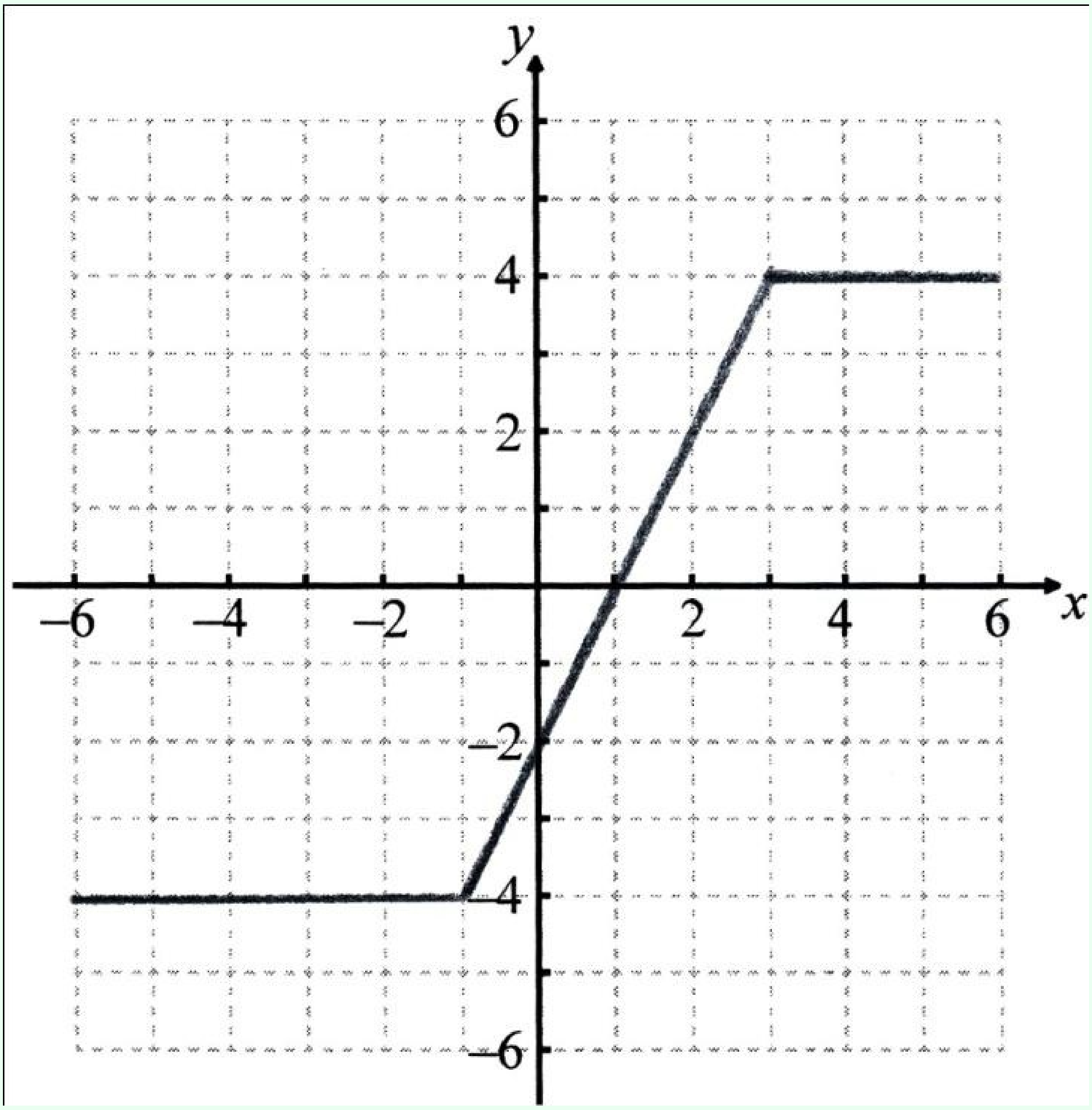
Topic 2 Part 3 *[438 marks]*

1a. *[4 marks]*

Markscheme



M1A1A1A1

Note: Award ***M1*** for any of the three sections completely correct, ***A1*** for each correct segment of the graph. ***[4 marks]***

Examiners report

Most candidates were able to produce a good graph, and many were able to interpret that to get correct answers to part (b). The most common error was to give 4 as the answer to (b) (iii). Some candidates did not recognise that the "hence" in the question meant that they had to use their graph to obtain their answers to part (b).

1b. *[4 marks]*

Markscheme

(i) 0 ***A1***
(ii) 2 ***A1***
(iii) finding area of rectangle ***(M1)***
-4 ***A1***

Note: Award ***M1A0*** for the answer 4.
[4 marks]

Examiners report

Most candidates were able to produce a good graph, and many were able to interpret that to get correct answers to part (b). The most common error was to give 4 as the answer to (b) (iii). Some candidates did not recognise that the "hence" in the question meant that they had to use their graph to obtain their answers to part (b).

2. *[5 marks]*

Markscheme

$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 12x + k$ ***M1A1***

For use of discriminant
$b^2 - 4ac = 0$ or completing the square
$3(x-2)^2 + k - 12$ ***(M1)***

$144 - 12k = 0$ ***(A1)***

Note: Accept trial and error, sketches of parabolas with vertex (2,0) or use of second derivative.

$k = 12$ ***A1***

[5 marks]

Examiners report

Generally candidates answer this question well using a diversity of methods. Surprisingly, a small number of candidates were successful in answering this question using the discriminant of the quadratic and in many cases reverted to trial and error to obtain the correct answer.

3. *[5 marks]*

Markscheme

$h(x) = f(x-3) - 2 = \ln(x-3) - 2$ ***(M1)(A1)***

$g(x) = -h(x) = 2 - \ln(x-3)$ ***M1***

Note: Award **M1** only if it is clear the effect of the reflection in the x-axis:

the expression is correct ***OR***
there is a change of signs of the previous expression ***OR***
there's a graph or an explanation making it explicit

$= \ln \mathrm{e}^2 - \ln(x-3)$ ***M1***

$= \ln\left(\frac{\mathrm{e}^2}{x-3}\right)$ ***A1***

[5 marks]

Examiners report

This question was well attempted but many candidates could have scored better had they written down all the steps to obtain the final expression. In some cases, as the final expression was incorrect and the middle steps were missing, candidates scored just 1 mark. That could be a consequence of a small mistake, but the lack of working prevented them from scoring at least all method marks. Some candidates performed the transformations well but were not able to use logarithms properties to transform the answer and give it as a single logarithm.

4a. *[1 mark]*

Markscheme

$(3.79, -5)$ ***A1***

[1 mark]

Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

4b. *[2 marks]*

Markscheme

$p = 1.57$ or $\frac{\pi}{2}$, $q = 6.00$ ***A1A1***

[2 marks]

Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

4c. *[4 marks]*

Markscheme

$f'(x) = 3\cos x - 4\sin x$ ***(M1)(A1)***

$3\cos x - 4\sin x = 3 \Rightarrow x = 4.43\ldots$ ***(A1)***

$(y = -4)$ ***A1***

Coordinates are
$(4.43, -4)$

[4 marks]

Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

4d.

[7 marks]

Markscheme

$m_{\text{normal}} = \frac{1}{m_{\text{tangent}}}$ ***(M1)***

gradient at P is
-4 so gradient of normal at P is
$\frac{1}{4}$ ***(A1)***

gradient at Q is 4 so gradient of normal at Q is
$-\frac{1}{4}$ ***(A1)***

equation of normal at P is
$y - 3 = \frac{1}{4}(x - 1.570...)$ (or $y = 0.25x + 2.60...$) ***(M1)***

equation of normal at Q is
$y - 3 = \frac{1}{4}(x - 5.999...)$ (or $y = -0.25x + \underbrace{4.499...}$) ***(M1)***

Note: Award the previous two ***M1*** even if the gradients are incorrect in
$y - b = m(x - a)$ where
(a, b) are coordinates of P and Q (or in
$y = mx + c$ with c determined using coordinates of P and Q.

intersect at
$(3.79,\ 3.55)$ ***A1A1***

Note: Award ***N2*** for 3.79 without other working.

[7 marks]

Examiners report

Candidates answered parts (a) and (b) of this question well and, although many were also successful in part (c), just a few candidates gave answers to the required level of accuracy. Part d) was rather challenging for many candidates. The most common errors among the candidates who attempted this question were the confusion between tangents and normals and incorrect final answers due to premature rounding.

5.

[6 marks]

Markscheme

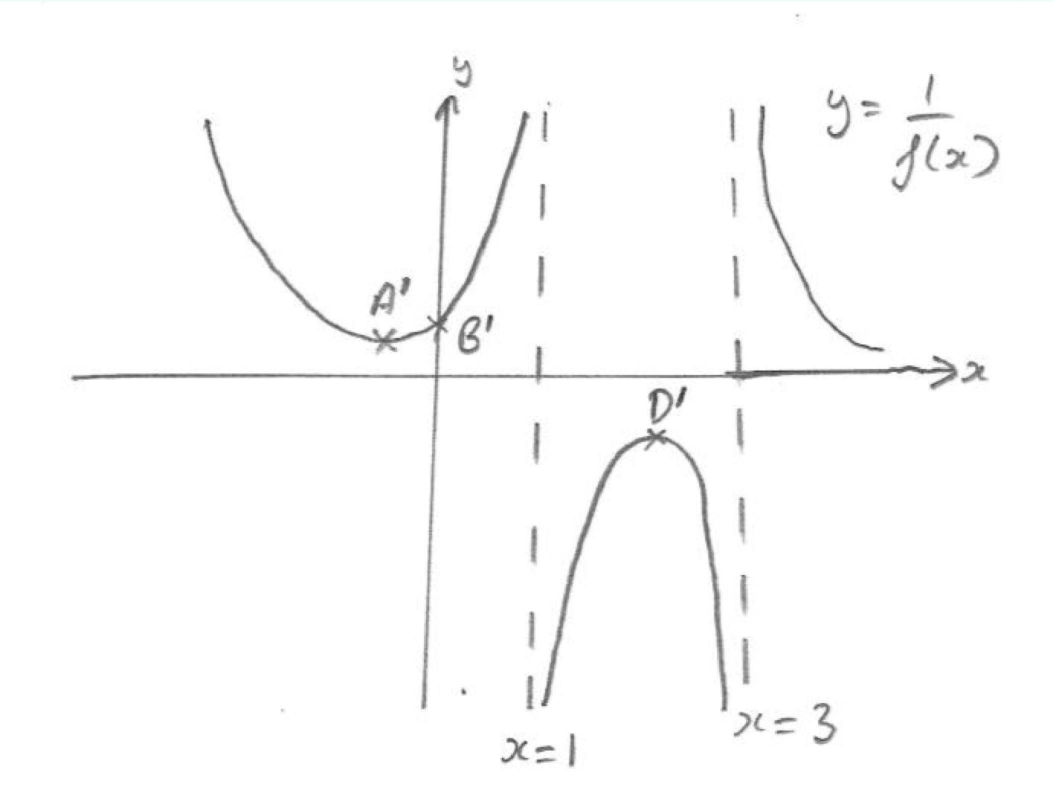
let
$f(x) = 2x^3 + kx^2 + 6x + 32$
let
$g(x) = x^4 - 6x^2 - k^2x + 9$
$f(-1) = -2 + k - 6 + 32 (= 24 + k)$ ***A1***
$g(-1) = 1 - 6 + k^2 + 9 (= 4 + k^2)$ ***A1***
$\Rightarrow 24 + k = 4 + k^2$ ***M1***
$\Rightarrow k^2 - k - 20 = 0$
$\Rightarrow (k - 5)(k + 4) = 0$ ***(M1)***
$\Rightarrow k = 5, -4$ ***A1A1***
[6 marks]

Examiners report

Candidates who used the remainder theorem usually went on to find the two possible values of k. Some candidates, however, attempted to find the remainders using long division. While this is a valid method, the algebra involved proved to be too difficult for most of these candidates.

6a. *[3 marks]*

Markscheme



A1A1A1

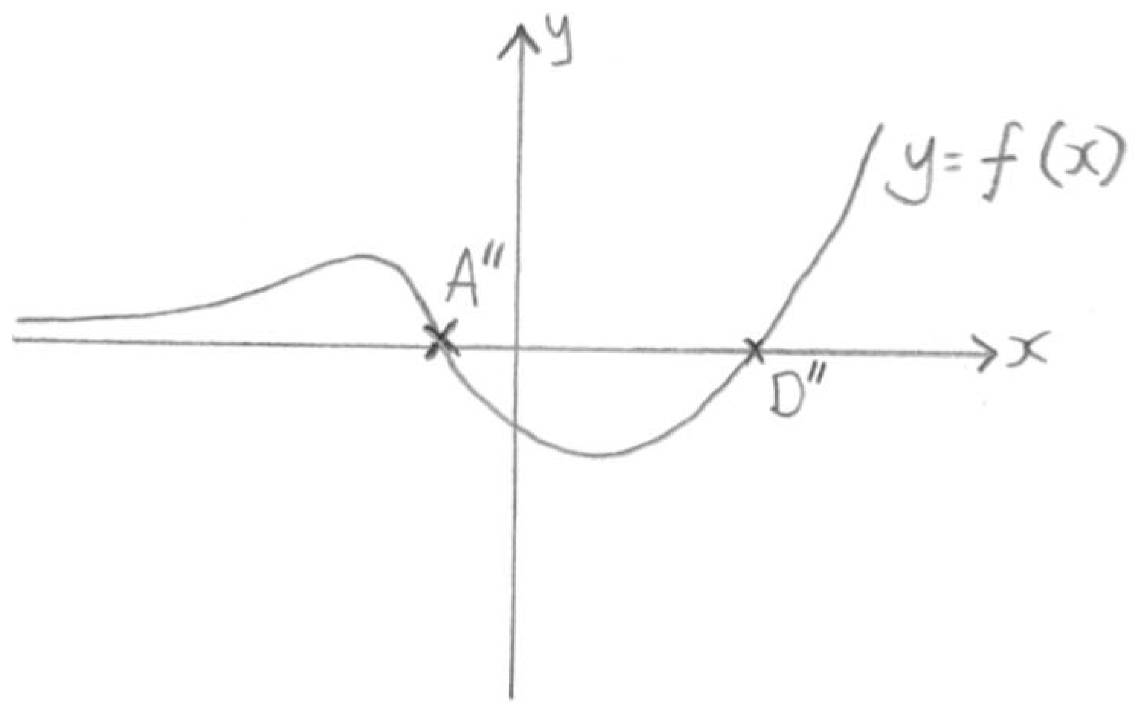
Note: Award ***A1*** for correct shape.
Award ***A1*** for two correct asymptotes, and
$x = 1$ and
$x = 3$.
Award ***A1*** for correct coordinates,
A$'$ $\left(-1, \frac{1}{4}\right)$, B$'$ $\left(0, \frac{1}{3}\right)$ and D$'$ $\left(2, -\frac{1}{3}\right)$.
[3 marks]

Examiners report

Solutions to this question were generally disappointing. In (a), the shape of the graph was often incorrect and many candidates failed to give the equations of the asymptotes and the coordinates of the image points. In (b), many candidates produced incorrect graphs although the coordinates of the image points were often given correctly.

6b. *[3 marks]*

Markscheme



A1A1A1

Note: Award ***A1*** for correct general shape including the horizontal asymptote.

Award ***A1*** for recognition of 1 maximum point and 1 minimum point.

Award ***A1*** for correct coordinates,

$A''(-1,0)$ and

$D''(2,0)$.

[3 marks]

Examiners report

Solutions to this question were generally disappointing. In (a), the shape of the graph was often incorrect and many candidates failed to give the equations of the asymptotes and the coordinates of the image points. In (b), many candidates produced incorrect graphs although the coordinates of the image points were often given correctly.

7a. *[1 mark]*

Markscheme

$e^{-x}\cos x = 0$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1***

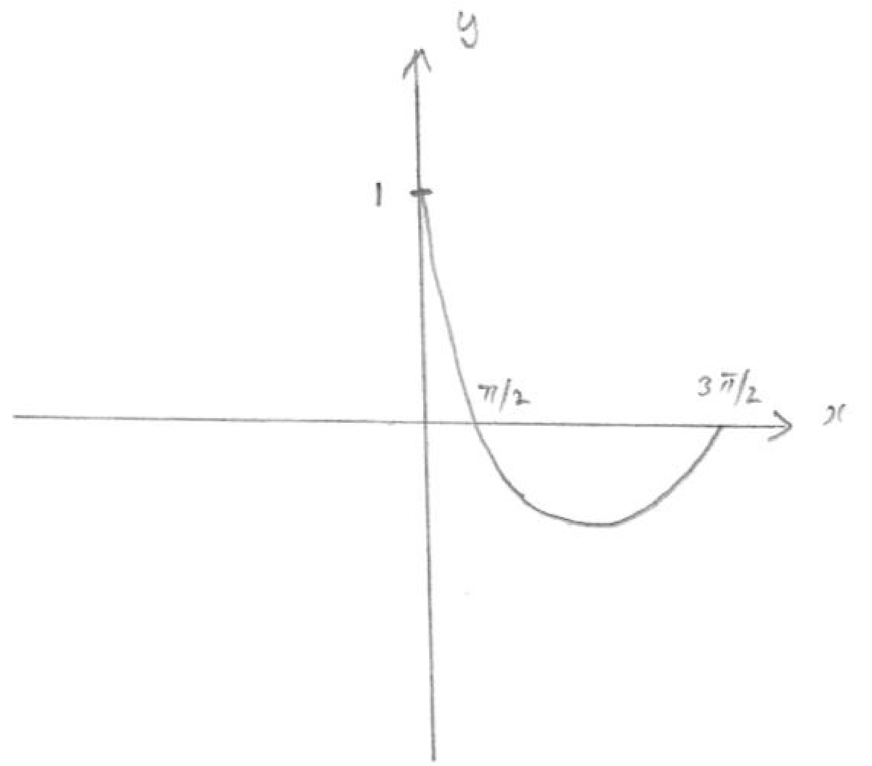
[1 mark]

Examiners report

Many candidates stated the two zeros of *f* correctly but the graph of *f* was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

7b. *[1 mark]*

Markscheme



A1

Note: Accept any form of concavity for
$x \in \left[0, \frac{\pi}{2}\right]$.

Note: Do not penalize unmarked zeros if given in part (a).

Note: Zeros written on diagram can be used to allow the mark in part (a) to be awarded retrospectively.

[1 mark]

Examiners report

Many candidates stated the two zeros of *f* correctly but the graph of *f* was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

7c. *[7 marks]*

Markscheme

attempt at integration by parts *M1*

EITHER

$I = \int \mathrm{e}^{-x} \cos x \mathrm{d}x = -\mathrm{e}^{-x} \cos x \mathrm{d}x - \int \mathrm{e}^{-x} \sin x \mathrm{d}x$ *A1*

$\Rightarrow I = -\mathrm{e}^{-x} \cos x \mathrm{d}x - [-\mathrm{e}^{-x} \sin x + \int \mathrm{e}^{-x} \cos x \mathrm{d}x]$ *A1*

$\Rightarrow I = \frac{\mathrm{e}^{-x}}{2}(\sin x - \cos x) + C$ *A1*

Note: Do not penalize absence of C.

OR

$I = \int \mathrm{e}^{-x} \cos x \mathrm{d}x = \mathrm{e}^{-x} \sin x + \int \mathrm{e}^{-x} \sin x \mathrm{d}x$ *A1*

$I = \mathrm{e}^{-x} \sin x - \mathrm{e}^{-x} \cos x - \int \mathrm{e}^{-x} \cos x \mathrm{d}x$ *A1*

$\Rightarrow I = \frac{\mathrm{e}^{-x}}{2}(\sin x - \cos x) + C$ *A1*

Note: Do not penalize absence of C.

THEN

$\int_0^{\frac{\pi}{2}} \mathrm{e}^{-x} \cos x \mathrm{d}x = \left[\frac{\mathrm{e}^{-x}}{2}(\sin x - \cos x)\right]_0^{\frac{\pi}{2}} = \frac{\mathrm{e}^{-\frac{\pi}{2}}}{2} + \frac{1}{2}$ *A1*

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \mathrm{e}^{-x} \cos x \mathrm{d}x = \left[\frac{\mathrm{e}^{-x}}{2}(\sin x - \cos x)\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = -\frac{\mathrm{e}^{-\frac{3\pi}{2}}}{2} - \frac{\mathrm{e}^{-\frac{\pi}{2}}}{2}$ *A1*

ratio of A:B is

$\frac{\frac{\mathrm{e}^{-\frac{\pi}{2}}}{2}+\frac{1}{2}}{\frac{\mathrm{e}^{-\frac{3\pi}{2}}}{2}+\frac{\mathrm{e}^{-\frac{\pi}{2}}}{2}}$

$= \frac{\mathrm{e}^{\frac{3\pi}{2}}\left(\mathrm{e}^{-\frac{\pi}{2}}+1\right)}{\mathrm{e}^{\frac{3\pi}{2}}\left(\mathrm{e}^{-\frac{3\pi}{2}}+\mathrm{e}^{-\frac{\pi}{2}}\right)}$ *M1*

$= \frac{\mathrm{e}^{\pi}\left(\mathrm{e}^{\frac{\pi}{2}}+1\right)}{\mathrm{e}^{\pi}+1}$ *AG*

[7 marks]

Examiners report

Many candidates stated the two zeros of *f* correctly but the graph of *f* was often incorrectly drawn. In (c), many candidates failed to realise that integration by parts had to be used twice here and even those who did that often made algebraic errors, usually due to the frequent changes of sign.

8a. *[2 marks]*

Markscheme

$f(x) \geqslant \frac{1}{25}$ *A1*

$g(x) \in \mathbb{R},\ g(x) \geqslant 0$ *A1*

[2 marks]

Examiners report

In (a), the ranges were often given incorrectly, particularly the range of *g* where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for $f \circ g(x)$. Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of *X*.

8b. *[4 marks]*

Markscheme

$f \circ g(x) = \frac{2\left(\frac{3x-4}{10}\right)^2+3}{75}$ *M1A1*

$= \frac{\frac{2(9x^2-24x+16)}{100}+3}{75}$ *(A1)*

$= \frac{9x^2-24x+166}{3750}$ *A1*

[4 marks]

Examiners report

In (a), the ranges were often given incorrectly, particularly the range of g where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for $f \circ g(x)$. Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of X.

8c. *[4 marks]*

Markscheme

(i) **METHOD 1**

$y = \frac{2x^2+3}{75}$

$x^2 = \frac{75y-3}{2}$ ***M1***

$x = \sqrt{\frac{75y-3}{2}}$ ***(A1)***

$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}}$ ***A1***

Note: Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***.
Award the ***A1*** only if written in the form
$f^{-1}(x) = .$

METHOD 2

$y = \frac{2x^2+3}{75}$

$x = \frac{2y^2+3}{75}$ ***M1***

$y = \sqrt{\frac{75x-3}{2}}$ ***(A1)***

$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}}$ ***A1***

Note: Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***.
Award the ***A1*** only if written in the form
$f^{-1}(x) = .$

(ii) domain:
$x \geqslant \frac{1}{25}$; range:
$f^{-1}(x) \geqslant 0$ ***A1***

[4 marks]

Examiners report

In (a), the ranges were often given incorrectly, particularly the range of g where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for $f \circ g(x)$. Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of X.

8d. *[6 marks]*

Markscheme

probabilities from
$f(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{3}{75}$	$\frac{5}{75}$	$\frac{11}{75}$	$\frac{21}{75}$	$\frac{35}{75}$

A2

Note: Award ***A1*** for one error, ***A0*** otherwise.

probabilities from
$g(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{8}{10}$

A2

Note: Award ***A1*** for one error, ***A0*** otherwise.

only in the case of
$f(x)$ does
$\sum P(X=x)=1$, hence only
$f(x)$ can be used as a probability mass function ***A2***
[6 marks]

Examiners report

In (a), the ranges were often given incorrectly, particularly the range of g where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for
$f \circ g(x)$. Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of X.

8e. *[2 marks]*

Markscheme

$E(x)=\sum x \cdot \mathrm{P}(X=x)$ ***M1***
$=\frac{5}{75}+\frac{22}{75}+\frac{63}{75}+\frac{140}{75}=\frac{230}{75}\left(=\frac{46}{15}\right)$ ***A1***
[2 marks]

Examiners report

In (a), the ranges were often given incorrectly, particularly the range of g where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for
$f \circ g(x)$. Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of X.

9a. *[2 marks]*

Markscheme

(i)
$(x+\mathrm{i}y)^2=-5+12\mathrm{i}$
$x^2+2\mathrm{i}xy+\mathrm{i}^2y^2=-5+12\mathrm{i}$ ***A1***
(ii) equating real and imaginary parts ***M1***
$x^2-y^2=-5$ ***AG***
$xy=6$ ***AG***
[2 marks]

Examiners report

Since (a) was a ‘show that’ question, it was essential for candidates to give a convincing explanation of how the quoted results were obtained. Many candidates just wrote

$$(x+\mathrm{i}y)^2 = x^2 - y^2 + 2\mathrm{i}xy = -5 + 12\mathrm{i}$$

$$\text{Therefore } x^2 - y^2 = -5 \text{ and } xy = 6$$

This was not given full credit since it simply repeated what was given in the question. Candidates were expected to make it clear that they were equating real and imaginary parts. In (b), candidates who attempted to use de Moivre’s Theorem to find the square roots were given no credit since the question stated ‘hence’.

9b. *[5 marks]*

Markscheme

substituting ***M1***

EITHER

$x^2 - \frac{36}{x^2} = -5$

$x^4 + 5x^2 - 36 = 0$ ***A1***

$x^2 = 4, -9$ ***A1***

$x = \pm 2$ and

$y = \pm 3$ ***(A1)***

OR

$\frac{36}{y^2} - y^2 = -5$

$y^4 - 5y^2 - 36 = 0$ ***A1***

$y^2 = 9, -4$ ***A1***

$y^2 = \pm 3$ **and**

$x = \pm 2$ ***(A1)***

Note: Accept solution by inspection if completely correct.

THEN

the square roots are

$(2 + 3\mathrm{i})$ and

$(-2 - 3\mathrm{i})$ ***A1***

[5 marks]

Examiners report

Since (a) was a ‘show that’ question, it was essential for candidates to give a convincing explanation of how the quoted results were obtained. Many candidates just wrote

$$(x+\mathrm{i}y)^2 = x^2 - y^2 + 2\mathrm{i}xy = -5 + 12\mathrm{i}$$

$$\text{Therefore } x^2 - y^2 = -5 \text{ and } xy = 6$$

This was not given full credit since it simply repeated what was given in the question. Candidates were expected to make it clear that they were equating real and imaginary parts. In (b), candidates who attempted to use de Moivre’s Theorem to find the square roots were given no credit since the question stated ‘hence’.

9c. *[3 marks]*

Markscheme

EITHER

consider

$z = x + \mathrm{i}y$

$z^* = x - \mathrm{i}y$

$(z^*)^2 = x^2 - y^2 - 2\mathrm{i}xy$ ***A1***

$(z^2) = x^2 - y^2 + 2\mathrm{i}xy$ ***A1***

$(z^2)^* = x^2 - y^2 - 2\mathrm{i}xy$ ***A1***

$(z^*)^2 = (z^2)^*$ ***AG***

OR

$z^* = r\mathrm{e}^{-\mathrm{i}\theta}$

$(z^*)^2 = r^2\mathrm{e}^{-2\mathrm{i}\theta}$ ***A1***

$z^2 = r^2\mathrm{e}^{2\mathrm{i}\theta}$ ***A1***

$(z^2)^* = r^2\mathrm{e}^{-2\mathrm{i}\theta}$ ***A1***

$(z^*)^2 = (z^2)^*$ ***AG***

[3 marks]

Examiners report

Since (a) was a ‘show that’ question, it was essential for candidates to give a convincing explanation of how the quoted results were obtained. Many candidates just wrote

$$(x + \mathrm{i}y)^2 = x^2 - y^2 + 2\mathrm{i}xy = -5 + 12\mathrm{i}$$

$$\text{Therefore } x^2 - y^2 = -5 \text{ and } xy = 6$$

This was not given full credit since it simply repeated what was given in the question. Candidates were expected to make it clear that they were equating real and imaginary parts. In (b), candidates who attempted to use de Moivre’s Theorem to find the square roots were given no credit since the question stated ‘hence’.

9d. *[2 marks]*

Markscheme

$(2 - 3\mathrm{i})$ and

$(-2 + 3\mathrm{i})$ ***A1A1***

[2 marks]

Examiners report

Since (a) was a ‘show that’ question, it was essential for candidates to give a convincing explanation of how the quoted results were obtained. Many candidates just wrote

$$(x + \mathrm{i}y)^2 = x^2 - y^2 + 2\mathrm{i}xy = -5 + 12\mathrm{i}$$

$$\text{Therefore } x^2 - y^2 = -5 \text{ and } xy = 6$$

This was not given full credit since it simply repeated what was given in the question. Candidates were expected to make it clear that they were equating real and imaginary parts. In (b), candidates who attempted to use de Moivre’s Theorem to find the square roots were given no credit since the question stated ‘hence’.

9e. *[2 marks]*

Markscheme

the graph crosses the x-axis twice, indicating two real roots ***R1***

since the quartic equation has four roots and only two are real, the other two roots must be complex ***R1***

[2 marks]

Examiners report

In (a), the explanations were often unconvincing. Candidates were expected to make it clear that the two intersections with the x-axis gave two real roots and, since the polynomial was a quartic and therefore had four zeros, the other two roots must be complex. Candidates who made vague statements such as 'the graph shows two real roots' were not given full credit. In (b), most candidates stated the values of a and b correctly but algebraic errors often led to incorrect values for the other parameters. Candidates who failed to solve (b) correctly were unable to solve (c), (d) and (e) correctly although follow through was used where possible.

9f. *[5 marks]*

Markscheme

$f(x) = (x+4)(x-2)(x^2+cx+d)$ ***A1A1***
$f(0) = -32 \Rightarrow d = 4$ ***A1***
Since the curve passes through
$(-1, -18)$,
$-18 = 3 \times (-3)(5-c)$ ***M1***
$c = 3$ ***A1***
Hence
$f(x) = (x+4)(x-2)(x^2+3x+4)$
[5 marks]

Examiners report

In (a), the explanations were often unconvincing. Candidates were expected to make it clear that the two intersections with the x-axis gave two real roots and, since the polynomial was a quartic and therefore had four zeros, the other two roots must be complex. Candidates who made vague statements such as 'the graph shows two real roots' were not given full credit. In (b), most candidates stated the values of a and b correctly but algebraic errors often led to incorrect values for the other parameters. Candidates who failed to solve (b) correctly were unable to solve (c), (d) and (e) correctly although follow through was used where possible.

9g. *[2 marks]*

Markscheme

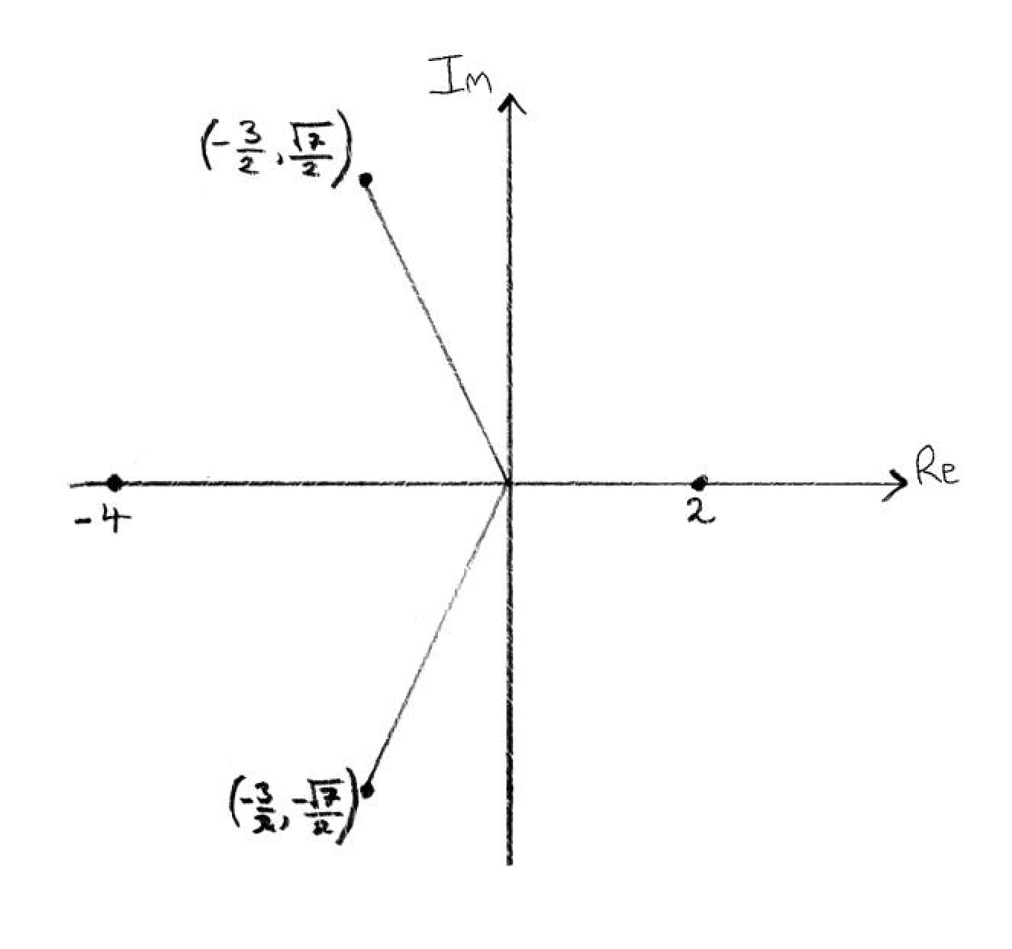
$x = \frac{-3 \pm \sqrt{9-16}}{2}$ ***(M1)***
$\Rightarrow x = -\frac{3}{2} \pm \mathrm{i}\frac{\sqrt{7}}{2}$ ***A1***
[2 marks]

Examiners report

In (a), the explanations were often unconvincing. Candidates were expected to make it clear that the two intersections with the x-axis gave two real roots and, since the polynomial was a quartic and therefore had four zeros, the other two roots must be complex. Candidates who made vague statements such as 'the graph shows two real roots' were not given full credit. In (b), most candidates stated the values of a and b correctly but algebraic errors often led to incorrect values for the other parameters. Candidates who failed to solve (b) correctly were unable to solve (c), (d) and (e) correctly although follow through was used where possible.

9h. *[2 marks]*

Markscheme



A1A1

Note: Accept points or vectors on complex plane.
Award ***A1*** for two real roots and ***A1*** for two complex roots.

[2 marks]

Examiners report

In (a), the explanations were often unconvincing. Candidates were expected to make it clear that the two intersections with the *x*-axis gave two real roots and, since the polynomial was a quartic and therefore had four zeros, the other two roots must be complex. Candidates who made vague statements such as ‘the graph shows two real roots’ were not given full credit. In (b), most candidates stated the values of *a* and *b* correctly but algebraic errors often led to incorrect values for the other parameters. Candidates who failed to solve (b) correctly were unable to solve (c), (d) and (e) correctly although follow through was used where possible.

9i. *[6 marks]*

Markscheme

real roots are

$4e^{i\pi}$ and

$2e^{i0}$ ***A1A1***

considering

$-\frac{3}{2} \pm i\frac{\sqrt{7}}{2}$

$r = \sqrt{\frac{9}{4} + \frac{7}{4}} = 2$ ***A1***

finding

θ using

$\arctan\left(\frac{\sqrt{7}}{3}\right)$ ***M1***

$\theta = \arctan\left(\frac{\sqrt{7}}{3}\right) + \pi$ or $\theta = \arctan\left(-\frac{\sqrt{7}}{3}\right) + \pi$ ***A1***

$\Rightarrow z = 2e^{i\left(\arctan\left(\frac{\sqrt{7}}{3}\right) + \pi\right)}$ or $\Rightarrow z = 2e^{i\left(\arctan\left(\frac{-\sqrt{7}}{3}\right) + \pi\right)}$ ***A1***

Note: Accept arguments in the range

$-\pi$ to π or 0 to 2π.

Accept answers in degrees.

[6 marks]

Examiners report

In (a), the explanations were often unconvincing. Candidates were expected to make it clear that the two intersections with the x-axis gave two real roots and, since the polynomial was a quartic and therefore had four zeros, the other two roots must be complex. Candidates who made vague statements such as 'the graph shows two real roots' were not given full credit. In (b), most candidates stated the values of a and b correctly but algebraic errors often led to incorrect values for the other parameters. Candidates who failed to solve (b) correctly were unable to solve (c), (d) and (e) correctly although follow through was used where possible.

10a. *[4 marks]*

Markscheme

METHOD 1

$f(x) = (x + 1)(x - 1)(x - 2)$ ***M1***

$= x^3 - 2x^2 - x + 2$ ***A1A1A1***

$a = -2$, $b = -1$ and $c = 2$

METHOD 2

from the graph or using $f(0) = 2$

$c = 2$ ***A1***

setting up linear equations using $f(1) = 0$ and $f(-1) = 0$ (or $f(2) = 0$) ***M1***

obtain $a = -2$, $b = -1$ ***A1A1***

[4 marks]

Examiners report

This question was well answered in general. Part b(ii) was often the most problematic, usually because of candidates going to the trouble of finding an explicit and sometimes incorrect expression for $f(x - 2)$.

10b. *[3 marks]*

Markscheme

(i) $(1, 0)$, $(3, 0)$ and $(4, 0)$ ***A1***

(ii) $g(0)$ occurs at $3f(-2)$ ***(M1)***

$= -36$ ***A1***

[3 marks]

Examiners report

This question was well answered in general. Part b(ii) was often the most problematic, usually because of candidates going to the trouble of finding an explicit and sometimes incorrect expression for $f(x - 2)$.

11a. *[3 marks]*

Markscheme

$(f \circ f)(x) = f\left(\frac{x}{2-x}\right) = \frac{\frac{x}{2-x}}{2-\frac{x}{2-x}}$ ***M1A1***

$(f \circ f)(x) = \frac{x}{4-3x}$ ***A1***

[3 marks]

Examiners report

Part a) proved to be an easy 3 marks for most candidates.

11b. *[8 marks]*

Markscheme

$P(n): \underbrace{(f \circ f \circ \ldots \circ f)}_{n \text{ times}}(x) = F_n(x)$

$P(1): \ f(x) = F_1(x)$

$LHS = f(x) = \frac{x}{2-x}$ and $RHS = F_1(x) = \frac{x}{2^1-(2^1-1)x} = \frac{x}{2-x}$ ***A1A1***

$\therefore P(1)$ true

assume that $P(k)$ is true, *i.e.*,

$\underbrace{(f \circ f \circ \ldots \circ f)}_{\text{k times}}(x) = F_k(x)$ ***M1***

consider

$P(k+1)$

EITHER

$\underbrace{(f \circ f \circ \ldots \circ f)}_{\text{k+1 times}}(x) = \left(f \circ \underbrace{f \circ f \circ \ldots \circ f}_{\text{k times}}\right)(x) = f(F_k(x))$ ***(M1)***

$= f\left(\frac{x}{2^k-(2^k-1)x}\right) = \frac{\frac{x}{2^k-(2^k-1)x}}{2-\frac{x}{2^k-(2^k-1)x}}$ ***A1***

$= \frac{x}{2(2^k-(2^k-1)x)-x} = \frac{x}{2^{k+1}-(2^{k+1}-2)x-x}$ ***A1***

OR

$\underbrace{(f \circ f \circ \ldots \circ f)}_{\text{k+1 times}}(x) = \left(f \circ \underbrace{f \circ f \circ \ldots \circ f}_{\text{k times}}\right)(x) = F_k(f(x))$ ***(M1)***

$= F_k\left(\frac{x}{2-x}\right) = \frac{\frac{x}{2-x}}{2^k-(2^k-1)\frac{x}{2-x}}$ ***A1***

$= \frac{x}{2^{k+1}-2^k x-2^k x+x}$ ***A1***

THEN

$= \frac{x}{2^{k+1}-(2^{k+1}-1)x} = F_{k+1}(x)$ ***A1***

$P(k)$ true implies $P(k+1)$ true, $P(1)$ true so $P(n)$ true for all

$n \in \mathbb{Z}^+$ ***R1***

[8 marks]

Examiners report

Part b) was often answered well, and candidates were well prepared in this session for this type of question. Candidates still need to take care when showing explicitly that P(1) is true, and some are still writing 'Let $n = k$' which gains no marks. The inductive step was often well argued, and given in clear detail, though the final inductive reasoning step was incorrect, or appeared rushed, even from the better candidates. 'True for $n = 1$, $n = k$ and $n = k + 1$' is still disappointingly seen, as were some even more unconvincing variations.

11c. *[6 marks]*

Markscheme

METHOD 1

$x = \frac{y}{2^n - (2^n - 1)y} \Rightarrow 2^n x - (2^n - 1)xy = y$ ***M1A1***

$\Rightarrow 2^n x = ((2^n - 1)x + 1)y \Rightarrow y = \frac{2^n x}{(2^n - 1)x + 1}$ ***A1***

$F_n^{-1}(x) = \frac{2^n x}{(2^n - 1)x + 1}$ ***A1***

$F_n^{-1}(x) = \frac{x}{\frac{2^n - 1}{2^n}x + \frac{1}{2^n}}$ ***M1***

$F_n^{-1}(x) = \frac{x}{(1 - 2^{-n})x + 2^{-n}}$ ***A1***

$F_n^{-1}(x) = \frac{x}{2^{-n} - (2^{-n} - 1)x}$ ***AG***

METHOD 2

attempt

$F_{-n}(F_n(x))$ ***M1***

$= F_{-n}\left(\frac{x}{2^n - (2^n - 1)x}\right) = \frac{\frac{x}{2^n - (2^n - 1)x}}{2^{-n} - (2^{-n} - 1)\frac{x}{2^n - (2^n - 1)x}}$ ***A1A1***

$= \frac{x}{2^{-n}(2^n - (2^n - 1)x) - (2^{-n} - 1)x}$ ***A1A1***

Note: Award ***A1*** marks for numerators and denominators.

$= \frac{x}{1} = x$ ***A1AG***

METHOD 3

attempt

$F_n(F_{-n}(x))$ **M1**

$= F_n\left(\frac{x}{2^{-n} - (2^{-n} - 1)x}\right) = \frac{\frac{x}{2^{-n} - (2^{-n} - 1)x}}{2^n - (2^n - 1)\frac{x}{2^{-n} - (2^{-n} - 1)x}}$ ***A1A1***

$= \frac{x}{2^n(2^{-n} - (2^{-n} - 1)x) - (2^n - 1)x}$ ***A1A1***

Note: Award ***A1*** marks for numerators and denominators.

$= \frac{x}{1} = x$ ***A1AG***

[6 marks]

Examiners report

Part c) was again very well answered by the majority. A few weaker candidates attempted to find an inverse for the individual case n = 1 , but gained no credit for this.

11d. *[6 marks]*

$F_n(0) = 0,\ F_n(1) = 1$

$2^n - (2^n - 1)x - 1 = (2^n - 1)(1 - x)$
> 0 if $0 < x < 1$ and $n \in \mathbb{Z}^+$

$2^n - (2^n - 1)x > 1$ and $F_n(x) = \frac{x}{2^n - (2^n - 1)x} < \frac{x}{1}(< x)$
$F_n(x) = \frac{x}{2^n - (2^n - 1)x} < x$ for $0 < x < 1$ and $n \in \mathbb{Z}^+$

$\frac{x}{2^n - (2^n - 1)x} < x \Leftrightarrow 2^n - (2^n - 1)x > 1$
$\Leftrightarrow (2^n - 1)x < 2^n - 1$
$\Leftrightarrow x < \frac{2^n - 1}{2^n - 1} = 1$
$]0,\ 1[$

$B_n = 2\left(A_n - \frac{1}{2}\right)\ (= 2A_n - 1)$

Examiners report

Part d) was not at all well understood, with virtually no candidates able to tie together the hints given by connecting the different parts of the question. Rash, and often thoughtless attempts were made at each part, though by this stage some seemed to be struggling through lack of time. The inequality part of the question tended to be 'fudged', with arguments seen by examiners being largely unconvincing and lacking clarity. A tiny number of candidates provided the correct answer to the final part, though a surprising number persisted with what should have been recognised as fruitless working – usually in the form of long-winded integration attempts.

12.

[4 marks]

Markscheme

$\Delta = (5-k)^2 + 4(k+2)$ ***M1A1***

$= k^2 - 6k + 33$ ***(A1)***

$= (k-3)^2 + 24$ which is positive for all k ***R1***

Note: Accept analytical, graphical or other correct methods. In all cases only award ***R1*** if a reason is given in words or graphically. Award ***M1A1A0R1*** if mistakes are made in the simplification but the argument given is correct.

[4 marks]

Examiners report

Overall the question was pretty well answered but some candidates seemed to have mixed up the terms determinant with discriminant. In some cases a lack of quality mathematical reasoning and understanding of the discriminant was evident. Many worked with the quadratic formula rather than just the discriminant, conveying a lack of understanding of the strategy required. Errors in algebraic simplification (expanding terms involving negative signs) prevented many candidates from scoring well in this question. Many candidates were not able to give a clear reason why the quadratic has always two distinct real solutions; in some cases a vague explanation was given, often referring to a graph which was not sketched.

13a.

[2 marks]

Markscheme

$\frac{c}{2} * \frac{3c}{4} = \frac{\frac{c}{2}+\frac{3c}{4}}{1+\frac{1}{2}\cdot\frac{3}{4}}$ ***M1***

$= \frac{\frac{5c}{4}}{\frac{11}{8}} = \frac{10c}{11}$ ***A1***

[2 marks]

Examiners report

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term "state" in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

13b.

[1 mark]

Markscheme

identity is 0 ***A1***

[1 mark]

Examiners report

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term "state" in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

13c. *[1 mark]*

Markscheme

inverse is $-x$ ***A1***

[1 mark]

Examiners report

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term "state" in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

13d. *[2 marks]*

Markscheme

$x * y = \frac{x+y}{1+\frac{xy}{c^2}}$, $y * x = \frac{y+x}{1+\frac{yx}{c^2}}$ ***M1***

(since ordinary addition and multiplication are commutative)

$x * y = y * x$ so $*$ is commutative ***R1***

Note: Accept arguments using symmetry.

[2 marks]

Examiners report

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term “state” in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

13e. *[4 marks]*

Markscheme

$$(x*y)*z=\frac{x+y}{1+\frac{xy}{c^2}}*z=\frac{\left(\frac{x+y}{1+\frac{xy}{c^2}}\right)+z}{1+\left(\frac{x+y}{1+\frac{xy}{c^2}}\right)\frac{z}{c^2}} \quad \textbf{\textit{M1}}$$

$$=\frac{\frac{\left(x+y+z+\frac{xyz}{c^2}\right)}{\left(1+\frac{xy}{c^2}\right)}}{\frac{\left(1+\frac{xy}{c^2}+\frac{xz}{c^2}+\frac{yz}{c^2}\right)}{\left(1+\frac{xy}{c^2}\right)}}=\frac{\left(x+y+z+\frac{xyz}{c^2}\right)}{\left(1+\left(\frac{xy+xz+yz}{c^2}\right)\right)} \quad \textbf{\textit{A1}}$$

$$x*(y*z)=x*\left(\frac{y+z}{1+\frac{yz}{c^2}}\right)=\frac{x+\left(\frac{y+z}{1+\frac{yz}{c^2}}\right)}{1+\frac{x}{c^2}\left(\frac{y+z}{1+\frac{yz}{c^2}}\right)}$$

$$=\frac{\frac{\left(x+\frac{xyz}{c^2}+y+z\right)}{\left(1+\frac{yz}{c^2}\right)}}{\frac{\left(1+\frac{yz}{c^2}+\frac{xy}{c^2}+\frac{xz}{c^2}\right)}{\left(1+\frac{yz}{c^2}\right)}}=\frac{\left(x+y+z+\frac{xyz}{c^2}\right)}{\left(1+\left(\frac{xy+xz+yz}{c^2}\right)\right)} \quad \textbf{\textit{A1}}$$

since both expressions are the same

$*$ is associative ***R1***

Note: After the initial ***M1A1***, correct arguments using symmetry also gain full marks.

[4 marks]

Examiners report

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term “state” in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

13f. *[2 marks]*

Markscheme

(i)

$c > x$ and $c > y \Rightarrow c - x > 0$ and $c - y > 0 \Rightarrow (c - x)(c - y) > 0$ ***R1AG***

(ii)

$c^2 - cx - cy + xy > 0 \Rightarrow c^2 + xy > cx + cy \Rightarrow c + \frac{xy}{c} > x + y$ (as $c > 0$)

so

$x + y < c + \frac{xy}{c}$ ***M1AG***

[2 marks]

Examiners report

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term “state” in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

13g. *[2 marks]*

Markscheme

if

$x,\, y \in G$ then $-c - \frac{xy}{c} < x + y < c + \frac{xy}{c}$

thus

$-c\left(1 + \frac{xy}{c^2}\right) < x + y < c\left(1 + \frac{xy}{c^2}\right)$ and $-c < \frac{x+y}{1+\frac{xy}{c^2}} < c$ ***M1***

(as $1 + \frac{xy}{c^2} > 0$) so $-c < x * y < c$ ***A1***

proving that G is closed under

$*$ ***AG***

[2 marks]

Examiners report

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term “state” in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

13h.

[2 marks]

Markscheme

as

$\{G, *\}$ is closed, is associative, has an identity and all elements have an inverse ***R1***

it is a group ***AG***

as

$*$ is commutative ***R1***

it is an Abelian group ***AG***

[2 marks]

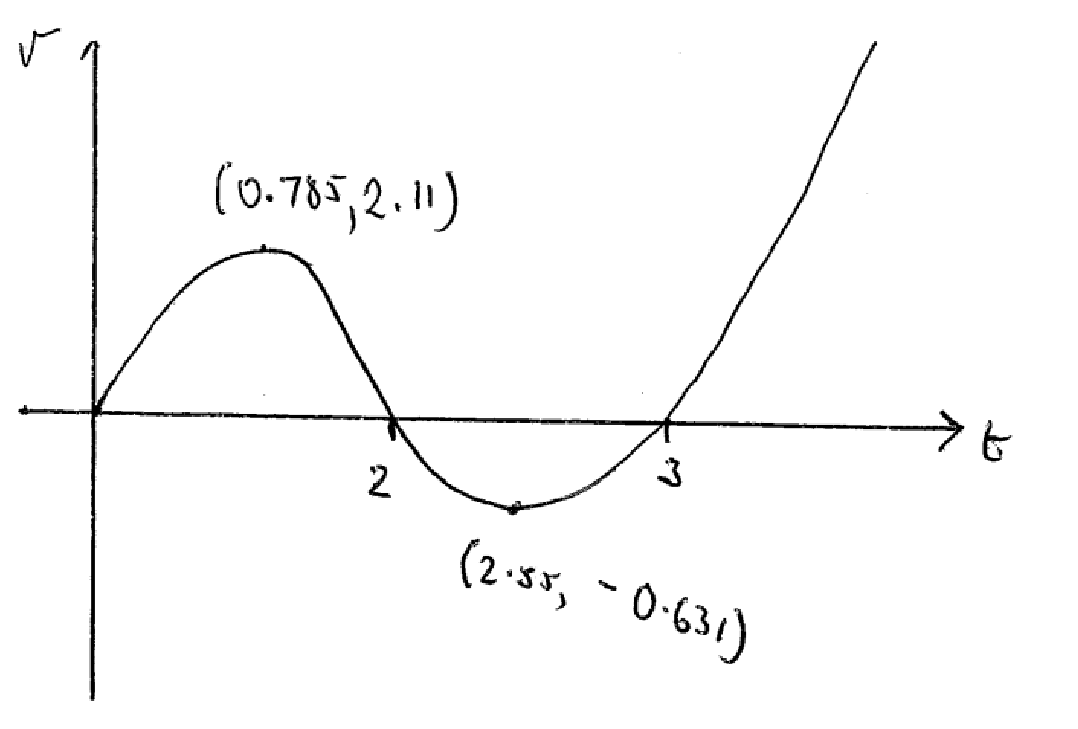
Examiners report

Most candidates were able to answer part (a) indicating preparation in such questions. Many students failed to identify the command term "state" in parts (b) and (c) and spent a lot of time – usually unsuccessfully - with algebraic methods. Most students were able to offer satisfactory solutions to part (d) and although most showed that they knew what to do in part (e), few were able to complete the proof of associativity. Surprisingly few managed to answer parts (f) and (g) although many who continued to this stage, were able to pick up at least one of the marks for part (h), regardless of what they had done before. Many candidates interpreted the question as asking to prove that the group was Abelian, rather than proving that it was an Abelian group. Few were able to fully appreciate the significance in part (i) although there were a number of reasonable solutions.

14a.

[3 marks]

Markscheme



A1A1A1

Note: Award ***A1*** for general shape, ***A1*** for correct maximum and minimum, ***A1*** for intercepts.

Note: Follow through applies to (b) and (c).

[3 marks]

Examiners report

Part (a) was generally well done, although correct accuracy was often a problem.

14b.

[2 marks]

Markscheme

$0 \leqslant t < 0.785$, $\left(\text{or } 0 \leqslant t < \frac{5-\sqrt{7}}{3}\right)$ ***A1***

(allow

$t < 0.785$)

and

$t > 2.55$ $\left(\text{or } t > \frac{5+\sqrt{7}}{3}\right)$ ***A1***

[2 marks]

Examiners report

Parts (b) and (c) were also generally quite well done.

14c.

[3 marks]

Markscheme

$0 \leqslant t < 0.785$, $\left(\text{or } 0 \leqslant t < \frac{5-\sqrt{7}}{3}\right)$ ***A1***

(allow

$t < 0.785$)

$2 < t < 2.55$, $\left(\text{or } 2 < t < \frac{5+\sqrt{7}}{3}\right)$ ***A1***

$t > 3$ ***A1***

[3 marks]

Examiners report

Parts (b) and (c) were also generally quite well done.

14d.

[3 marks]

Markscheme

position of A:

$x_A = \int t^3 - 5t^2 + 6t \,\mathrm{d}t$ ***(M1)***

$x_A = \frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2 \quad (+c)$ ***A1***

when

$t = 0,\ x_A = 0$, so

$c = 0$ ***R1***

[3 marks]

Examiners report

A variety of approaches were seen in part (d) and many candidates were able to obtain at least 2 out of 3. A number missed to consider the
$+c$, thereby losing the last mark.

14e.

[4 marks]

Markscheme

$\frac{\mathrm{d}v_B}{\mathrm{d}t} = -2v_B \Rightarrow \int \frac{1}{v_B}\mathrm{d}v_B = \int -2\mathrm{d}t$ ***(M1)***

$\ln|v_B| = -2t + c$ ***(A1)***

$v_B = Ae^{-2t}$ ***(M1)***

$v_B = -20$ when $t = 0$ so

$v_B = -20e^{-2t}$ ***A1***

[4 marks]

Examiners report

Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

14f. *[6 marks]*

Markscheme

$x_B = 10e^{-2t}(+c)$ ***(M1)(A1)***

$x_B = 20$ when $t = 0$ so $x_B = 10e^{-2t} + 10$ ***(M1)A1***

meet when

$\frac{1}{4}t^4 - \frac{5}{3}t^3 + 3t^2 = 10e^{-2t} + 10$ ***(M1)***

$t = 4.41(290\ldots)$ ***A1***

[6 marks]

Examiners report

Surprisingly few candidates were able to solve part (e) correctly. Very few could recognise the easy variable separable differential equation. As a consequence part (f) was frequently left.

15a. *[6 marks]*

Markscheme

$f(2) = 9$ ***(A1)***

$f^{-1}(x) = (x-1)^{\frac{1}{3}}$ ***A1***

$(f^{-1})'(x) = \frac{1}{3}(x-1)^{-\frac{2}{3}}$ ***(M1)***

$(f^{-1})'(9) = \frac{1}{12}$ ***A1***

$f'(x) = 3x^2$ ***(M1)***

$\frac{1}{f'(2)} = \frac{1}{3\times4} = \frac{1}{12}$ ***A1***

Note: The last ***M1*** and ***A1*** are independent of previous marks.

[6 marks]

Examiners report

There were many good attempts at parts (a) and (b), although in (b) many were unable to give a thorough justification.

15b. *[3 marks]*

Markscheme

$g'(x) = \mathrm{e}^{x^2} + 2x^2\mathrm{e}^{x^2}$ ***M1A1***

$g'(x) > 0$ as each part is positive ***R1***

[3 marks]

Examiners report

There were many good attempts at parts (a) and (b), although in (b) many were unable to give a thorough justification.

15c. *[4 marks]*

Markscheme

to find the x-coordinate on

$y = g(x)$ solve

$2 = x\mathrm{e}^{x^2}$ ***(M1)***

$x = 0.89605022078\ldots$ ***(A1)***

gradient

$= (g^{-1})'(2) = \frac{1}{g'(0.896\ldots)}$ ***(M1)***

$= \frac{1}{\mathrm{e}^{(0.896\ldots)^2}\left(1+2\times(0.896\ldots)^2\right)} = 0.172$ to 3sf ***A1***

(using the

$\frac{\mathrm{d}y}{\mathrm{d}x}$ function on gdc

$g'(0.896\ldots) = 5.7716028\ldots$

$\frac{1}{g'(0.896\ldots)} = 0.173$

[4 marks]

Examiners report

Few good solutions to parts (c) and (d)(ii) were seen although many were able to answer (d)(i) correctly.

15d. *[6 marks]*

Markscheme

(i)

$(x^3+1)\mathrm{e}^{(x^3+1)^2} = 2$ ***A1***

$x = -0.470191\ldots$ ***A1***

(ii) **METHOD 1**

$(g \circ f)'(x) = 3x^2\mathrm{e}^{(x^3+1)^2}\left(2(x^3+1)^2+1\right)$ ***(M1)(A1)***

$(g \circ f)'(-0.470191\ldots) = 3.85755\ldots$ ***(A1)***

$h'(2) = \frac{1}{3.85755\ldots} = 0.259\ (232\ldots)$ ***A1***

Note: The solution can be found without the student obtaining the explicit form of the composite function.

METHOD 2

$h(x) = (f^{-1} \circ g^{-1})(x)$ ***A1***

$h'(x) = (f^{-1})'\left(g^{-1}(x)\right) \times (g^{-1})'(x)$ ***M1***

$= \frac{1}{3}\left(g^{-1}(x) - 1\right)^{-\frac{2}{3}} \times (g^{-1})'(x)$ ***M1***

$h'(2) = \frac{1}{3}\left(g^{-1}(2) - 1\right)^{-\frac{2}{3}} \times (g^{-1})'(2)$

$= \frac{1}{3}(0.89605\ldots - 1)^{-\frac{2}{3}} \times 0.171933\ldots$

$= 0.259\ (232\ldots)$ ***A1*** ***N4***

[6 marks]

Examiners report

Few good solutions to parts (c) and (d)(ii) were seen although many were able to answer (d)(i) correctly.

16a.

[4 marks]

Markscheme

$\int x\sec^2 x\,\mathrm{d}x = x\tan x - \int 1 \times \tan x\,\mathrm{d}x$ ***M1A1***

$= x\tan x + \ln|\cos x|(+c)\ \ (= x\tan x - \ln|\sec x|(+c))$ ***M1A1***

[4 marks]

Examiners report

In part (a), a large number of candidates were able to use integration by parts correctly but were unable to use integration by substitution to then find the indefinite integral of tan x. In part (b), a large number of candidates attempted to solve the equation without direct use of a GDC's numerical solve command. Some candidates stated more than one solution for m and some specified m correct to two significant figures only.

16b.

[2 marks]

Markscheme

attempting to solve an appropriate equation *eg*

$m\tan m + \ln(\cos m) = 0.5$ ***(M1)***

$m = 0.822$ ***A1***

Note: Award ***A1*** if $m = 0.822$ is specified with other positive solutions.

[2 marks]

Examiners report

In part (a), a large number of candidates were able to use integration by parts correctly but were unable to use integration by substitution to then find the indefinite integral of tan x. In part (b), a large number of candidates attempted to solve the equation without direct use of a GDC's numerical solve command. Some candidates stated more than one solution for m and some specified m correct to two significant figures only.

17a.

[2 marks]

Markscheme

$u_n - v_n = 1.6 + (n-1) \times 1.5 - 3 \times 1.2^{n-1}\ (= 1.5n + 0.1 - 3 \times 1.2^{n-1})$ ***A1A1***

[2 marks]

Examiners report

In part (a), most candidates were able to express
u_n and
v_n correctly and hence obtain a correct expression for
$u_n - v_n$. Some candidates made careless algebraic errors when unnecessarily simplifying
u_n while other candidates incorrectly stated
v_n as
$3(1.2)^n$.

17b. *[3 marks]*

Markscheme

attempting to solve

$u_n > v_n$ numerically or graphically. ***(M1)***

$n = 2.621\ldots, 9.695\ldots$ ***(A1)***

So

$3 \leqslant n \leqslant 9$ ***A1***

[3 marks]

Examiners report

In parts (b) and (c), most candidates treated n as a continuous variable rather than as a discrete variable. Candidates should be aware that a GDC's table feature can be extremely useful when attempting such question types.

17c. *[1 mark]*

Markscheme

The greatest value of

$u_n - v_n$ is 1.642. ***A1***

Note: Do not accept 1.64.

[1 mark]

Examiners report

In parts (b) and (c), most candidates treated n as a continuous variable rather than as a discrete variable. Candidates should be aware that a GDC's table feature can be extremely useful when attempting such question types. In part (c), a number of candidates attempted to find the maximum value of n rather than attempting to find the maximum value of $u_n - v_n$.

18a. *[4 marks]*

Markscheme

(i)

$\sum\limits_{k=1}^{n}(2k-1)$ (or equivalent) ***A1***

Note: Award ***A0*** for

$\sum\limits_{n=1}^{n}(2n-1)$ or equivalent.

(ii) **EITHER**

$2\times\frac{n(n+1)}{2}-n$ ***M1A1***

OR

$\frac{n}{2}(2+(n-1)2)$ (using $S_n=\frac{n}{2}(2u_1+(n-1)d)$) ***M1A1***

OR

$\frac{n}{2}(1+2n-1)$ (using $S_n=\frac{n}{2}(u_1+u_n)$) ***M1A1***

THEN

$=n^2$ ***AG***

(iii)

$47^2-14^2=2013$ ***A1***

[4 marks]

Examiners report

In part (a) (i), a large number of candidates were unable to correctly use sigma notation to express the sum of the first n positive odd integers. Common errors included summing $2n-1$ from 1 to n and specifying sums with incorrect limits. Parts (a) (ii) and (iii) were generally well done.

18b. *[7 marks]*

Markscheme

(i) **EITHER**

a pentagon and five diagonals ***A1***

OR

five diagonals (circle optional) ***A1***

(ii) Each point joins to n – 3 other points. ***A1***

a correct argument for

$n(n-3)$ ***R1***

a correct argument for

$\frac{n(n-3)}{2}$ ***R1***

(iii) attempting to solve

$\frac{1}{2}n(n-3) > 1\,000\,000$ for n. ***(M1)***

$n > 1415.7$ ***(A1)***

$n = 1416$ ***A1***

[7 marks]

Examiners report

Parts (b) (i) and (iii) were generally well done. In part (b) (iii), many candidates unnecessarily simplified their quadratic when direct GDC use could have been employed. A few candidates gave $n > 1416$ as their final answer. While some candidates displayed sound reasoning in part (b) (ii), many candidates unfortunately adopted a 'proof by example' approach.

18c. *[8 marks]*

Markscheme

(i) np = 4 and npq = 3 ***(A1)***

attempting to solve for n and p ***(M1)***

$n = 16$ and

$p = \frac{1}{4}$ ***A1***

(ii)

$X \sim B(16, 0.25)$ ***(A1)***

$P(X = 1) = 0.0534538...(= \binom{16}{1}(0.25)(0.75)^{15})$ ***(A1)***

$P(X = 3) = 0.207876...(= \binom{16}{3}(0.25)^3(0.75)^{13})$ ***(A1)***

$\mathrm{P}(X = 1) + \mathrm{P}(X = 3)$ ***(M1)***

= 0.261 ***A1***

[8 marks]

Examiners report

Part (c) was generally well done. In part (c) (ii), some candidates multiplied the two probabilities rather than adding the two probabilities.

19a. *[2 marks]*

Markscheme

EITHER

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent) ***M1A1***

Note: Accept

$\theta = 180^\circ - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent).

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$ (or equivalent) ***M1A1***

[2 marks]

Examiners report

Part (a) was reasonably well done. While many candidates exhibited sound trigonometric knowledge to correctly express θ in terms of x, many other candidates were not able to use elementary trigonometry to formulate the required expression for θ.

19b. *[2 marks]*

Markscheme

(i)

$\theta = 0.994\ \left(= \arctan\frac{20}{13}\right)$ ***A1***

(ii)

$\theta = 1.19\ \left(= \arctan\frac{5}{2}\right)$ ***A1***

[2 marks]

Examiners report

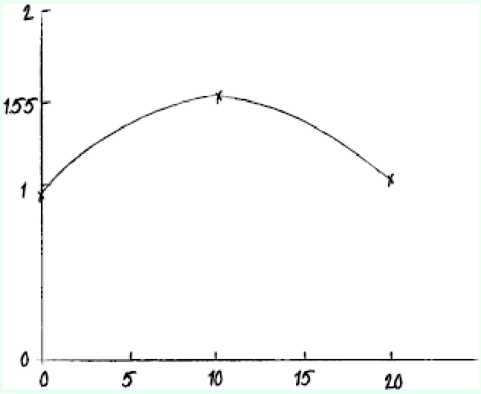
In part (b), a large number of candidates did not realize that θ could only be acute and gave obtuse angle values for θ. Many candidates also demonstrated a lack of insight when substituting endpoint x-values into θ.

19c. *[2 marks]*

Markscheme

correct shape. ***A1***

correct domain indicated. ***A1***



[2 marks]

Examiners report

In part (c), many candidates sketched either inaccurate or implausible graphs.

19d. *[6 marks]*

Markscheme

attempting to differentiate one

$\arctan(f(x))$ term ***M1***

EITHER

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{8}{x^2} \times \frac{1}{1+\left(\frac{8}{x}\right)^2} - \frac{13}{(20-x)^2} \times \frac{1}{1+\left(\frac{13}{20-x}\right)^2}$ ***A1A1***

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{\frac{1}{8}}{1+\left(\frac{x}{8}\right)^2} + \frac{-\frac{1}{13}}{1+\left(\frac{20-x}{13}\right)^2}$ ***A1A1***

THEN

$= \frac{8}{x^2+64} - \frac{13}{569-40x+x^2}$ ***A1***

$= \frac{8(569-40x+x^2)-13(x^2+64)}{(x^2+64)(x^2-40x+569)}$ ***M1A1***

$= \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}$ ***AG***

[6 marks]

Examiners report

In part (d), a large number of candidates started their differentiation incorrectly by failing to use the chain rule correctly.

19e.

[3 marks]

Markscheme

Maximum light intensity at P occurs when

$\frac{d\theta}{dx} = 0.$ ***(M1)***

either attempting to solve

$\frac{d\theta}{dx} = 0$ for x or using the graph of either

θ or

$\frac{d\theta}{dx}$ ***(M1)***

$x = 10.05$ (m) ***A1***

[3 marks]

Examiners report

For a question part situated at the end of the paper, part (e) was reasonably well done. A large number of candidates demonstrated a sound knowledge of finding where the maximum value of θ occurred and rejected solutions that were not physically feasible.

19f.

[4 marks]

Markscheme

$\frac{dx}{dt} = 0.5$ ***(A1)***

At $x = 10$,

$\frac{d\theta}{dx} = 0.000453\ \left(= \frac{5}{11029}\right).$ ***(A1)***

use of

$\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$ ***M1***

$\frac{d\theta}{dt} = 0.000227\ \left(= \frac{5}{22058}\right)\ (\text{rad s}^{-1})$ ***A1***

Note: Award ***(A1)*** for

$\frac{dx}{dt} = -0.5$ and ***A1*** for

$\frac{d\theta}{dt} = -0.000227\ \left(= -\frac{5}{22058}\right)$.

Note: Implicit differentiation can be used to find

$\frac{d\theta}{dt}$. Award as above.

[4 marks]

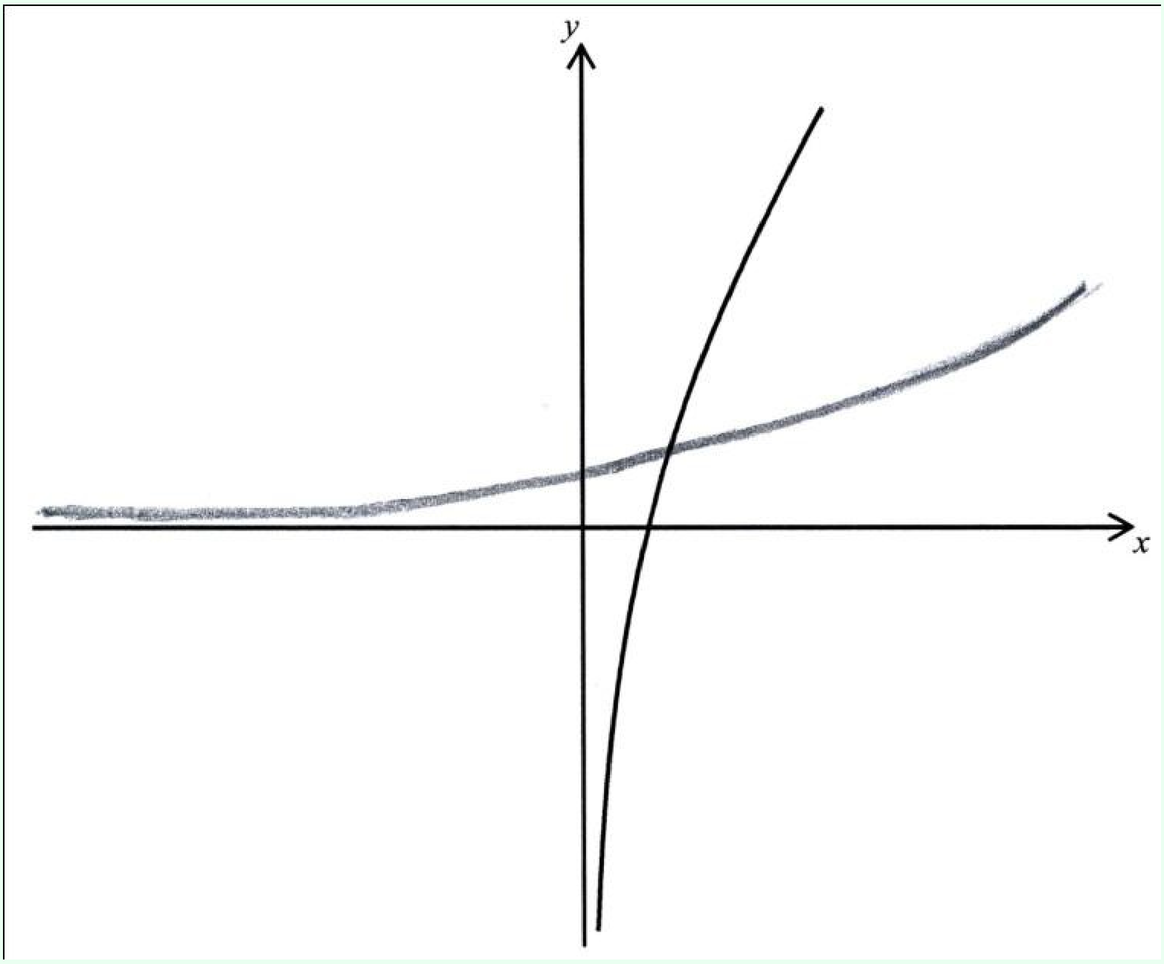
Examiners report

In part (f), many candidates were able to link the required rates, however only a few candidates were able to successfully apply the chain rule in a related rates context.

20. *[6 marks]*

Markscheme

(a)



A1A1

Note: Award ***A1*** for correct asymptote with correct behaviour and ***A1*** for shape.

[2 marks]

(b) intersect on

$y = x$ ***(M1)***

$x + \ln x = x \Rightarrow \ln x = 0$ ***(A1)***

intersect at (1, 1) ***A1*** ***A1***

[4 marks]

Total [6 marks]

Examiners report

Most students were able to sketch the correct graph, but then many failed to recognise that they could use their solution to determine the solution of part (b). Those who did were generally successful and those who embarked on attempts to find the inverse function did not realise that this was leading them nowhere.

21. *[5 marks]*

Markscheme

using the factor theorem or long division ***(M1)***

$-A+B-1+6=0 \Rightarrow A-B=5$ ***(A1)***

$8A+4B+2+6=0 \Rightarrow 2A+B=-2$ ***(A1)***

$3A=3 \Rightarrow A=1$ ***(A1)***

$B=-4$ ***(A1)*** ***(N3)***

Note: Award ***M1A0A0A1A1*** for using $(x-3)$ as the third factor, without justification that the leading coefficient is 1.

[5 marks]

Examiners report

Most candidates attempted this question and it was the best done question on the paper with many fully correct answers. It was good to see a range of approaches used (mainly factor theorem or long division). A number of candidates assumed $(x-3)$ was the missing factor without justification.

22. *[8 marks]*

Markscheme

(a)

$h(x) = g \circ f(x) = \frac{1}{e^{x^2}+3}, \ (x \geqslant 0)$ ***(M1)A1***

(b)

$0 < x \leqslant \frac{1}{4}$ ***A1A1***

Note: Award ***A1*** for limits and ***A1*** for correct inequality signs.

(c)

$y = \frac{1}{e^{x^2}+3}$

$ye^{x^2} + 3y = 1$ ***M1***

$e^{x^2} = \frac{1-3y}{y}$ ***A1***

$x^2 = \ln\frac{1-3y}{y}$ ***M1***

$x = \pm\sqrt{\ln\frac{1-3y}{y}}$

$\Rightarrow h^{-1}(x) = \sqrt{\ln\frac{1-3x}{x}} \ \left(= \sqrt{\ln\left(\frac{1}{x}-3\right)}\right)$ ***A1***

[8 marks]

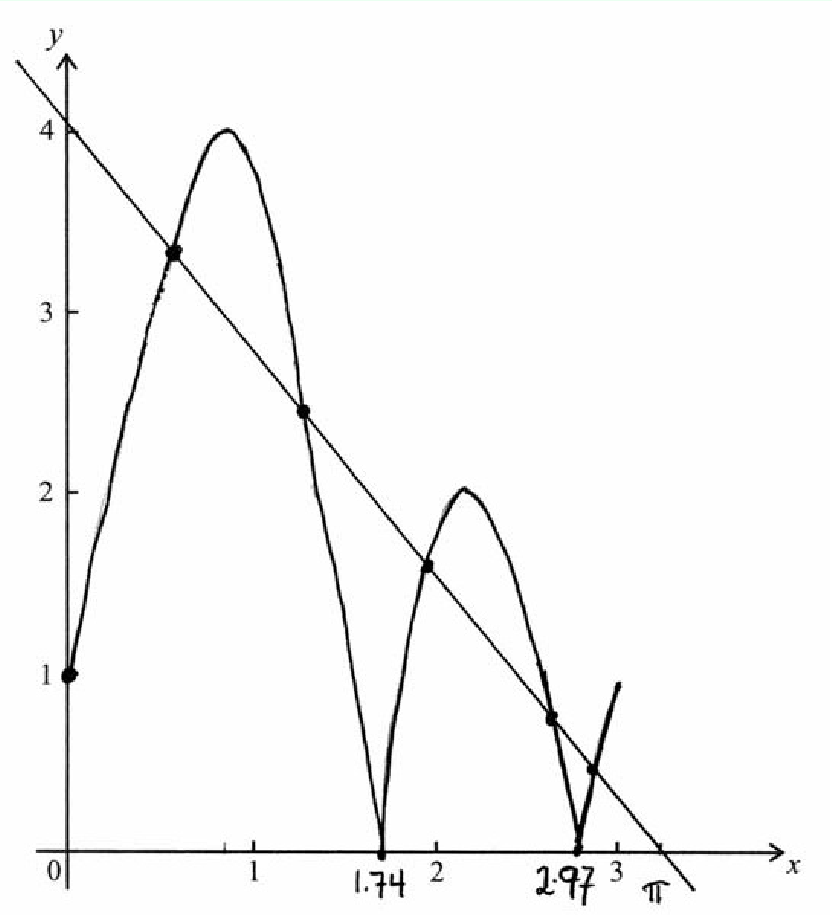
Examiners report

Part (a) was correctly done by the vast majority of candidates. In contrast, only the very best students gave the correct answer to part (b). Part (c) was correctly started by a majority of candidates, but many did not realise that they needed to use logarithms and were careless about the use of notation

23. [6 marks]

Markscheme

(a)



A1A1A1A1

Note: Award ***A1*** for y-intercept

A1A1 for x-intercepts

A1 for shape

(b) correct line ***A1***

5 solutions ***A1***

[6 marks]

Examiners report

Part (a) was well executed by the majority of candidates. Most candidates had the correct graph with the correct x and y intercepts.

For part (b), some candidates had the straight line intersect the x-axis at 3 rather than at

π , and hence did not observe that there were 5 points of intersection.

24. *[6 marks]*

Markscheme

METHOD 1

As $(x+1)$ is a factor of $P(x)$, then $P(-1) = 0$ ***(M1)***

$\Rightarrow a - b + 1 = 0$ (or equivalent) ***A1***

As $(x - 2)$ is a factor of $P(x)$, then $P(2) = 0$ ***(M1)***

$\Rightarrow 4a + 2b + 10 = 0$ (or equivalent) ***A1***

Attempting to solve for a and b ***M1***

$a = -2$ and $b = -1$ ***A1 N1***

[6 marks]

METHOD 2

By inspection third factor must be $x - 1$. ***(M1)A1***

$(x+1)(x-2)(x-1) = x^3 - 2x^2 - x + 2$ ***(M1)A1***

Equating coefficients $a = -2$, $b = -1$ ***(M1)A1 N1***

[6 marks]

METHOD 3

Considering

$\frac{P(x)}{x^2-x-2}$ or equivalent ***(M1)***

$\frac{P(x)}{x^2-x-2} = (x+a+1) + \frac{(a+b+3)x+2(a+2)}{x^2-x-2}$ ***A1A1***

Recognising that

$(a+b+3)x + 2(a+2) = 0$ ***(M1)***

Attempting to solve for a and b ***M1***

$a = -2$ and $b = -1$ ***A1 N1***

[6 marks]

Examiners report

Most candidates successfully answered this question. The majority used the factor theorem, but a few employed polynomial division or a method based on inspection to determine the third linear factor.

25.

[6 marks]

Markscheme

(a)

$h(x) = g\left(\frac{4}{x+2}\right)$ ***(M1)***

$= \frac{4}{x+2} - 1 \quad \left(= \frac{2-x}{2+x}\right)$ ***A1***

(b) **METHOD 1**

$x = \frac{4}{y+2} - 1$ (interchanging x and y) ***M1***

Attempting to solve for y ***M1***

$(y+2)(x+1) = 4 \quad \left(y + 2 = \frac{4}{x+1}\right)$ ***(A1)***

$h^{-1}(x) = \frac{4}{x+1} - 2 \quad (x \neq -1)$ ***A1 N1***

METHOD 2

$x = \frac{2-y}{2+y}$ (interchanging x and y) ***M1***

Attempting to solve for y ***M1***

$xy + y = 2 - 2x \quad (y(x+1) = 2(1-x))$ ***(A1)***

$h^{-1}(x) = \frac{2(1-x)}{x+1} \quad (x \neq -1)$ ***A1 N1***

Note: In either **METHOD 1** or **METHOD 2** rearranging first and interchanging afterwards is equally acceptable.

[6 marks]

Examiners report

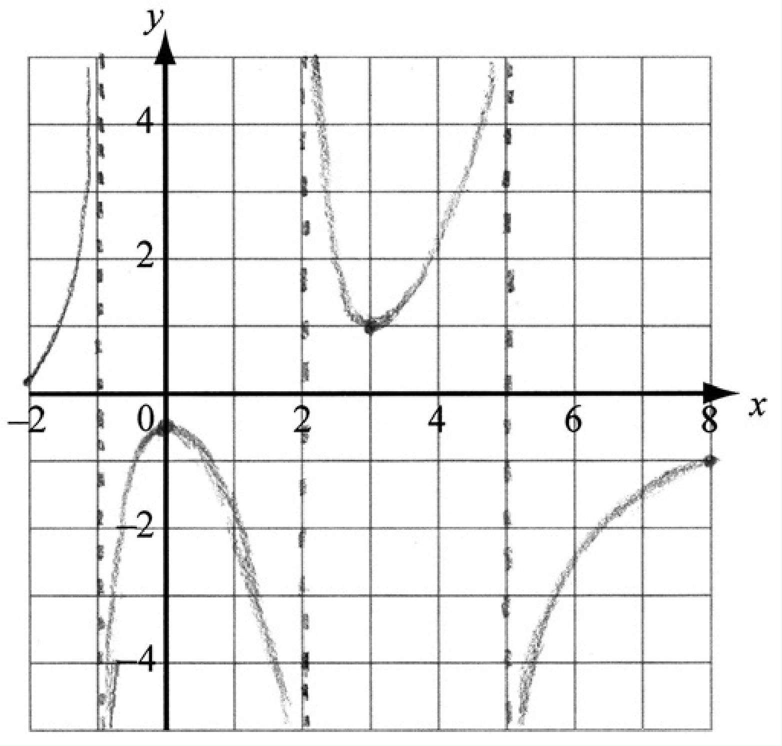
This question was generally well done, with very few candidates calculating

$f \circ g$ rather than

$g \circ f$.

26. *[5 marks]*

Markscheme



A1A1A1A1A1

Notes: Award ***A1*** for vertical asymptotes at $x = -1$, $x = 2$ and $x = 5$.

A1 for

$x \to -2,\ \frac{1}{f(x)} \to 0^+$

A1 for

$x \to 8,\ \frac{1}{f(x)} \to -1$

A1 for local maximum at

$\left(0, -\frac{1}{2}\right)$ (branch containing local max. must be present)

A1 for local minimum at (3, 1) (branch containing local min. must be present)

In each branch, correct asymptotic behaviour must be displayed to obtain the ***A1***.

Disregard any stated horizontal asymptotes such as $y = 0$ or $y = -1$.

[5 marks]

Examiners report

A large number of candidates had difficulty graphing the reciprocal function. Most candidates were able to locate the vertical asymptotes but experienced difficulties graphing the four constituent branches. A common error was to specify incorrect coordinates of the local maximum *i.e.* (0,–1) or (0,–2) instead of

$\left(0, -\frac{1}{2}\right)$. A few candidates attempted to sketch the inverse while others had difficulty using the scaled grid.

27. *[6 marks]*

Markscheme

Attempting to solve

$|0.1x^2 - 2x + 3| = \log_{10} x$ numerically or graphically. ***(M1)***

$x = 1.52,\ 1.79$ ***(A1)(A1)***

$x = 17.6,\ 19.1$ ***(A1)***

$(1.52 < x < 1.79) \cup (17.6 < x < 19.1)$ ***A1A1*** ***N2***

[6 marks]

Examiners report

This question was generally not well done. A number of candidates attempted an 'ill-fated' algebraic approach. Most candidates who used their GDC were able to correctly locate one inequality. The few successful candidates were able to employ a suitable window or suitable window(s) to correctly locate both inequalities.

28. *[6 marks]*

Markscheme

$f(2) = 16 + 24 + 4p - 4 + q = 15$ ***M1***

$\Rightarrow 4p + q = -21$ ***A1***

$f(-3) = 81 - 81 + 9p + 6 + q = 0$ ***M1***

$\Rightarrow 9p + q = -6$ ***A1***

$\Rightarrow p = 3$ and $q = -33$ ***A1A1*** ***N0***

[6 marks]

Examiners report

Most candidates made a meaningful attempt at this question. Weaker candidates often made arithmetic errors and a few candidates tried using long division, which also often resulted in arithmetic errors. Overall there were many fully correct solutions.

29. *[5 marks]*

Markscheme

$\ln(x^2 - 1) - \ln(x + 1)^2 + \ln x(x + 1)$ ***(A1)***

$= \ln \frac{x(x^2-1)(x+1)}{(x+1)^2}$ ***(M1)A1***

$= \ln \frac{x(x+1)(x-1)(x+1)}{(x+1)^2}$ ***(A1)***

$= \ln x(x - 1) \quad \left(= \ln(x^2 - x)\right)$ ***A1***

[5 marks]

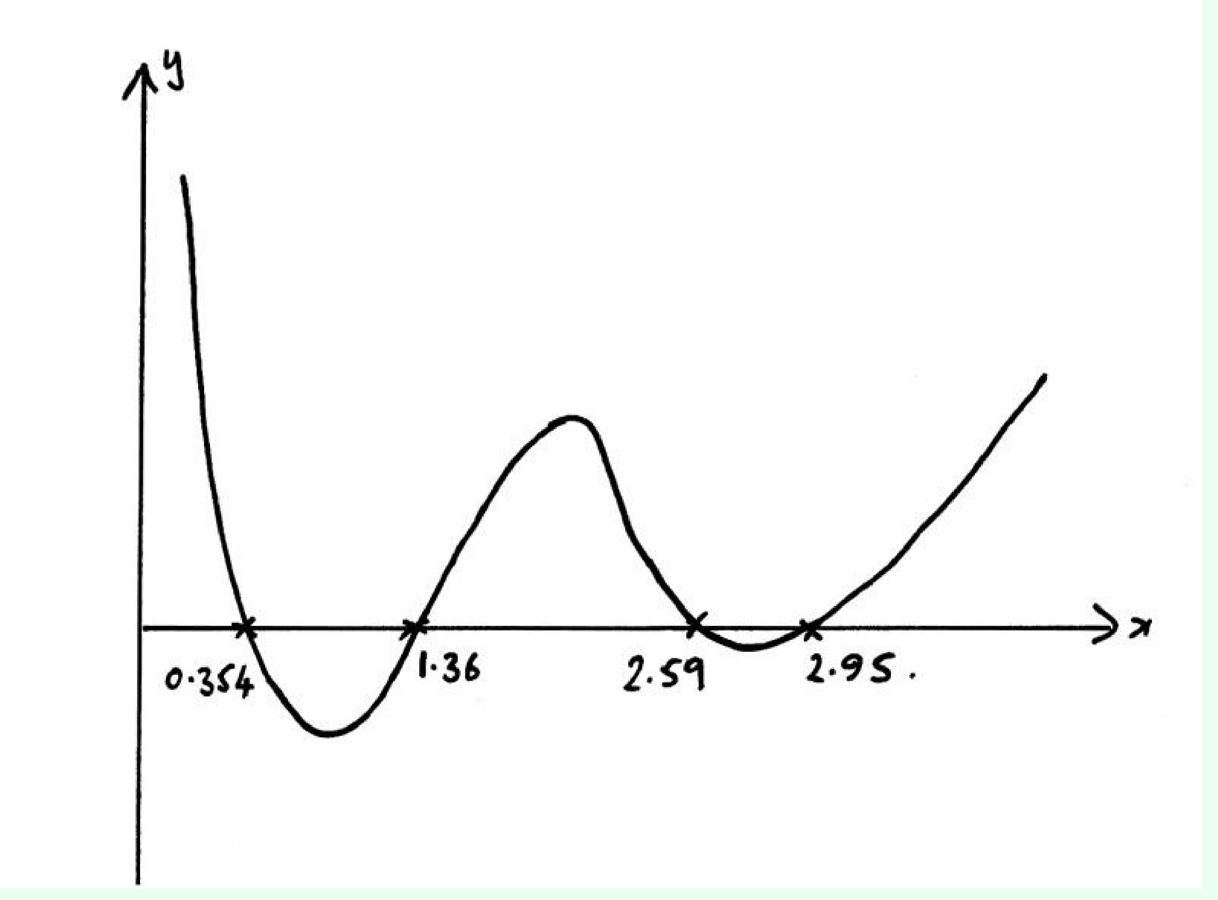
Examiners report

There were fewer correct solutions to this question than might be expected. A significant number of students managed to combine the terms to form one logarithm, but rather than factorising, then expanded the brackets, which left them unable to gain an answer in its simplest form.

30. *[7 marks]*

Markscheme

(a)



A1

Note: Award ***A1*** for shape.

x-intercepts 0.354, 1.36, 2.59, 2.95 ***A2***

Note: Award ***A1*** for three correct, ***A0*** otherwise.

maximum = (1.57, 0.352) =
$\left(\frac{\pi}{2}, 0.352\right)$ ***A1***
minimum = (1, – 0.640) and (2.77, – 0.0129) ***A1***

(b)
$0 < x < 0.354,\ 1.36 < x < 2.59,\ 2.95 < x < 4$ ***A2***
Note: Award ***A1*** if two correct regions given.

[7 marks]

Examiners report

Solutions to this question were extremely disappointing with many candidates doing the sketch in degree mode instead of radian mode. The two adjacent intercepts at 2.59 and 2.95 were often missed due to an unsatisfactory window. Some sketches were so small that a magnifying glass was required to read some of the numbers; candidates would be well advised to draw sketches large enough to be easily read.

31. *[5 marks]*

Markscheme

$g(x) = 0$

$\log_5 |2\log_3 x| = 0$ ***(M1)***

$|2\log_3 x| = 1$ ***A1***

$\log_3 x = \pm\frac{1}{2}$ ***(A1)***

$x = 3^{\pm\frac{1}{2}}$ ***A1***

so the product of the zeros of g is

$3^{\frac{1}{2}} \times 3^{-\frac{1}{2}} = 1$ ***A1*** ***N0***

[5 marks]

Examiners report

There were many candidates showing difficulties in manipulating logarithms and the absolute value to solve the equation.

32. [8 marks]

Markscheme

(a) **METHOD 1**

$V = a^3 - \frac{1}{a^3}$ ***A1***

$x^3 = \left(a - \frac{1}{a}\right)^3$ ***M1***

$= a^3 - 3a + \frac{3}{a} - \frac{1}{a^3}$

$= a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right)$ (or equivalent) ***(A1)***

$\Rightarrow a^3 - \frac{1}{a^3} = x^3 + 3x$

$V = x^3 + 3x$ ***A1*** ***N0***

METHOD 2

$V = a^3 - \frac{1}{a^3}$ ***A1***

attempt to use difference of cubes formula,

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ ***M1***

$V = \left(a - \frac{1}{a}\right)\left(a^2 + 1 + \left(\frac{1}{a}\right)^2\right)$

$= \left(a - \frac{1}{a}\right)\left(\left(a - \frac{1}{a}\right)^2 + 3\right)$ ***(A1)***

$= x(x^2 + 3)$ or $x^3 + 3x$ ***A1*** ***N0***

METHOD 3

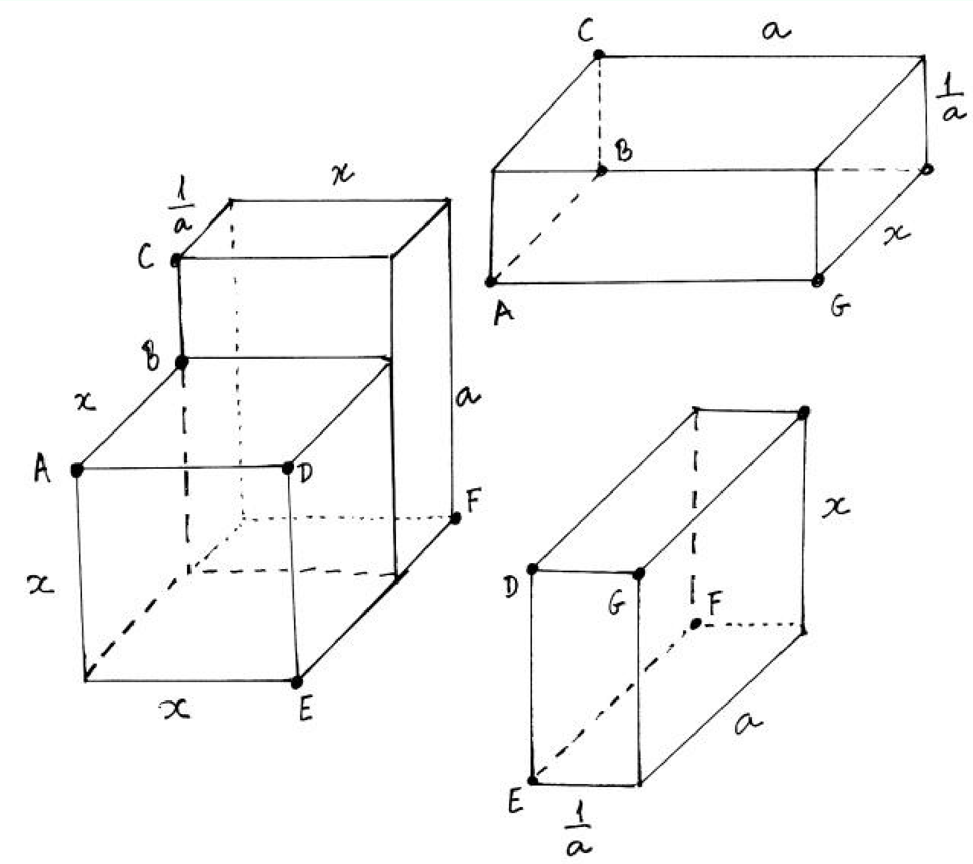


diagram showing that the solid can be decomposed ***M1***

into three congruent

$x \times a \times \frac{1}{a}$ cuboids with volume x ***A1***

and a cube with edge x with volume

x^3 ***A1***

so,

$V = x^3 + 3x$ ***A1*** ***N0***

(b)

Note: Do not accept any method where candidate substitutes the given value of a into
$x = a - \frac{1}{a}$.

METHOD 1

$V = 4x \Leftrightarrow x^3 + 3x = 4x \Leftrightarrow x^3 - x = 0$ ***M1***

$\Leftrightarrow x(x-1)(x+1) = 0$

$\Rightarrow x = 1$ as

$x > 0$ ***A1***

so,

$a - \frac{1}{a} = 1 \Rightarrow a^2 - a - 1 = 0 \Rightarrow a = \frac{1 \pm \sqrt{5}}{2}$ ***M1A1***

as

$a > 1$,

$a = \frac{1+\sqrt{5}}{2}$ ***AG*** ***N0***

METHOD 2

$a^3 - \frac{1}{a^3} = 4\left(a - \frac{1}{a}\right) \Rightarrow a^6 - 4a^4 + 4a^2 - 1 = 0 \Leftrightarrow (a^2 - 1)(a^4 - 3a^2 + 1) = 0$ ***M1A1***

as

$a > 1 \Rightarrow a^2 > 1,$

$a^2 = \frac{3+\sqrt{5}}{2} \Leftrightarrow a^2 = \sqrt{\left(\frac{1+\sqrt{5}}{2}\right)^2}$ ***M1A1***

$\Rightarrow a = \frac{1+\sqrt{5}}{2}$ ***AG*** ***N0***

[8 marks]

Examiners report

A fair amount of candidates had difficulties with this question. In part (a) many candidates were able to write down an expression for the volume in terms of a, but thereafter were largely unsuccessful. There is evidence that many candidates have lack of algebraic skills to manipulate the expression and obtain the volume in terms of x. In part (b) some candidates started with what they were trying to show to be true.

33. *[5 marks]*

Markscheme

$q(-1) = k + 9$ ***M1A1***

$q(-2) = 4k + 9$ ***A1***

$k + 9 = 7(4k + 9)$ ***M1***

$k = -2$ ***A1***

Notes: The first ***M1*** is for one substitution and the consequent equations.

Accept expressions for

$q(-1)$ and

$q(-2)$ that are not simplified.

[5 marks]

Examiners report

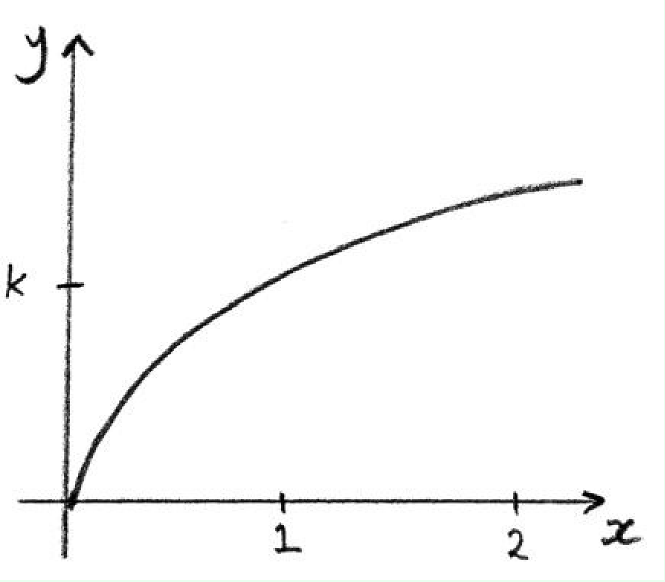
Most candidates were able to access this question although the number who used either synthetic division or long division was surprising as this often lead to difficulty and errors. The most common error was in applying the factor of 7 to the wrong side of the equation. It was also disappointing the number of students who made simple algebraic errors late in the question.

34.

[16 marks]

Markscheme

(a)



A1

Note: Award ***A1*** for correct concavity, passing through (0, 0) and increasing.

Scales need not be there.

[1 mark]

(b) a statement involving the application of the Horizontal Line Test or equivalent ***A1***

[1 mark]

(c)

$y = k\sqrt{x}$

for either

$x = k\sqrt{y}$ or

$x = \frac{y^2}{k^2}$ ***A1***

$f^{-1}(x) = \frac{x^2}{k^2}$ ***A1***

$\text{dom}\left(f^{-1}(x)\right) = [0, \infty[$ ***A1***

[3 marks]

(d)

$\frac{x^2}{k^2} = k\sqrt{x}$ or equivalent method ***M1***

$k = \sqrt{x}$

$k = 2$ ***A1***

[2 marks]

(e) (i)

$A = \int_a^b (y_1 - y_2)\mathrm{d}x$ ***(M1)***

$A = \int_0^4 \left(2x^{\frac{1}{2}} - \frac{1}{4}x^2\right)\mathrm{d}x$ ***A1***

$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{12}x^3\right]_0^4$ ***A1***

$= \frac{16}{3}$ ***A1***

(ii) attempt to find either

$f'(x)$ or

$(f^{-1})'(x)$ ***M1***

$f'(x) = \frac{1}{\sqrt{x}}$, $\left((f^{-1})'(x) = \frac{x}{2}\right)$ ***A1A1***

$\frac{1}{\sqrt{c}} = \frac{c}{2}$ ***M1***

$c = 2^{\frac{2}{3}}$ ***A1***

[9 marks]

Total [16 marks]

Examiners report

Many students could not sketch the function. There was confusion between the vertical and horizontal line test for one-to-one functions. A significant number of students gave long and inaccurate explanations for a one-to-one function. Finding the inverse was done very well by most students although the notation used was generally poor. The domain of the inverse was ignored by many or done incorrectly even if the sketch was correct. Many did not make the connections between the parts of the question. An example of this was the number of students who spent time finding the point of intersection in part e) even though it was given in d).

35.

[5 marks]

Markscheme

(a)

$f(1) = 3 - a + b$ ***(A1)***

$f(-1) = -3 + a + b$ ***(A1)***

$3 - a + b = -3 + a + b$ ***M1***

$2a = 6$

$a = 3$ ***A1*** ***N4***

(b) b is any real number ***A1***

[5 marks]

Examiners report

Many candidates answered part (a) successfully. For part (b), some candidates did not consider that the entire set of real numbers was asked for.

36. *[6 marks]*

Markscheme

(a)

$-1 \leqslant \ln x \leqslant 1$ ***(M1)***

$\Rightarrow \frac{1}{\mathrm{e}} \leqslant x \leqslant \mathrm{e}$ ***A1A1***

(b)

$y = \arcsin(\ln x) \Rightarrow \ln x = \sin y$ ***(M1)***

$\ln y = \sin x \Rightarrow y = \mathrm{e}^{\sin x}$ ***(M1)***

$\Rightarrow f^{-1}(x) = \mathrm{e}^{\sin x}$ ***A1***

[6 marks]

Examiners report

Very few candidates attempted part (a), and of those that did, few were successful. Part (b) was answered fairly well by most candidates.

37. *[4 marks]*

Markscheme

$g(x) = 0$ or 3 ***(M1)(A1)***

$x = -1$ or 4 or 1 or 2 ***A1A1***

Notes: Award ***A1A1*** for all four correct values,

A1A0 for two or three correct values,

A0A0 for less than two correct values.

Award ***M1*** and corresponding ***A*** marks for correct attempt to find expressions for f and g.

[4 marks]

Examiners report

A small number of candidates gave correct and well explained answers. Many candidates answered the question without showing any kind of work and in many cases it was clear that candidates were guessing and clearly did not know about composition of functions. A number of candidates attempted to find expressions for both functions but made little progress and wasted time.

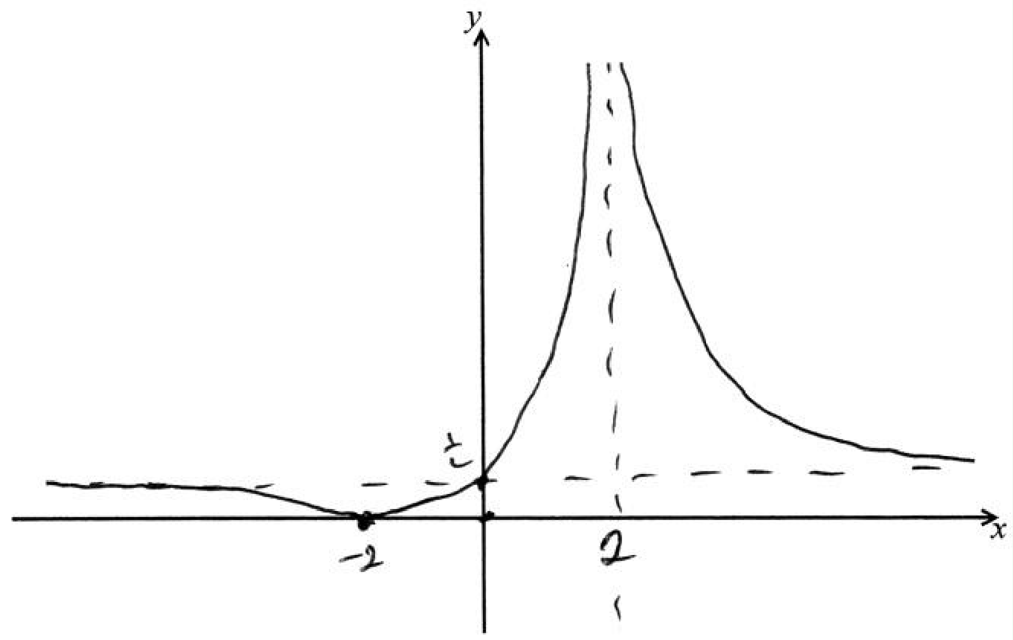
38. *[8 marks]*

Markscheme

(a) an attempt to use either asymptotes or intercepts ***(M1)***

$a = -2,\ b = 1,\ c = \frac{1}{2}$ ***A1A1A1***

(b)



A4

Note: Award ***A1*** for both asymptotes,

A1 for both intercepts,

A1, ***A1*** for the shape of each branch, ignoring shape at

$(x = -2)$.

[8 marks]

Examiners report

It was pleasing to see a lot of good work with part (a), though some candidates lost marks due to problems with the algebra which led to one or more incorrect values. Regarding part (b), most candidates did not succeed in finding the new intercepts and asymptotes and were unable to apply the absolute value function. A significant number of candidates misread part (b) and took it as the modulus of the graph in part (a).

39. *[6 marks]*

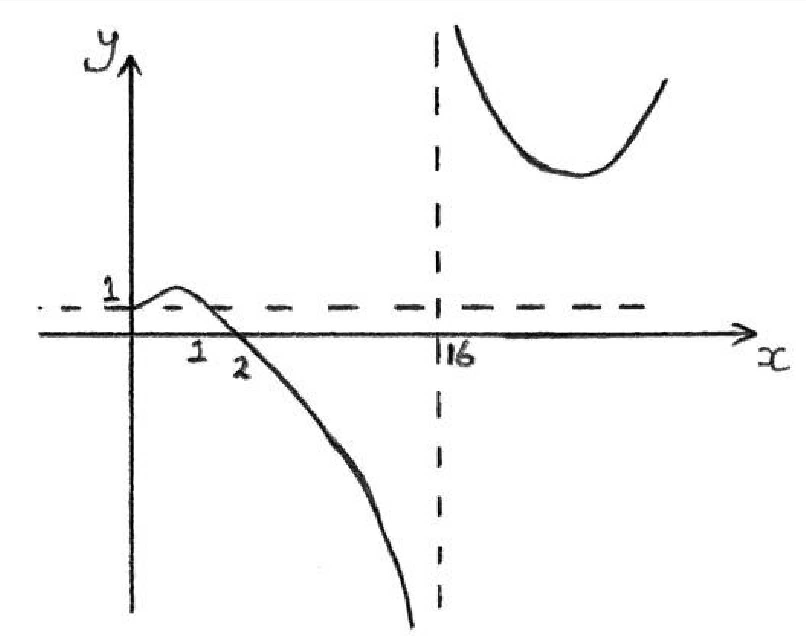
Markscheme

(a)

$x \geqslant 0$ and

$x \neq 16$ ***A1A1***

(b)



graph not to scale

finding crossing points ***(M1)***

e.g.

$4 - x^2 = 4 - \sqrt{x}$

$x = 0$ or $x = 1$ ***(A1)***

$0 \leqslant x \leqslant 1$ or

$x > 16$ ***A1A1***

Note: Award ***M1A1A1A0*** for solving the inequality only for the case

$x < 16$.

[6 marks]

Examiners report

Most students were able to obtain partial marks, but there were very few completely correct answers.

40. *[6 marks]*

Markscheme

(a) attempt at completing the square *(M1)*

$3x^2 - 6x + 5 = 3(x^2 - 2x) + 5 = 3(x-1)^2 - 1 + 5$ *(A1)*

$= 3(x-1)^2 + 2$ ***A1***

$(a = 3,\ b = -1,\ c = 2)$

(b) definition of suitable basic transformations:

$\mathrm{T}_1 =$ stretch in *y* direction scale factor 3 ***A1***

$\mathrm{T}_2 =$ translation

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ***A1***

$\mathrm{T}_3 =$ translation

$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ ***A1***

[6 marks]

Examiners report

There were fewer correct solutions to this question than might be expected with a significant minority of candidates unable to complete the square successfully and a number of candidates unable to describe the transformations. A minority of candidates knew the correct terminology for the transformations and this potentially highlights the need for teachers to teach students appropriate terminology.

41. *[6 marks]*

Markscheme

(a) **Note:** Interchange of variables may take place at any stage.

for the inverse, solve for x in

$y = \frac{2x-3}{x-1}$

$y(x-1) = 2x - 3$ ***M1***

$yx - 2x = y - 3$

$x(y-2) = y - 3$ ***(A1)***

$x = \frac{y-3}{y-2}$

$\Rightarrow f^{-1}(x) = \frac{x-3}{x-2} \ \ (x \neq 2)$ ***A1***

Note: Do not award final ***A1*** unless written in the form

$f^{-1}(x) = \ldots$

(b)

$\pm f^{-1}(x) = 1 + f^{-1}(x)$ leads to

$2\frac{x-3}{x-2} = -1$ ***(M1)***

$x = \frac{8}{3}$ ***A1***

[6 marks]

Examiners report

Many candidates gained the correct answer to part (a), although a significant minority left the answer in the form

$y = \ldots$ or $x = \ldots$ rather than

$f^{-1}(x) = \ldots$. Only the better candidates were able to make significant progress in part (b).

42.

[6 marks]

Markscheme

(a) rewrite the equation as

$(4x-1)\ln 2 = (x+5)\ln 8 + (1-2x)\log_2 16$ ***(M1)***

$(4x-1)\ln 2 = (3x+15)\ln 2 + 4 - 8x$ ***(M1)(A1)***

$x = \frac{4+16\ln 2}{8+\ln 2}$ ***A1***

(b)

$x = a^2$ ***(M1)***

$a = 1.318$ ***A1***

Note: Treat 1.32 as an ***AP***.

Award ***A0*** for ±.

[6 marks]

Examiners report

A more difficult question. Many candidates failed to read the question carefully so did not express x in terms of $\ln 2$.

43.

[6 marks]

Markscheme

(a) the expression is

$\frac{n!}{(n-3)!3!} - \frac{(2n)!}{(2n-2)!2!}$ ***(A1)***

$\frac{n(n-1)(n-2)}{6} - \frac{2n(2n-1)}{2}$ ***M1A1***

$= \frac{n(n^2-15n+8)}{6} \left(= \frac{n^3-15n^2+8n}{6}\right)$ ***A1***

(b) the inequality is

$\frac{n^3-15n^2+8n}{6} > 32n$

attempt to solve cubic inequality or equation ***(M1)***

$n^3 - 15n^2 - 184n > 0 \quad n(n-23)(n+8) > 0$

$n > 23 \quad (n \geqslant 24)$ ***A1***

[6 marks]

Examiners report

Part(a) - Although most understood the notation, few knew how to simplify the binomial coefficients.

Part(b) - Many were able to solve the cubic, but some failed to report their answer as an integer inequality.

44. *[14 marks]*

Markscheme

(a) solving to obtain one root: 1, – 2 or – 5 ***A1***

obtain other roots ***A1***

[2 marks]

(b)

$D = x \in [-5,\ -2] \cup [1,\ \infty)$ (or equivalent) ***M1A1***

Note: ***M1*** is for 1 finite and 1 infinite interval.

[2 marks]

(c) coordinates of local maximum

$-3.73 - 2 - \sqrt{3},\ 3.22\sqrt{6\sqrt{3}}$ ***A1A1***

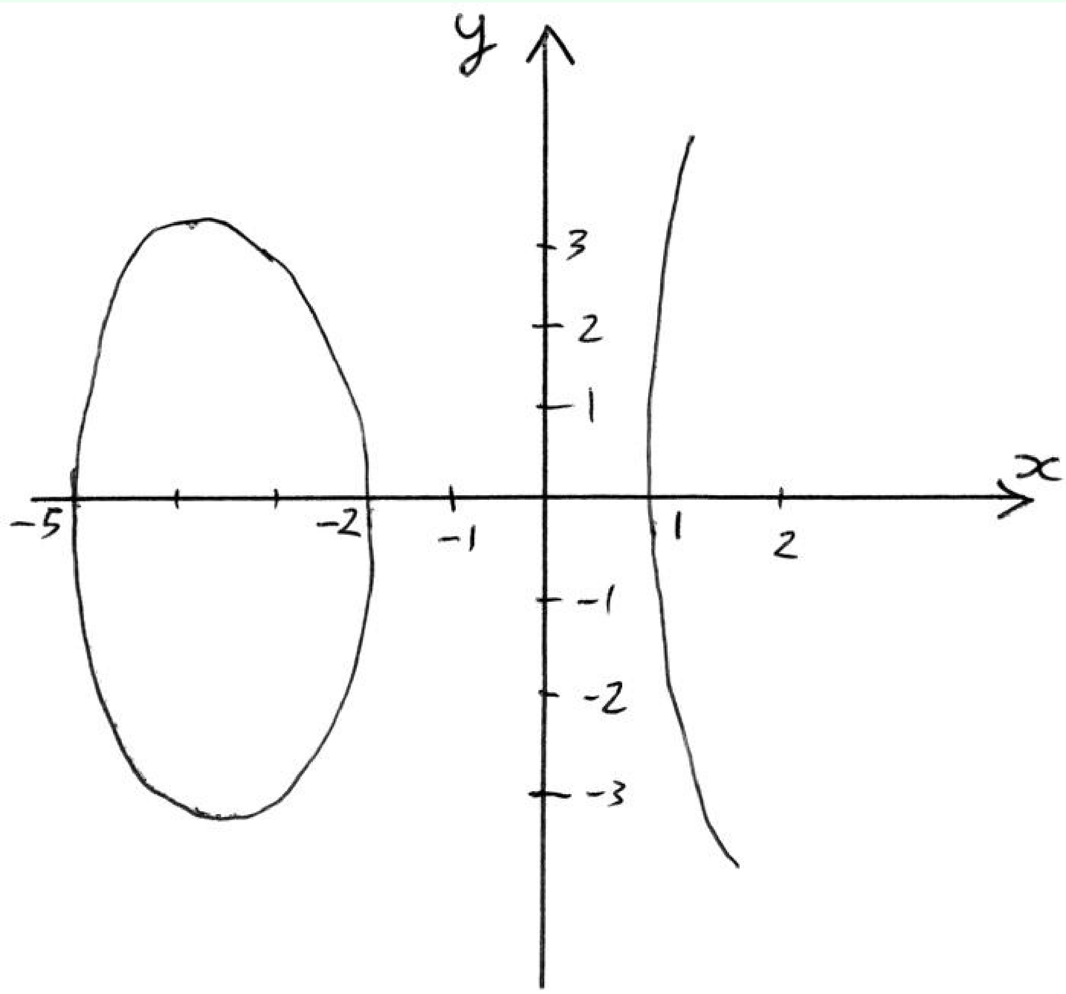
[2 marks]

(d) use GDC to obtain one root: 1.41, – 3.18 or – 4.23 ***A1***

obtain other roots ***A1***

[2 marks]

(e)



A1A1A1

Note: Award ***A1*** for shape, ***A1*** for max and for min clearly in correct places, ***A1*** for all intercepts.

Award ***A1A0A0*** if only the complete top half is shown.

[3 marks]

(f) required area is twice that of

$y = f(x)$ between – 5 and – 2 ***M1A1***

answer 14.9 ***A1*** ***N3***

Note: Award ***M1A0A0*** for

$\int_{-5}^{-2} f(x)\mathrm{d}x = 7.47\ldots$ or ***N1*** for 7.47.

[3 marks]

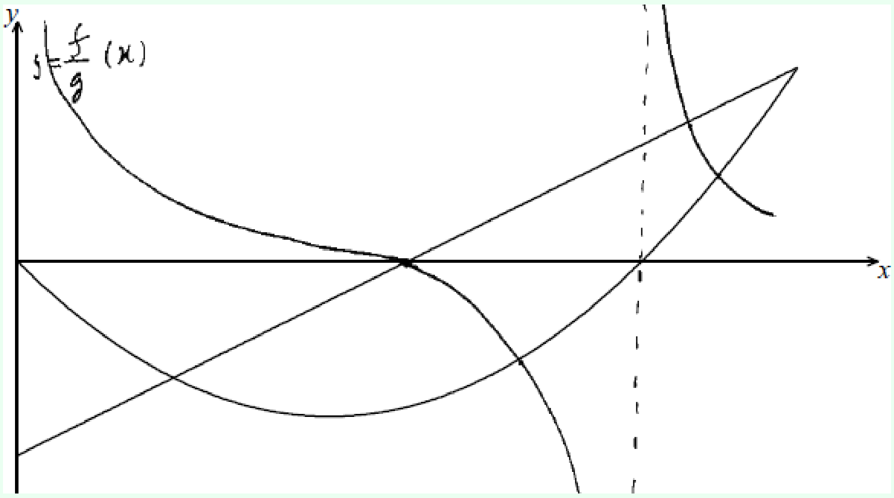
Total [14 marks]

Examiners report

This was a multi-part question that was well answered by many candidates. The main difficulty was sketching the graph and this meant that the last part was not well answered.

45. *[5 marks]*

Markscheme



correct concavities ***A1A1***

Note: Award ***A1*** for concavity of each branch of the curve.

correct x-intercept of

$\frac{f}{g}$ (which is EXACTLY the x-intercept of f) ***A1***

correct vertical asymptotes of

$\frac{f}{g}$ (which ONLY occur when x equals the x-intercepts of g) ***A1A1***

[5 marks]

Examiners report

Many candidates answered well this question. Full marks were often achieved. Many other candidates did not attempt it at all.

46.

[5 marks]

Markscheme

from GDC, sketch a relevant graph ***A1***

maximum:
$y = 3$ or (–1, 3) ***A1***

minimum:
$y = 1.81$ or (0.333, 1.81)
$\left(\text{or } y = \frac{49}{27} \text{ or } \left(\frac{1}{3}, \frac{49}{27}\right)\right)$ ***A1***

hence,
$1.81 < k < 3$ ***A1A1*** ***N3***

Note: Award ***A1*** for
$1.81 \leqslant k \leqslant 3$.

[5 marks]

Examiners report

Responses to this question were surprisingly poor. Few candidates recognised that the easier way to answer the question was to use a graph on the GDC. Many candidates embarked on fruitless algebraic manipulation which led nowhere.

47.

[8 marks]

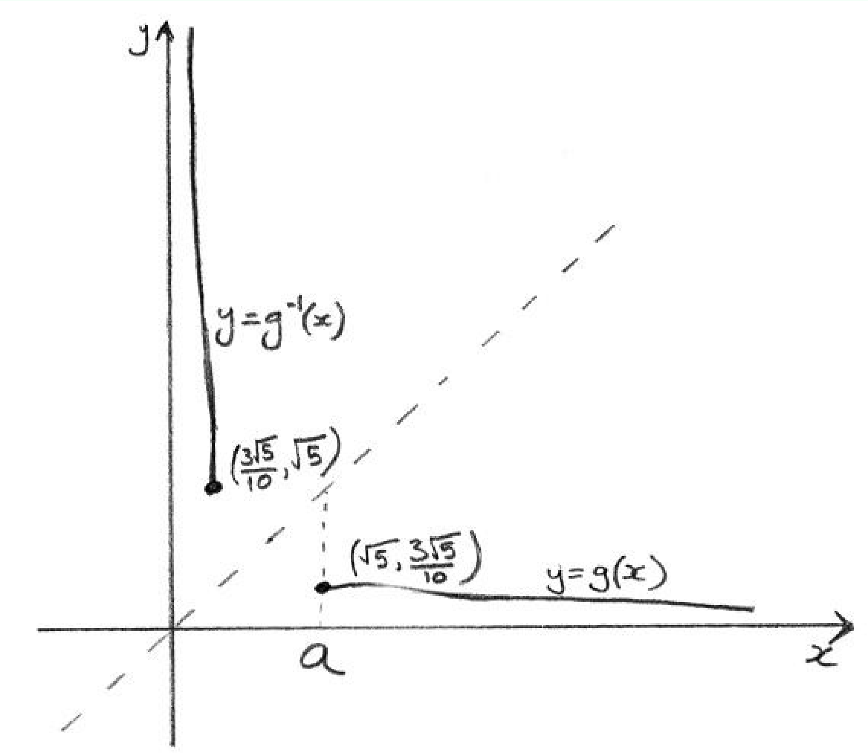
Markscheme

(a)

$a = 2.24$

$\sqrt{5}$ ***A1***

(b) (i)



A2

Note: Award ***A1*** for end point

A1 for its asymptote.

(ii) sketch of

g^{-1} (see above) ***A2***

Note: Award ***A1*** for end point

A1 for its asymptote.

(c)

$y = \frac{3x}{5+x^2} \Rightarrow yx^2 - 3x + 5y = 0$ ***M1***

$\Rightarrow x = \frac{3\pm\sqrt{9-20y^2}}{2y}$ ***A1***

$g^{-1}(x) = \frac{3\pm\sqrt{9-20x^2}}{2x}$ ***A1***

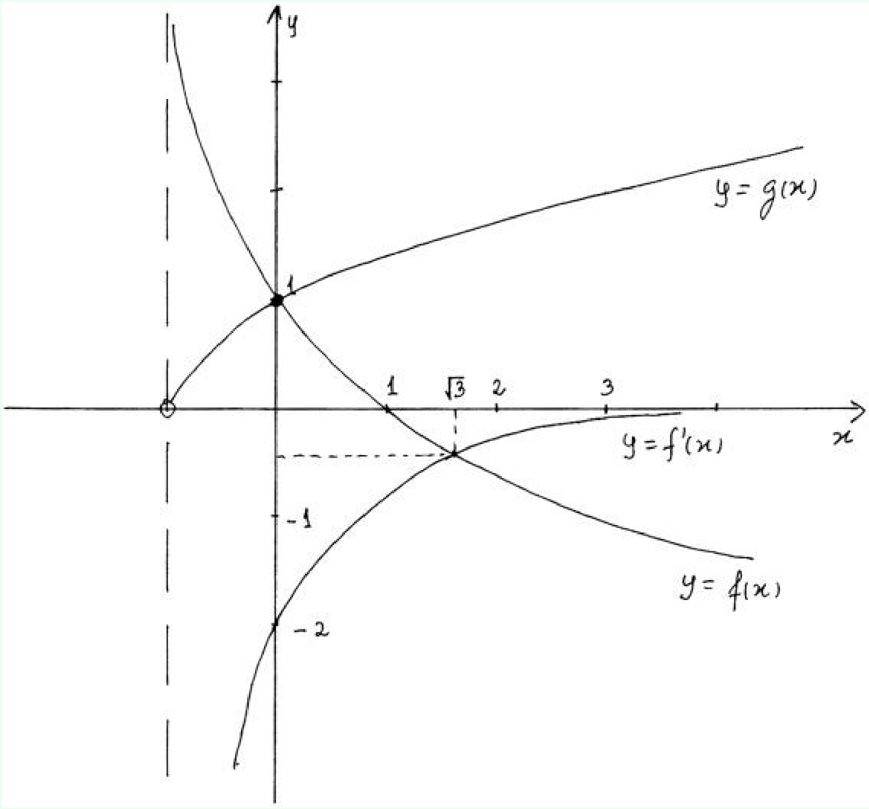
[8 marks]

Examiners report

Very few completely correct answers were given to this question. Many students found a to be 0 and many failed to provide adequate sketches. There were very few correct answers to part (c) although many students were able to obtain partial marks.

48. [7 marks]

Markscheme



$f'(x) = \frac{-2}{(1+x)^2}$ ***M1A1***

Note: Alternatively, award ***M1A1*** for correct sketch of the derivative.

find at least one point of intersection of graphs ***(M1)***

$y = f(x)$ and
$y = f'(x)$ for
$x = \sqrt{3}$ or
1.73 ***(A1)***

$y = f(x)$ and
$y = g(x)$ for
$x = 0$ ***(A1)***

forming inequality
$0 \leqslant x \leqslant \sqrt{3}$ (or
$0 \leqslant x \leqslant 1.73$) ***A1A1*** ***N4***

Note: Award ***A1*** for correct limits and ***A1*** for correct inequalities.

[7 marks]

Examiners report

Most students were able to find the derived function correctly, although attempts to solve the inequality algebraically were often unsuccessful. This was a question where students prepared in good use of GDC were able to easily obtain good marks.

49a. *[2 marks]*

Markscheme

EITHER

translation of $-\frac{1}{2}$ parallel to the x-axis

stretch of a scale factor of $\frac{1}{2}$ parallel to the x-axis ***A1A1***

OR

stretch of a scale factor of $\frac{1}{2}$ parallel to the x-axis

translation of -1 parallel to the x-axis ***A1A1***

Note: Accept clear alternative terminologies for either transformation.

[2 marks]

Examiners report

This question was well done by many candidates. It would appear, however, that few candidates were aware of the standard terminology – *Stretch* and *Translation* - used to describe the relevant graph transformations. Most made good use of a GDC to find the critical points and to help in deciding on the correct intervals. A significant minority failed to note $x = 10$ as an endpoint.

49b. *[4 marks]*

Markscheme

EITHER

$1.16 < x < 5.71 \cup 6.75 < x \leqslant 10$ ***A1A1A1A1***

OR

$]1.16, 5.71[\cup]6.75, 10]$ ***A1A1A1A1***

Note: Award ***A1*** for 1 intersection value, ***A1*** for the other 2, ***A1A1*** for the intervals.

[6 marks]

Examiners report

This question was well done by many candidates. It would appear, however, that few candidates were aware of the standard terminology – *Stretch* and *Translation* - used to describe the relevant graph transformations. Most made good use of a GDC to find the critical points and to help in deciding on the correct intervals. A significant minority failed to note $x = 10$ as an endpoint.