report

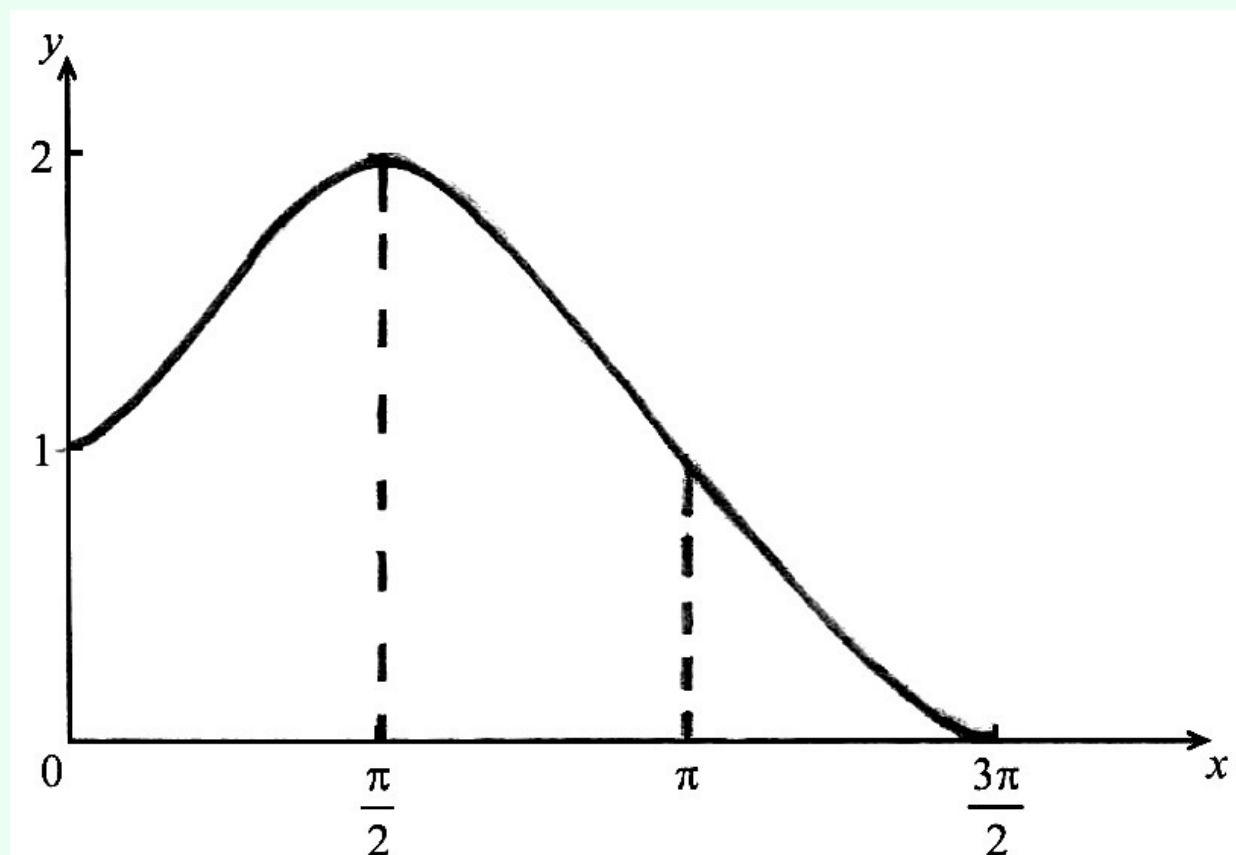
Most candidates attempted this question and part (a) was answered correctly by most candidates but in (b), despite the wording of the question, the obtuse angle was often omitted leading to only one solution.

In many cases early rounding led to inaccuracy in the final answers and many candidates failed to round their answers to two decimal places as required.

42a.

[1 mark]

Markscheme



A1

[1 mark]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of $\cos(2x)$ and the insertion of the limits.

42b.

[1 mark]

Markscheme

$$(1 + \sin x)^2 = 1 + 2 \sin x + \sin^2 x$$

$$\stackrel{A1}{=} 1 + 2 \sin x + \frac{1}{2}(1 - \cos 2x)$$

$$\stackrel{AG}{=} \frac{3}{2} + 2 \sin x - \frac{1}{2} \cos 2x$$

[1 mark]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of $\cos(2x)$ and the insertion of the limits.

42c. [4 marks]

Markscheme

$$\begin{aligned}
 V &= \pi \int_0^{\frac{3\pi}{2}} (1 + \sin x)^2 dx \\
 &= \pi \int_0^{\frac{3\pi}{2}} \left(\frac{3}{2} + 2 \sin x - \frac{1}{2} \cos 2x \right) dx \\
 &= \pi \left[\frac{3}{2}x - 2 \cos x - \frac{\sin 2x}{4} \right]_0^{\frac{3\pi}{2}} \\
 &= \frac{9\pi^2}{4} + 2\pi
 \end{aligned}$$

[4 marks]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of $\cos(2x)$ and the insertion of the limits.

43a. [3 marks]

Markscheme

angle APB is a right angle

$$\begin{aligned}
 &\text{AI} \\
 &\Rightarrow \cos \theta = \frac{AP}{4} \Rightarrow AP = 4 \cos \theta
 \end{aligned}$$

Note: Allow correct use of cosine rule.

$$\begin{aligned}
 &\text{AI} \\
 \text{arc PB} &= 2 \times 2\theta = 4\theta
 \end{aligned}$$

$$t = \frac{AP}{3} + \frac{PB}{6}$$

Note: Allow use of their AP and their PB for the **AI**.

$$\begin{aligned}
 &\text{AG} \\
 &\Rightarrow t = \frac{4 \cos \theta}{3} + \frac{4\theta}{6} = \frac{4 \cos \theta}{3} + \frac{2\theta}{3} = \frac{2}{3}(2 \cos \theta + \theta)
 \end{aligned}$$

[3 marks]

Examiners report

The fairly easy trigonometry challenged a large number of candidates.

43b. [2 marks]

Markscheme

$$\begin{aligned}
 \frac{dt}{d\theta} &= \frac{2}{3}(-2 \sin \theta + 1) \\
 &\text{(or 30 degrees)} \\
 \frac{2}{3}(-2 \sin \theta + 1) &= 0 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}
 \end{aligned}$$

[2 marks]

Examiners report

Part (b) was very well done.

43c. [3 marks]

Markscheme

$$\frac{d^2t}{d\theta^2} = -\frac{4}{3}\cos\theta < 0 \quad \left(\text{at } \theta = \frac{\pi}{6}\right)$$

is maximized at
 $\Rightarrow t$

$$\theta = \frac{\pi}{6}$$

time needed to walk along arc AB is

$$\frac{2\pi}{6} \quad (\approx 1 \text{ hour})$$

time needed to row from A to B is

$$\frac{4}{3} \quad (\approx 1.33 \text{ hour})$$

hence, time is minimized in walking from A to B **RI**

[3 marks]

Examiners report

Satisfactory answers were very rarely seen for (c). Very few candidates realised that a minimum can occur at the beginning or end of an interval.

44a. [3 marks]

Markscheme

$$\tan\left(\arctan\frac{1}{2} - \arctan\frac{1}{3}\right) = \tan(\arctan a)$$

$$a = 0.14285\dots = \frac{1}{7}$$

[3 marks]

Examiners report

Many candidates failed to give the answer for (a) in rational form. The GDC can render the answer in this form as well as the decimal approximation, but this was obviously missed by many candidates.

44b. [2 marks]

Markscheme

$$\arctan\left(\frac{1}{7}\right) = \arcsin(x) \Rightarrow x = \sin\left(\arctan\frac{1}{7}\right) \approx 0.141$$

Note: Accept exact value of

$$\left(\frac{1}{\sqrt{50}}\right)$$

[2 marks]

Examiners report

(b) was generally answered successfully.

45a. [2 marks]

Markscheme

$$\frac{\sin 2\theta}{1 + \cos 2\theta} = \frac{2 \sin \theta \cos \theta}{1 + 2 \cos^2 \theta - 1}$$

Note: Award *MI* for use of double angle formulae.

$$= \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta}$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta$$

[2 marks]

Examiners report

The performance in this question was generally good with most candidates answering (a) well; (b) caused more difficulties, in particular the rationalization of the denominator. A number of misconceptions were identified, for example

$$\cot \frac{\pi}{8} = \tan \frac{8}{\pi}$$

45b. [3 marks]

Markscheme

$$\tan \frac{\pi}{8} = \frac{\sin \frac{\pi}{4}}{1 + \cos \frac{\pi}{4}}$$

$$\cot \frac{\pi}{8} = \frac{1 + \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}}$$

$$= \frac{1 + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$= 1 + \sqrt{2}$$

[3 marks]

Examiners report

The performance in this question was generally good with most candidates answering (a) well; (b) caused more difficulties, in particular the rationalization of the denominator. A number of misconceptions were identified, for example

$$\cot \frac{\pi}{8} = \tan \frac{8}{\pi}$$

46a. [2 marks]

Markscheme

$$(\cos \theta + i \sin \theta)^3 = \cos^3 \theta + 3 \cos^2 \theta (i \sin \theta) + 3 \cos \theta (i \sin \theta)^2 + (i \sin \theta)^3$$

$$= \cos^3 \theta - 3 \cos \theta \sin^2 \theta + i (3 \cos^2 \theta \sin \theta - \sin^3 \theta)$$

[2 marks]

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

46b.

[3 marks]

Markscheme

from De Moivre's theorem

$$\stackrel{(MI)}{(\cos \theta + i \sin \theta)^3 = \cos 3\theta + i \sin 3\theta}$$

$$\cos 3\theta + i \sin 3\theta = (\cos^3 \theta - 3 \cos \theta \sin^2 \theta) + i (3 \cos^2 \theta \sin \theta - \sin^3 \theta)$$

equating real parts MI

$$\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta$$

$$\stackrel{AI}{=} \cos^3 \theta - 3 \cos \theta (1 - \cos^2 \theta)$$

$$= \cos^3 \theta - 3 \cos \theta + 3 \cos^3 \theta$$

$$\stackrel{AG}{=} 4 \cos^3 \theta - 3 \cos \theta$$

Note: Do not award marks if part (a) is not used.

[3 marks]

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

46c.

[3 marks]

Markscheme

$$(\cos \theta + i \sin \theta)^5 =$$

$$\stackrel{(AI)}{\cos^5 \theta + 5 \cos^4 \theta (i \sin \theta) + 10 \cos^3 \theta (i \sin \theta)^2 + 10 \cos^2 \theta (i \sin \theta)^3 + 5 \cos \theta (i \sin \theta)^4 + (i \sin \theta)^5}$$

from De Moivre's theorem

$$\stackrel{MI}{\cos 5\theta = \cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta}$$

$$\stackrel{AI}{=} \cos^5 \theta - 10 \cos^3 \theta (1 - \cos^2 \theta) + 5 \cos \theta (1 - \cos^2 \theta)^2$$

$$= \cos^5 \theta - 10 \cos^3 \theta + 10 \cos^5 \theta + 5 \cos \theta - 10 \cos^3 \theta + 5 \cos^5 \theta$$

$$\stackrel{AG}{\therefore \cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta}$$

Note: If compound angles used in (b) and (c), then marks can be allocated in (c) only.

[3 marks]

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

46d.

[6 marks]

Markscheme

$$\begin{aligned}
 &\cos 5\theta + \cos 3\theta + \cos \theta \\
 &\quad \text{M1} \\
 &= (16\cos^5\theta - 20\cos^3\theta + 5\cos\theta) + (4\cos^3\theta - 3\cos\theta) + \cos\theta = 0 \\
 &\quad \text{A1} \\
 &16\cos^5\theta - 16\cos^3\theta + 3\cos\theta = 0 \\
 &\cos\theta (16\cos^4\theta - 16\cos^2\theta + 3) = 0 \\
 &\quad \text{A1} \\
 &\cos\theta (4\cos^2\theta - 3) (4\cos^2\theta - 1) = 0 \\
 &\vdots \\
 &\therefore \cos\theta = 0 \\
 &\pm \frac{\sqrt{3}}{2} \\
 &\quad \text{A1} \\
 &\pm \frac{1}{2} \\
 &\vdots \\
 &\therefore \theta = \pm \frac{\pi}{6} \\
 &\pm \frac{\pi}{3} \\
 &\quad \text{A2} \\
 &\pm \frac{\pi}{2} \\
 &[6 \text{ marks}]
 \end{aligned}$$

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

46e.

[8 marks]

Markscheme

$$\begin{aligned}
 &\cos 5\theta = 0 \\
 &\vdots \\
 &5\theta = \dots \frac{\pi}{2} \\
 &\left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) \\
 &\frac{7\pi}{2} \text{ (M1)} \\
 &\dots \\
 &\vdots \\
 &\theta = \dots \frac{\pi}{10} \\
 &\left(\frac{3\pi}{10}, \frac{5\pi}{10}\right) \\
 &\frac{7\pi}{10} \text{ (M1)} \\
 &\dots
 \end{aligned}$$

Note: These marks can be awarded for verifications later in the question.

now consider

$$\begin{aligned}
 &\quad \text{M1} \\
 &16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0 \\
 &\cos\theta (16\cos^4\theta - 20\cos^2\theta + 5) = 0 \\
 &\vdots \\
 &\cos^2\theta = \frac{20 \pm \sqrt{400 - 4(16)(5)}}{32} \\
 &\quad \text{A1} \\
 &\cos\theta = 0 \\
 &\cos\theta = \pm \sqrt{\frac{20 \pm \sqrt{400 - 4(16)(5)}}{32}} \\
 &\text{since max value of cosine is } 1 \\
 &\text{angle closest to zero} \quad \text{RT} \\
 &\Rightarrow \\
 &\quad \text{A1} \\
 &\cos \frac{\pi}{10} = \sqrt{\frac{4.5 + 4\sqrt{25 - 4(5)}}{4.8}} = \sqrt{\frac{5 + \sqrt{5}}{8}} \\
 &\quad \text{A1A1} \\
 &\cos \frac{7\pi}{10} = -\sqrt{\frac{5 - \sqrt{5}}{8}} \\
 &[8 \text{ marks}]
 \end{aligned}$$

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

Markscheme

METHOD 1

$$\frac{\sin C}{7} = \frac{\sin 40}{5}$$

AI

$$\angle BCD = 64.14\dots^\circ$$

MI

$$CD = 2 \times 5 \cos 64.14\dots$$

Note: Also allow use of sine or cosine rule.

AI

$$CD = 4.36$$

[5 marks]

METHOD 2

let

$$AC = x$$

cosine rule

$$5^2 = 7^2 + x^2 - 2 \times 7 \times x \cos 40$$

$$x^2 - 10.7\dots x + 24 = 0$$

$$x = \frac{10.7\dots \pm \sqrt{(10.7\dots)^2 - 4 \times 24}}{2}$$

;

$$x = 7.54$$

AI

$$3.18$$

CD is the difference in these two values

AI

$$= 4.36$$

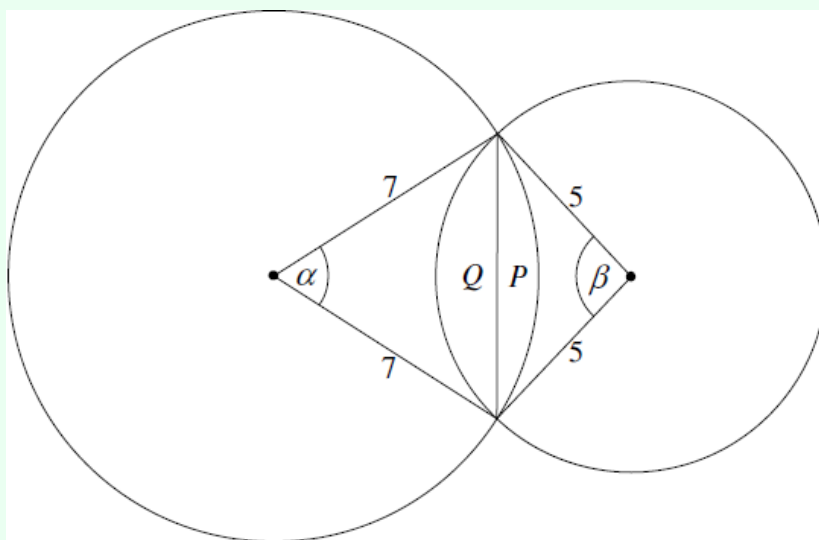
Note: Other methods may be seen.

[5 marks]

Examiners report

This was an accessible question to most candidates although care was required when calculating the angles. Candidates who did not annotate the diagram or did not take care with the notation for the angles and sides often had difficulty recognizing when an angle was acute or obtuse. This prevented the candidate from obtaining a correct solution. There were many examples of candidates rounding answers prematurely and thus arriving at a final answer that was to the correct degree of accuracy but incorrect.

Markscheme



$$\alpha = 2 \arcsin\left(\frac{4.5}{7}\right)$$

M1(A1)
 $\Rightarrow \alpha = 1.396... = 80.010^\circ \dots$

$$\beta = 2 \arcsin\left(\frac{4.5}{5}\right)$$

A1
 $\Rightarrow \beta = 2.239... = 128.31^\circ \dots$
Note: Allow use of cosine rule.

area

$$P = \frac{1}{2} \times 7^2 \times (\alpha - \sin \alpha) = 10.08...$$

M1(A1)
area

$$Q = \frac{1}{2} \times 5^2 \times (\beta - \sin \beta) = 18.18...$$

(A1)

Note: The **M1** is for an attempt at area of sector minus area of triangle.

Note: The use of degrees correctly converted is acceptable.

$$\text{area} = 28.3 \text{ (cm}^2\text{)} \quad \textbf{A1}$$

[7 marks]

Examiners report

Whilst most candidates were able to make the correct construction to solve the problem some candidates seemed unable to find the area of a segment. In a number of cases candidates used degrees in a formula that required radians. There were a number of candidates who followed a completely correct method but due to premature approximation were unable to obtain a correct solution.

Markscheme

(a) shaded area area of triangle area of sector, *i.e.* **(M1)**

$$\left(\frac{1}{2} \times 4^2 \sin x\right) - \left(\frac{1}{2} 2^2 x\right) = 8 \sin x - 2x$$

(b) **EITHER**

any method from GDC gaining

(M1)(A1)

$$x \approx 1.32$$

maximum value for given domain is

A2

5.11

OR

$$\frac{dA}{dx} = 8 \cos x - 2$$

hence

$$\frac{dA}{dx} = 0$$

$$8 \cos x - 2 = 0$$

A1

$$\cos x = \frac{1}{4} \Rightarrow x \approx 1.32$$

hence

$$A_{\max} = 5.11$$

[7 marks]

Examiners report

Generally a well answered question.

Markscheme

(a) PQ

and non-intersecting **RI**

= 50

[1 mark]

(b) a construction QT (where T is on the radius MP), parallel to MN, so that

(angle between tangent and radius

$\angle QTM = 90^\circ$

= 90°

lengths

,

and angle

$x = 10$

marked on a diagram, or equivalent **RI**

Note: Other construction lines are possible.

[2 marks]

(c) (i) MN

AI

= $\sqrt{50^2 - (x - 10)^2}$

(ii) maximum for MN occurs when

AI

$x = 10$

[2 marks]

(d) (i)

MI

$\alpha = 2\pi - 2\theta$

AI

(ii) $= 2\pi - 2\arccos\left(\frac{x-10}{50}\right)$

(

$\beta = 2\pi - \alpha$

= 2θ

AI

[4 marks] $= 2\left(\cos^{-1}\left(\frac{x-10}{50}\right)\right)$

(e) (i)

AI

$b(x) = x\alpha + 10\beta + 2\sqrt{50^2 - (x - 10)^2}$

MI

(ii) $= x\left(2\pi - 2\left(\cos^{-1}\left(\frac{x-10}{50}\right)\right)\right) + 20\left(\left(\cos^{-1}\left(\frac{x-10}{50}\right)\right)\right) + 2\sqrt{50^2 - (x - 10)^2}$

maximum value of perimeter

A2

= 276

(iii) perimeter of

cm

200

(MI)

$b(x) = 200$

when

AI

$x = 21.2$

[9 marks]

Total [18 marks]

Examiners report

This is not an inherently difficult question, but candidates either made heavy weather of it or avoided it almost entirely. The key to answering the question is in obtaining the displayed answer to part (b), for which a construction line parallel to MN through Q is required. Diagrams seen by examiners on some scripts tend to suggest that the perpendicularity property of a tangent to a circle and the associated radius is not as firmly known as they had expected. Some candidates mixed radians and degrees in their expressions.

