Topic 4 Part 1 *[437 marks]*

1a. Show that the points $\mathrm{O}(0,\ 0,\ 0)$, $\mathrm{A}(6,\ 0,\ 0)$, $\mathrm{B}(6,\ -\sqrt{24},\ \sqrt{12})$, $\mathrm{C}(0,\ -\sqrt{24},\ \sqrt{12})$ form a square. *[3 marks]*

1b. Find the coordinates of M, the mid-point of [OB]. *[1 mark]*

1c. Show that an equation of the plane Π, containing the square OABC, is $y+\sqrt{2}z=0$. *[3 marks]*

1d. Find a vector equation of the line L, through M, perpendicular to the plane Π. *[3 marks]*

1e. Find the coordinates of D, the point of intersection of the line L with the plane whose equation is $y=0$. *[3 marks]*

1f. Find the coordinates of E, the reflection of the point D in the plane Π. *[3 marks]*

1g. (i) Find the angle $\mathrm{O\hat{D}A}$.

(ii) State what this tells you about the solid OABCDE. *[6 marks]*

2. PQRS is a rhombus. Given that $\overrightarrow{\mathrm{PQ}}=$ a and $\overrightarrow{\mathrm{QR}}=$ b, *[6 marks]*

(a) express the vectors $\overrightarrow{\mathrm{PR}}$ and $\overrightarrow{\mathrm{QS}}$ in terms of a and b;

(b) hence show that the diagonals in a rhombus intersect at right angles.

3a. Given the points A(1, 0, 4), B(2, 3, −1) and C(0, 1, − 2) , find the vector equation of the line L_1 passing through the points A and B. *[2 marks]*

3b. The line L_2 has Cartesian equation $\frac{x-1}{3} = \frac{y+2}{1} = \frac{z-1}{-2}$.

Show that L_1 and L_2 are skew lines. *[5 marks]*

Consider the plane Π_1, parallel to both lines L_1 and L_2. Point C lies in the plane Π_1.

3c. Find the Cartesian equation of the plane Π_1. *[4 marks]*

The line L_3 has vector equation

$$r = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}.$$

The plane Π_2 has Cartesian equation $x + y = 12$.

The angle between the line L_3 and the plane Π_2 is $60°$.

3d. (i) Find the value of k. *[7 marks]*

(ii) Find the point of intersection P of the line L_3 and the plane Π_2.

Consider the points $\mathrm{A}(1, 0, 0)$, $\mathrm{B}(2, 2, 2)$ and $\mathrm{C}(0, 2, 1)$.

4a. Find the vector $\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}$. *[4 marks]*

4b. Find an exact value for the area of the triangle ABC. *[3 marks]*

4c. Show that the Cartesian equation of the plane Π_1, containing the triangle ABC, is $2x + 3y - 4z = 2$. *[3 marks]*

4d. A second plane Π_2 is defined by the Cartesian equation $\Pi_2 : 4x - y - z = 4$. *[5 marks]*

L_1 is the line of intersection of the planes Π_1 and Π_2.

Find a vector equation for L_1.

A third plane Π_3 is defined by the Cartesian equation $16x + \alpha y - 3z = \beta$.

4e. Find the value of α if all three planes contain L_1. *[3 marks]*

4f. Find conditions on α and β if the plane Π_3 does **not** intersect with L_1. *[2 marks]*

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a} = (3\cos\theta + 6)\boldsymbol{i} + 7\boldsymbol{j}$ and $\boldsymbol{b} = (\cos\theta - 2)\boldsymbol{i} + (1 + \sin\theta)\boldsymbol{j}$.

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular,

5a. show that $3\sin^2\theta - 7\sin\theta + 2 = 0$; *[3 marks]*

5b. find the smallest possible positive value of θ. *[3 marks]*

6. A line L_1 has equation $\boldsymbol{r} = \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$. *[7 marks]*

A line L_2 passing through the origin intersects L_1 and is perpendicular to L_1.

(a) Find a vector equation of L_2.

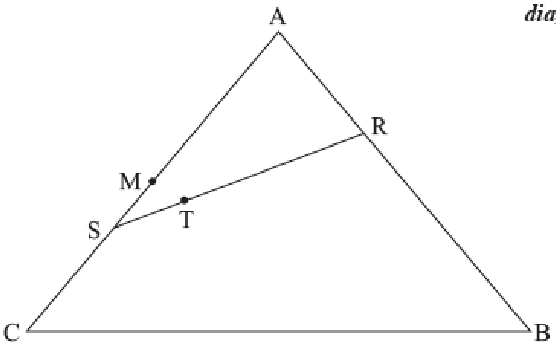
(b) Determine the shortest distance from the origin to L_1.

7. A point P, relative to an origin O, has position vector $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 1+s \\ 3+2s \\ 1-s \end{pmatrix},\ s \in \mathbb{R}$. *[5 marks]*

Find the minimum length of $\overrightarrow{\mathrm{OP}}$.

The position vectors of the points A, B and C are a, b and c respectively, relative to an origin O. The following diagram shows the triangle ABC and points M, R, S and T.

diagram not to scale



M is the midpoint of [AC].

R is a point on [AB] such that $\overrightarrow{\mathrm{AR}} = \frac{1}{3}\overrightarrow{\mathrm{AB}}$.

S is a point on [AC] such that $\overrightarrow{\mathrm{AS}} = \frac{2}{3}\overrightarrow{\mathrm{AC}}$.

T is a point on [RS] such that $\overrightarrow{\mathrm{RT}} = \frac{2}{3}\overrightarrow{\mathrm{RS}}$.

8a. (i) Express $\overrightarrow{\mathrm{AM}}$ in terms of a and c. *[4 marks]*

(ii) Hence show that $\overrightarrow{\mathrm{BM}} = \frac{1}{2}a - b + \frac{1}{2}c$.

8b. (i) Express $\overrightarrow{\mathrm{RA}}$ in terms of a and b. [5 marks]

(ii) Show that $\overrightarrow{RT} = -\frac{2}{9}a - \frac{2}{9}b + \frac{4}{9}c$.

8c. Prove that T lies on $[BM]$. [5 marks]

9. Consider the two planes [6 marks]

$\pi_1 : 4x + 2y - z = 8$

$\pi_2 : x + 3y + 3z = 3.$

Find the angle between π_1 and π_2, giving your answer correct to the nearest degree.

The lines l_1 and l_2 are defined as

$$l_1 : \frac{x-1}{3} = \frac{y-5}{2} = \frac{z-12}{-2}$$

$$l_2 : \frac{x-1}{8} = \frac{y-5}{11} = \frac{z-12}{6}.$$

The plane π contains both l_1 and l_2.

10a. Find the Cartesian equation of π. *[4 marks]*

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. . . .

10b. The line l_3 passing through the point $(4,\ 0,\ 8)$ is perpendicular to π. *[4 marks]*

Find the coordinates of the point where l_3 meets π.

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. . . .

. . . .

A curve is defined $x^2 - 5xy + y^2 = 7$.

11a. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5y-2x}{2y-5x}$. *[3 marks]*

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. . . .

11b. Find the equation of the normal to the curve at the point $(6,\ 1)$. *[4 marks]*

11c. Find the distance between the two points on the curve where each tangent is parallel to the line $y = x$. *[8 marks]*

The equations of the lines L_1 and L_2 are

$$L_1 : r_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$L_2 : r_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix}.$$

12a. Show that the lines L_1 and L_2 are skew. *[4 marks]*

12b. Find the acute angle between the lines L_1 and L_2. *[4 marks]*

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13. The planes $2x + 3y - z = 5$ and $x - y + 2z = k$ intersect in the line $5x + 1 = 9 - 5y = -5z$. Find the value of k. *[5 marks]*

The coordinates of points A, B and C are given as $(5, -2, 5)$, $(5, 4, -1)$ and $(-1, -2, -1)$ respectively.

14a. Show that AB = AC and that $\mathrm{B\hat{A}C} = 60^\circ$. *[4 marks]*

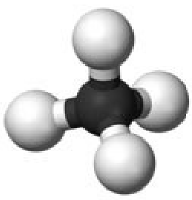
14b. Find the Cartesian equation of Π, the plane passing through A, B, and C. *[4 marks]*

14c. (i) Find the Cartesian equation of Π_1, the plane perpendicular to (AB) passing through the midpoint of [AB]. *[4 marks]*

(ii) Find the Cartesian equation of Π_2, the plane perpendicular to (AC) passing through the midpoint of [AC].

14d. Find the vector equation of L, the line of intersection of Π_1 and Π_2, and show that it is perpendicular to Π. *[3 marks]*

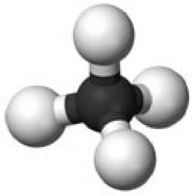
14e. A methane molecule consists of a carbon atom with four hydrogen atoms symmetrically placed around it in three dimensions. *[3 marks]*



The positions of the centres of three of the hydrogen atoms are A, B and C as given. The position of the centre of the fourth hydrogen atom is D.

Using the fact that $\mathrm{AB} = \mathrm{AD}$, show that the coordinates of one of the possible positions of the fourth hydrogen atom is $(-1, 4, 5)$.

14f. A methane molecule consists of a carbon atom with four hydrogen atoms symmetrically placed around it in three dimensions. *[6 marks]*



The positions of the centres of three of the hydrogen atoms are A, B and C as given. The position of the centre of the fourth hydrogen atom is D.

Letting D be
$(-1, 4, 5)$, show that the coordinates of G, the position of the centre of the carbon atom, are
$(2, 1, 2)$. Hence calculate
$\mathrm{D\hat{G}A}$, the bonding angle of carbon.

15. Find the values of *x* for which the vectors *[5 marks]*
$\begin{pmatrix} 1 \\ 2\cos x \\ 0 \end{pmatrix}$ and
$\begin{pmatrix} -1 \\ 2\sin x \\ 1 \end{pmatrix}$ are perpendicular,
$0 \leqslant x \leqslant \frac{\pi}{2}$.

16a. Find the values of *k* for which the following system of equations has no solutions and the value of *k* for the system to have an infinite number of solutions. *[5 marks]*

$$x - 3y + z = 3$$

$$x + 5y - 2z = 1$$

$$16y - 6z = k$$

16b. Given that the system of equations can be solved, find the solutions in the form of a vector equation of a line, $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, where the components of $\boldsymbol{b}$ are integers. *[7 marks]*

16c. The plane *[5 marks]*
÷ is parallel to both the line in part (b) and the line
$\frac{x-4}{3} = \frac{y-6}{-2} = \frac{z-2}{0}$.
Given that
÷ contains the point (1, 2, 0) , show that the Cartesian equation of ÷ is $16x + 24y - 11z = 64$.

16d. The *z*-axis meets the plane *[2 marks]*
÷ at the point P. Find the coordinates of P.

16e. Find the angle between the line *[5 marks]*
$\frac{x-2}{3} = \frac{y+5}{4} = \frac{z}{2}$ and the plane
÷ .

Two boats, *A* and *B* , move so that at time *t* hours, their position vectors, in kilometres, are $\boldsymbol{r}_A = (9t)\boldsymbol{i} + (3 - 6t)\boldsymbol{j}$ and $\boldsymbol{r}_B = (7 - 4t)\boldsymbol{i} + (7t - 6)\boldsymbol{j}$.

17a. Find the coordinates of the common point of the paths of the two boats. *[4 marks]*

17b. Show that the boats do not collide. *[2 marks]*

Consider the planes
$\pi_1 : x - 2y - 3z = 2$ and $\pi_2 : 2x - y - z = k$.

18a. Find the angle between the planes $\pi_1$and π_2 . *[4 marks]*

18b. The planes π_1 and π_2 intersect in the line L_1 . Show that the vector equation of L_1 is *[5 marks]*

$r = \begin{pmatrix} 0 \\ 2-3k \\ 2k-2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$

18c. The line L_2 has Cartesian equation $5 - x = y + 3 = 2 - 2z$. The lines L_1 and L_2 intersect at a point X. Find the coordinates of X. *[5 marks]*

18d. Determine a Cartesian equation of the plane π_3 containing both lines L_1 and L_2 . *[5 marks]*

18e. Let Y be a point on L_1 and Z be a point on L_2 such that XY is perpendicular to YZ and the area of the triangle XYZ is 3. Find the perimeter of the triangle XYZ. *[5 marks]*

Consider the points A(1, 2, 3), B(1, 0, 5) and C(2, –1, 4).

19a. Find *[4 marks]*

$\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$.

19b. Hence find the area of the triangle ABC. *[2 marks]*

Consider the points P(–3, –1, 2) and Q(5, 5, 6).

20a. Find a vector equation for the line, *[3 marks]*

L_1, which passes through the points P and Q.

The line

L_2 has equation

$$r = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} + s\begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix}.$$

20b. Show that

L_1 and

L_2 intersect at the point R(1, 2, 4).

[4 marks]

20c. Find the acute angle between

L_1 and

L_2.

[3 marks]

20d. Let S be a point on

L_2 such that

$\left|\overrightarrow{\mathrm{RP}}\right| = \left|\overrightarrow{\mathrm{RS}}\right|$.

Show that one of the possible positions for S is

$\mathrm{S}_1(-4, 0, 4)$ and find the coordinates of the other possible position,

S_2.

[6 marks]

20e. Let S be a point on

L_2 such that

$\left|\overrightarrow{\mathrm{RP}}\right| = \left|\overrightarrow{\mathrm{RS}}\right|$.

Find a vector equation of the line which passes through R and bisects

$\mathrm{P\hat{R}S_1}$.

[4 marks]

The vertices of a triangle ABC have coordinates given by A(−1, 2, 3), B(4, 1, 1) and C(3, −2, 2).

21a. (i) Find the lengths of the sides of the triangle.

(ii) Find

$\cos \mathrm{B\hat{A}C}$.

[6 marks]

21b. (i) Show that

$\overrightarrow{\mathrm{BC}} \times \overrightarrow{\mathrm{CA}} = -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$.

(ii) Hence, show that the area of the triangle ABC is

$\frac{1}{2}\sqrt{314}$.

[5 marks]

21c. Find the Cartesian equation of the plane containing the triangle ABC.

[3 marks]

21d. Find a vector equation of (AB).

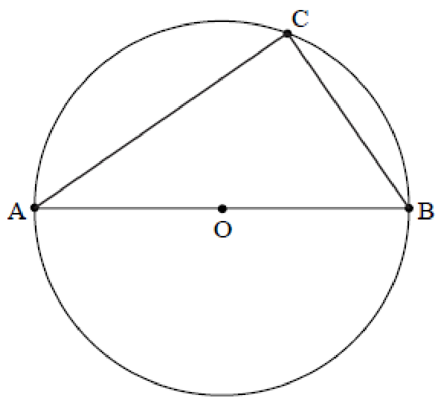
[2 marks]

21e. The point D on (AB) is such that $\overrightarrow{\mathrm{OD}}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$ where O is the origin. *[5 marks]*

(i) Find the coordinates of D.

(ii) Show that D does not lie between A and B.

In the diagram below, [AB] is a diameter of the circle with centre O. Point C is on the circumference of the circle. Let $\overrightarrow{\mathrm{OB}} = b$ and $\overrightarrow{\mathrm{OC}} = c$.



22a. Find an expression for $\overrightarrow{\mathrm{CB}}$ and for $\overrightarrow{\mathrm{AC}}$ in terms of b and c . *[2 marks]*

22b. Hence prove that $\mathrm{A\hat{C}B}$ is a right angle. *[3 marks]*

The points P(−1, 2, − 3), Q(−2, 1, 0), R(0, 5, 1) and S form a parallelogram, where S is diagonally opposite Q.

23a. Find the coordinates of S. *[2 marks]*

23b. The vector product $\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PS}} = \begin{pmatrix} -13 \\ 7 \\ m \end{pmatrix}$. Find the value of m . *[2 marks]*

23c. Hence calculate the area of parallelogram PQRS. *[2 marks]*

23d. Find the Cartesian equation of the plane, $\prod_1$, containing the parallelogram PQRS. *[3 marks]*

23e. Write down the vector equation of the line through the origin (0, 0, 0) that is perpendicular to the plane [1 mark]

$\prod_1$.

23f. Hence find the point on the plane that is closest to the origin. [3 marks]

23g. A second plane, [4 marks]

$\prod_2$, has equation $x - 2y + z = 3$. Calculate the angle between the two planes.

24. (a) Show that the two planes [7 marks]

$$\pi_1 : x + 2y - z = 1$$

$$\pi_2 : x + z = -2$$

are perpendicular.

(b) Find the equation of the plane

π_3 that passes through the origin and is perpendicular to both

π_1 and

π_2.

25. Consider the vectors [4 marks]

$\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$,

$\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and

$\overrightarrow{\mathrm{OC}} = \boldsymbol{a} + \boldsymbol{b}$. Show that if

$|\boldsymbol{a}$

$| =$

$|\boldsymbol{b}$

$|$ then $(\boldsymbol{a} + \boldsymbol{b})$

$\cdot(\boldsymbol{a} - \boldsymbol{b}) = 0$. Comment on what this tells us about the parallelogram OACB.

26. A plane [20 marks]

π has vector equation $\boldsymbol{r} = (-2\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}) +$

$\lambda(2\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}) +$

$\mu(6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k})$.

(a) Show that the Cartesian equation of the plane

π is $3x + 2y - 6z = 12$.

(b) The plane

π meets the x, y and z axes at A, B and C respectively. Find the coordinates of A, B and C.

(c) Find the volume of the pyramid OABC.

(d) Find the angle between the plane

π and the x-axis.

(e) **Hence**, or otherwise, find the distance from the origin to the plane

π.

(f) Using your answers from (c) and (e), find the area of the triangle ABC.

27. [5 marks]

The three vectors

a,

b and

c are given by

$$a = \begin{pmatrix} 2y \\ -3x \\ 2x \end{pmatrix}, b = \begin{pmatrix} 4x \\ y \\ 3 - x \end{pmatrix}, c = \begin{pmatrix} 4 \\ -7 \\ 6 \end{pmatrix} \text{ where } x, y \in \mathbb{R} .$$

(a) If $\boldsymbol{a} + 2\boldsymbol{b} - \boldsymbol{c} = 0$, find the value of x and of y.

(b) Find the exact value of

$|\boldsymbol{a} + 2\boldsymbol{b}$

$|$.

28a. Consider the vectors $\boldsymbol{a} = 6\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 4\boldsymbol{k}$. [11 marks]

(i) Find the cosine of the angle between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(ii) Find $\boldsymbol{a}$

$\times \boldsymbol{b}$.

(iii) **Hence** find the Cartesian equation of the plane

$\prod$ containing the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ and passing through the point $(1, 1, -1)$.

(iv) The plane

$\prod$ intersects the x-y plane in the line l. Find the area of the finite triangular region enclosed by l, the x-axis and the y-axis.

28b. Given two vectors $\boldsymbol{p}$ and $\boldsymbol{q}$, *[8 marks]*

(i) show that $\boldsymbol{p} \cdot \boldsymbol{p} = |\boldsymbol{p}|^2$;

(ii) hence, or otherwise, show that $|\boldsymbol{p} + \boldsymbol{q}|^2 = |\boldsymbol{p}|^2 + 2\boldsymbol{p} \cdot \boldsymbol{q} + |\boldsymbol{q}|^2$;

(iii) deduce that $|\boldsymbol{p} + \boldsymbol{q}| \le |\boldsymbol{p}| + |\boldsymbol{q}|$.

29. Consider the plane with equation *[8 marks]*

$4x - 2y - z = 1$ and the line given by the parametric equations

$x = 3 - 2\lambda$

$y = (2k - 1) + \lambda$

$z = -1 + k\lambda$.

Given that the line is perpendicular to the plane, find

(a) the value of k;

(b) the coordinates of the point of intersection of the line and the plane.

30. Let

α be the angle between the unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, where

$0 \leqslant \alpha \leqslant \pi$.

[5 marks]

(a) Express

$|\boldsymbol{a} - \boldsymbol{b}$

$|$ and

$|\boldsymbol{a} + \boldsymbol{b}$

$|$ in terms of

α.

(b) Hence determine the value of

$\cos\alpha$ for which

$|\boldsymbol{a} + \boldsymbol{b}$

$| = 3$

$|\boldsymbol{a} - \boldsymbol{b}$

$|$.

31. Consider the vectors $\boldsymbol{a}$

[8 marks]

$= \sin(2\alpha)\boldsymbol{i}$

$- \cos(2\alpha)\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b}$

$= \cos\alpha\boldsymbol{i}$

$- \sin\alpha\boldsymbol{j} - \boldsymbol{k}$, where

$0 < \alpha < 2\pi$.

Let

θ be the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) Express

$\cos\theta$ in terms of

α.

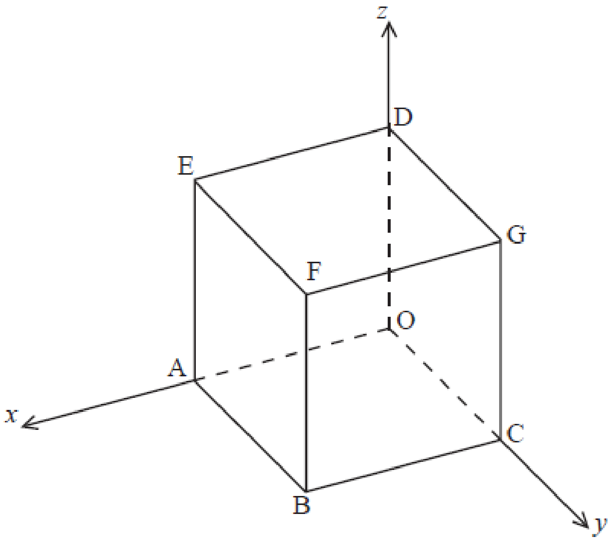
(b) Find the acute angle

α for which the two vectors are perpendicular.

(c) For

$\alpha = \frac{7\pi}{6}$, determine the vector product of $\boldsymbol{a}$ and $\boldsymbol{b}$ and comment on the geometrical significance of this result.

32. The diagram shows a cube OABCDEFG. *[20 marks]*



Let O be the origin, (OA) the x-axis, (OC) the y-axis and (OD) the z-axis.

Let M, N and P be the midpoints of [FG], [DG] and [CG], respectively.

The coordinates of F are (2, 2, 2).

(a) Find the position vectors

$\overrightarrow{\text{OM}}$,

$\overrightarrow{\text{ON}}$ and

$\overrightarrow{\text{OP}}$ in component form.

(b) Find

$\overrightarrow{\text{MP}} \times \overrightarrow{\text{MN}}$.

(c) **Hence**,

(i) calculate the area of the triangle MNP;

(ii) show that the line (AG) is perpendicular to the plane MNP;

(iii) find the equation of the plane MNP.

(d) Determine the coordinates of the point where the line (AG) meets the plane MNP.

33a. For non-zero vectors *[8 marks]*

a and

b, show that

(i) if

$|a - b| = |a + b|$, then

a and

b are perpendicular;

(ii)

$|a \times b|^{2} = |a|^{2}|b|^{2} - (a \cdot b)^{2}$.

33b. The points A, B and C have position vectors [7 marks]

a,

b and

c.

(i) Show that the area of triangle ABC is

$\frac{1}{2}|a \times b + b \times c + c \times a|$.

(ii) Hence, show that the shortest distance from B to AC is

$$\frac{|a \times b + b \times c + c \times a|}{|c - a|}.$$

Two planes Π_1 and Π_2 have equations $2x + y + z = 1$ and $3x + y - z = 2$ respectively.

34a. Find the vector equation of L, the line of intersection of [6 marks]

Π_1 and

Π_2.

34b. Show that the plane [4 marks]

Π_3 which is perpendicular to

Π_1 and contains L, has equation

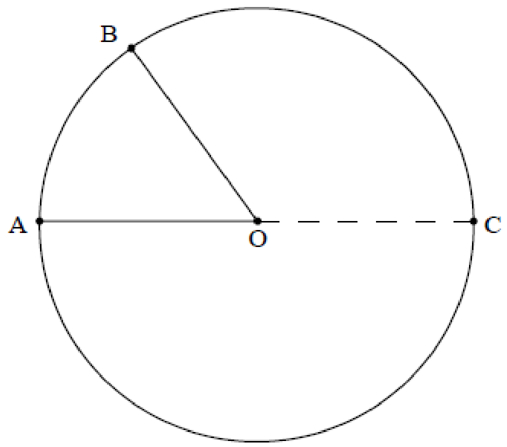
$x - 2z = 1$.

34c. The point P has coordinates (–2, 4, 1) , the point Q lies on [6 marks]

Π_3 and PQ is perpendicular to

Π_2. Find the coordinates of Q.

The diagram below shows a circle with centre O. The points A, B, C lie on the circumference of the circle and [AC] is a diameter.



Let $\overrightarrow{\mathrm{OA}} = a$ and $\overrightarrow{\mathrm{OB}} = b$.

35a. Write down expressions for $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{CB}}$ in terms of the vectors a and b . *[2 marks]*

35b. Hence prove that angle $\mathrm{A\hat{B}C}$ is a right angle. *[3 marks]*

The points A(1, 2, 1) , B(−3, 1, 4) , C(5, −1, 2) and D(5, 3, 7) are the vertices of a tetrahedron.

36a. Find the vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. *[2 marks]*

36b. Find the Cartesian equation of the plane $\prod$ that contains the face ABC. *[4 marks]*

37. Port A is defined to be the origin of a set of coordinate axes and port B is located at the point (70, 30), where distances are measured in kilometres. A ship S_1 sails from port A at 10:00 in a straight line such that its position t hours after 10:00 is given by $r = t\begin{pmatrix}10\\20\end{pmatrix}$. *[7 marks]*

A speedboat S_2 is capable of three times the speed of S_1 and is to meet S_1 by travelling the shortest possible distance. What is the latest time that S_2 can leave port B?