Topic 2 Part 2 *[375 marks]*

[2 marks]

Markscheme

$4(x - 0.5)^2 + 4$ *A1A1*

Note: *A1* for two correct parameters, *A2* for all three correct.

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e).

[3 marks]

Markscheme

translation

$\begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$ (allow "0.5 to the right")   *A1*

stretch parallel to *y*-axis, scale factor 4 (allow vertical stretch or similar)   *A1*

translation

$\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ (allow "4 up")   *A1*

Note: All transformations must state magnitude and direction.

Note: First two transformations can be in either order.

It could be a stretch followed by a single translation of

$\begin{pmatrix} 0.5 \\ 4 \end{pmatrix}$. If the vertical translation is before the stretch it is

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

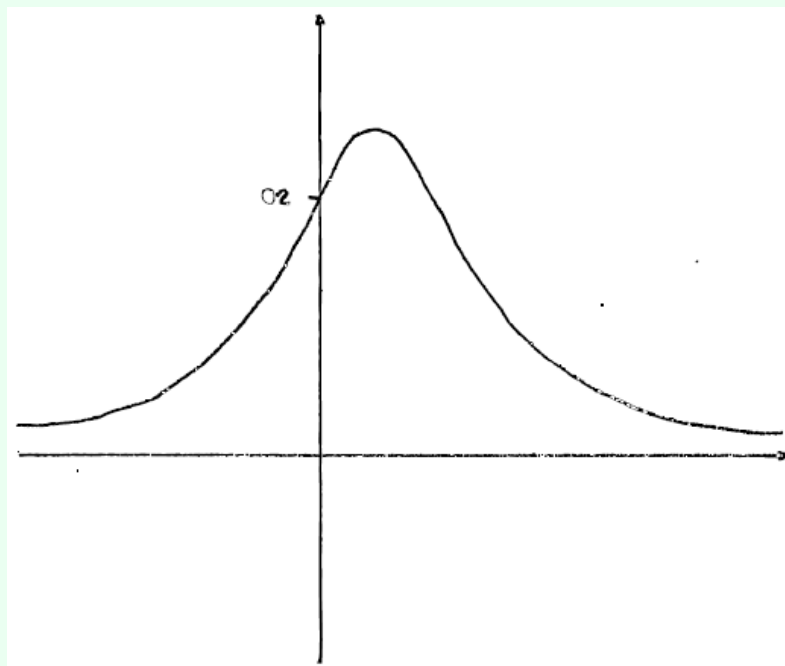*[3 marks]*

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(b) Exam technique would have helped those candidates who could not get part (a) correct as any solution of the form given in the question could have led to full marks in part (b). Several candidates obtained expressions which were not of this form in (a) and so were unable to receive any marks in (b) Many missed the fact that if a vertical translation is performed before the vertical stretch it has a different magnitude to if it is done afterwards. Though on this occasion the markscheme was fairly flexible in the words it allowed to be used by candidates to describe the transformations it would be less risky to use the correct expressions.

1c.

Markscheme

general shape (including asymptote and single maximum in first quadrant), **A1**

intercept

$\left(0, \frac{1}{5}\right)$ or maximum

$\left(\frac{1}{2}, \frac{1}{4}\right)$ shown   **A1**

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(c) Generally the sketches were poor. The general rule for all sketch questions should be that any asymptotes or intercepts should be clearly labelled. Sketches do not need to be done on graph paper, but a ruler should be used, particularly when asymptotes are involved.

1d.

Markscheme

$0 < f(x) \leqslant \frac{1}{4}$ **A1A1**

Note: A1 for

$\leqslant \frac{1}{4}$, **A1** for

$0 <$.

[2 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e).

Markscheme

let

$u = x - \frac{1}{2}$ *A1*

$\frac{du}{dx} = 1$ (or $du = dx$)   *A1*

$\int \frac{1}{4x^2 - 4x + 5} dx = \int \frac{1}{4\left(x - \frac{1}{2}\right)^2 + 4} dx$ *A1*

$\int \frac{1}{4u^2 + 4} du = \frac{1}{4} \int \frac{1}{u^2 + 1} du$ *AG*

Note: If following through an incorrect answer to part (a), do not award final *A1* mark.

[3 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(e) and (f) were well done up to the final part of (f), in which candidates did not realise they needed to use the compound angle formula.

Markscheme

$\int_1^{3.5} \frac{1}{4x^2 - 4x + 5} dx = \frac{1}{4} \int_{0.5}^3 \frac{1}{u^2 + 1} du$ *A1*

Note: *A1* for correct change of limits. Award also if they do not change limits but go back to *x* values when substituting the limit (even if there is an error in the integral).

$\frac{1}{4} \left[\arctan(u) \right]_{0.5}^3$ *(M1)*

$\frac{1}{4} \left(\arctan(3) - \arctan\left(\frac{1}{2}\right) \right)$ *A1*

let the integral = *I*

$\tan 4I = \tan\left(\arctan(3) - \arctan\left(\frac{1}{2}\right) \right)$ *M1*

$\frac{3 - 0.5}{1 + 3 \times 0.5} = \frac{2.5}{2.5} = 1$ *(M1)A1*

$4I = \frac{\pi}{4} \Rightarrow I = \frac{\pi}{16}$ *A1AG*

[7 marks]

Examiners report

This question covered many syllabus areas, completing the square, transformations of graphs, range, integration by substitution and compound angle formulae. There were many good solutions to parts (a) – (e) but the following points caused some difficulties.

(e) and (f) were well done up to the final part of (f), in which candidates did not realise they needed to use the compound angle formula.

2.

Markscheme

other root is $2 - i$ *(A1)*

a quadratic factor is therefore

$(x - 2 + i)(x - 2 - i)$ *(M1)*

$= x^2 - 4x + 5$ *A1*

$x + 1$ is a factor   *A1*

$(x - 2)^2$ is a factor   *A1*

$p(x) = a(x + 1)(x - 2)^2(x^2 - 4x + 5)$ *(M1)*

$p(0) = 4 \Rightarrow a = \frac{1}{5}$ *A1*

$p(x) = \frac{1}{5}(x + 1)(x - 2)^2(x^2 - 4x + 5)$

[7 marks]

Examiners report

Whilst most candidates knew that another root was
$2 - i$, much fewer were able to find the quadratic factor. Surprisingly few candidates knew that
$(x - 2)$ must be a repeated factor and less surprisingly many did not recognise that the whole expression needed to be multiplied by
$\frac{1}{5}$.

3a.

Markscheme

$\Delta = b^2 - 4ac = 4k^2 - 4 \times 3 \times (k - 1) = 4k^2 - 12k + 12$ *M1A1*
Note: Award *M1A1* if expression seen within quadratic formula.

EITHER
$144 - 4 \times 4 \times 12 < 0$ *M1*
Δ always positive, therefore the equation always has two distinct real roots   *R1*
(and cannot be always negative as
$a > 0$)
OR
sketch of
$y = 4k^2 - 12k + 12$ or
$y = k^2 - 3k + 3$ not crossing the x-axis   *M1*
Δ always positive, therefore the equation always has two distinct real roots   *R1*
OR
write
Δ as
$4(k - 1.5)^2 + 3$ *M1*
Δ always positive, therefore the equation always has two distinct real roots   *R1*
[4 marks]

Examiners report

Most candidates were able to find the discriminant (sometimes only as part of the quadratic formula) but fewer were able to explain
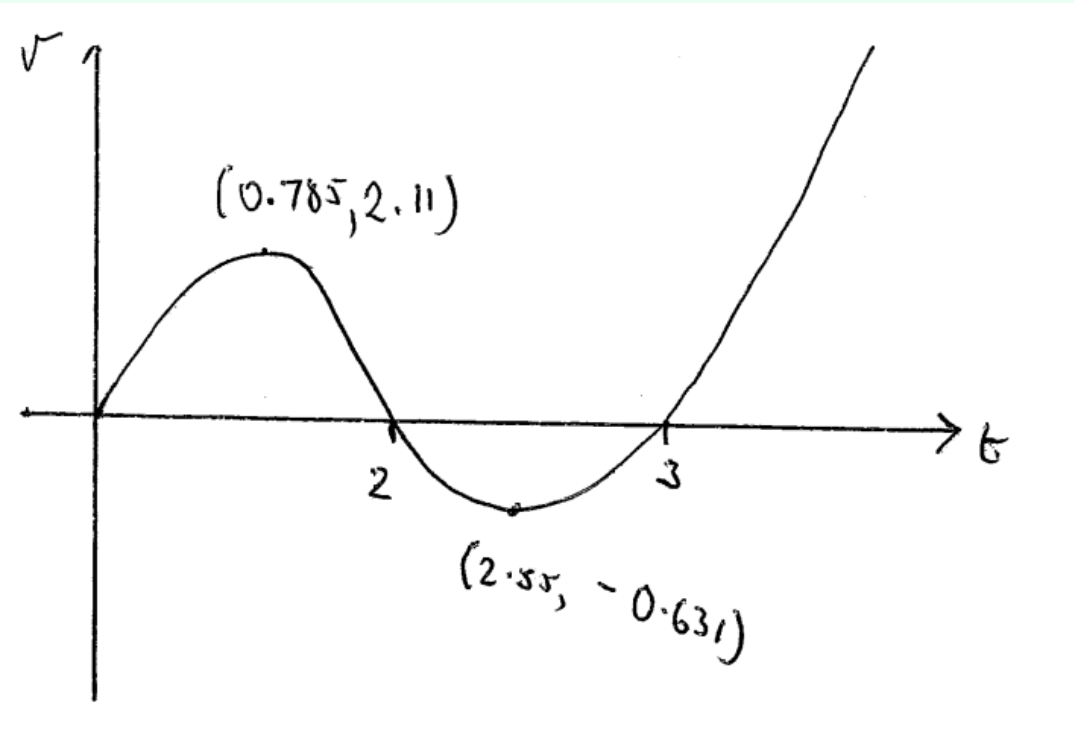satisfactorily why there were two distinct roots.

3b.

Markscheme

closest together when

Δ is least   *(M1)*

minimum value occurs when $k = 1.5$ *(M1)A1*

[3 marks]

Examiners report

Most candidates were able to find the discriminant (sometimes only as part of the quadratic formula) but fewer were able to explain satisfactorily why there were two distinct roots. Only the better candidates were able to give good answers to part (b).

4a.

Markscheme

A1A1A1

Note: Award *A1* for general shape, *A1* for correct maximum and minimum, *A1* for intercepts.

Note: Follow through applies to (b) and (c).

[3 marks]

Examiners report

Part (a) was generally well done, although correct accuracy was often a problem.

4b.

Markscheme

$0 \leqslant t < 0.785, \ \left(\text{or } 0 \leqslant t < \frac{5 - \sqrt{7}}{3} \right)$ **A1**

(allow

$t < 0.785$)

and

$t > 2.55 \ \left(\text{or } t > \frac{5 + \sqrt{7}}{3} \right)$ **A1**

[2 marks]

Examiners report

Parts (b) and (c) were also generally quite well done.

4c.

Markscheme

$0 \leqslant t < 0.785, \ \left(\text{or } 0 \leqslant t < \frac{5 - \sqrt{7}}{3} \right)$ **A1**

(allow

$t < 0.785$)

$2 < t < 2.55, \ \left(\text{or } 2 < t < \frac{5 + \sqrt{7}}{3} \right)$ **A1**

$t > 3$ **A1**

[3 marks]

Examiners report

Parts (b) and (c) were also generally quite well done.

4d.

Markscheme

position of A:

$x_A = \int t^3 - 5t^2 + 6t \ \mathrm{d}t$ **(M1)**

$x_A = \frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2 \ (+c)$ **A1**

when

$t = 0, \ x_A = 0$, so

$c = 0$ **R1**

[3 marks]

Examiners report

A variety of approaches were seen in part (d) and many candidates were able to obtain at least 2 out of 3. A number missed to consider the
$+c$, thereby losing the last mark.

4e.

Markscheme

$\frac{dv_B}{dt} = -2v_B \Rightarrow \int \frac{1}{v_B} dv_B = \int -2dt$ **(M1)**

$\ln|v_B| = -2t + c$ **(A1)**

$v_B = Ae^{-2t}$ **(M1)**

$v_B = -20$ when $t = 0$ so

$v_B = -20e^{-2t}$ **A1**

[4 marks]

Examiners report

Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

4f.

Markscheme

$x_B = 10e^{-2t}(+c)$ **(M1)(A1)**

$x_B = 20$ when $t = 0$ so $x_B = 10e^{-2t} + 10$ **(M1)A1**

meet when

$\frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2 = 10e^{-2t} + 10$ **(M1)**

$t = 4.41(290\ldots)$ **A1**

[6 marks]

Examiners report

Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

5a.

Markscheme

$|z_1| = \sqrt{10}$; $\arg(z_2) = -\frac{3\pi}{4}$ $\left(\text{accept } \frac{5\pi}{4}\right)$ **A1A1**

[2 marks]

Examiners report

Disappointingly, few candidates obtained the correct argument for the second complex number, mechanically using arctan(1) but not thinking about the position of the number in the complex plane.

$\frac{dv_B}{dt} = -2v_B \Rightarrow \int \frac{1}{v_B} dv_B = \int -2dt$ **(M1)**

Markscheme

$|z_1 + \alpha z_2| = \sqrt{(1-\alpha)^2 + (3-\alpha)^2}$ or the squared modulus   *(M1)(A1)*

attempt to minimise

$2\alpha^2 - 8\alpha + 10$ or their quadratic or its half or its square root   *M1*

obtain

$\alpha = 2$ at minimum   *(A1)*

state

$\sqrt{2}$ as final answer   *A1*

[5 marks]

Examiners report

Most candidates obtained the correct quadratic or its square root, but few knew how to set about minimising it.

Markscheme

EITHER

$f(x) - 1 = \frac{1 + 3^{-x}}{3^x - 3^{-x}}$ *M1A1*

> 0 as both numerator and denominator are positive   *R1*

OR

$3^x + 1 > 3^x > 3^x - 3^{-x}$ *M1A1*

Note: Accept a convincing valid argument the numerator is greater than the denominator.

numerator and denominator are positive   *R1*

hence

$f(x) > 1$ *AG*

[3 marks]

Examiners report

(a) This is a question where carefully organised reasoning is crucial. It is important to state that both the numerator and the denominator are positive for
$x > 0$. Candidates were more successful with part (b) than with part (a).

Markscheme

one line equation to solve, for example,

$4(3^x - 3^{-x}) = 3^x + 1$, or equivalent   *A1*

$(3y^2 - y - 4 = 0)$

attempt to solve a three-term equation   *M1*

obtain

$y = \frac{4}{3}$ *A1*

$x = \log_3\left(\frac{4}{3}\right)$ or equivalent   *A1*

Note: Award *A0* if an extra solution for x is given.

[4 marks]

Examiners report

(a) This is a question where carefully organised reasoning is crucial. It is important to state that both the numerator and the denominator are positive for
$x > 0$. Candidates were more successful with part (b) than with part (a).

Markscheme

by division or otherwise

$f(x) = 2 - \frac{5}{x+2}$ *A1A1*

[2 marks]

Examiners report

Generally well done.

Markscheme

$f'(x) = \frac{5}{(x+2)^2}$ *A1*

> 0 as

$(x+2)^2 > 0$ (on D)   *R1AG*

Note: Do not penalise candidates who use the original form of the function to compute its derivative.

[2 marks]

Examiners report

In their answers to Part (b), most candidates found the derivative, but many assumed it was obviously positive.

7c.

Markscheme

$S = \left[-3, \frac{3}{2}\right]$ *A2*

Note: Award *A1A0* for the correct endpoints and an open interval.

[2 marks]

Examiners report

[N/A]

Markscheme

$S = \left[-3, \frac{3}{2}\right]$ *A2*

Note: Award *A1A0* for the correct endpoints and an open interval.

Markscheme

(i) **EITHER**

rearrange

$y = f(x)$ to make x the subject   *M1*

obtain one-line equation, *e.g.*

$2x - 1 = xy + 2y$ *A1*

$x = \frac{2y+1}{2-y}$ *A1*

OR

interchange x and y *M1*

obtain one-line equation, *e.g.*

$2y - 1 = xy + 2x$ *A1*

$y = \frac{2x+1}{2-x}$ *A1*

THEN

$f^{-1}(x) = \frac{2x+1}{2-x}$ *A1*

Note: Accept

$\frac{5}{2-x} - 2$

(ii), (iii)

A1A1A1A1

[8 marks]

Note: Award *A1* for correct shape of

$y = f(x)$.

Award **A1** for x intercept

$\frac{1}{2}$ seen. Award **A1** for y intercept

$-\frac{1}{2}$ seen.

Award **A1** for the graph of

$y = f^{-1}(x)$ being the reflection of

$y = f(x)$ in the line

$y = x$. Candidates are not required to indicate the full domain, but

$y = f(x)$ should not be shown approaching

$x = -2$. Candidates, in answering (iii), can **FT** on their sketch in (ii).
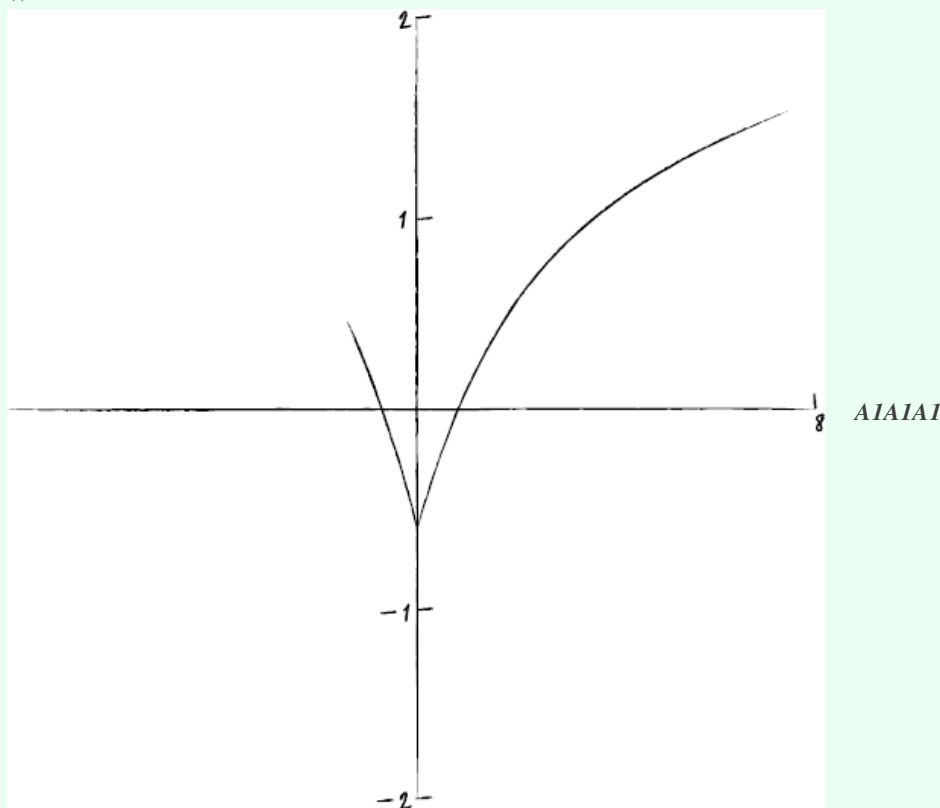
Examiners report

Part (d)(i) Generally well done, but some candidates failed to label their final expression as $f^{-1}(x)$. Part (d)(ii) Marks were lost by candidates who failed to mark the intercepts with values.

Markscheme

(i)

AlAlAl

Note: *A1* for correct sketch

$x > 0$, *A1* for symmetry, *A1* for correct domain (from –1 to +8).

Note: Candidates can **FT** on their sketch in (d)(ii).

(ii)   attempt to solve

$f(x) = -\frac{1}{4}$ *(M1)*

obtain

$x = \frac{2}{9}$ *A1*

use of symmetry or valid algebraic approach   *(M1)*

obtain

$x = -\frac{2}{9}$ *A1*

[7 marks]

Examiners report

Marks were also lost in this part and in part (e)(i) for graphs that went beyond the explicitly stated domain.

Markscheme

(i)

$z_1 = 2\text{cis}\left(\frac{\pi}{6}\right)$, $z_2 = 2\text{cis}\left(\frac{5\pi}{6}\right)$, $z_3 = 2\text{cis}\left(-\frac{\pi}{2}\right)$ or $2\text{cis}\left(\frac{3\pi}{2}\right)$ **A1A1A1**

Note: Accept modulus and argument given separately, or the use of exponential (Euler) form.

Note: Accept arguments given in rational degrees, except where exponential form is used.

(ii)   the points lie on a circle of radius 2 centre the origin   **A1**

differences are all

$\frac{2\pi}{3}(\text{ mod } 2\pi)$ **A1**

$\Rightarrow$ points equally spaced

$\Rightarrow$ triangle is equilateral   **R1AG**

Note: Accept an approach based on a clearly marked diagram.

(iii)

$z_1^{3n} + z_2^{3n} = 2^{3n}\text{cis}\left(\frac{n\pi}{2}\right) + 2^{3n}\text{cis}\left(\frac{5n\pi}{2}\right)$ **M1**

$= 2 \times 2^{3n}\text{cis}\left(\frac{n\pi}{2}\right)$ **A1**

$2z_3^{3n} = 2 \times 2^{3n}\text{cis}\left(\frac{9n\pi}{2}\right) = 2 \times 2^{3n}\text{cis}\left(\frac{n\pi}{2}\right)$ **A1AG**

[9 marks]

Examiners report

(i) A disappointingly large number of candidates were unable to give the correct arguments for the three complex numbers. Such errors undermined their efforts to tackle parts (ii) and (iii).

Markscheme

(i) attempt to obtain **seven** solutions in modulus argument form *M1*

$z = \text{cis}\left(\frac{2k\pi}{7}\right)$, $k = 0, 1 \dots 6$ *A1*

(ii) w has argument

$\frac{2\pi}{7}$ and $1 + w$ has argument

ϕ,

then

$\tan(\phi) = \dfrac{\sin\left(\frac{2\pi}{7}\right)}{1 + \cos\left(\frac{2\pi}{7}\right)}$ *M1*

$= \dfrac{2\sin\left(\frac{\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)}{2\cos^2\left(\frac{\pi}{7}\right)}$ *A1*

$= \tan\left(\frac{\pi}{7}\right) \Rightarrow \phi = \frac{\pi}{7}$ *A1*

Note: Accept alternative approaches.

(iii) since roots occur in conjugate pairs, *(R1)*

$z^7 - 1$ has a quadratic factor

$\left(z - \text{cis}\left(\frac{2\pi}{7}\right)\right) \times \left(z - \text{cis}\left(-\frac{2\pi}{7}\right)\right)$ *A1*

$= z^2 - 2z\cos\left(\frac{2\pi}{7}\right) + 1$ *AG*

other quadratic factors are

$z^2 - 2z\cos\left(\frac{4\pi}{7}\right) + 1$ *A1*

and

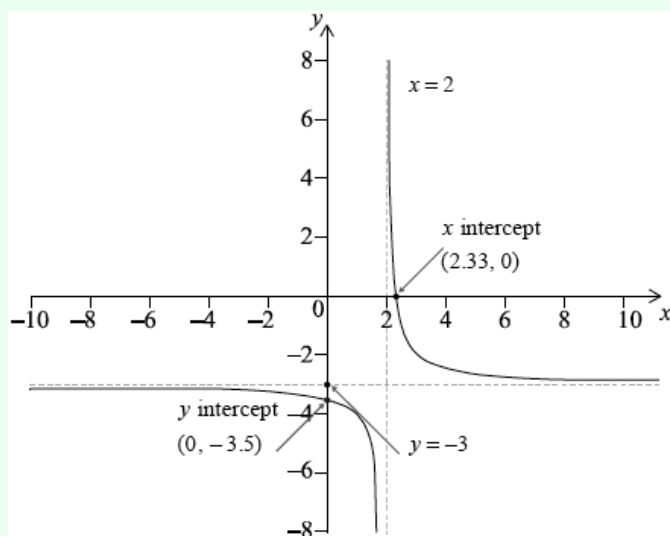$z^2 - 2z\cos\left(\frac{6\pi}{7}\right) + 1$ *A1*

[9 marks]

Examiners report

Many candidates were successful in part (i), but failed to capitalise on that – in particular, few used the fact that roots of $z^7 - 1 = 0$ come in complex conjugate pairs.

Markscheme

A1A1A1

Note: Award **A1** for correct shape, **A1** for
$x = 2$ clearly stated and **A1** for
$y = -3$ clearly stated.

x intercept (2.33, 0) and y intercept (0, –3.5)   **A1**

Note: Accept –3.5 and 2.33 (7/3) marked on the correct axes.

[4 marks]

Examiners report

[N/A]

Markscheme

$x = -3 + \frac{1}{y-2}$ **M1**

Note: Award **M1** for interchanging x and y (can be done at a later stage).

$x + 3 = \frac{1}{y-2}$
$y - 2 = \frac{1}{x+3}$ **M1**

Note: Award **M1** for attempting to make y the subject.

$f^{-1}(x) = 2 + \frac{1}{x+3} \left(= \frac{2x+7}{x+3} \right)$, $x \neq -3$ **A1A1**

Note: Award **A1** only if
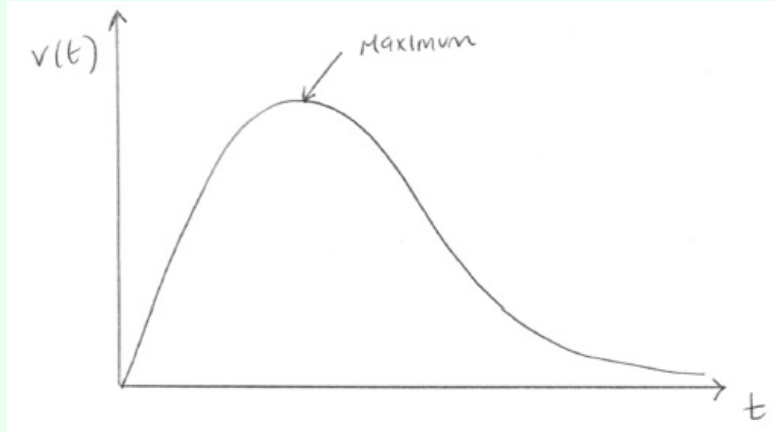$f^{-1}(x)$ is seen. Award **A1** for the domain.

[4 marks]

10a. *[2 marks]*

Markscheme

(a)

A1

A1 for correct shape and correct domain

$(1.41,\ 0.0884)$ $\left(\sqrt{2},\ \frac{\sqrt{2}}{16}\right)$ **A1**

[2 marks]

10b. *[4 marks]*

Markscheme

EITHER

$u = t^2$

$\frac{du}{dt} = 2t$ **A1**

OR

$t = u^{\frac{1}{2}}$

$\frac{dt}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ **A1**

THEN

$\int \frac{t}{12+t^4}dt = \frac{1}{2}\int \frac{du}{12+u^2}$ **M1**

$= \frac{1}{2\sqrt{12}}\arctan\left(\frac{u}{\sqrt{12}}\right)(+c)$ **M1**

$= \frac{1}{4\sqrt{3}}\arctan\left(\frac{t^2}{2\sqrt{3}}\right)(+c)$ or equivalent   **A1**

[4 marks]

10c.

Markscheme

$\int_0^6 \frac{t}{12+t^4} dt$ *(M1)*

$= \left[\frac{1}{4\sqrt{3}} \arctan\left(\frac{t^2}{2\sqrt{3}}\right) \right]_0^6$ *M1*

$= \frac{1}{4\sqrt{3}} \left(\arctan\left(\frac{36}{2\sqrt{3}}\right) \right) \left(= \frac{1}{4\sqrt{3}} \left(\arctan\left(\frac{18}{\sqrt{3}}\right) \right) \right)$ (m)   *A1*

Note: Accept

$\frac{\sqrt{3}}{12} \arctan\left(6\sqrt{3}\right)$ or equivalent.

[3 marks]

Examiners report

[N/A]

10d.

Markscheme

$\frac{dv}{ds} = \frac{1}{2\sqrt{s(1-s)}}$ *(A1)*

$a = v\frac{dv}{ds}$

$a = \arcsin\left(\sqrt{s}\right) \times \frac{1}{2\sqrt{s(1-s)}}$ *(M1)*

$a = \arcsin\left(\sqrt{0.1}\right) \times \frac{1}{2\sqrt{0.1 \times 0.9}}$

$a = 0.536\,(\text{ms}^{-2})$ *A1*

[3 marks]

Examiners report

[N/A]

11.

Markscheme

$f(-2) = 0\ (\Rightarrow -24 + 4p - 2q - 2 = 0)$ *M1*

$f(-1) = 4\ (\Rightarrow -3 + p - q - 2 = 4)$ *M1*

Note: In each case award the *M* marks if correct substitution attempted and right-hand side correct.

attempt to solve simultaneously

$(2p - q = 13,\ p - q = 9)$ *M1*

$p = 4$ *A1*
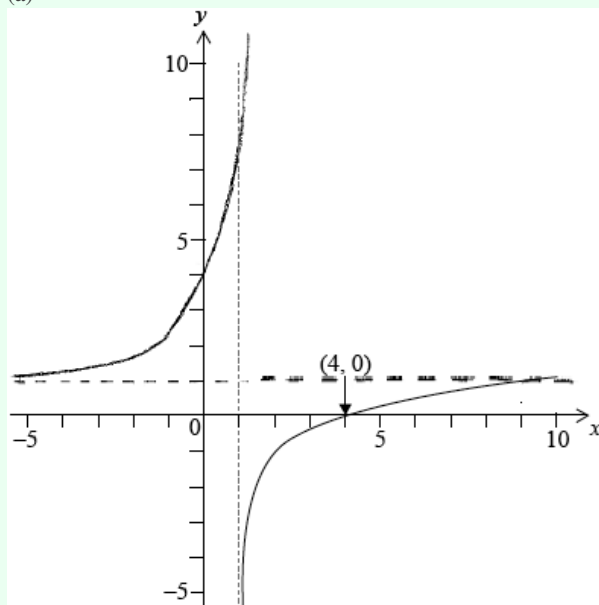
$q = -5$ *A1*

[5 marks]

Examiners report

Many candidates scored full marks on what was thought to be an easy first question. However, a number of candidates wrote down two correct equations but proceeded to make algebraic errors and thus found incorrect values for *p* and *q*. A small number also attempted to answer this question using long division, but fully correct answers using this technique were rarely seen.

12a.

Markscheme

(a)

shape with y-axis intercept $(0, 4)$ **A1**

Note: Accept curve with an asymptote at
$x = 1$ suggested.

correct asymptote
$y = 1$ **A1**
[2 marks]

Examiners report

A number of candidates were able to answer a) and b) correctly but found part c) more challenging. Correct sketches for the inverse were seen, but with a few missing a horizontal asymptote. The range in part b) was usually seen correctly. In part c), only a small number of very good candidates were able to gain full marks. A large number used the point
$(4, 0)$ to form the equation
$4a + b = 1$ but were unable (or did not recognise the need) to use the asymptote to form a second equation.

12b.

Markscheme

range is
$f^{-1}(x) > 1$ (or $]1, \infty[$)   **A1**

Note: Also accept
$]1, 10]$ or
$]1, 10[$.

Note: Do not allow follow through from incorrect asymptote in (a).

[1 mark]

12c.

Markscheme

$(4, 0) \Rightarrow \ln(4a + b) = 0$ *M1*

$\Rightarrow 4a + b = 1$ *A1*

asymptote at

$x = 1 \Rightarrow a + b = 0$ *M1*

$\Rightarrow a = \frac{1}{3}, \ b = -\frac{1}{3}$ *A1*

[4 marks]

Examiners report

A number of candidates were able to answer a) and b) correctly but found part c) more challenging. Correct sketches for the inverse were seen, but with a few missing a horizontal asymptote. The range in part b) was usually seen correctly. In part c), only a small number of very good candidates were able to gain full marks. A large number used the point $(4, 0)$ to form the equation $4a + b = 1$ but were unable (or did not recognise the need) to use the asymptote to form a second equation.

13.

Markscheme

(a)

$\log_2(x-2) = \log_4(x^2-6x+12)$

EITHER

$\log_2(x-2) = \frac{\log_2(x^2-6x+12)}{\log_2 4}$ *M1*

$2\log_2(x-2) = \log_2(x^2-6x+12)$

OR

$\frac{\log_4(x-2)}{\log_4 2} = \log_4(x^2-6x+12)$ *M1*

$2\log_4(x-2) = \log_4(x^2-6x+12)$

THEN

$(x-2)^2 = x^2 - 6x + 12$ *A1*

$x^2 - 4x + 4 = x^2 - 6x + 12$

$x = 4$ *A1*   *N1*

[3 marks]

(b)

$x^{\ln x} = e^{(\ln x)^3}$

taking ln of both sides or writing

$x = e^{\ln x}$ *M1*

$(\ln x)^2 = (\ln x)^3$ *A1*

$(\ln x)^2(\ln x - 1) = 0$ *(A1)*

$x = 1, \; x = e$ *A1A1*   *N2*

Note: Award second (*A1*) only if factorisation seen or if two correct solutions are seen.

[5 marks]

Total [8 marks]

Examiners report

Part a) was answered well, and a very large proportion of candidates displayed familiarity and confidence with this type of change-of base equation.

In part b), good candidates were able to solve this proficiently. A number obtained only one solution, either through observation or mistakenly cancelling a

$\ln x$ term. An incorrect solution

$x = e^3$ was somewhat prevalent amongst the weaker candidates.

14a.

Markscheme

(i)

$f'(x) = e^{-x} - xe^{-x}$ *M1A1*

(ii)

$f'(x) = 0 \Rightarrow x = 1$

coordinates

$(1, \, e^{-1})$ *A1*

[3 marks]

14b.

Markscheme

$f''(x) = -e^{-x} - e^{-x} + xe^{-x} \ (= -e^{-x}(2 - x))$ *A1*

substituting

$x = 1$ into

$f''(x)$ *M1*

$f''(1) \ (= -e^{-1}) < 0$ hence maximum   *R1AG*

[3 marks]

Examiners report

Full marks for part b) were again likewise seen, though a small number shied away from considering the sign of their second derivative, despite the question asking them to do so.

Part c) again proved to be an easily earned 2 marks.

14c.

Markscheme

$f''(x) = 0 \ (\Rightarrow x = 2)$ *M1*

coordinates

$(2, \ 2e^{-2})$ *A1*

[2 marks]

Examiners report

Full marks for part b) were again likewise seen, though a small number shied away from considering the sign of their second derivative, despite the question asking them to do so.

Part c) again proved to be an easily earned 2 marks.

14d.

Markscheme

(i)

$g(x) = \frac{x}{2}e^{-\frac{x}{2}}$ **A1**

(ii)   coordinates of maximum

$(2, e^{-1})$ **A1**

(iii)   equating

$f(x) = g(x)$ and attempting to solve

$xe^{-x} = \frac{x}{2}e^{-\frac{x}{2}}$

$\Rightarrow x\left(2e^{\frac{x}{2}} - e^{x}\right) = 0$ **(A1)**

$\Rightarrow x = 0$ **A1**

or

$2e^{\frac{x}{2}} = e^{x}$

$\Rightarrow e^{\frac{x}{2}} = 2$

$\Rightarrow x = 2\ln 2$

$(\ln 4)$ **A1**

Note: Award first **(A1)** only if factorisation seen or if two correct
solutions are seen.

Examiners report

Many candidates lost their way in part d). A variety of possibilities for
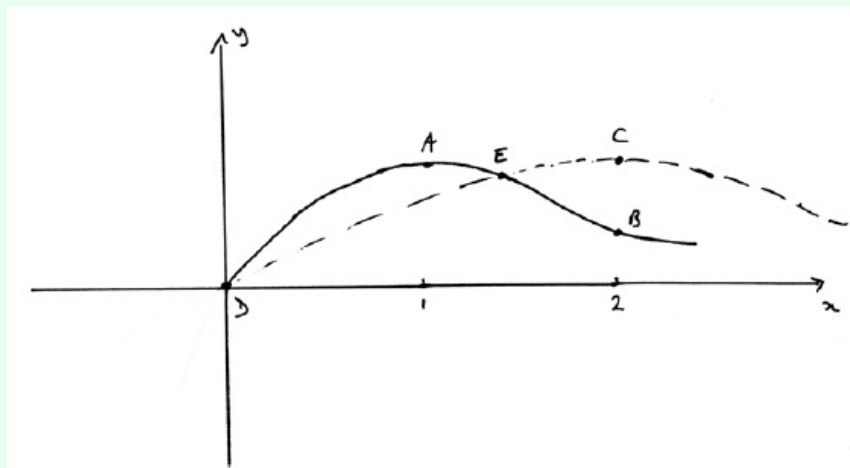$g(x)$ were suggested, commonly
$2xe^{-2x}$,
$\frac{xe^{-1}}{2}$ or similar variations. Despite section ii) being worth only one mark, (and 'state' being present in the question), many laborious
attempts at further differentiation were seen. Part diii was usually answered well by those who gave the correct function for
$g(x)$.

Markscheme

A4

Note: Award *A1* for shape of
f, including domain extending beyond
$x = 2$.
　Ignore any graph shown for
$x < 0$.
　Award *A1* for A and B correctly identified.
　Award *A1* for shape of
g, including domain extending beyond
$x = 2$.
　Ignore any graph shown for
$x < 0$. Allow follow through from
f.
　Award *A1* for C, D and E correctly identified (D and E are interchangeable).

[4 marks]

Examiners report

Part e) was also answered well by those who had earned full marks up to that point.

Markscheme

$A = \int_0^1 \frac{x}{2} e^{-\frac{x}{2}} \, \mathrm{d}x$ *M1*
$= \left[-x e^{-\frac{x}{2}} \right]_0^1 - \int_0^1 -e^{-\frac{x}{2}} \, \mathrm{d}x$ *A1*

Note: Condone absence of limits or incorrect limits.

$= -e^{-\frac{1}{2}} - \left[2e^{-\frac{x}{2}} \right]_0^1$
$= -e^{-\frac{1}{2}} - \left(2e^{-\frac{1}{2}} - 2 \right) = 2 - 3e^{-\frac{1}{2}}$ *A1*
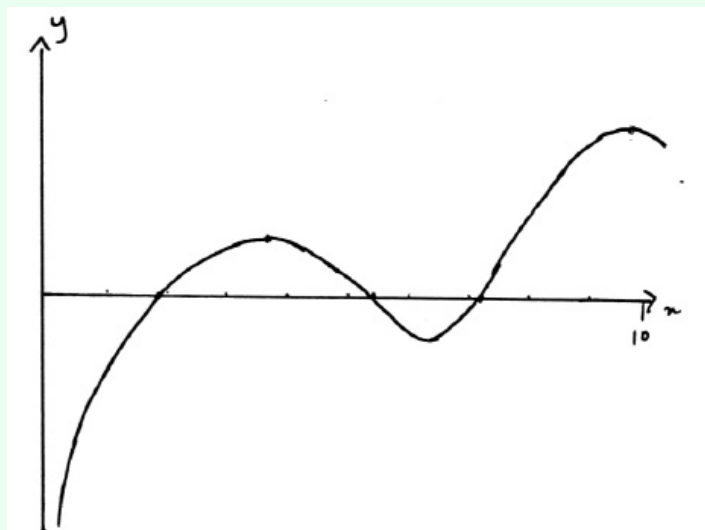
[3 marks]

Examiners report

While the integration by parts technique was clearly understood, it was somewhat surprising how many careless slips were seen in this part of the question. Only a minority gained full marks for part f).

Markscheme

A correct graph shape for

$0 < x \leqslant 10$ *A1*

maxima (3.78, 0.882) and (9.70, 1.89)   *A1*

minimum (6.22, –0.885)   *A1*

x-axis intercepts (1.97, 0), (5.24, 0) and (7.11, 0)   *A2*

Note: Award *A1* if two x-axis intercepts are correct.

[5 marks]

Examiners report

Part (a) was reasonably well done although more care was required when showing correct endpoint behaviour. A number of sketch graphs suggested the existence of either a vertical axis intercept or displayed an open circle on the vertical axis. A large number of candidates did not state the coordinates of the various key features correct to three significant figures. A large number of candidates did not locate the maximum near

$x = 10$. Most candidates were able to locate the x-axis intercepts and the minimum. A few candidates unfortunately sketched a graph from a GDC set in degrees.

Markscheme

$0 < x \leqslant 1.97$ *A1*

$5.24 \leqslant x \leqslant 7.11$ *A1*

[2 marks]

Examiners report

In part (b), a number of candidates identified the correct critical values but used incorrect inequality signs. Some candidates attempted to solve the inequality algebraically.

16a.

Markscheme

attempting to form

$(3\cos\theta + 6)(\cos\theta - 2) + 7(1 + \sin\theta) = 0$ *M1*

$3\cos^2\theta - 12 + 7\sin\theta + 7 = 0$ *A1*

$3(1 - \sin^2\theta) + 7\sin\theta - 5 = 0$ *M1*

$3\sin^2\theta - 7\sin\theta + 2 = 0$ *AG*

[3 marks]

Examiners report

Part (a) was very well done. Most candidates were able to use the scalar product and $\cos^2\theta = 1 - \sin^2\theta$ to show the required result.

16b.

Markscheme

attempting to solve algebraically (including substitution) or graphically for

$\sin\theta$ *(M1)*

$\sin\theta = \frac{1}{3}$ *(A1)*

$\theta = 0.340 \, (= 19.5°)$ *A1*

[3 marks]

Examiners report

Part (b) was reasonably well done. A few candidates confused 'smallest possible positive value' with a minimum function value. Some candidates gave $\theta = 0.34$ as their final answer.

17a.

Markscheme

$A = \frac{1}{2} \times 10^2 \times \theta - \frac{1}{2} \times 10^2 \times \sin\theta$ *M1A1*

Note: Award *M1* for use of area of segment = area of sector – area of triangle.

$= 50\theta - 50\sin\theta$ *AG*

[2 marks]

Examiners report

Part (a) was very well done. Most candidates knew how to calculate the area of a segment. A few candidates used $r = 20$.

Markscheme

METHOD 1

unshaded area

$= \frac{\pi \times 10^2}{2} - 50(\theta - \sin\theta)$

(or equivalent *eg*

$50\pi - 50\theta + 50\sin\theta)$ **(M1)**

$50\theta - 50\sin\theta = \frac{1}{2}(50\pi - 50\theta + 50\sin\theta)$ **(A1)**

$3\theta - 3\sin\theta - \pi = 0$

$\Rightarrow \theta = 1.969 \,(\text{rad})$ **A1**

METHOD 2

$50\theta - 50\sin\theta = \frac{1}{3}\left(\frac{\pi \times 10^2}{2}\right)$ **(M1)(A1)**

$3\theta - 3\sin\theta - \pi = 0$

$\Rightarrow \theta = 1.969 \,(\text{rad})$ **A1**

[3 marks]

Examiners report

Part (b) challenged a large proportion of candidates. A common error was to equate the unshaded area and the shaded area. Some candidates expressed their final answer correct to three significant figures rather than to the four significant figures specified.

Markscheme

$S_n = \frac{n}{2}[2a + (n-1)d]$

$212 = \frac{16}{2}(2a + 15d) \;\; (= 16a + 120d)$ **A1**

n^{th} term is $a + (n-1)d$

$8 = a + 4d$ **A1**

solving simultaneously: **(M1)**

$d = 1.5, \, a = 2$ **A1**

[4 marks]

Examiners report

This proved to be a good start to the paper for most candidates. The vast majority made a meaningful attempt at this question with many gaining the correct answers. Candidates who lost marks usually did so because of mistakes in the working. In part (b) the most efficient way of gaining the answer was to use the calculator once the initial inequality was set up. A small number of candidates spent valuable time unnecessarily manipulating the algebra before moving to the calculator.

Markscheme

$\frac{n}{2}[4 + 1.5(n-1)] > 600$ **(M1)**

$\Rightarrow 3n^2 + 5n - 2400 > 0$ **(A1)**

$\Rightarrow n > 27.4..., \, (n < -29.1...)$

Note: Do not penalize improper use of inequalities.

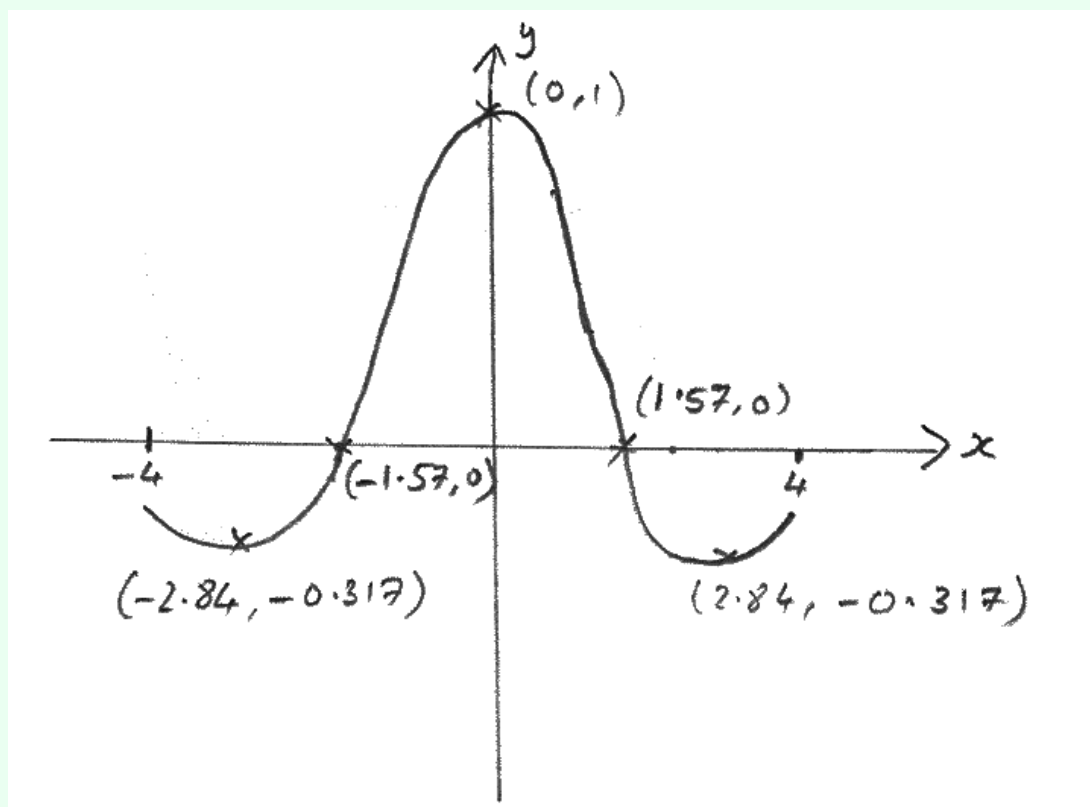$\Rightarrow n = 28$ **A1**

[3 marks]

Examiners report

This proved to be a good start to the paper for most candidates. The vast majority made a meaningful attempt at this question with many gaining the correct answers. Candidates who lost marks usually did so because of mistakes in the working. In part (b) the most efficient way of gaining the answer was to use the calculator once the initial inequality was set up. A small number of candidates spent valuable time unnecessarily manipulating the algebra before moving to the calculator.

Markscheme

A1A1A1A1

Note: Award *A1* for correct shape. Do not penalise if too large a domain is used,
A1 for correct *x*-intercepts,
A1 for correct coordinates of two minimum points,
A1 for correct coordinates of maximum point.
Accept answers which correctly indicate the position of the intercepts, maximum point and minimum points.

[4 marks]

Examiners report

Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

Markscheme

gradient at $x = 1$ is –0.786   *A1*
[1 mark]

Examiners report

Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

Markscheme

gradient of normal is

$\frac{-1}{-0.786}(= 1.272...)$ *(A1)*

when $x = 1$, $y = 0.3820...$ *(A1)*

Equation of normal is $y - 0.382 = 1.27(x - 1)$ *A1*

$(\Rightarrow y = 1.27x - 0.890)$

[3 marks]

Examiners report

Most candidates were able to make a meaningful start to this question, but many made errors along the way and hence only a relatively small number of candidates gained full marks for the question. Common errors included trying to use degrees, rather than radians, trying to use algebraic methods to find the gradient in part (b) and trying to find the equation of the tangent rather than the equation of the normal in part (c).

Markscheme

$\frac{dv}{dt} = -v^2 - 1$

attempt to separate the variables   *M1*

$\int \frac{1}{1+v^2} dv = \int -1 dt$ *A1*

$\arctan v = -t + k$ *A1A1*

Note: Do not penalize the lack of constant at this stage.

when $t = 0$, $v = 1$ *M1*

$\Rightarrow k = \arctan 1 = \left(\frac{\pi}{4}\right) = (45°)$ *A1*

$\Rightarrow v = \tan\left(\frac{\pi}{4} - t\right)$ *A1*
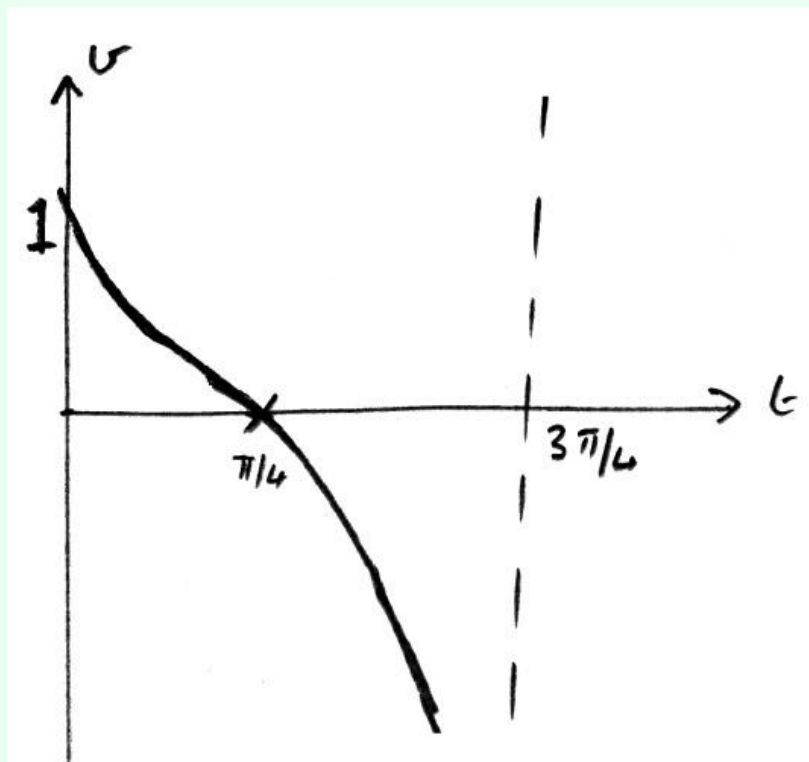
[7 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

20b.

Markscheme

A1A1A1

Note: Award *A1* for general shape,
      A1 for asymptote,
      A1 for correct t and v intercept.

Note: Do not penalise if a larger domain is used.

[3 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

20c.

Markscheme

(i)
$T = \frac{\pi}{4}$ *A1*
(ii)   area under curve
$= \int_0^{\frac{\pi}{4}} \tan\left(\frac{\pi}{4} - t\right) \mathrm{d}t$ *(M1)*
$= 0.347 \left(= \frac{1}{2}\ln 2\right)$ *A1*
[3 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

Markscheme

$v = \tan\left(\frac{\pi}{4} - t\right)$

$s = \int \tan\left(\frac{\pi}{4} - t\right) \mathrm{d}t$ ***M1***

$\int \dfrac{\sin\left(\frac{\pi}{4} - t\right)}{\cos\left(\frac{\pi}{4} - t\right)} \, \mathrm{d}t$ ***(M1)***

$= \ln\cos\left(\frac{\pi}{4} - t\right) + k$ ***A1***

when

$t = 0, \; s = 0$

$k = -\ln\cos\frac{\pi}{4}$ ***A1***

$s = \ln\cos\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4}\left(= \ln\left[\sqrt{2}\cos\left(\frac{\pi}{4} - t\right)\right]\right)$ ***A1***

[5 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.
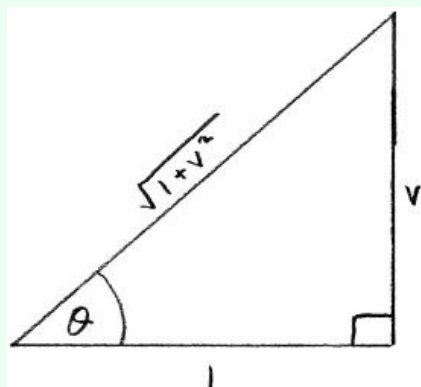
Markscheme

METHOD 1

$\frac{\pi}{4} - t = \arctan v$ ***M1***

$t = \frac{\pi}{4} - \arctan v$

$s = \ln\left[\sqrt{2}\cos\left(\frac{\pi}{4} - \frac{\pi}{4} + \arctan v\right)\right]$

$s = \ln\left[\sqrt{2}\cos(\arctan v)\right]$ ***M1A1***

$s = \ln\left[\sqrt{2}\cos\left(\arccos\frac{1}{\sqrt{1+v^2}}\right)\right]$ ***A1***

$= \ln\frac{\sqrt{2}}{\sqrt{1+v^2}}$

$= \frac{1}{2}\ln\frac{2}{1+v^2}$ ***AG***

METHOD 2

$s = \ln\cos\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4}$

$= -\ln\sec\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4}$ ***M1***

$= -\ln\sqrt{1 + \tan^2\left(\frac{\pi}{4} - t\right)} - \ln\cos\frac{\pi}{4}$ ***M1***

$= -\ln\sqrt{1 + v^2} - \ln\cos\frac{\pi}{4}$ ***A1***

$= \ln\frac{1}{\sqrt{1+v^2}} + \ln\sqrt{2}$ ***A1***

$= \frac{1}{2}\ln\frac{2}{1+v^2}$ ***AG***

METHOD 3

$v\frac{dv}{ds} = -v^2 - 1$ ***M1***

$\int\frac{v}{v^2+1}dv = -\int 1 ds$ ***M1***

$\frac{1}{2}\ln(v^2 + 1) = -s + k$ ***A1***

when

$s = 0$, $t = 0 \Rightarrow v = 1$

$\Rightarrow k = \frac{1}{2}\ln 2$ ***A1***

$\Rightarrow s = \frac{1}{2}\ln\frac{2}{1+v^2}$ ***AG***

[4 marks]

Examiners report

This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

Markscheme

METHOD 1

$f'(x) = q - 2x = 0$ ***M1***

$f'(3) = q - 6 = 0$

$q = 6$ ***A1***

$f(3) = p + 18 - 9 = 5$ ***M1***

$p = -4$ ***A1***

METHOD 2

$f(x) = -(x - 3)^2 + 5$ ***M1A1***

$= -x^2 + 6x - 4$

$q = 6, p = -4$ ***A1A1***

[4 marks]

Examiners report

In general candidates handled this question well although a number equated the derivative to the function value rather than zero. Most recognised the shift in the second part although a number shifted only the squared value and not both x values.

Markscheme

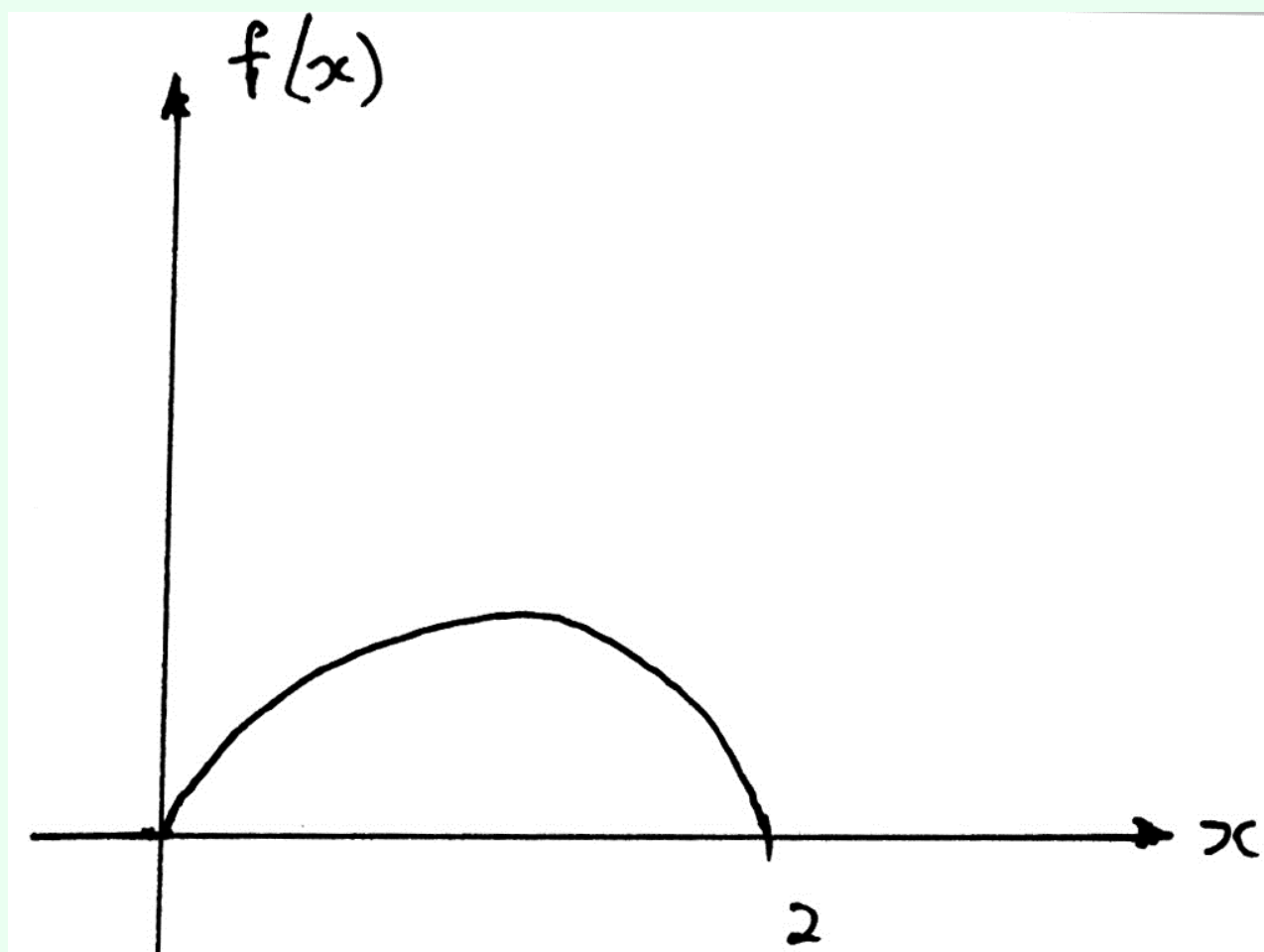$g(x) = -4 + 6(x - 3) - (x - 3)^2 \; (= -31 + 12x - x^2)$ ***M1A1***

Note: Accept any alternative form which is correct.

Award ***M1A0*** for a substitution of $(x + 3)$.

[2 marks]

Examiners report

In general candidates handled this question well although a number equated the derivative to the function value rather than zero. Most recognised the shift in the second part although a number shifted only the squared value and not both x values.

Markscheme

A1

Note: Award *A1* for intercepts of 0 and 2 and a concave down curve in the given domain .

Note: Award *A0* if the cubic graph is extended outside the domain [0, 2] .

[1 mark]

Examiners report

Most candidates completed this question well. A number extended the graph beyond the given domain.

22b.

Markscheme

$\int_0^2 kx(x+1)(2-x)\mathrm{d}x = 1$ **(M1)**

Note: The correct limits and =1 must be seen but may be seen later.

$k\int_0^2 (-x^3 + x^2 + 2x)\mathrm{d}x = 1$ **A1**

$k\left[-\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2\right]_0^2 = 1$ **M1**

$k\left(-4 + \frac{8}{3} + 4\right) = 1$ **(A1)**
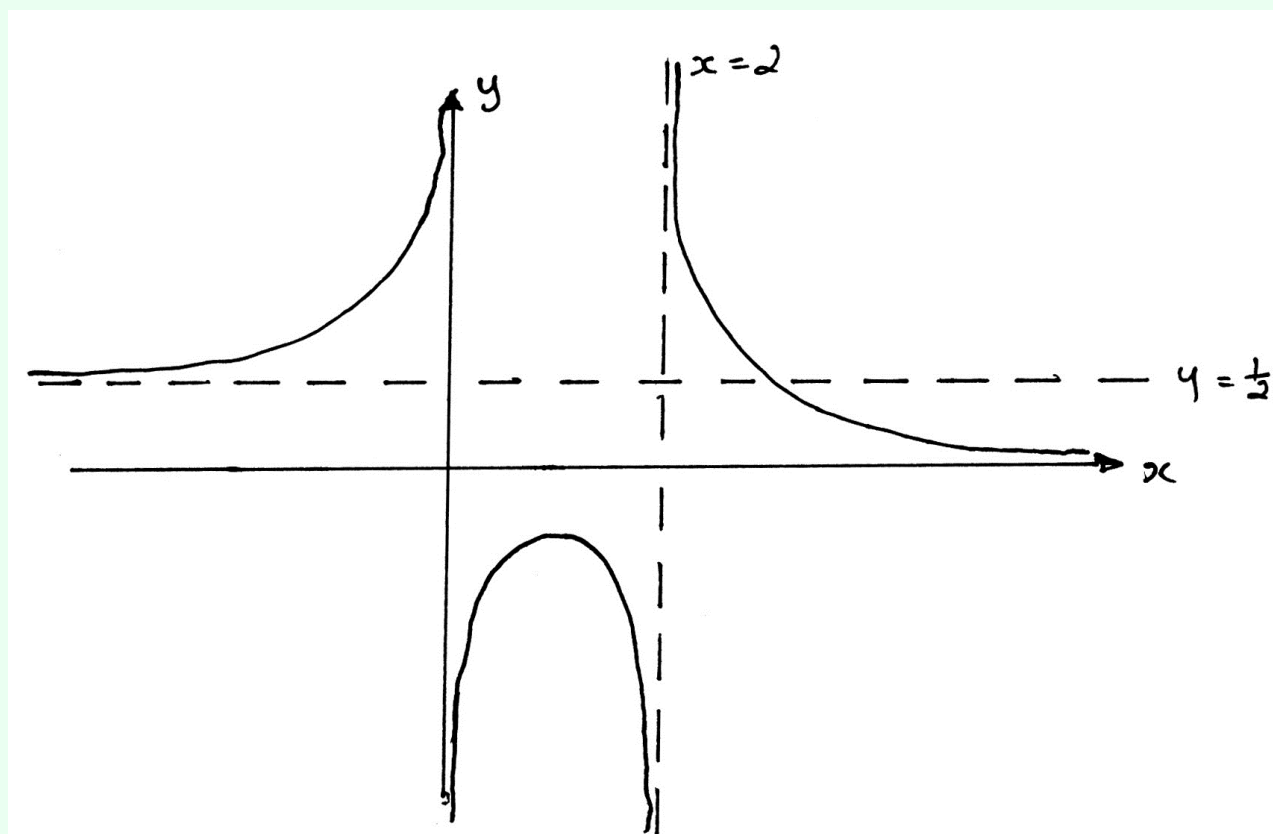
$k = \frac{3}{8}$ **A1**

[5 marks]

Examiners report

Most candidates completed this question well. A number extended the graph beyond the given domain.

23a.

Markscheme

A3

Note: Award *A1* for each correct branch with position of asymptotes clearly indicated.

If $x = 2$ is not indicated, only penalise once.

[3 marks]

Examiners report

Many candidates were able to find the reciprocal but many struggled with the second part. Sketches were quite poor in detail.

23b. [3 marks]

Markscheme

A3

Note: Award **A1** for behaviour at

$x = 0$, **A1** for intercept at

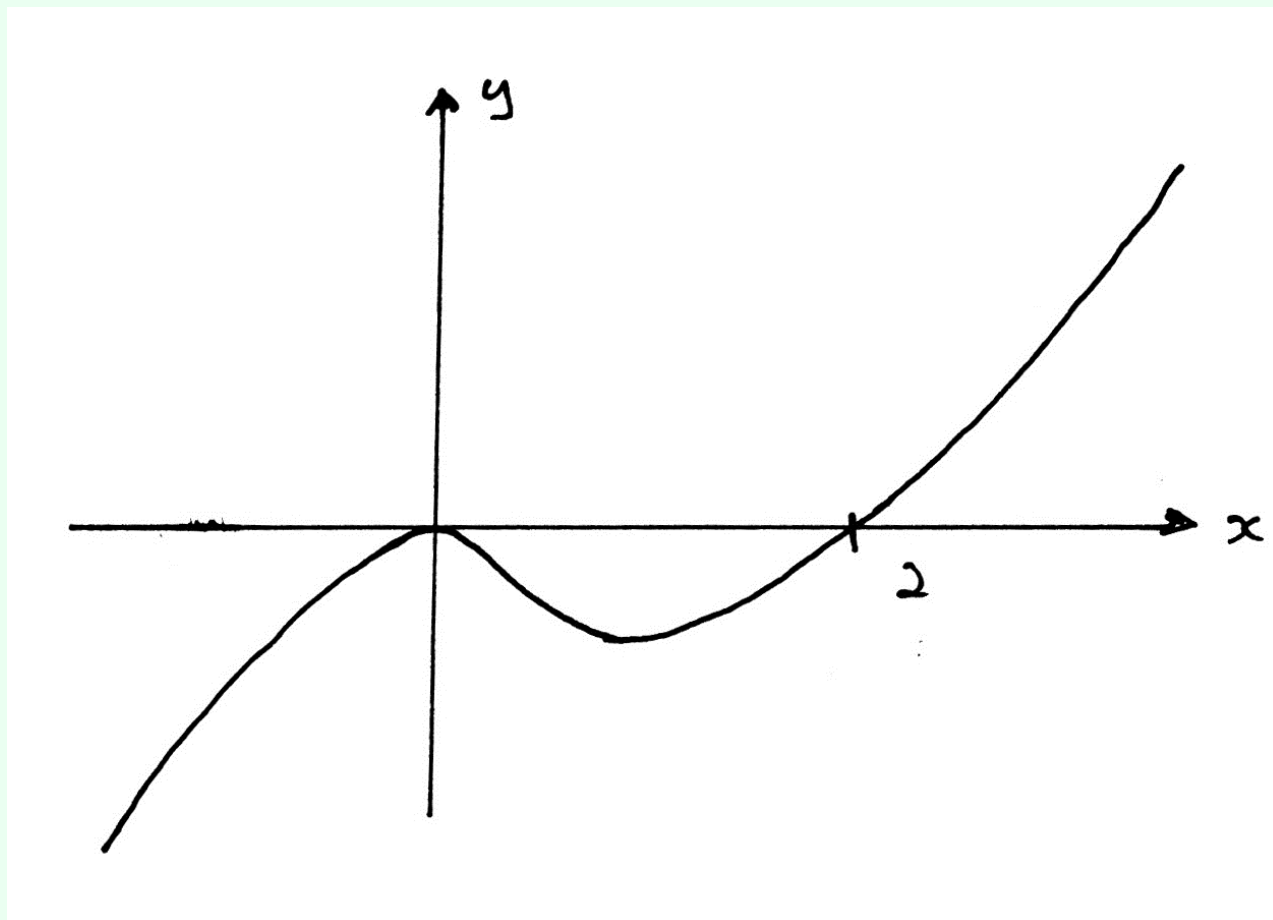$x = 2$, **A1** for behaviour for large

$|x|$.

[3 marks]

Examiners report

Many candidates were able to find the reciprocal but many struggled with the second part. Sketches were quite poor in detail.

24.

Markscheme

$x = 2e^y - \frac{1}{e^y}$ *M1*

Note: The *M1* is for switching the variables and may be awarded at any stage in the process and is awarded independently. Further marks do not rely on this mark being gained.

$xe^y = 2e^{2y} - 1$

$2e^{2y} - xe^y - 1 = 0$ *A1*

$e^y = \frac{x \pm \sqrt{x^2 + 8}}{4}$ *M1A1*

$y = \ln\left(\frac{x \pm \sqrt{x^2 + 8}}{4}\right)$

therefore

$h^{-1}(x) = \ln\left(\frac{x + \sqrt{x^2 + 8}}{4}\right)$ *A1*

since ln is undefined for the second solution *R1*

Note: Accept

$y = \ln\left(\frac{x + \sqrt{x^2 + 8}}{4}\right)$.

Note: The *R1* may be gained by an appropriate comment earlier.

[6 marks]

Examiners report

A significant number of candidates did not recognise the need for the quadratic formula in order to find the inverse. Even when they did most candidates who got this far did not recognise the need to limit the solution to the positive only. This question was done well by a very limited number of candidates.

25a.

Markscheme

using the factor theorem $z + 1$ is a factor *(M1)*

$z^3 + 1 = (z + 1)(z^2 - z + 1)$ *A1*

[2 marks]

Examiners report

In part a) the factorisation was, on the whole, well done.

Markscheme

(i)   **METHOD 1**

$z^3 = -1 \Rightarrow z^3 + 1 = (z+1)(z^2 - z + 1) = 0$ *(M1)*

solving

$z^2 - z + 1 = 0$ *M1*

$z = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$ *A1*

therefore one cube root of -1 is

γ *AG*

METHOD 2

$\gamma^2 = \left(\frac{1+i\sqrt{3}}{2} \right)^2 = \frac{-1+i\sqrt{3}}{2}$ *M1A1*

$\gamma^2 = \frac{-1+i\sqrt{3}}{2} \times \frac{1+i\sqrt{3}}{2} = \frac{-1-3}{4}$ *A1*

$= -1$ AG

METHOD 3

$\gamma = \frac{1+i\sqrt{3}}{2} = e^{i\frac{\pi}{3}}$ *M1A1*

$\gamma^3 = e^{i\pi} = -1$ *A1*

(ii)   **METHOD 1**

as

γ is a root of

$z^2 - z + 1 = 0$ then

$\gamma^2 - \gamma + 1 = 0$ *M1R1*

$\therefore \gamma^2 = \gamma - 1$ *AG*

Note: Award *M1* for the use of

$z^2 - z + 1 = 0$ in any way.

Award *R1* for a correct reasoned approach.

METHOD 2

$\gamma^2 = \frac{-1+i\sqrt{3}}{2}$ *M1*

$\gamma - 1 = \frac{1+i\sqrt{3}}{2} - 1 = \frac{-1+i\sqrt{3}}{2}$ *A1*

(iii)   **METHOD 1**

$(1-\gamma)^6 = (-\gamma^2)^6$ *(M1)*

$= (\gamma)^{12}$ *A1*

$= (\gamma^3)^4$ *(M1)*

$= (-1)^4$

$= 1$ *A1*

METHOD 2

$(1-\gamma)^6$

$= 1 - 6\gamma + 15\gamma^2 - 20\gamma^3 + 15\gamma^4 - 6\gamma^5 + \gamma^6$ *M1A1*

Note: Award *M1* for attempt at binomial expansion.

use of any previous result *e.g.*

$= 1 - 6\gamma + 15\gamma^2 + 20 - 15\gamma + 6\gamma^2 + 1$ *M1*

$= 1$ *A1*

Note: As the question uses the word 'hence', other methods that do not use previous results are awarded no marks.
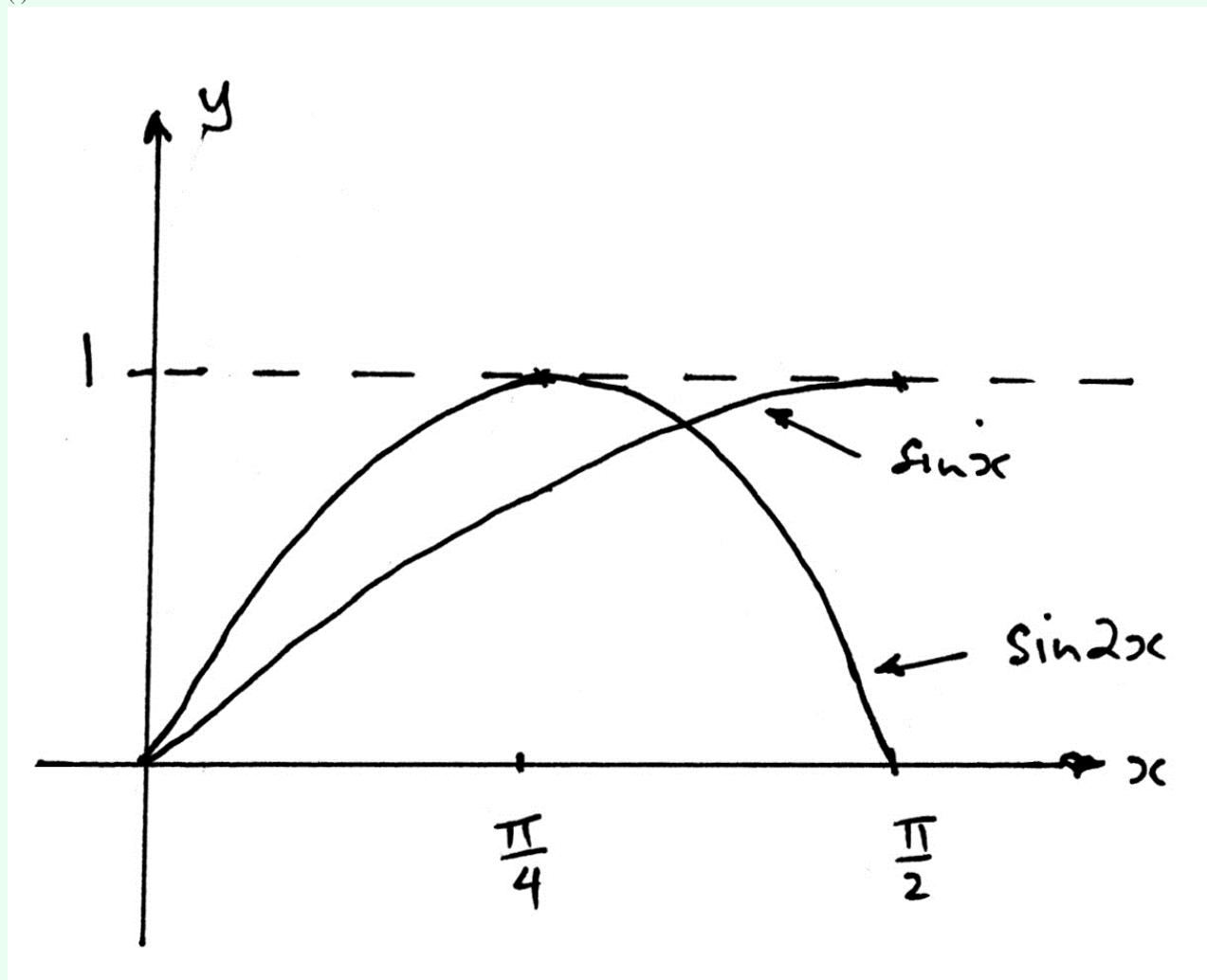
[9 marks]

Examiners report

Part (b) was done well by most although using a substitution method rather than the result above. This used much m retime than was necessary but was successful. A number of candidates did not use the previous results in part (iii) and so seemed to not understand the use of the 'hence'.

Markscheme

(i)

A2

Note: Award *A1* for correct
$\sin x$, *A1* for correct
$\sin 2x$.

Note: Award *A1A0* for two correct shapes with
$\frac{\pi}{2}$ and/or 1 missing.

Note: Condone graph outside the domain.

(ii)
$\sin 2x = \sin x$,
$0 \leqslant x \leqslant \frac{\pi}{2}$
$2\sin x \cos x - \sin x = 0$ *M1*
$\sin x(2\cos x - 1) = 0$
$x = 0, \frac{\pi}{3}$ *A1A1*   *N1N1*

(iii)   area
$= \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)\mathrm{d}x$ *M1*
Note: Award *M1* for an integral that contains limits, not necessarily correct, with
$\sin x$ and
$\sin 2x$ subtracted in either order.

$$= \left[-\tfrac{1}{2}\cos 2x + \cos x\right]_0^{\frac{\pi}{3}} \quad \textbf{\textit{A1}}$$
$$= \left(-\tfrac{1}{2}\cos \tfrac{2\pi}{3} + \cos \tfrac{\pi}{3}\right) - \left(-\tfrac{1}{2}\cos 0 + \cos 0\right) \quad \textbf{\textit{(M1)}}$$
$$= \tfrac{3}{4} - \tfrac{1}{2}$$
$$= \tfrac{1}{4} \quad \textbf{\textit{A1}}$$

[9 marks]

Examiners report

A significant number of candidates did not seem to have the time required to attempt this question satisfactorily.

Part (a) was done quite well by most but a number found sketching the functions difficult, the most common error being poor labelling of the axes.

Part (ii) was done well by most the most common error being to divide the equation by $\sin x$ and so omit the $x = 0$ value. Many recognised the value from the graph and corrected this in their final solution.

The final part was done well by many candidates.

Many candidates found (b) challenging. Few were able to substitute the dx expression correctly and many did not even seem to recognise the need for this term. Those that did tended to be able to find the integral correctly. Most saw the need for the double angle expression although many did not change the limits successfully.

Few candidates attempted part c). Those who did get this far managed the sketch well and were able to explain the relationship required. Among those who gave a response to this many were able to get the result although a number made errors in giving the inverse function. On the whole those who got this far did it well.

26b. *[8 marks]*

Markscheme

$$\int_0^1 \sqrt{\tfrac{x}{4-x}}\,\mathrm{d}x = \int_0^{\frac{\pi}{6}} \sqrt{\tfrac{4\sin^2\theta}{4-4\sin^2\theta}} \times 8\sin\theta\cos\theta\,\mathrm{d}\theta \quad \textbf{\textit{M1A1A1}}$$

Note: Award **M1** for substitution and reasonable attempt at finding expression for dx in terms of dθ, first **A1** for correct limits, second **A1** for correct substitution for dx.

$$\int_0^{\frac{\pi}{6}} 8\sin^2\theta\,\mathrm{d}\theta \quad \textbf{\textit{A1}}$$
$$\int_0^{\frac{\pi}{6}} 4 - 4\cos 2\theta\,\mathrm{d}\theta \quad \textbf{\textit{M1}}$$
$$= \left[4\theta - 2\sin 2\theta\right]_0^{\frac{\pi}{6}} \quad \textbf{\textit{A1}}$$
$$= \left(\tfrac{2\pi}{3} - 2\sin \tfrac{\pi}{3}\right) - 0 \quad \textbf{\textit{(M1)}}$$
$$= \tfrac{2\pi}{3} - \sqrt{3} \quad \textbf{\textit{A1}}$$

[8 marks]

Examiners report

A significant number of candidates did not seem to have the time required to attempt this question satisfactorily.

Part (a) was done quite well by most but a number found sketching the functions difficult, the most common error being poor labelling of the axes.
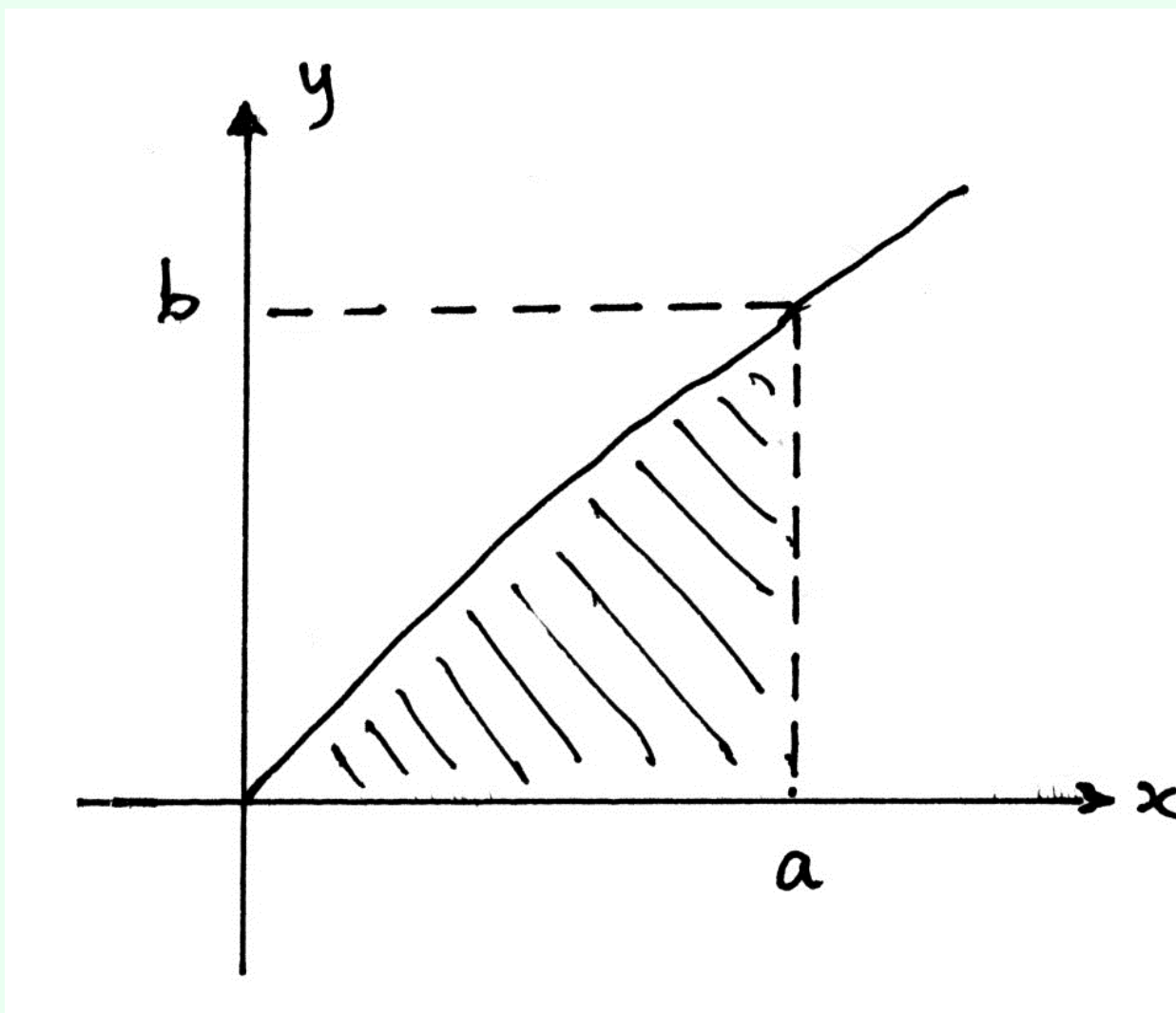
Part (ii) was done well by most the most common error being to divide the equation by $\sin x$ and so omit the $x = 0$ value. Many recognised the value from the graph and corrected this in their final solution.

The final part was done well by many candidates.

Many candidates found (b) challenging. Few were able to substitute the dx expression correctly and many did not even seem to recognise the need for this term. Those that did tended to be able to find the integral correctly. Most saw the need for the double angle expression although many did not change the limits successfully.

Few candidates attempted part c). Those who did get this far managed the sketch well and were able to explain the relationship required. Among those who gave a response to this many were able to get the result although a number made errors in giving the inverse function. On the whole those who got this far did it well.

Markscheme

(i)

M1

from the diagram above

the shaded area

$= \int_0^a f(x)\mathrm{d}x = ab - \int_0^b f^{-1}(y)\mathrm{d}y$ **R1**

$= ab - \int_0^b f^{-1}(x)\mathrm{d}x$ **AG**

(ii)

$f(x) = \arcsin\frac{x}{4} \Rightarrow f^{-1}(x) = 4\sin x$ **A1**

$\int_0^2 \arcsin\left(\frac{x}{4}\right)\mathrm{d}x = \frac{\pi}{3} - \int_0^{\frac{\pi}{6}} 4\sin x\mathrm{d}x$ **M1A1A1**

Note: Award **A1** for the limit

$\frac{\pi}{6}$ seen anywhere, **A1** for all else correct.

$= \frac{\pi}{3} - [-4\cos x]_0^{\frac{\pi}{6}}$ **A1**

$= \frac{\pi}{3} - 4 + 2\sqrt{3}$ **A1**

Note: Award no marks for methods using integration by parts.

[8 marks]

Examiners report

A significant number of candidates did not seem to have the time required to attempt this question satisfactorily.
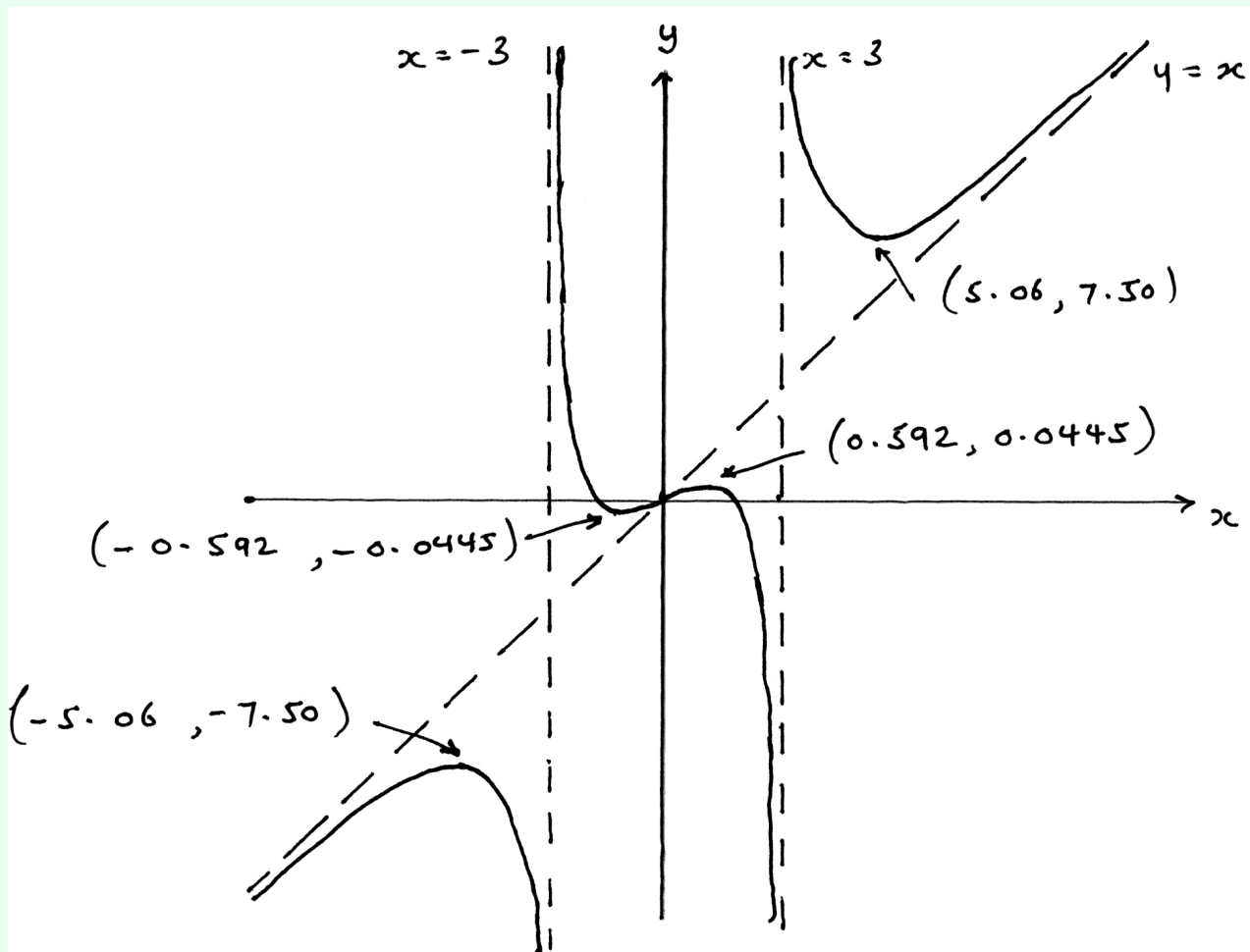
Part (a) was done quite well by most but a number found sketching the functions difficult, the most common error being poor labelling of the axes.

Part (ii) was done well by most the most common error being to divide the equation by

$\sin x$ and so omit the $x = 0$ value. Many recognised the value from the graph and corrected this in their final solution.

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Few candidates attempted part c). Those who did get this far managed the sketch well and were able to explain the relationship required. Among those who gave a response to this many were able to get the result although a number made errors in giving the inverse function. On the whole those who got this far did it well.

27.

Markscheme

M1A1A1A1A1A1A1

Note: Award *A1* for both vertical asymptotes correct,

M1 for recognizing that there are two turning points near the origin,

A1 for both turning points near the origin correct, (only this *A* mark is dependent on the *M* mark)

A1 for the other pair of turning points correct,

A1 for correct positioning of the oblique asymptote,

A1 for correct equation of the oblique asymptote,

A1 for correct asymptotic behaviour in all sections.

[7 marks]

Examiners report

This question was generally well done, except for the behaviour near the origin. The questions alerted candidates to the existence of four turning points and an oblique asymptote, but not all reported back on this information.

Markscheme

$x^3 + 1 = \frac{1}{x^3+1}$

$(-1.26, -1)$ $\left(= \left(-\sqrt[3]{2}, -1\right)\right)$ *A1*

[1 mark]

Examiners report

In part (a) almost all candidates obtained the correct answer, either in numerical form or in exact form. Although many candidates scored one mark in (b), for one gradient, few scored any more. Successful candidates almost always adopted a vector approach to finding the angle between the two tangents, rather than using trigonometry.

Markscheme

$f'(-1.259...) = 4.762...$

$\left(3 \times 2^{\frac{2}{3}}\right)$ *A1*

$g'(-1.259...) = -4.762...$

$\left(-3 \times 2^{\frac{2}{3}}\right)$ *A1*

required angle

$= 2\arctan\left(\frac{1}{4.762...}\right)$ *M1*

$= 0.414$ (accept 23.7)   *A1*

Note: Accept alternative methods including finding the obtuse angle first.

[4 marks]

Examiners report

In part (a) almost all candidates obtained the correct answer, either in numerical form or in exact form. Although many candidates scored one mark in (b), for one gradient, few scored any more. Successful candidates almost always adopted a vector approach to finding the angle between the two tangents, rather than using trigonometry.

29.

Markscheme

EITHER

$|x-1| > |2x-1| \Rightarrow (x-1)^2 > (2x-1)^2$ *M1*

$x^2 - 2x + 1 > 4x^2 - 4x + 1$

$3x^2 - 2x < 0$ *A1*

$0 < x < \frac{2}{3}$ *A1A1*   *N2*

Note: Award *A1A0* for incorrect inequality signs.

OR

$|x-1| > |2x-1|$

$x - 1 = 2x - 1$

$x - 1 = 1 - 2x$ *M1A1*

$-x = 0$

$3x = 2$

$x = 0$

$x = \frac{2}{3}$

Note: Award *M1* for any attempt to find a critical value. If graphical methods are used, award *M1* for correct graphs, *A1* for correct values of *x*.

$0 < x < \frac{2}{3}$ *A1A1*   *N2*

Note: Award *A1A0* for incorrect inequality signs.

[4 marks]

Examiners report

This question turned out to be more difficult than expected. Candidates who squared both sides or drew a graph generally gave better solutions than those who relied on performing algebraic operations on terms involving modulus signs.

30.

Markscheme

(a)

$\frac{\pi}{4} - \arccos x \geqslant 0$

$\arccos x \leqslant \frac{\pi}{4}$ **(M1)**

$x \geqslant \frac{\sqrt{2}}{2}$ $\left(\text{accept } x \geqslant \frac{1}{\sqrt{2}}\right)$ **(A1)**

since

$-1 \leqslant x \leqslant 1$ **(M1)**

$\Rightarrow \frac{\sqrt{2}}{2} \leqslant x \leqslant 1$ $\left(\text{accept } \frac{1}{\sqrt{2}} \leqslant x \leqslant 1\right)$ **A1**

Note: Penalize the use of

$<$ instead of

$\leqslant$ only once.

(b)

$y = \sqrt{\frac{\pi}{4} - \arccos x} \Rightarrow x = \cos\left(\frac{\pi}{4} - y^2\right)$ **M1A1**

$f^{-1} : x \rightarrow \cos\left(\frac{\pi}{4} - x^2\right)$ **A1**

$0 \leqslant x \leqslant \sqrt{\frac{\pi}{4}}$ **A1**

[8 marks]

Examiners report

Very few correct solutions were seen to (a). Many candidates realised that
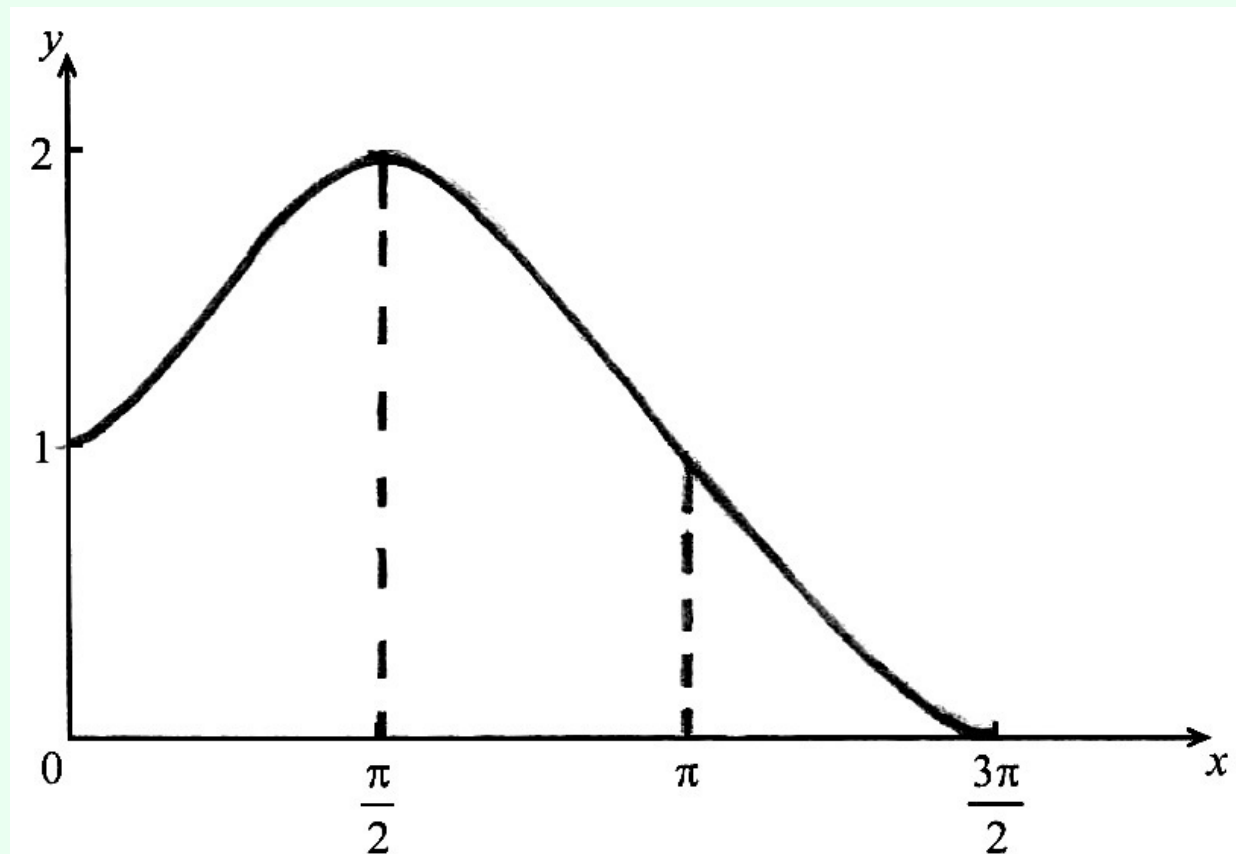
$\arccos x \leqslant \frac{\pi}{4}$ but then concluded incorrectly, not realising that cos is a decreasing function, that

$x \leqslant \cos\left(\frac{\pi}{4}\right)$. In (b) candidates often gave an incorrect domain.

31a.

Markscheme

A1

[1 mark]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of $\cos(2x)$ and the insertion of the limits.

31b.

Markscheme

$(1 + \sin x)^2 = 1 + 2\sin x + \sin^2 x$

$= 1 + 2\sin x + \frac{1}{2}(1 - \cos 2x)$ *A1*

$= \frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x$ *AG*

[1 mark]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of $\cos(2x)$ and the insertion of the limits.

31c. [4 marks]

Markscheme

$V = \pi \int_0^{\frac{3\pi}{2}} (1 + \sin x)^2 \, dx$ *(M1)*

$= \pi \int_0^{\frac{3\pi}{2}} \left(\frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x \right) dx$

$= \pi \left[\frac{3}{2}x - 2\cos x - \frac{\sin 2x}{4} \right]_0^{\frac{3\pi}{2}}$ *A1*

$= \frac{9\pi^2}{4} + 2\pi$ *A1A1*

[4 marks]

Examiners report

Parts (a) and (b) were almost invariably correctly answered by candidates. In (c), most errors involved the integration of $\cos(2x)$ and the insertion of the limits.

32a. [4 marks]

Markscheme

for the equation to have real roots

$(y - 1)^2 - 4y(y - 1) \geqslant 0$ *M1*

$\Rightarrow 3y^2 - 2y - 1 \leqslant 0$

(by sign diagram, or algebraic method)   *M1*

$-\frac{1}{3} \leqslant y \leqslant 1$ *A1A1*

Note: Award first *A1* for

$-\frac{1}{3}$ and 1, and second *A1* for inequalities. These are independent marks.

[4 marks]

Examiners report

(a) The best answered part of the question. The critical points were usually found, but the inequalities were often incorrect. Few candidates were convincing regarding the connection between (a) and (b). This had consequences for (c).

32b. [3 marks]

Markscheme

$f : x \to \frac{x+1}{x^2+x+1} \Rightarrow x + 1 = yx^2 + yx + y$ *(M1)*

$\Rightarrow 0 = yx^2 + (y-1)x + (y-1)$ *A1*

hence, from (a) range is

$-\frac{1}{3} \leqslant y \leqslant 1$ *A1*

[3 marks]

Examiners report

(a) The best answered part of the question. The critical points were usually found, but the inequalities were often incorrect. Few candidates were convincing regarding the connection between (a) and (b). This had consequences for (c).

32c.

Markscheme

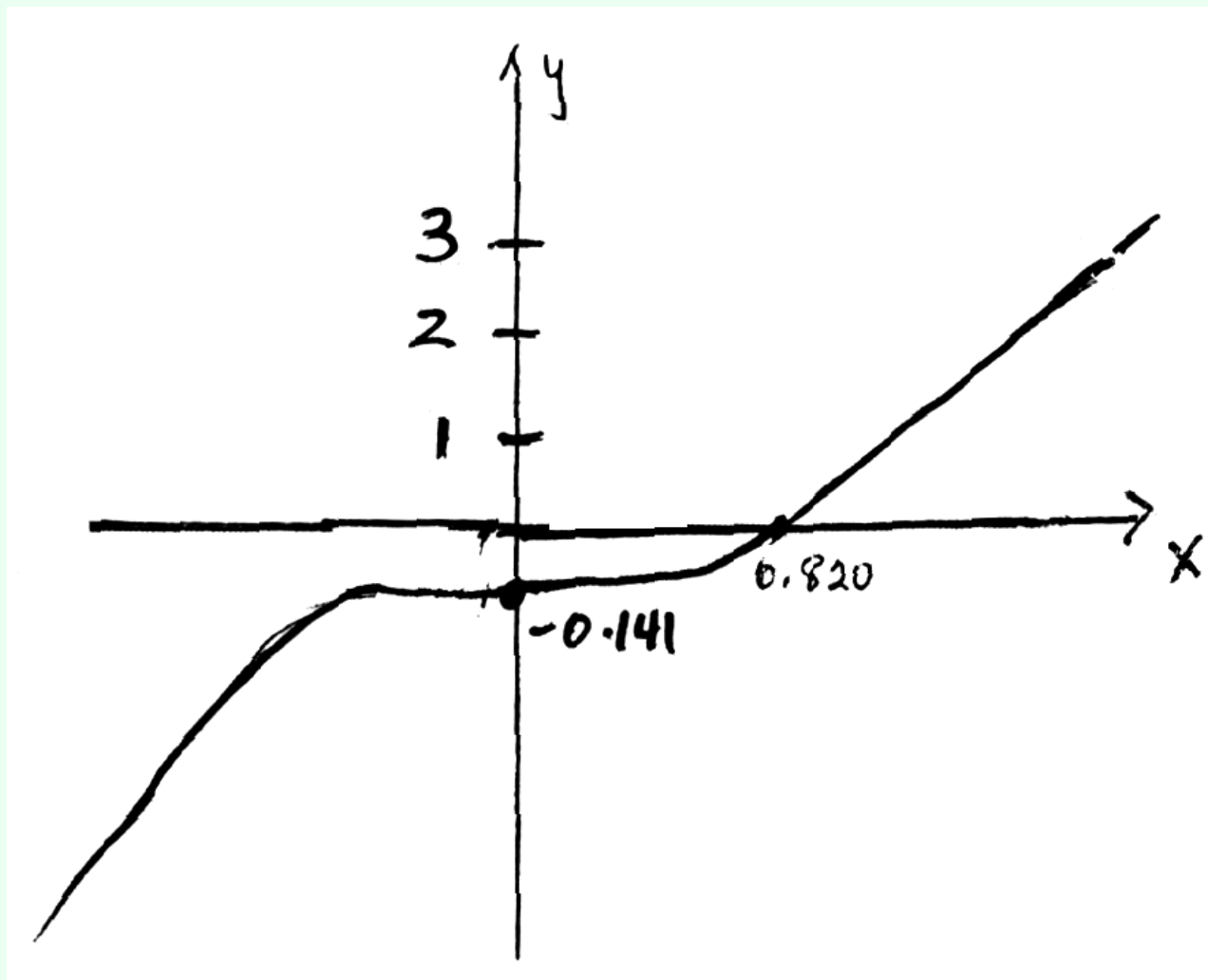a value for y would lead to 2 values for x from (a)   **R1**

Note: Do not award **R1** if (b) has not been tackled.

[1 mark]

Examiners report

(a) The best answered part of the question. The critical points were usually found, but the inequalities were often incorrect. Few candidates were convincing regarding the connection between (a) and (b). This had consequences for (c).

Markscheme

A1A1A1

Note: Award *A1* for shape,

A1 for x-intercept is 0.820, accept

$\sin(-3)$ or $-\sin(3)$

A1 for y-intercept is −0.141.

[3 marks]

Examiners report

Many candidates attempted this question successfully. In (a), however, a large number of candidates did not use the zoom feature of the GDC to draw an accurate sketch of the given function. In (b), some candidates used the domain as the limits of the integral. Other candidates did not take the absolute value of the integral.

33b.

Markscheme

$A = \int_0^{0.8202} |x + \sin(x - 3)| \, dx \approx 0.0816$ sq units   **(M1)A1**

[2 marks]

Examiners report

Many candidates attempted this question successfully. In (a), however, a large number of candidates did not use the zoom feature of the GDC to draw an accurate sketch of the given function. In (b), some candidates used the domain as the limits of the integral. Other candidates did not take the absolute value of the integral.

34a.

Markscheme

$y = \frac{1}{1+e^{-x}}$

$y(1 + e^{-x}) = 1$ **M1**

$1 + e^{-x} = \frac{1}{y} \Rightarrow e^{-x} = \frac{1}{y} - 1$ **A1**

$\Rightarrow x = -\ln\left(\frac{1}{y} - 1\right)$ **A1**

$f^{-1}(x) = -\ln\left(\frac{1}{x} - 1\right) \ \left(= \ln\left(\frac{x}{1-x}\right)\right)$ **A1**

domain: $0 < x < 1$ **A1A1**

Note: Award **A1** for endpoints and **A1** for strict inequalities.

[6 marks]

Examiners report

Finding the inverse function was done successfully by a very large number of candidates. The domain, however, was not always correct. Some candidates failed to use the GDC correctly to find (b), while other candidates had unsuccessful attempts at an analytic solution.

34b.

Markscheme

0.659   **A1**

[1 mark]

Examiners report

Finding the inverse function was done successfully by a very large number of candidates. The domain, however, was not always correct. Some candidates failed to use the GDC correctly to find (b), while other candidates had unsuccessful attempts at an analytic solution.

35a. [2 marks]

Markscheme

(i)

$(g \circ f)(x) = \frac{1}{2x+3}$,

$x \neq -\frac{3}{2}$ (or equivalent)   *A1*

(ii)

$(f \circ g)(x) = \frac{2}{x} + 3$,

$x \neq 0$ (or equivalent)   *A1*

[2 marks]

Examiners report

Part (a) was in general well answered and part (b) well attempted. Some candidates had difficulties with the order of composition and in using correct notation to represent the domains of the functions.

35b. [4 marks]

Markscheme

EITHER

$f(x) = \left(g^{-1} \circ f \circ g\right)(x) \Rightarrow (f \circ g)(x)$ *(M1)*

$\frac{1}{2x+3} = \frac{2}{x} + 3$ *A1*

OR

$\left(g^{-1} \circ f \circ g\right)(x) = \frac{1}{\frac{2}{x}+3}$ *A1*

$2x + 3 = \frac{1}{\frac{2}{x}+3}$ *M1*

THEN

$6x^2 + 12x + 6 = 0$ (or equivalent)   *A1*

$x = -1$,

$y = 1$ (coordinates are $(-1, 1)$)   *A1*

[4 marks]

Examiners report

Part (a) was in general well answered and part (b) well attempted. Some candidates had difficulties with the order of composition and in using correct notation to represent the domains of the functions.

36a. [2 marks]

Markscheme

$f(x - a) \neq b$ *(M1)*

$x \neq 0$ and
$x \neq 2a$ (or equivalent)   *A1*

[2 marks]

Examiners report

A significant number of candidates did not answer this question. Among the candidates who attempted it there were many who had difficulties in connecting vertical asymptotes and the domain of the function and dealing with transformations of graphs. In a few cases candidates managed to answer (a) but provided an answer to (b) which was inconsistent with the domain found.

Markscheme

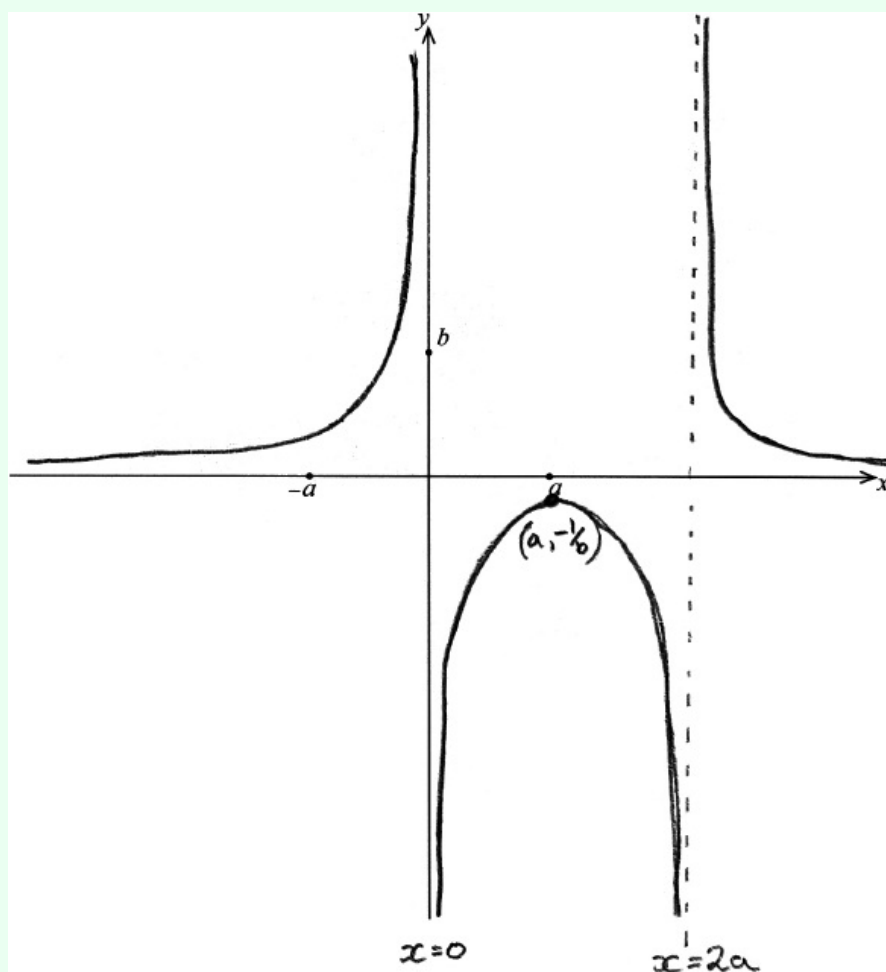vertical asymptotes
$x = 0,$
$x = 2a$ **A1**

horizontal asymptote
$y = 0$ **A1**

Note: Equations must be seen to award these marks.

maximum

$\left(a, -\frac{1}{b}\right)$ **A1A1**

Note: Award **A1** for correct *x*-coordinate and **A1** for correct *y*-coordinate.

one branch correct shape   **A1**

other 2 branches correct shape   **A1**

[6 marks]

Examiners report

A significant number of candidates did not answer this question. Among the candidates who attempted it there were many who had difficulties in connecting vertical asymptotes and the domain of the function and dealing with transformations of graphs. In a few cases candidates managed to answer (a) but provided an answer to (b) which was inconsistent with the domain found.

Markscheme

(i)

$f'(x) = \frac{x\frac{1}{x} - \ln x}{x^2}$ **M1A1**

$= \frac{1 - \ln x}{x^2}$

so

$f'(x) = 0$ when

$\ln x = 1$, *i.e.*

$x = e$ **A1**

(ii)

$f'(x) > 0$ when

$x < e$ and

$f'(x) < 0$ when

$x > e$ **R1**

hence local maximum **AG**

Note: Accept argument using correct second derivative.

(iii)

$y \leqslant \frac{1}{e}$ **A1**

[5 marks]

Examiners report

Most candidates attempted parts (a), (b) and (c) and scored well, although many did not gain the reasoning marks for the justification of the existence of local maximum and inflexion point. The graph sketching was poorly done. A wide selection of range shapes were seen, in some cases showing little understanding of the relation between the derivatives of the function and its graph and difficulties with transformation of graphs. In some cases candidates sketched graphs consistent with their previous calculations but failed to label them properly.

37b.

Markscheme

$f''(x) = \frac{x^2 \frac{-1}{x} - (1 - \ln x)2x}{x^4}$ **M1**

$= \frac{-x - 2x + 2x \ln x}{x^4}$

$= \frac{-3 + 2\ln x}{x^3}$ **A1**

Note: May be seen in part (a).

$f''(x) = 0$ **(M1)**

$-3 + 2\ln x = 0$

$x = e^{\frac{3}{2}}$

since

$f''(x) < 0$ when

$x < e^{\frac{3}{2}}$ and

$f''(x) > 0$ when

$x > e^{\frac{3}{2}}$ **R1**

then point of inflexion

$\left(e^{\frac{3}{2}}, \frac{3}{2e^{\frac{3}{2}}} \right)$ **A1**
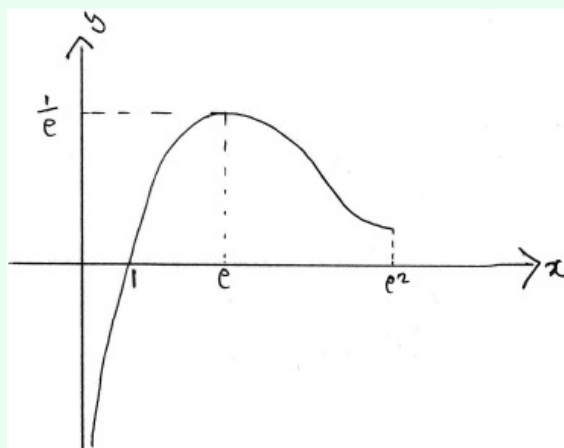
[5 marks]

Examiners report

Most candidates attempted parts (a), (b) and (c) and scored well, although many did not gain the reasoning marks for the justification of the existence of local maximum and inflexion point. The graph sketching was poorly done. A wide selection of range shapes were seen, in some cases showing little understanding of the relation between the derivatives of the function and its graph and difficulties with transformation of graphs. In some cases candidates sketched graphs consistent with their previous calculations but failed to label them properly.

37c.

Markscheme

A1A1A1

Note: Award *A1* for the maximum and intercept, *A1* for a vertical asymptote and *A1* for shape (including turning concave up).
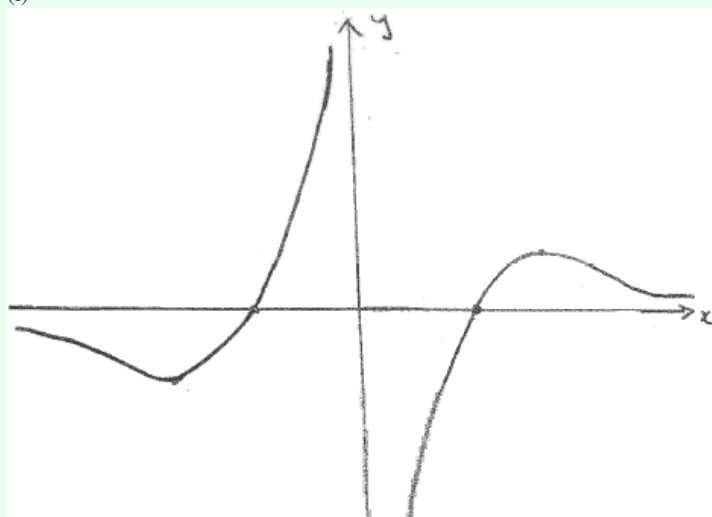
[3 marks]

Examiners report

Most candidates attempted parts (a), (b) and (c) and scored well, although many did not gain the reasoning marks for the justification of the existence of local maximum and inflexion point. The graph sketching was poorly done. A wide selection of range shapes were seen, in some cases showing little understanding of the relation between the derivatives of the function and its graph and difficulties with transformation of graphs. In some cases candidates sketched graphs consistent with their previous calculations but failed to label them properly.

Markscheme

(i)

A1A1

Note: Award *A1* for each correct branch.

(ii) all real values   *A1*

(iii)

(M1)(A1)

Note: Award *(M1)(A1)* for sketching the graph of h, ignoring any graph of g.

$-e^2 < x < -1$ (accept
$x < -1$)   *A1*

[6 marks]

Most candidates attempted parts (a), (b) and (c) and scored well, although many did not gain the reasoning marks for the justification of the existence of local maximum and inflexion point. The graph sketching was poorly done. A wide selection of range shapes were seen, in some cases showing little understanding of the relation between the derivatives of the function and its graph and difficulties with transformation of graphs. In some cases candidates sketched graphs consistent with their previous calculations but failed to label them properly.

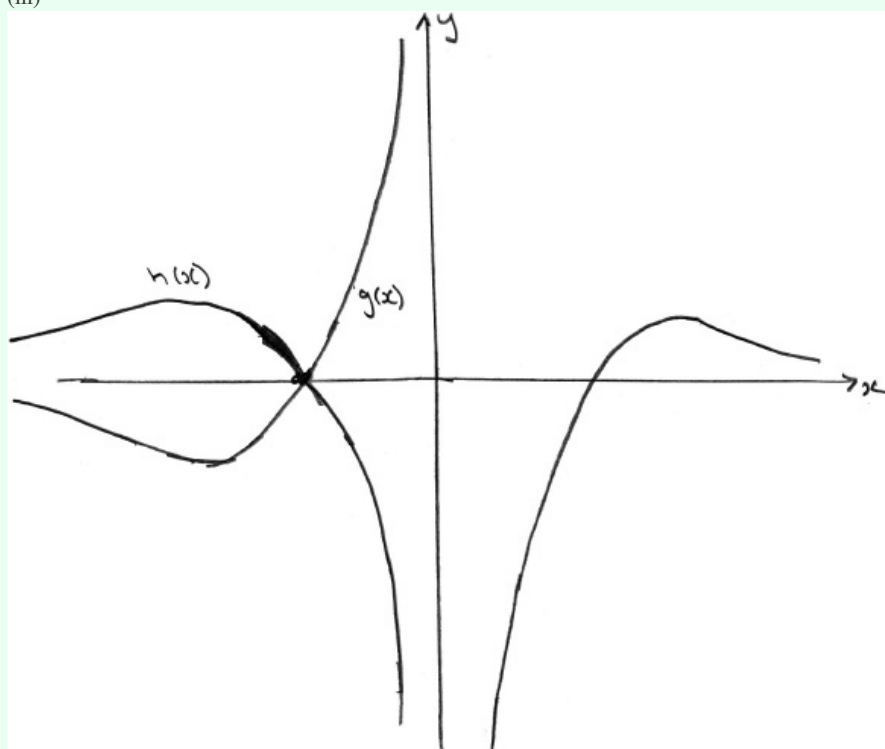**38a.**   *[3 marks]*

Markscheme

$f(a) = 4a^3 + 2a^2 - 7a = -10$ *M1*

$\quad 4a^3 + 2a^2 - 7a + 10 = 0$

$(a+2)\left(4a^2 - 6a + 5\right) = 0$ or sketch or GDC   *(M1)*

$a = -2$ *A1*

[3 marks]

Examiners report

Candidates found this question surprisingly challenging. The most straightforward approach was use of the Remainder Theorem but a significant number of candidates seemed unaware of this technique. This lack of knowledge led many candidates to attempt an algebraically laborious use of long division. In (b) a number of candidates did not seem to appreciate the significance of the word unique and hence found it difficult to provide sufficient detail to make a meaningful argument. However, most candidates did recognize that they needed a technological approach when attempting (b).

38b.   *[2 marks]*

Markscheme

substituting
$a = -2$ into
$f(x)$

$f(x) = 4x^3 - 4x + 14 = 0$ *A1*

EITHER

graph showing unique solution which is indicated (must include max and min)   *R1*

OR

convincing argument that only one of the solutions is real   *R1*

$(-1.74, 0.868 \pm 1.12i)$

[5 marks]

Examiners report

Candidates found this question surprisingly challenging. The most straightforward approach was use of the Remainder Theorem but a significant number of candidates seemed unaware of this technique. This lack of knowledge led many candidates to attempt an algebraically laborious use of long division. In (b) a number of candidates did not seem to appreciate the significance of the word unique and hence found it difficult to provide sufficient detail to make a meaningful argument. However, most candidates did recognize that they needed a technological approach when attempting (b).

Markscheme

$2x^2 + x - 3 = (2x + 3)(x - 1)$ **A1**

Note: Accept
$2\left(x + \frac{3}{2}\right)(x - 1)$.

Note: Either of these may be seen in (b) and if so **A1** should be awarded.

[1 mark]

Examiners report

Many candidates struggled to find an efficient approach to this problem by applying the Binomial Theorem. A disappointing number of candidates attempted the whole expansion which was clearly an unrealistic approach when it is noted that the expansion is to the 8th power. The fact that some candidates wrote down Pascal's Triangle suggested that they had not studied the Binomial Theorem in enough depth or in a sufficient variety of contexts.

Markscheme

EITHER

$\left(2x^2 + x - 3\right)^8 = (2x + 3)^8(x - 1)^8$ **M1**

$= \left(3^8 + 8\left(3^7\right)(2x) + \ldots\right)\left((-1)^8 + 8(-1)^7(x) + \ldots\right)$ **(A1)**

coefficient of
$x = 3^8 \times 8 \times (-1)^7 + 3^7 \times 8 \times 2 \times (-1)^8$ **M1**

$= -17\,496$ **A1**

Note: Under ft, final **A1** can only be achieved for an integer answer.

OR

$\left(2x^2 + x - 3\right)^8 = \left(3 - \left(x - 2x^2\right)\right)^8$ **M1**

$= 3^8 + 8\left(-\left(x - 2x^2\right)\left(3^7\right) + \ldots\right)$ **(A1)**

coefficient of
$x = 8 \times (-1) \times 3^7$ **M1**

$= -17\,496$ **A1**

Note: Under ft, final **A1** can only be achieved for an integer answer.

[4 marks]

Examiners report

Many candidates struggled to find an efficient approach to this problem by applying the Binomial Theorem. A disappointing number of candidates attempted the whole expansion which was clearly an unrealistic approach when it is noted that the expansion is to the 8th power. The fact that some candidates wrote down Pascal's Triangle suggested that they had not studied the Binomial Theorem in enough depth or in a sufficient variety of contexts.