Topic 5 Part 1 *[523 marks]*

The random variable X has the distribution
$\mathrm{B}(30,\ p)$. Given that
$\mathrm{E}(X) = 10$, find

1a. the value of p ; *[1 mark]*

1b. $\mathrm{P}(X = 10)$; *[2 marks]*

1c. $\mathrm{P}(X \geqslant 15)$. *[2 marks]*

The random variable X has the distribution
$\mathrm{Po}(m)$.
Given that
$\mathrm{P}(X = 5) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$, find

2a. the value of m ; *[3 marks]*

2b. $\mathrm{P}\ (X > 2)$. *[2 marks]*

The probability density function of a continuous random variable X is given by
$f(x) = \frac{1}{1+x^4}$,
0
"
x
"
a .

3a. Find the value of a . *[3 marks]*

3b. Find the mean of X . *[2 marks]*

A market stall sells apples, pears and plums.

4a. The weights of the apples are normally distributed with a mean of 200 grams and a standard deviation of 25 grams. *[5 marks]*
(i) Given that there are 450 apples on the stall, what is the expected number of apples with a weight of more than 225 grams?
(ii) Given that 70 % of the apples weigh less than m grams, find the value of m .

4b. The weights of the pears are normally distributed with a mean of ∝ grams and a standard deviation of σ grams. Given that 8 % of these pears have a weight of more than 270 grams and 15 % have a weight less than 250 grams, find ∝ and
σ . *[6 marks]*

4c. The weights of the plums are normally distributed with a mean of 80 grams and a standard deviation of 4 grams. 5 plums are chosen at random. What is the probability that exactly 3 of them weigh more than 82 grams? *[3 marks]*

Each week the management of a football club recorded the number of injuries suffered by their playing staff in that week. The results for a 52-week period were as follows:

Number of injuries per week	0	1	2	3	4	5	6
Number of weeks	6	14	15	9	5	2	1

5a. Calculate the mean and variance of the number of injuries per week. *[2 marks]*

5b. Explain why these values provide supporting evidence for using a Poisson distribution model. *[1 mark]*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 2x, & 0 \leqslant x \leqslant 0.5, \\ \frac{4}{3} - \frac{2}{3}x, & 0.5 \leqslant x \leqslant 2 \\ 0, & \text{otherwise.} \end{cases}$$

6a. Sketch the function f and show that the lower quartile is 0.5. *[3 marks]*

6b. (i) Determine $\mathrm{E}(X)$. *[4 marks]*
(ii) Determine
$\mathrm{E}(X^2)$.

6c. Two independent observations are made from X and the values are added. *[5 marks]*
The resulting random variable is denoted Y.
(i) Determine
$\mathrm{E}(Y - 2X)$.
(ii) Determine
$\mathrm{Var}(Y - 2X)$.

6d. (i) Find the cumulative distribution function for X. *[7 marks]*
(ii) Hence, or otherwise, find the median of the distribution.

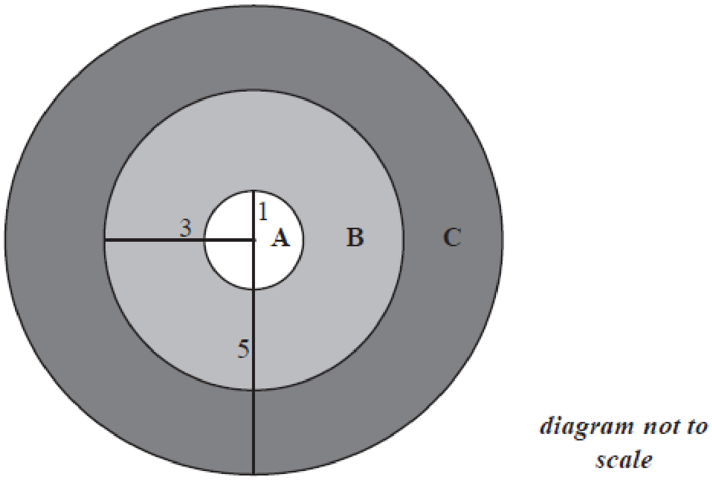
The random variable
$X \sim \mathrm{Po}(m)$. Given that $\mathrm{P}(X = k - 1) = \mathrm{P}(X = k + 1)$, where k is a positive integer,

7a. show that *[2 marks]*
$m^2 = k(k+1)$;

7b. hence show that the mode of X is k. *[6 marks]*

8. In a particular city 20 % of the inhabitants have been immunized against a certain disease. The probability of infection from the disease among those immunized is *[6 marks]*
$\frac{1}{10}$, and among those not immunized the probability is
$\frac{3}{4}$. If a person is chosen at random and found to be infected, find the probability that this person has been immunized.

9. A target consists of three concentric circles of radii 1 m, 3 m and 5 m respectively, as shown in the diagram. *[6 marks]*



Nina shoots an arrow at the target. She has a probability of $\frac{1}{2}$ of hitting the target. If the arrow hits the target it does so at a random point on the target. Ten points are scored for hitting region A, six points for hitting region B, and three points for hitting region C. Find the expected number of points Nina scores each time she shoots an arrow at the target.

A continuous random variable X has the probability density function

$$f(x) = \begin{cases} k \sin x, & 0 \leqslant x \leqslant \frac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases}$$

10a. Find the value of k. *[2 marks]*

10b. Find *[5 marks]*

$\mathrm{E}(X)$.

10c. Find the median of X. *[3 marks]*

The number of vehicles passing a particular junction can be modelled using the Poisson distribution. Vehicles pass the junction at an average rate of 300 per hour.

11a. Find the probability that no vehicles pass in a given minute. *[2 marks]*

11b. Find the expected number of vehicles which pass in a given two minute period. *[1 mark]*

11c. Find the probability that more than this expected number actually pass in a given two minute period. *[2 marks]*

The probability that the 08:00 train will be delayed on a work day (Monday to Friday) is

$\frac{1}{10}$. Assuming that delays occur independently,

12a. find the probability that the 08:00 train is delayed exactly twice during any period of five work days; *[2 marks]*

12b. find the minimum number of work days for which the probability of the 08:00 train being delayed at least once exceeds 90 %. *[3 marks]*

Jan and Sia have been selected to represent their country at an international discus throwing competition. Assume that the distance thrown by each athlete is normally distributed. The mean distance thrown by Jan in the past year was 60.33 metres with a standard deviation of 1.95 metres.

13a. In the past year, 80 % of Jan's throws have been longer than x metres. Find x correct to two decimal places. *[2 marks]*

13b. In the past year, 80 % of Sia's throws have been longer than 56.52 metres. If the mean distance of her throws was 59.39 metres, find the standard deviation of her throws. *[3 marks]*

13c. This year, Sia's throws have a mean of 59.50 metres and a standard deviation of 3.00 metres. The mean and standard deviation of Jan's throws have remained the same. In the competition, an athlete must have at least one throw of 65 metres or more in the first round to qualify for the final round. Each athlete is allowed three throws in the first round. *[10 marks]*

(i) Determine whether Jan or Sia is more likely to qualify for the final on their first throw.

(ii) Find the probability that both athletes qualify for the final.

The weight of tea in *Supermug* tea bags has a normal distribution with mean 4.2 g and standard deviation 0.15 g. The weight of tea in *Megamug* tea bags has a normal distribution with mean 5.6 g and standard deviation 0.17 g.

14a. Find the probability that a randomly chosen *Supermug* tea bag contains more than 3.9 g of tea. *[2 marks]*

14b. Find the probability that, of two randomly chosen *Megamug* tea bags, one contains more than 5.4 g of tea and one contains less than 5.4 g of tea. *[4 marks]*

14c. Find the probability that five randomly chosen *Supermug* tea bags contain a total of less than 20.5 g of tea. *[4 marks]*

14d. Find the probability that the total weight of tea in seven randomly chosen *Supermug* tea bags is more than the total weight in five randomly chosen *Megamug* tea bags. *[5 marks]*

Events
A and
B are such that
$\mathrm{P}(A) = 0.3$ and
$\mathrm{P}(B) = 0.4$.

15a. Find the value of $\mathrm{P}(A \cup B)$ when *[4 marks]*

(i)
A and
B are mutually exclusive;
(ii)
A and
B are independent.

15b. Given that $\mathrm{P}(A \cup B) = 0.6$, find *[3 marks]*
$\mathrm{P}(A|B)$.

The random variable *X* has probability density function *f* where

$$f(x) = \begin{cases} kx(x+1)(2-x), & 0 \leqslant x \leqslant 2 \\ 0, & \text{otherwise} . \end{cases}$$

16a. Sketch the graph of the function. You are not required to find the coordinates of the maximum. *[1 mark]*

16b. Find the value of *k* . *[5 marks]*

A batch of 15 DVD players contains 4 that are defective. The DVD players are selected at random, one by one, and examined. The ones that are checked are not replaced.

17a. What is the probability that there are exactly 3 defective DVD players in the first 8 DVD players examined? *[4 marks]*

17b. What is the probability that the *[3 marks]*
9^{th} DVD player examined is the
4^{th} defective one found?

The fish in a lake have weights that are normally distributed with a mean of 1.3 kg and a standard deviation of 0.2 kg.

18a. Determine the probability that a fish which is caught weighs less than 1.4 kg. *[1 mark]*

18b. John catches 6 fish. Calculate the probability that at least 4 of the fish weigh more than 1.4 kg. *[3 marks]*

18c. Determine the probability that a fish which is caught weighs less than 1 kg, given that it weighs less than 1.4 kg. *[2 marks]*

The number of accidents that occur at a large factory can be modelled by a Poisson distribution with a mean of 0.5 accidents per month.

19a. Find the probability that no accidents occur in a given month. *[1 mark]*

19b. Find the probability that no accidents occur in a given 6 month period. *[2 marks]*

19c. Find the length of time, in complete months, for which the probability that at least 1 accident occurs is greater than 0.99. *[6 marks]*

19d. To encourage safety the factory pays a bonus of $1000 into a fund for workers if no accidents occur in any given month, a bonus of $500 if 1 or 2 accidents occur and no bonus if more than 2 accidents occur in the month. *[9 marks]*

(i) Calculate the expected amount that the company will pay in bonuses each month.

(ii) Find the probability that in a given 3 month period the company pays a total of exactly $2000 in bonuses.

The weights of the oranges produced by a farm may be assumed to be normally distributed with mean 205 grams and standard deviation 10 grams.

20a. Find the probability that a randomly chosen orange weighs more than 200 grams. *[2 marks]*

20b. Five of these oranges are selected at random to be put into a bag. Find the probability that the combined weight of the five oranges is less than 1 kilogram. *[4 marks]*

20c. The farm also produces lemons whose weights may be assumed to be normally distributed with mean 75 grams and standard deviation 3 grams. Find the probability that the weight of a randomly chosen orange is more than three times the weight of a randomly chosen lemon. *[5 marks]*

21. Two players, A and B, alternately throw a fair six-sided dice, with A starting, until one of them obtains a six. Find the probability that A obtains the first six. *[7 marks]*

22. The ten numbers *[6 marks]*

$x_1,\ x_2,\ \ldots,\ x_{10}$ have a mean of 10 and a standard deviation of 3.

Find the value of

$\sum\limits_{i=1}^{10}(x_i-12)^2$.

23. A continuous random variable X has probability density function [20 marks]

$$f(x) = \begin{cases} 0, & x < 0 \\ a\mathrm{e}^{-ax}, & x \geqslant 0. \end{cases}$$

It is known that

$\mathrm{P}(X < 1) = 1 - \frac{1}{\sqrt{2}}$.

(a) Show that

$a = \frac{1}{2}\ln 2$.

(b) Find the median of X.

(c) Calculate the probability that $X < 3$ given that $X > 1$.

24. A continuous random variable X has the probability density function f given by [5 marks]

$$f(x) = \begin{cases} c(x - x^2), & 0 \leqslant x \leqslant 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine c.

(b) Find

$\mathrm{E}(X)$.

25. A biased coin is weighted such that the probability of obtaining a head is [4 marks]

$\frac{4}{7}$. The coin is tossed 6 times and X denotes the number of heads observed. Find the value of the ratio

$\frac{\mathrm{P}(X=3)}{\mathrm{P}(X=2)}$.

26. Jenny goes to school by bus every day. When it is not raining, the probability that the bus is late is [5 marks]

$\frac{3}{20}$. When it is raining, the probability that the bus is late is

$\frac{7}{20}$. The probability that it rains on a particular day is

$\frac{9}{20}$. On one particular day the bus is late. Find the probability that it is not raining on that day.

27. The weight loss, in kilograms, of people using the slimming regime *SLIM3M* for a period of three months is modelled by a random variable X. Experimental data showed that 67 % of the individuals using *SLIM3M* lost up to five kilograms and 12.4 % lost at least seven kilograms. Assuming that X follows a normal distribution, find the expected weight loss of a person who follows the *SLIM3M* regime for three months. [5 marks]

28. The random variable X follows a Poisson distribution with mean m and satisfies [6 marks]

$$\mathrm{P}(X = 1) + \mathrm{P}(X = 3) = \mathrm{P}(X = 0) + \mathrm{P}(X = 2).$$

(a) Find the value of m correct to four decimal places.

(b) For this value of m, calculate

$\mathrm{P}(1 \leqslant X \leqslant 2)$.

29. Tim throws two identical fair dice simultaneously. Each die has six faces: two faces numbered 1, two faces numbered 2 and two faces numbered 3. His score is the sum of the two numbers shown on the dice. *[21 marks]*

(a) (i) Calculate the probability that Tim obtains a score of 6.

(ii) Calculate the probability that Tim obtains a score of at least 3.

Tim plays a game with his friend Bill, who also has two dice numbered in the same way. Bill's score is the sum of the two numbers shown on his dice.

(b) (i) Calculate the probability that Tim and Bill **both** obtain a score of 6.

(ii) Calculate the probability that Tim and Bill obtain the same score.

(c) Let X denote the largest number shown on the four dice.

(i) Show that

$\mathrm{P}(X \leqslant 2) = \frac{16}{81}$.

(ii) Copy and complete the following probability distribution table.

x	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{81}$		

(iii) Calculate

$\mathrm{E}(X)$ and

$\mathrm{E}(X^2)$ and hence find

$\mathrm{Var}(X)$.

(d) Given that $X = 3$, find the probability that the sum of the numbers shown on the four dice is 8.

In a population of rabbits,
1% are known to have a particular disease. A test is developed for the disease that gives a positive result for a rabbit that **does** have the disease in
99% of cases. It is also known that the test gives a positive result for a rabbit that **does not** have the disease in
0.1% of cases. A rabbit is chosen at random from the population.

30a. Find the probability that the rabbit tests positive for the disease. *[2 marks]*

30b. Given that the rabbit tests positive for the disease, show that the probability that the rabbit does not have the disease is less than 10 %. *[3 marks]*

A continuous random variable
X has a probability density function given by the function
$f(x)$, where

$$f(x) = \begin{cases} k(x+2)^2, & -2 \leqslant x < 0 \\ k, & 0 \leqslant x \leqslant \frac{4}{3} \\ 0, & \text{otherwise.} \end{cases}$$

31a. Find the value of
k. *[2 marks]*

31b. Hence find *[5 marks]*

(i) the mean of X;

(ii) the median of X.

A student arrives at a school X minutes after 08:00, where X may be assumed to be normally distributed. On a particular day it is observed that 40% of the students arrive before 08:30 and 90% arrive before 08:55.

32a. Find the mean and standard deviation of X. *[5 marks]*

32b. The school has 1200 students and classes start at 09:00. Estimate the number of students who will be late on that day. *[3 marks]*

32c. Maelis had not arrived by 08:30. Find the probability that she arrived late. *[2 marks]*

Consider the function $f(x) = \frac{\ln x}{x}$, $0 < x < \mathrm{e}^2$.

32d. At 15:00 it is the end of the school day and it is assumed that the departure of the students from school can be modelled by a Poisson distribution. On average 24 students leave the school every minute. *[3 marks]*

Find the probability that at least 700 students leave school before 15:30.

32e. At 15:00 it is the end of the school day and it is assumed that the departure of the students from school can be modelled by a Poisson distribution. On average 24 students leave the school every minute. *[4 marks]*

There are 200 days in a school year. Given that Y denotes the number of days in the year that at least 700 students leave before 15:30, find

(i) $\mathrm{E}(Y)$;

(ii) $P(Y > 150)$.

33. The random variable T has the probability density function *[7 marks]*

$$f(t) = \frac{\pi}{4}\cos\left(\frac{\pi t}{2}\right), \quad -1 \leqslant t \leqslant 1.$$

Find

(a) $\mathrm{P}(T = 0)$;

(b) the interquartile range.

34. A company produces computer microchips, which have a life expectancy that follows a normal distribution with a mean of 90 months and a standard deviation of 3.7 months. *[6 marks]*

(a) If a microchip is guaranteed for 84 months find the probability that it will fail before the guarantee ends.

(b) The probability that a microchip does not fail before the end of the guarantee is required to be 99 %. For how many months should it be guaranteed?

(c) A rival company produces microchips where the probability that they will fail after 84 months is 0.88. Given that the life expectancy also follows a normal distribution with standard deviation 3.7 months, find the mean.

35. Only two international airlines fly daily into an airport. UN Air has 70 flights a day and IS Air has 65 flights a day. Passengers flying with UN Air have an 18 % probability of losing their luggage and passengers flying with IS Air have a 23 % probability of losing their luggage. You overhear someone in the airport complain about her luggage being lost. *[6 marks]*

Find the probability that she travelled with IS Air.

36. The lifts in the office buildings of a small city have occasional breakdowns. The breakdowns at any given time are independent of one another and can be modelled using a Poisson Distribution with mean 0.2 per day. *[13 marks]*

(a) Determine the probability that there will be exactly four breakdowns during the month of June (June has 30 days).

(b) Determine the probability that there are more than 3 breakdowns during the month of June.

(c) Determine the probability that there are no breakdowns during the first five days of June.

(d) Find the probability that the first breakdown in June occurs on June
3rd.

(e) It costs 1850 Euros to service the lifts when they have breakdowns. Find the expected cost of servicing lifts for the month of June.

(f) Determine the probability that there will be no breakdowns in exactly 4 out of the first 5 days in June.

37. The probability distribution of a discrete random variable *X* is defined by *[6 marks]*

$\mathrm{P}(X=x)=cx(5-x),\ x=1,2,3,4$.

(a) Find the value of *c*.

(b) Find $E(X)$.

38. Let *A* and *B* be events such that *[6 marks]*

$\mathrm{P}(A)=0.6,\ \mathrm{P}(A\cup B)=0.8$ and $\mathrm{P}(A|B)=0.6$.

Find $\mathrm{P}(B)$.

39. Consider the data set *[5 marks]*

$\{k-2,\ k,\ k+1,\ k+4\}$, where $k\in\mathbb{R}$.

(a) Find the mean of this data set in terms of *k*.

Each number in the above data set is now **decreased** by 3.

(b) Find the mean of this **new** data set in terms of *k*.

40. A continuous random variable *X* has probability density function *[6 marks]*

$$f(x)=\begin{cases}12x^2(1-x), & \text{for } 0\leqslant x\leqslant 1,\\ 0, & \text{otherwise.}\end{cases}$$

Find the probability that *X* lies between the mean and the mode.

41. John removes the labels from three cans of tomato soup and two cans of chicken soup in order to enter a competition, and puts the cans away. He then discovers that the cans are identical, so that he cannot distinguish between cans of tomato soup and chicken soup. Some weeks later he decides to have a can of chicken soup for lunch. He opens the cans at random until he opens a can of chicken soup. Let Y denote the number of cans he opens. *[7 marks]*

Find

(a) the possible values of Y ,

(b) the probability of each of these values of Y ,

(c) the expected value of Y .

42. A continuous random variable X has a probability density function given by *[7 marks]*

$$f(x) = \begin{cases} \frac{(x+1)^3}{60}, & \text{for } 1 \leqslant x \leqslant 3 \\ 0, & \text{otherwise.} \end{cases},$$

Find

(a)

$\mathrm{P}(1.5 \leqslant X \leqslant 2.5)$;

(b) $\mathrm{E}(X)$;

(c) the median of X .

43. (a) Ahmed is typing Section A of a mathematics examination paper. The number of mistakes that he makes, X , can be modelled by a Poisson distribution with mean 3.2 . Find the probability that Ahmed makes exactly four mistakes. *[8 marks]*

(b) His colleague, Levi, is typing Section B of this paper. The number of mistakes that he makes, Y , can be modelled by a Poisson distribution with mean m.

(i) If

$\mathrm{E}(Y^2) = 5.5$, find the value of m.

(ii) Find the probability that Levi makes exactly three mistakes.

(c) Given that X and Y are independent, find the probability that Ahmed makes exactly four mistakes and Levi makes exactly three mistakes.

44a. (a) A box of biscuits is considered to be underweight if it weighs less than 228 grams. It is known that the weights of these boxes of biscuits are normally distributed with a mean of 231 grams and a standard deviation of 1.5 grams. *[11 marks]*

What is the probability that a box is underweight?

(b) The manufacturer decides that the probability of a box being underweight should be reduced to 0.002.

(i) Bill's suggestion is to increase the mean and leave the standard deviation unchanged. Find the value of the new mean.

(ii) Sarah's suggestion is to reduce the standard deviation and leave the mean unchanged. Find the value of the new standard deviation.

(c) After the probability of a box being underweight has been reduced to 0.002, a group of customers buys 100 boxes of biscuits. Find the probability that at least two of the boxes are underweight.

44b. There are six boys and five girls in a school tennis club. A team of two boys and two girls will be selected to represent the school in a tennis competition. *[10 marks]*

(a) In how many different ways can the team be selected?

(b) Tim is the youngest boy in the club and Anna is the youngest girl. In how many different ways can the team be selected if it must include both of them?

(c) What is the probability that the team includes both Tim and Anna?

(d) Fred is the oldest boy in the club. Given that Fred is selected for the team, what is the probability that the team includes Tim or Anna, but not both?

45. A random variable has a probability density function given by [6 marks]

$$f(x) = \begin{cases} kx(2-x), & 0 \leqslant x \leqslant 2 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Show that

$k = \frac{3}{4}$.

(b) Find

$\mathrm{E}(X)$.

46. At a nursing college, 80 % of incoming students are female. College records show that 70 % of the incoming females graduate and 90 % of the incoming males graduate. A student who graduates is selected at random. Find the probability that the student is male, giving your answer as a fraction in its lowest terms. [5 marks]

47. The random variable X has the distribution [13 marks]

$\mathrm{B}(n\ ,p)$.

(a) (i) Show that

$\frac{\mathrm{P}(X=x)}{\mathrm{P}(X=x-1)} = \frac{(n-x+1)p}{x(1-p)}$.

(ii) Deduce that if

$\mathrm{P}(X=x) > \mathrm{P}(X=x-1)$ then

$x < (n+1)p$.

(iii) Hence, determine the value of x which maximizes

$\mathrm{P}(X=x)$ when

$(n+1)p$ is not an integer.

(b) Given that $n = 19$, find the set of values of p for which X has a unique mode of 13.

48. The random variable X is assumed to have probability density function f, where [3 marks]

$$f(x) = \begin{cases} \frac{x}{18}, & 0 \leqslant x \leqslant 6 \\ 0, & \text{otherwise.} \end{cases}$$

Show that if the assumption is correct, then

$$\mathrm{P}(a \leqslant X \leqslant b) = \frac{b^2 - a^2}{36}, \text{ for } 0 \leqslant a \leqslant b \leqslant 6.$$

49. In a factory producing glasses, the weights of glasses are known to have a mean of 160 grams. It is also known that the interquartile range of the weights of glasses is 28 grams. Assuming the weights of glasses to be normally distributed, find the standard deviation of the weights of glasses. [6 marks]

50. Casualties arrive at an accident unit with a mean rate of one every 10 minutes. Assume that the number of arrivals can be modelled by a Poisson distribution. *[15 marks]*

(a) Find the probability that there are no arrivals in a given half hour period.

(b) A nurse works for a two hour period. Find the probability that there are fewer than ten casualties during this period.

(c) Six nurses work consecutive two hour periods between 8am and 8pm. Find the probability that no more than three nurses have to attend to less than ten casualties during their working period.

(d) Calculate the time interval during which there is a 95 % chance of there being at least two casualties.

51. A discrete random variable X has a probability distribution given in the following table. *[6 marks]*

x	0.5	1.5	2.5	3.5	4.5	5.5
$\mathrm{P}(X = x)$	0.15	0.21	p	q	0.13	0.07

(a)

$\mathrm{E}(X) = 2.61$, determine the value of p and of q.

(b) Calculate

$\mathrm{Var}(X)$ to three significant figures.

52. After being sprayed with a weedkiller, the survival time of weeds in a field is normally distributed with a mean of 15 days. *[5 marks]*

(a) If the probability of survival after 21 days is 0.2 , find the standard deviation of the survival time.

When another field is sprayed, the survival time of weeds is normally distributed with a mean of 18 days.

(b) If the standard deviation of the survival time is unchanged, find the probability of survival after 21 days.

53. The random variable X follows a Poisson distribution with mean *[6 marks]*

λ.

(a) Find

λ if

$\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.123$.

(b) With this value of

λ, find

$\mathrm{P}(0 < X < 9)$.

54. The annual weather-related loss of an insurance company is modelled by a random variable X with probability density function *[8 marks]*

$$f(x) = \begin{cases} \frac{2.5(200)^{2.5}}{x^{3.5}}, & x \geqslant 200 \\ 0, & \text{otherwise.} \end{cases}$$

Find the median.

Tim goes to a popular restaurant that does not take any reservations for tables. It has been determined that the waiting times for a table are normally distributed with a mean of
18 minutes and standard deviation of
4 minutes.

55. (a) Tim says he will leave if he is not seated at a table within 25 minutes of arriving at the restaurant. Find the probability that Tim will leave without being seated. *[6 marks]*

(b) Tim has been waiting for 15 minutes. Find the probability that he will be seated within the next five minutes.

In each round of two different games Ying tosses three fair coins and Mario tosses two fair coins.

56. (a) The first game consists of one round. If Ying obtains more heads than Mario, she receives \$5 from Mario. If Mario obtains more heads than Ying, he receives \$10 from Ying. If they obtain the same number of heads, then Mario receives \$2 from Ying. Determine Ying's expected winnings. *[20 marks]*

(b) They now play the second game, where the winner will be the player who obtains the larger number of heads in a round. If they obtain the same number of heads, they play another round until there is a winner. Calculate the probability that Ying wins the game.

57. Bob measured the heights of 63 students. After analysis, he conjectured that the height, H, of the students could be modelled by a normal distribution with mean 166.5 cm and standard deviation 5 cm. *[6 marks]*

(a) Based on this assumption, estimate the number of these students whose height is at least 170 cm.

Later Bob noticed that the tape he had used to measure the heights was faulty as it started at the 5 cm mark and not at the zero mark.

(b) What are the correct values of the mean and variance of the distribution of the heights of these students?

Mr Lee is planning to go fishing this weekend. Assuming that the number of fish caught per hour follows a Poisson distribution with mean
0.6, find

58. (a) the probability that he catches at least one fish in the first hour; *[7 marks]*

(b) the probability that he catches exactly three fish if he fishes for four hours;

(c) the number of complete hours that Mr Lee needs to fish so that the probability of catching more than two fish exceeds 80 %.

In a class of
20 students,
12 study Biology,
15 study History and
2 students study neither Biology nor History.

59. (a) Illustrate this information on a Venn diagram. *[4 marks]*

(b) Find the probability that a randomly selected student from this class is studying both Biology and History.

(c) Given that a randomly selected student studies Biology, find the probability that this student also studies History.

60. (a) Find the percentage of the population that has been vaccinated. *[5 marks]*

(b) A randomly chosen person catches the virus. Find the probability that this person has been vaccinated.

Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of
998 ml and a standard deviation of
2.5 ml.

61. (a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212. *[20 marks]*

(b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water.

(c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water, is greater than 0.99.

(d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B.

(e) The company that makes the mineral water receives, on average, m phone calls every 10 minutes. The number of phone calls, X, follows a Poisson distribution such that $\mathrm{P}(X=2)=\mathrm{P}(X=3)+\mathrm{P}(X=4)$.

(i) Find the value of m.

(ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period.