

Topic 2 Part 4 [47 marks]

Consider the equation

$$9x^3 - 45x^2 + 74x - 40 = 0 .$$

1a. Write down the numerical value of the sum and of the product of the roots of this equation. [1 mark]

1b. The roots of this equation are three consecutive terms of an arithmetic sequence. [6 marks]

Taking the roots to be

α , $\alpha \pm \beta$, solve the equation.

The function f is defined, for

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2} , \text{ by}$$

$$f(x) = 2 \cos x + x \sin x .$$

2a. Determine whether f is even, odd or neither even nor odd. [3 marks]

2b. Show that [2 marks]

$$f''(0) = 0 .$$

2c. John states that, because [2 marks]

$f''(0) = 0$, the graph of f has a point of inflexion at the point $(0, 2)$. Explain briefly whether John's statement is correct or not.

The function f is defined on the domain

$$x \geq 0 \text{ by}$$

$$f(x) = e^x - x^e .$$

3a. (i) Find an expression for [3 marks]

$$f'(x) .$$

(ii) Given that the equation

$f'(x) = 0$ has two roots, state their values.

3b. Sketch the graph of f , showing clearly the coordinates of the maximum and minimum. [3 marks]

3c. Hence show that [1 mark]

$$e^\pi > \pi^e .$$

The function f is defined by

$$f(x) = \begin{cases} 2x - 1, & x \leq 2 \\ ax^2 + bx - 5, & 2 < x < 3 \end{cases}$$

where a ,

$b \in \mathbb{R}$.

4a. Given that f and its derivative, [6 marks]
 f' , are continuous for all values in the domain of f , find the values of a and b .

4b. Show that f is a one-to-one function. [3 marks]

4c. Obtain expressions for the inverse function [5 marks]
 f^{-1} and state their domains.

5. Given that $(x - 2)$ is a factor of [6 marks]
 $f(x) = x^3 + ax^2 + bx - 4$ and that division
 $f(x)$ by $(x - 1)$ leaves a remainder of -6 , find the value of a and the value of b .

6. The function f is of the form [6 marks]
 $f(x) = \frac{x+a}{bx+c}$,
 $x \neq -\frac{c}{b}$. Given that the graph of f has asymptotes $x = -4$ and $y = -2$, and that the point
 $(\frac{2}{3}, 1)$ lies on the graph, find the values of a , b and c .