Topic 9 Part 1 *[597 marks]*

Consider the functions

f and

g given by

$f(x) = \frac{e^x + e^{-x}}{2}$ and $g(x) = \frac{e^x - e^{-x}}{2}$.

1a. Show that

$f'(x) = g(x)$ and

$g'(x) = f(x)$.

[2 marks]

1b. Find the first three non-zero terms in the Maclaurin expansion of $f(x)$.

[5 marks]

1c. Hence find the value of

$\lim\limits_{x \to 0} \frac{1 - f(x)}{x^2}$.

[3 marks]

1d. Find the value of the improper integral

$\int_0^\infty \frac{g(x)}{[f(x)]^2} \mathrm{d}x$.

[6 marks]

2a. Consider the functions

$f(x) = (\ln x)^2$, $x > 1$ and

$g(x) = \ln(f(x))$, $x > 1$.

(i)   Find

$f'(x)$.

(ii)   Find

$g'(x)$.

(iii)   Hence, show that

$g(x)$ is increasing on

$]1, \infty[$.

[5 marks]

2b. Consider the differential equation

$$(\ln x)\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2}{x}y = \frac{2x - 1}{(\ln x)}, \ x > 1.$$

(i)   Find the general solution of the differential equation in the form

$y = h(x)$.

(ii)   Show that the particular solution passing through the point with coordinates

(e, e^2) is given by

$y = \frac{x^2 - x + e}{(\ln x)^2}$.

(iii)   Sketch the graph of your solution for

$x > 1$, clearly indicating any asymptotes and any maximum or minimum points.

[12 marks]

3. Each term of the power series

$\frac{1}{1\times2} + \frac{1}{4\times5}x + \frac{1}{7\times8}x^2 + \frac{1}{10\times11}x^3 + \dots$ has the form

$\frac{1}{b(n)\times c(n)}x^n$, where

$b(n)$ and

$c(n)$ are linear functions of

n.

(a) Find the functions

$b(n)$ and

$c(n)$.

(b) Find the radius of convergence.

(c) Find the interval of convergence.

The function f is defined by

$$f(x) = \begin{cases} e^{-x^3}(-x^3 + 2x^2 + x), x \le 1 \\ ax + b, x > 1 \end{cases}, \text{ where}$$

a and

b are constants.

4a. Find the exact values of

a and

b if

f is continuous and differentiable at

$x = 1$.

4b. (i) Use Rolle's theorem, applied to

f, to prove that

$2x^4 - 4x^3 - 5x^2 + 4x + 1 = 0$ has a root in the interval

$]-1, 1[$.

(ii) Hence prove that

$2x^4 - 4x^3 - 5x^2 + 4x + 1 = 0$ has at least two roots in the interval

$]-1, 1[$.

5. Consider the infinite series

$\sum\limits_{n=1}^{\infty} \frac{2}{n^2+3n}$.

Use a comparison test to show that the series converges.

6. The general term of a sequence

$\{a_n\}$ is given by the formula

$a_n = \frac{e^n + 2^n}{2e^n}$, $n \in \mathbb{Z}^+$.

(a) Determine whether the sequence

$\{a_n\}$ is decreasing or increasing.

(b) Show that the sequence

$\{a_n\}$ is convergent and find the limit L.

(c) Find the smallest value of

$N \in \mathbb{Z}^+$ such that

$|a_n - L| < 0.001$, for all

$n \ge N$.

7. Consider the differential equation

$\frac{dy}{dx} = \frac{y}{x+\sqrt{xy}}$, for

$x, y > 0$.

(a)   Use Euler's method starting at the point

$(x, y) = (1, 2)$, with interval

$h = 0.2$, to find an approximate value of y when

$x = 1.6$.

(b)   Use the substitution

$y = vx$ to show that

$x\frac{dv}{dx} = \frac{v}{1+\sqrt{v}} - v$.

(c)   (i)   Hence find the solution of the differential equation in the form

$f(x, y) = 0$, given that

$y = 2$ when

$x = 1$.

(ii)   Find the value of

y when

$x = 1.6$.

[19 marks]

8a. Use an integrating factor to show that the general solution for $\frac{dx}{dt} - \frac{x}{t} = -\frac{2}{t}$, $t > 0$ is $x = 2 + ct$, where c is a constant.

[4 marks]

The weight in kilograms of a dog,
t weeks after being bought from a pet shop, can be modelled by the following function:

$$w(t) = \begin{cases} 2 + ct & 0 \le t \le 5 \\ 16 - \frac{35}{t} & t > 5 \end{cases}.$$

8b. Given that $w(t)$ is continuous, find the value of c.

[2 marks]

8c. Write down

[2 marks]

(i)   the weight of the dog when bought from the pet shop;

(ii)   an upper bound for the weight of the dog.

Consider the differential equation $\frac{dy}{dx} = f(x, y)$ where $f(x, y) = y - 2x$.

9a. Sketch, on one diagram, the four isoclines corresponding to $f(x, y) = k$ where k takes the values $-1, -0.5, 0$ and 1. Indicate clearly where each isocline crosses the y axis.

[2 marks]

9b. A curve, C, passes through the point $(0, 1)$ and satisfies the differential equation above.

[3 marks]

Sketch C on your diagram.

9c. A curve, C, passes through the point $(0, 1)$ and satisfies the differential equation above. *[1 mark]*

State a particular relationship between the isocline $f(x, y) = -0.5$ and the curve C, at their point of intersection.

....

....

....

9d. A curve, C, passes through the point $(0, 1)$ and satisfies the differential equation above. *[4 marks]*

Use Euler's method with a step interval of 0.1 to find an approximate value for y on C, when $x = 0.5$.

....

....

....

In this question you may assume that $\arctan x$ is continuous and differentiable for $x \in \mathbb{R}$.

10a. Consider the infinite geometric series *[1 mark]*

$$1 - x^2 + x^4 - x^6 + \dots \quad |x| < 1.$$

Show that the sum of the series is $\frac{1}{1+x^2}$.

....

....

....

10b. Hence show that an expansion of $\arctan x$ is $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ [4 marks]

10c. f is a continuous function defined on $[a, b]$ and differentiable on $]a, b[$ with $f'(x) > 0$ on $]a, b[$. [4 marks]
Use the mean value theorem to prove that for any $x, y \in [a, b]$, if $y > x$ then $f(y) > f(x)$.

10d. (i) Given $g(x) = x - \arctan x$, prove that $g'(x) > 0$, for $x > 0$. [4 marks]
(ii) Use the result from part (c) to prove that $\arctan x < x$, for $x > 0$.

10e. Use the result from part (c) to prove that $\arctan x > x - \frac{x^3}{3}$, for $x > 0$. [5 marks]

10f. Hence show that $\frac{16}{3\sqrt{3}} < \pi < \frac{6}{\sqrt{3}}$. [4 marks]

11. The function f is defined by $f(x) = \mathrm{e}^{-x}\cos x + x - 1$. [7 marks]

By finding a suitable number of derivatives of f, determine the first non-zero term in its Maclaurin series.

12a. Show that $y = \frac{1}{x} \int f(x) \mathrm{d}x$ is a solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + y = f(x), \; x > 0.$$

....

....

....

12b. Hence solve $x\frac{\mathrm{d}y}{\mathrm{d}x} + y = x^{-\frac{1}{2}}$, $x > 0$, given that $y = 2$ when $x = 4$.

....

....

....

13a. Show that the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$ converges.

....

....

....

13b. (i) State why the integral test can be used to determine the convergence or divergence of $\sum\limits_{n=2}^{\infty} \frac{1}{n \ln n}$.

[8 marks]

(ii) Hence determine the convergence or divergence of $\sum\limits_{n=2}^{\infty} \frac{1}{n \ln n}$.

14a. The mean value theorem states that if f is a continuous function on $[a,\,b]$ and differentiable on $]a,\,b[$ then $f'(c) = \frac{f(b)-f(a)}{b-a}$ for some $c \in]a,\,b[$.

[7 marks]

(i) Find the two possible values of c for the function defined by $f(x) = x^3 + 3x^2 - 2$ on the interval $[-3,\,1]$.

(ii) Illustrate this result graphically.

14b. (i) The function f is continuous on $[a,\,b]$, differentiable on $]a,\,b[$ and $f'(x) = 0$ for all $x \in]a,\,b[$. Show that $f(x)$ is constant on $[a,\,b]$.

[9 marks]

(ii) Hence, prove that for $x \in [0,\,1]$, $2\arccos x + \arccos(1 - 2x^2) = \pi$.

A differential equation is given by
$\frac{dy}{dx} = \frac{y}{x}$, where $x > 0$ and $y > 0$.

15a. Solve this differential equation by separating the variables, giving your answer in the form $y = f(x)$.

[3 marks]

15b. Solve the same differential equation by using the standard homogeneous substitution $y = vx$. [4 marks]

15c. Solve the same differential equation by the use of an integrating factor. [5 marks]

15d. If $y = 20$ when $x = 2$, find y when $x = 5$. [1 mark]

Let the differential equation

$\frac{dy}{dx} = \sqrt{x+y}$, $(x+y \geqslant 0)$ satisfying the initial conditions $y = 1$ when $x = 1$. Also let $y = c$ when $x = 2$.

16a. Use Euler's method to find an approximation for the value of c, using a step length of $h = 0.1$. Give your answer to four decimal places. [6 marks]

16b. You are told that if Euler's method is used with $h = 0.05$ then [3 marks]
$c \simeq 2.7921$, if it is used with $h = 0.01$ then
$c \simeq 2.8099$ and if it is used with $h = 0.005$ then
$c \simeq 2.8121$.
Plot on graph paper, with h on the horizontal axis and the approximation for c on the vertical axis, the four points (one of which you have calculated and three of which have been given). Use a scale of 1 cm = 0.01 on both axes. Take the horizontal axis from 0 to 0.12 and the vertical axis from 2.76 to 2.82.

16c. Draw, by eye, the straight line that best fits these four points, using a ruler. [1 mark]

16d. Use your graph to give the best possible estimate for c, giving your answer to three decimal places. [2 marks]

17a. Prove that [3 marks]

$\lim\limits_{H \to \infty} \int_a^H \frac{1}{x^2} dx$ exists and find its value in terms of

a (where $a \in \mathbb{R}^+$).

17b. Use the integral test to prove that [3 marks]

$\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$ converges.

17c. Let

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = L \ .$$

The diagram below shows the graph of

$$y = \frac{1}{x^2}.$$

diagram
not to scale
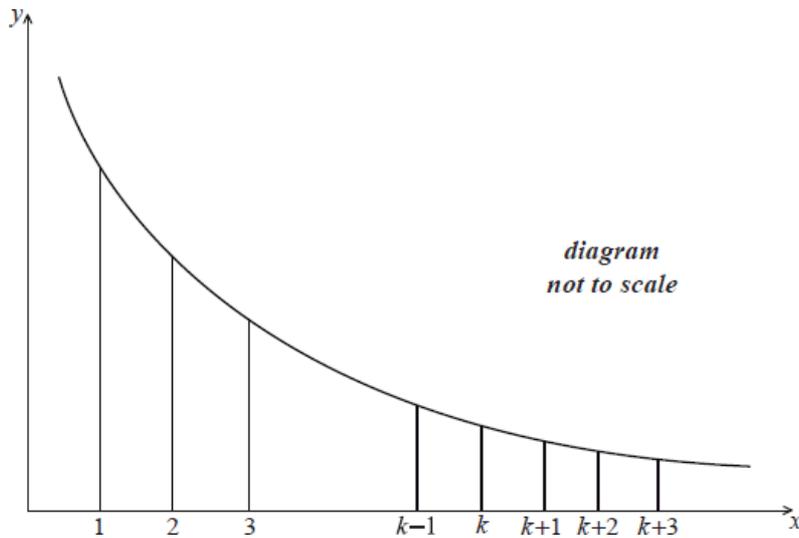
(i)   Shade suitable regions on a copy of the diagram above and show that

$$\sum_{n=1}^{k} \frac{1}{n^2} + \int_{k+1}^{\infty} \frac{1}{x^2} \mathrm{d}x < L \ .$$

(ii)   Similarly shade suitable regions on another copy of the diagram above and show that

$$L < \sum_{n=1}^{k} \frac{1}{n^2} + \int_{k}^{\infty} \frac{1}{x^2} \mathrm{d}x \ .$$

17d. Hence show that

$$\sum_{n=1}^{k} \frac{1}{n^2} + \frac{1}{k+1} < L < \sum_{n=1}^{k} \frac{1}{n^2} + \frac{1}{k}$$

17e. You are given that

$$L = \frac{\pi^2}{6}.$$

By taking $k = 4$, use the upper bound and lower bound for L to find an upper bound and lower bound for π . Give your bounds to three significant figures.

18a. Use the limit comparison test to prove that

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \text{ converges.}$$

18b. Using the Maclaurin series for

$\ln(1+x)$, show that the Maclaurin series for

$(1+x)\ln(1+x)$ is

$$x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+1}}{n(n+1)}.$$

The Taylor series of

$\sqrt{x}$ about $x = 1$ is given by

$$a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3 + \ldots$$

19a. Find the values of

a_0, a_1, a_2 and

a_3.

[6 marks]

19b. Hence, or otherwise, find the value of

$\lim\limits_{x \to 1} \frac{\sqrt{x}-1}{x-1}$.

[3 marks]

Consider the differential equation

$\frac{dy}{dx} + y\tan x = \cos^2 x$, given that $y = 2$ when $x = 0$.

20a. Use Euler's method with a step length of 0.1 to find an approximation to the value of y when $x = 0.3$.

[5 marks]

20b.
(i)   Show that the integrating factor for solving the differential equation is

$\sec x$.

(ii)   Hence solve the differential equation, giving your answer in the form

$y = f(x)$.

[10 marks]

Consider the infinite series

$\sum\limits_{n=1}^{\infty} \frac{n^2}{2^n} x^n$.

21a. Find the radius of convergence.

[4 marks]

21b. Find the interval of convergence.

[3 marks]

21c. Given that $x = -0.1$, find the sum of the series correct to three significant figures.

[4 marks]
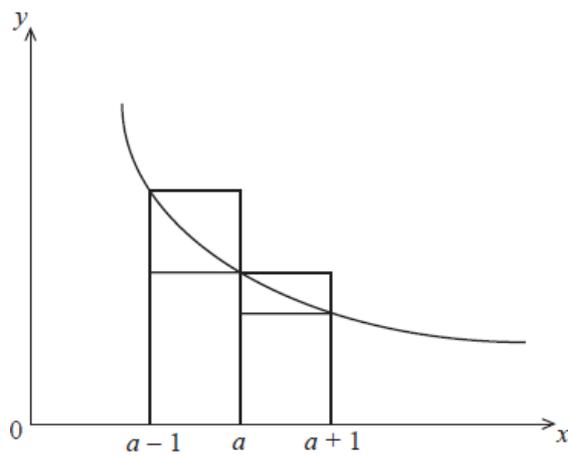
Figure 1

[9 marks]

22a. Figure 1 shows part of the graph of

$y = \frac{1}{x}$ together with line segments parallel to the coordinate axes.

(i)   By considering the areas of appropriate rectangles, show that

$$\frac{2a+1}{a(a+1)} < \ln\left(\frac{a+1}{a-1}\right) < \frac{2a-1}{a(a-1)}.$$

(ii)   Hence find lower and upper bounds for

$\ln(1.2)$.
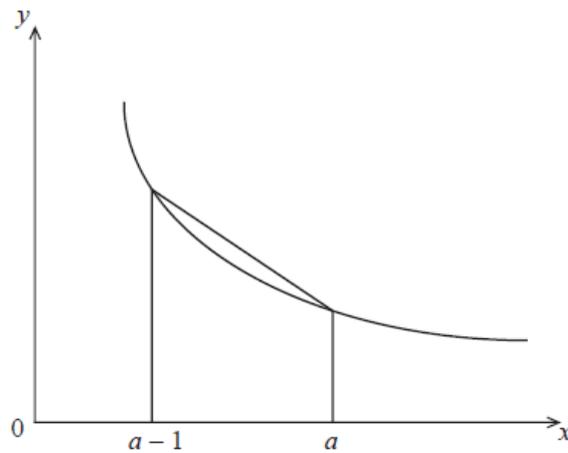
Figure 2

[5 marks]

22b. An improved upper bound can be found by considering Figure 2 which again shows part of the graph of

$y = \frac{1}{x}$.

(i)   By considering the areas of appropriate regions, show that

$$\ln\left(\frac{a}{a-1}\right) < \frac{2a-1}{2a(a-1)}.$$

(ii)   Hence find an upper bound for

$\ln(1.2)$.

23. The acceleration of a car is [6 marks]

$\frac{1}{40}(60 - v)$ ms^{-2}, when its velocity is

v ms^{-2}. Given the car starts from rest, find the velocity of the car after 30 seconds.

Consider the differential equation

$y\frac{dy}{dx} = \cos 2x.$

24a. (i)   Show that the function [10 marks]

$y = \cos x + \sin x$ satisfies the differential equation.

(ii)   Find the general solution of the differential equation. Express your solution in the form

$y = f(x)$, involving a constant of integration.

(iii)   For which value of the constant of integration does your solution coincide with the function given in part (i)?

24b. A different solution of the differential equation, satisfying $y = 2$ when [12 marks]

$x = \frac{\pi}{4}$, defines a curve C.

(i)   Determine the equation of C in the form

$y = g(x)$, and state the range of the function g.

A region R in the xy plane is bounded by C, the x-axis and the vertical lines $x = 0$ and

$x = \frac{\pi}{2}$.

(ii)   Find the area of R.

(iii)   Find the volume generated when that part of R above the line $y = 1$ is rotated about the x-axis through

2π radians.

Let
$g(x) = \sin x^2$, where
$x \in \mathbb{R}$.

25a. Using the result [4 marks]
$\lim_{t \to 0} \frac{\sin t}{t} = 1$, or otherwise, calculate
$\lim_{x \to 0} \frac{g(2x) - g(3x)}{4x^2}$.

25b. Use the Maclaurin series of [2 marks]
$\sin x$ to show that
$g(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$

25c. Hence determine the minimum number of terms of the expansion of $g(x)$ required to approximate the value of [7 marks]
$\int_0^1 g(x)dx$ to four decimal places.

26. A function

f is defined in the interval

$]-k,\ k[$, where

$k>0$. The gradient function

f' exists at each point of the domain of

f.

The following diagram shows the graph of

$y=f(x)$, its asymptotes and its vertical symmetry axis.
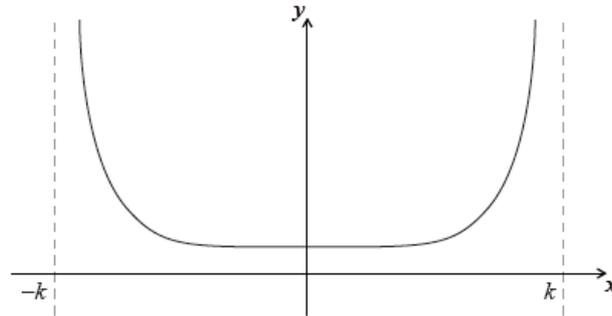
(a)   Sketch the graph of

$y=f'(x)$.

Let

$p(x)=a+bx+cx^2+dx^3+\ldots$ be the Maclaurin expansion of

$f(x)$.

(b)   (i)   Justify that

$a>0$.

(ii)   Write down a condition for the largest set of possible values for each of the parameters

b,

c and

d.

(c)   State, with a reason, an upper bound for the radius of convergence.

27. Solve the differential equation

$x^2\frac{dy}{dx}=y^2+3xy+2x^2$

given that $y=-1$ when $x=1$. Give your answer in the form

$y=f(x)$.

The exponential series is given by

$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$.

28a. Find the set of values of x for which the series is convergent.

28b. (i)   Show, by comparison with an appropriate geometric series, that

$$e^x-1<\frac{2x}{2-x},\ \text{for } 0<x<2.$$

(ii)   Hence show that

$e<\left(\frac{2n+1}{2n-1}\right)^n$, for

$n\in\mathbb{Z}^+$.

28c. (i)   Write down the first three terms of the Maclaurin series for

$1 - e^{-x}$ and explain why you are able to state that

$$1 - e^{-x} > x - \frac{x^2}{2}, \text{ for } 0 < x < 2.$$

(ii)   Deduce that

$e > \left(\frac{2n^2}{2n^2 - 2n + 1} \right)^n$, for

$n \in \mathbb{Z}^+$.

28d. Letting $n = 1000$, use the results in parts (b) and (c) to calculate the value of e correct to as many decimal places as possible.

29a. Find

$\lim\limits_{x \to 0} \frac{\tan x}{x + x^2}$;

29b. Find

$\lim\limits_{x \to 1} \frac{1 - x^2 + 2x^2 \ln x}{1 - \sin \frac{\pi x}{2}}$.

30. The variables x and y are related by

$\frac{dy}{dx} - y \tan x = \cos x$.

(a)   Find the Maclaurin series for y up to and including the term in

x^2 given that

$y = -\frac{\pi}{2}$ when $x = 0$.

(b)   Solve the differential equation given that $y = 0$ when

$x = \pi$. Give the solution in the form

$y = f(x)$.

31a. Determine whether the series

$\sum\limits_{n=1}^{\infty} \sin \frac{1}{n}$ is convergent or divergent.

31b. Show that the series

$\sum\limits_{n=2}^{\infty} \frac{1}{n (\ln n)^2}$ is convergent.

32a. If z is a non-zero complex number, we define

$L(z)$ by the equation

$$L(z) = \ln|z| + i\arg(z),\ 0 \leqslant \arg(z) < 2\pi.$$

(a)   Show that when z is a positive real number,

$L(z) = \ln z$.

(b)   Use the equation to calculate

(i)

$L(-1)$;

(ii)

$L(1 - i)$;

(iii)

$L(-1 + i)$.

(c)   Hence show that the property

$L(z_1 z_2) = L(z_1) + L(z_2)$ does not hold for all values of

z_1 and

z_2 .

32b. Let f be a function with domain

$\mathbb{R}$ that satisfies the conditions,

$f(x + y) = f(x)f(y)$, for all x and y and

$f(0) \neq 0$.

(a)   Show that

$f(0) = 1$.

(b)   Prove that

$f(x) \neq 0$, for all

$x \in \mathbb{R}$.

(c)   Assuming that

$f'(x)$ exists for all

$x \in \mathbb{R}$, use the definition of derivative to show that

$f(x)$ satisfies the differential equation

$f'(x) = k\, f(x)$, where

$k = f'(0)$.

(d)   Solve the differential equation to find an expression for

$f(x)$.

33. A certain population can be modelled by the differential equation

$\frac{dy}{dt} = ky\cos kt$, where y is the population at time t hours and k is a positive constant.

(a)   Given that

$y = y_0$ when $t = 0$, express y in terms of k , t and

y_0 .

(b)   Find the ratio of the minimum size of the population to the maximum size of the population.

34. Solve the differential equation

$\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$ (where x > 0)

given that $y = 2$ when $x = 1$. Give your answer in the form

$y = f(x)$.

35. The function f is defined by

$f(x) = e^{(e^x - 1)}$.

(a)   Assuming the Maclaurin series for

e^x , show that the Maclaurin series for

$f(x)$

is

$1 + x + x^2 + \frac{5}{6}x^3 + \dots$.

(b)   Hence or otherwise find the value of

$\lim\limits_{x \to 0} \frac{f(x) - 1}{f'(x) - 1}$.

36a. Find the radius of convergence of the infinite series

$$\frac{1}{2}x + \frac{1 \times 3}{2 \times 5}x^2 + \frac{1 \times 3 \times 5}{2 \times 5 \times 8}x^3 + \frac{1 \times 3 \times 5 \times 7}{2 \times 5 \times 8 \times 11}x^4 + \dots .$$

36b. Determine whether the series

$\sum\limits_{n=1}^{\infty} \sin\left(\frac{1}{n} + n\pi\right)$ is convergent or divergent.

37. Given that

$\frac{dy}{dx} - 2y^2 = e^x$ and $y = 1$ when $x = 0$, use Euler's method with a step length of 0.1 to find an approximation for the value of y when $x = 0.4$. Give all intermediate values with maximum possible accuracy.

38. (a)   Using integration by parts, show that

$\int_0^\infty e^{-x} \cos x \, dx = \int_0^\infty \sin x \, dx.$

(b)   Find the value of these two integrals.

39. Solve the differential equation

$$x^2\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 + xy + 4x^2,$$

given that $y = 2$ when $x = 1$. Give your answer in the form

$y = f(x)$.

40. (a) Using the Maclaurin series for

$(1 + x)^n$, write down and simplify the Maclaurin series approximation for

$(1 - x^2)^{-\frac{1}{2}}$ as far as the term in

x^4

(b) Use your result to show that a series approximation for $\arccos x$ is

$$\arccos x \approx \frac{\pi}{2} - x - \frac{1}{6}x^3 - \frac{3}{40}x^5.$$

(c) Evaluate

$$\lim_{x\to 0}\frac{\frac{\pi}{2}-\arccos(x^2)-x^2}{x^6}.$$

(d) Use the series approximation for

$\arccos x$ to find an approximate value for

$$\int_0^{0.2} \arccos(\sqrt{x})\mathrm{d}x,$$

giving your answer to 5 decimal places. Does your answer give the actual value of the integral to 5 decimal places?

41a. Consider the power series

$$\sum_{k=1}^{\infty} k\left(\frac{x}{2}\right)^k.$$

(i) Find the radius of convergence.

(ii) Find the interval of convergence.

41b. Consider the infinite series

$$\sum_{k=1}^{\infty} (-1)^{k+1} \times \frac{k}{2k^2+1}.$$

(i) Show that the series is convergent.

(ii) Show that the sum to infinity of the series is less than 0.25.

42. Find y in terms of x, given that

$(1 + x^3)\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^2 \tan y$ and

$y = \frac{\pi}{2}$ when $x = 0$.

43. Find

$$\lim_{x\to 0}\left(\frac{1-\cos x^6}{x^{12}}\right).$$

44. Determine whether or not the following series converge.

(a)

$$\sum_{n=0}^{\infty} \left(\sin \frac{n\pi}{2} - \sin \frac{(n+1)\pi}{2} \right)$$

(b)

$$\sum_{n=1}^{\infty} \frac{e^n - 1}{\pi^n}$$

(c)

$$\sum_{n=2}^{\infty} \frac{\sqrt{n+1}}{n(n-1)}$$

45. (a)   Using the Maclaurin series for the function

e^x, write down the first four terms of the Maclaurin series for

$e^{-\frac{x^2}{2}}$.

(b)   Hence find the first four terms of the series for

$\int_0^x e^{-\frac{u^2}{2}} \, du.$

(c)   Use the result from part (b) to find an approximate value for

$\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{x^2}{2}} \, dx.$

46. Solve the differential equation

$$(x-1)\frac{dy}{dx} + xy = (x-1)e^{-x}$$

given that $y = 1$ when $x = 0$. Give your answer in the form

$y = f(x).$

47. Consider the infinite series

$$\frac{1}{2\ln 2} - \frac{1}{3\ln 3} + \frac{1}{4\ln 4} - \frac{1}{5\ln 5} + \dots .$$

(a)   Show that the series converges.

(b)   Determine if the series converges absolutely or conditionally.

48. (a)   Solve the differential equation
$\frac{\cos^2 x}{e^y} - e^y \frac{dy}{dx} = 0$, given that
$y = 0$ when
$x = \pi.$

(b)   Find the value of y when
$x = \frac{\pi}{2}.$

49. The acceleration in ms^{-2} of a particle moving in a straight line at time
t seconds,
$t \geqslant 0$, is given by the formula
$a = -\frac{1}{2}v$. When
$t = 0$, the velocity is
40 ms^{-1} .

Find an expression for
v in terms of
t .

[6 marks]