Topic 1 Part 3 *[483 marks]*

1a. *[3 marks]*

Markscheme

$\mathrm{AB} = \sqrt{1^2 + (2 - \sqrt{3})^2}$ ***M1***

$= \sqrt{8 - 4\sqrt{3}}$ ***A1***

$= 2\sqrt{2 - \sqrt{3}}$ ***A1***

[3 marks]

Examiners report

It was disappointing to note the lack of diagram in many solutions. Most importantly the lack of understanding of the notation AB was apparent. Teachers need to make sure that students are aware of correct notation as given in the outline. A number used the cosine rule but then confused the required angle or sides.

1b. *[3 marks]*

Markscheme

METHOD 1

$\arg z_1 = -\frac{\pi}{4}$ $\arg z_2 = -\frac{\pi}{3}$ ***A1A1***

Note: Allow $\frac{\pi}{4}$ and $\frac{\pi}{3}$.

Note: Allow degrees at this stage.

$\mathrm{A\hat{O}B} = \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{\pi}{12}$ (accept $-\frac{\pi}{12}$) ***A1***

Note: Allow ***FT*** for final ***A1***.

METHOD 2

attempt to use scalar product or cosine rule ***M1***

$\cos \mathrm{A\hat{O}B} = \frac{1+\sqrt{3}}{2\sqrt{2}}$ ***A1***

$\mathrm{A\hat{O}B} = \frac{\pi}{12}$ ***A1***

[3 marks]

Examiners report

It was disappointing to note the lack of diagram in many solutions. Most importantly the lack of understanding of the notation AB was apparent. Teachers need to make sure that students are aware of correct notation as given in the outline. A number used the cosine rule but then confused the required angle or sides.

2a. *[3 marks]*

Markscheme

let the first three terms of the geometric sequence be given by u_1, $u_1 r$, $u_1 r^2$

$\therefore u_1 = a + 2d$, $u_1 r = a + 3d$ and $u_1 r^2 = a + 6d$ ***(M1)***

$\frac{a+6d}{a+3d} = \frac{a+3d}{a+2d}$ ***A1***

$a^2 + 8ad + 12d^2 = a^2 + 6ad + 9d^2$ ***A1***

$2a + 3d = 0$

$a = -\frac{3}{2}d$ ***AG***

[3 marks]

Examiners report

This question was done well by many students. Those who did not do it well often became involved in convoluted algebraic processes that complicated matters significantly. There were a number of different approaches taken which were valid.

2b. *[5 marks]*

Markscheme

$u_1 = \frac{d}{2}$, $u_1 r = \frac{3d}{2}$, $\left(u_1 r^2 = \frac{9d}{2}\right)$ ***M1***

r = 3 ***A1***

geometric 4^{th} term $u_1 r^3 = \frac{27d}{2}$ ***A1***

arithmetic 16^{th} term $a + 15d = -\frac{3}{2}d + 15d$ ***M1***

$= \frac{27d}{2}$ ***A1***

Note: Accept alternative methods.

[3 marks]

Examiners report

This question was done well by many students. Those who did not do it well often became involved in convoluted algebraic processes that complicated matters significantly. There were a number of different approaches taken which were valid.

3a. *[2 marks]*

Markscheme

using the factor theorem $z + 1$ is a factor ***(M1)***

$z^3 + 1 = (z+1)(z^2 - z + 1)$ ***A1***

[2 marks]

Examiners report

In part a) the factorisation was, on the whole, well done.

3b. *[9 marks]*

Markscheme

(i) **METHOD 1**

$z^3 = -1 \Rightarrow z^3 + 1 = (z+1)(z^2 - z + 1) = 0$ ***(M1)***

solving $z^2 - z + 1 = 0$ ***M1***

$z = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \mathrm{i}\sqrt{3}}{2}$ ***A1***

therefore one cube root of -1 is γ ***AG***

METHOD 2

$\gamma^2 = \left(\frac{1+i\sqrt{3}}{2}^2\right) = \frac{-1+i\sqrt{3}}{2}$ ***M1A1***

$\gamma^2 = \frac{-1+i\sqrt{3}}{2} \times \frac{1+i\sqrt{3}}{2} = \frac{-1-3}{4}$ ***A1***

$= -1$ AG

METHOD 3

$\gamma = \frac{1+i\sqrt{3}}{2} = e^{i\frac{\pi}{3}}$ ***M1A1***

$\gamma^3 = e^{i\pi} = -1$ ***A1***

(ii) **METHOD 1**

as γ is a root of $z^2 - z + 1 = 0$ then $\gamma^2 - \gamma + 1 = 0$ ***M1R1***

$\therefore \gamma^2 = \gamma - 1$ ***AG***

Note: Award ***M1*** for the use of $z^2 - z + 1 = 0$ in any way.

Award ***R1*** for a correct reasoned approach.

METHOD 2

$\gamma^2 = \frac{-1+i\sqrt{3}}{2}$ ***M1***

$\gamma - 1 = \frac{1+i\sqrt{3}}{2} - 1 = \frac{-1+i\sqrt{3}}{2}$ ***A1***

(iii) **METHOD 1**

$(1-\gamma)^6 = (-\gamma^2)^6$ ***(M1)***

$= (\gamma)^{12}$ ***A1***

$= (\gamma^3)^4$ ***(M1)***

$= (-1)^4$

$= 1$ ***A1***

METHOD 2

$(1-\gamma)^6$

$= 1 - 6\gamma + 15\gamma^2 - 20\gamma^3 + 15\gamma^4 - 6\gamma^5 + \gamma^6$ ***M1A1***

Note: Award ***M1*** for attempt at binomial expansion.

use of any previous result *e.g.* $= 1 - 6\gamma + 15\gamma^2 + 20 - 15\gamma + 6\gamma^2 + 1$ ***M1***

= 1 ***A1***

Note: As the question uses the word ‘hence’, other methods that do not use previous results are awarded no marks.

[9 marks]

Examiners report

Part (b) was done well by most although using a substitution method rather than the result above. This used much m retime than was necessary but was successful. A number of candidates did not use the previous results in part (iii) and so seemed to not understand the use of the ‘hence’.

4. *[5 marks]*

Markscheme

$u_4 = u_1 + 3d = 7$, $u_9 = u_1 + 8d = 22$ ***A1A1***

Note: $5d = 15$ gains both above marks

$u_1 = -2$, $d = 3$ ***A1***

$S_n = \frac{n}{2}\left(-4 + (n-1)3\right) > 10\,000$ ***M1***

$n = 83$ ***A1***

[5 marks]

Examiners report

This question was well answered by most candidates. A few did not realise that the answer had to be an integer.

5a. *[8 marks]*

Markscheme

prove that $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}$

for $n = 1$

$\text{LHS} = 1,\ \text{RHS} = 4 - \frac{1+2}{2^0} = 4 - 3 = 1$

so true for $n = 1$ ***R1***

assume true for $n = k$ ***M1***

so $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} = 4 - \frac{k+2}{2^{k-1}}$

now for $n = k + 1$

LHS: $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} + (k+1)\left(\frac{1}{2}\right)^k$ ***A1***

$= 4 - \frac{k+2}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k$ ***M1A1***

$= 4 - \frac{2(k+2)}{2^k} + \frac{k+1}{2^k}$ (or equivalent) ***A1***

$= 4 - \frac{(k+1)+2}{2^{(k+1)-1}}$ (accept $4 - \frac{k+3}{2^k}$) ***A1***

Therefore if it is true for $n = k$ it is true for $n = k + 1$. It has been shown to be true for $n = 1$ so it is true for all n $(\in \mathbb{Z}^+)$. ***R1***

Note: To obtain the final ***R*** mark, a reasonable attempt at induction must have been made.

[8 marks]

Examiners report

Part A: Given that this question is at the easier end of the ‘proof by induction’ spectrum, it was disappointing that so many candidates failed to score full marks. The $n = 1$ case was generally well done. The whole point of the method is that it involves logic, so ‘let n = k’ or ‘put n = k’, instead of ‘assume ... to be true for n = k’, gains no marks. The algebraic steps need to be more convincing than some candidates were able to show. It is astonishing that the R1 mark for the final statement was so often not awarded.

5b. *[17 marks]*

Markscheme

(a)

METHOD 1

$\int e^{2x} \sin x dx = -\cos x e^{2x} + \int 2e^{2x} \cos x dx$ ***M1A1A1***

$= -\cos x e^{2x} + 2e^{2x} \sin x - \int 4e^{2x} \sin x dx$ ***A1A1***

$5 \int e^{2x} \sin x dx = -\cos x e^{2x} + 2e^{2x} \sin x$ ***M1***

$\int e^{2x} \sin x dx = \frac{1}{5} e^{2x}(2 \sin x - \cos x) + C$ ***AG***

METHOD 2

$\int \sin x e^{2x} dx = \frac{\sin x e^{2x}}{2} - \int \cos x \frac{e^{2x}}{2} dx$ ***M1A1A1***

$= \frac{\sin x e^{2x}}{2} - \cos x \frac{e^{2x}}{4} - \int \sin x \frac{e^{2x}}{4} dx$ ***A1A1***

$\frac{5}{4} \int e^{2x} \sin x dx = \frac{e^{2x} \sin x}{2} - \frac{\cos x e^{2x}}{4}$ ***M1***

$\int e^{2x} \sin x dx = \frac{1}{5} e^{2x}(2 \sin x - \cos x) + C$ ***AG***

[6 marks]

(b)

$\int \frac{dy}{\sqrt{1-y^2}} = \int e^{2x} \sin x dx$ ***M1A1***

$\arcsin y = \frac{1}{5} e^{2x}(2 \sin x - \cos x)(+C)$ ***A1***

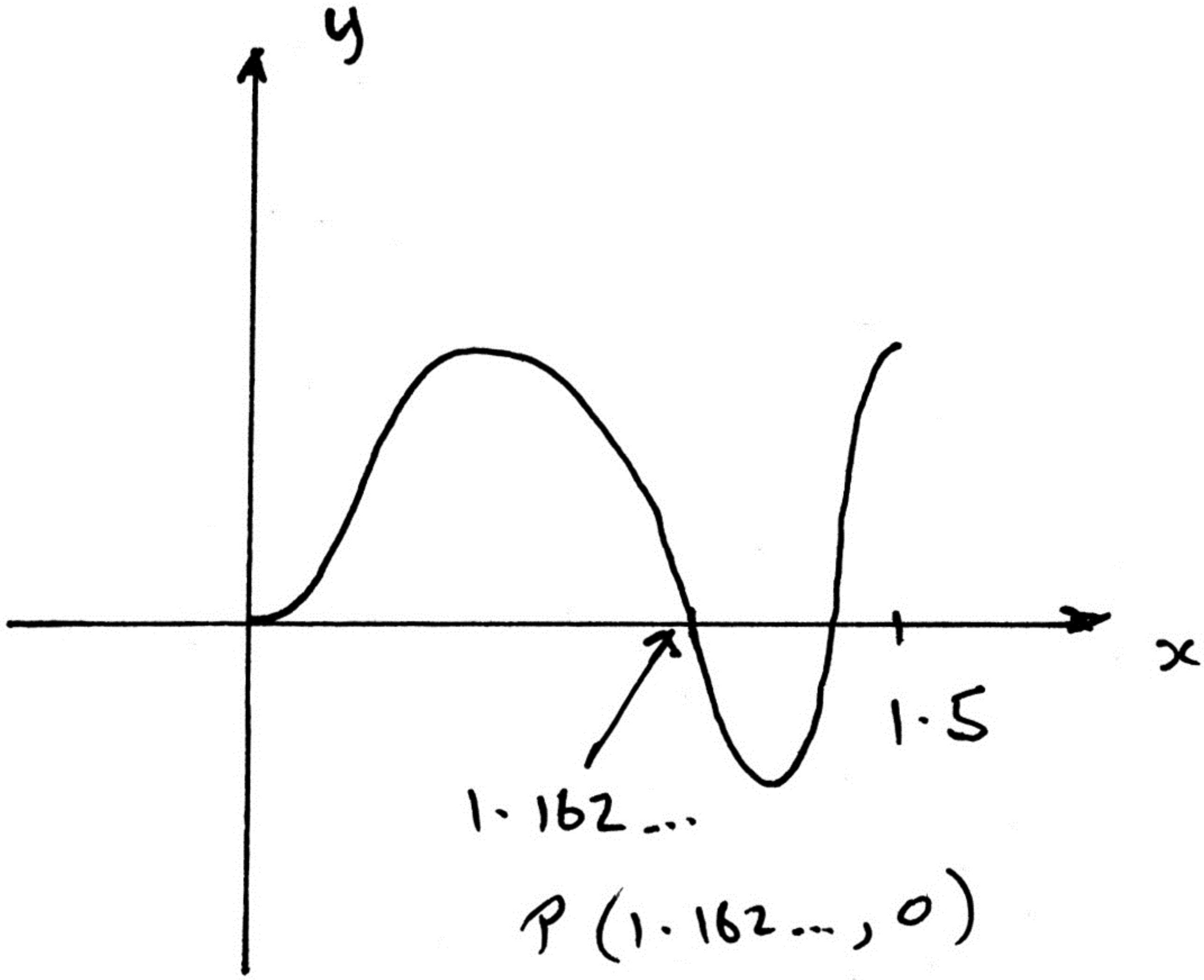
when $x = 0,\ y = 0 \Rightarrow C = \frac{1}{5}$ ***M1***

$y = \sin\left(\frac{1}{5} e^{2x}(2 \sin x - \cos x) + \frac{1}{5}\right)$ ***A1***

[5 marks]

(c)

(i)



A1

P is (1.16, 0) ***A1***

Note: Award ***A1*** for 1.16 seen anywhere, ***A1*** for complete sketch.

Note: Allow FT on their answer from (b)

(ii) $V = \int_0^{1.162...} \pi y^2 \mathrm{d}x$ ***M1A1***

$= 1.05$ ***A2***

Note: Allow FT on their answers from (b) and (c)(i).

[6 marks]

Examiners report

Part B: Part (a) was often well done, although some faltered after the first integration. Part (b) was also generally well done, although there were some errors with the constant of integration. In (c) the graph was often attempted, but errors in (b) usually led to manifestly incorrect plots. Many attempted the volume of integration and some obtained the correct value.

6a. *[6 marks]*

Markscheme

$I_0 = \int_0^\pi \mathrm{e}^{-x} \sin x \mathrm{d}x$ ***M1***

Note: Award ***M1*** for $I_0 = \int_0^\pi \mathrm{e}^{-x} |\sin x| \mathrm{d}x$

Attempt at integration by parts, even if inappropriate modulus signs are present. ***M1***

$= -[\mathrm{e}^{-x} \cos x]_0^\pi - \int_0^\pi \mathrm{e}^{-x} \cos x \mathrm{d}x$ **or** $= -[\mathrm{e}^{-x} \sin x]_0^\pi - \int_0^\pi \mathrm{e}^{-x} \cos x \mathrm{d}x$ ***A1***

$= -[\mathrm{e}^{-x} \cos x]_0^\pi - [\mathrm{e}^{-x} \sin x]_0^\pi - \int_0^\pi \mathrm{e}^{-x} \sin x \mathrm{d}x$ **or** $= -[\mathrm{e}^{-x} \sin x + \mathrm{e}^{-x} \cos x]_0^\pi - \int_0^\pi \mathrm{e}^{-x} \sin x \mathrm{d}x$ ***A1***

$= -[\mathrm{e}^{-x} \cos x]_0^\pi - [\mathrm{e}^{-x} \sin x]_0^\pi - I_0$ **or** $-[\mathrm{e}^{-x} \sin x + \mathrm{e}^{-x} \cos x]_0^\pi - I_0$ ***M1***

Note: Do not penalise absence of limits at this stage

$I_0 = \mathrm{e}^{-\pi} + 1 - I_0$ ***A1***

$I_0 = \frac{1}{2}\left(1 + \mathrm{e}^{-\pi}\right)$ ***AG***

Note: If modulus signs are used around $\cos x$, award no accuracy marks but do not penalise modulus signs around $\sin x$.

[6 marks]

Examiners report

Part (a) is essentially core work requiring repeated integration by parts and many candidates realised that. However, some candidates left the modulus signs in I_0 which invalidated their work. In parts (b) and (c) it was clear that very few candidates had a complete understanding of the significance of the modulus sign and what conditions were necessary for it to be dropped. Overall, attempts at (b) and (c) were disappointing with few correct solutions seen.

6b. *[4 marks]*

Markscheme

$I_n = \int_{n\pi}^{(n+1)\pi} e^{-x} |\sin x| dx$

Attempt to use the substitution $y = x - n\pi$ ***M1***

(putting $y = x - n\pi$, $dy = dx$ and $[n\pi,\ (n+1)\pi] \to [0,\ \pi]$)

so $I_n = \int_0^\pi e^{-(y+n\pi)} |\sin(y+n\pi)| dy$ ***A1***

$= e^{-n\pi} \int_0^\pi e^{-y} |\sin(y+n\pi)| dy$ ***A1***

$= e^{-n\pi} \int_0^\pi e^{-y} \sin y dy$ ***A1***

$= e^{-n\pi} I_0$ ***AG***

[4 marks]

Examiners report

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6c. *[5 marks]*

Markscheme

$\int_0^\infty e^{-x} |\sin x| dx = \sum_{n=0}^{\infty} I_n$ ***M1***

$= \sum_{n=0}^{\infty} e^{-n\pi} I_0$ ***(A1)***

the $\sum$ term is an infinite geometric series with common ratio $e^{-\pi}$ ***(M1)***

therefore

$\int_0^\infty e^{-x} |\sin x| dx = \frac{I_0}{1-e^{-\pi}}$ ***(A1)***

$= \frac{1+e^{-\pi}}{2(1-e^{-\pi})} \quad \left(= \frac{e^{\pi}+1}{2(e^{\pi}-1)}\right)$ ***A1***

[5 marks]

Examiners report

Part (a) is essentially core work requiring repeated integration by parts and many candidates realised that. However, some candidates left the modulus signs in I_0 which invalidated their work. In parts (b) and (c) it was clear that very few candidates had a complete understanding of the significance of the modulus sign and what conditions were necessary for it to be dropped. Overall, attempts at (b) and (c) were disappointing with few correct solutions seen.

7. *[5 marks]*

Markscheme

$2^{2x-2} = 2^x + 8$ ***(M1)***

$\frac{1}{4}\,2^{2x} = 2^x + 8$ ***(A1)***

$2^{2x} - 4 \times 2^x - 32 = 0$ ***A1***

$(2^x - 8)(2^x + 4) = 0$ ***(M1)***

$2^x = 8 \Rightarrow x = 3$ ***A1***

Notes: Do not award final ***A1*** if more than 1 solution is given.

[5 marks]

Examiners report

Very few candidates knew how to solve this equation. A significant number guessed the answer using trial and error after failed attempts to solve it. A number of misconceptions were identified involving properties of logarithms and exponentials.

8. *[20 marks]*

Markscheme

(a) $\sin(2n+1)x\cos x - \cos(2n+1)x\sin x = \sin(2n+1)x - x$ ***M1A1***

$= \sin 2nx$ ***AG***

[2 marks]

(b) if $n = 1$ ***M1***

$\text{LHS} = \cos x$

$\text{RHS} = \frac{\sin 2x}{2\sin x} = \frac{2\sin x\cos x}{2\sin x} = \cos x$ ***M1***

so LHS = RHS and the statement is true for $n = 1$ ***R1***

assume true for $n = k$ ***M1***

Note: Only award ***M1*** if the word **true** appears.

Do **not** award ***M1*** for 'let $n = k$' only.

Subsequent marks are independent of this ***M1***.

so $\cos x + \cos 3x + \cos 5x + \ldots + \cos(2k-1)x = \frac{\sin 2kx}{2\sin x}$

if $n = k + 1$ then

$\cos x + \cos 3x + \cos 5x + \ldots + \cos(2k-1)x + \cos(2k+1)x$ ***M1***

$= \frac{\sin 2kx}{2\sin x} + \cos(2k+1)x$ ***A1***

$= \frac{\sin 2kx + 2\cos(2k+1)x\sin x}{2\sin x}$ ***M1***

$= \frac{\sin(2k+1)x\cos x - \cos(2k+1)x\sin x + 2\cos(2k+1)x\sin x}{2\sin x}$ ***M1***

$= \frac{\sin(2k+1)x\cos x + \cos(2k+1)x\sin x}{2\sin x}$ ***A1***

$= \frac{\sin(2k+2)x}{2\sin x}$ ***M1***

$= \frac{\sin 2(k+1)x}{2\sin x}$ ***A1***

so if true for $n = k$, then also true for $n = k + 1$

as true for $n = 1$ then true for all $n \in \mathbb{Z}^+$ ***R1***

Note: Final ***R1*** is independent of previous work.

[12 marks]

(c) $\frac{\sin 4x}{2\sin x} = \frac{1}{2}$ ***M1A1***

$\sin 4x = \sin x$

$4x = x \Rightarrow x = 0$ but this is impossible

$4x = \pi - x \Rightarrow x = \frac{\pi}{5}$ ***A1***

$4x = 2\pi + x \Rightarrow x = \frac{2\pi}{3}$ ***A1***

$4x = 3\pi - x \Rightarrow x = \frac{3\pi}{5}$ ***A1***

for not including any answers outside the domain ***R1***

Note: Award the first ***M1A1*** for correctly obtaining $8\cos^3 x - 4\cos x - 1 = 0$ or equivalent and subsequent marks as appropriate including the answers $\left(-\frac{1}{2}, \frac{1\pm\sqrt{5}}{4}\right)$.

[6 marks]

Total [20 marks]

Examiners report

This question showed the weaknesses of many candidates in dealing with formal proofs and showing their reasoning in a logical manner. In part (a) just a few candidates clearly showed the result and part (b) showed that most candidates struggle with the formality of a proof by induction. The logic of many solutions was poor, though sometimes contained correct trigonometric work. Very few candidates were successful in answering part (c) using the unit circle. Most candidates attempted to manipulate the equation to obtain a cubic equation but made little progress. A few candidates guessed $\frac{2\pi}{3}$ as a solution but were not able to determine the other solutions.

9. *[5 marks]*

Markscheme

EITHER

using row reduction (or attempting to eliminate a variable) ***M1***

$$\begin{pmatrix} 2 & -1 & 3 & 2 \\ 3 & 1 & 2 & -2 \\ -1 & 2 & a & b \end{pmatrix} \begin{matrix} \\ \to 2R2 - 3R1 \\ \to 2R3 + R1 \end{matrix}$$

$$\begin{pmatrix} 2 & -1 & 3 & 2 \\ 0 & 5 & -5 & -10 \\ 0 & 3 & 2a+3 & 2b+2 \end{pmatrix} \to R2/5$$ ***A1***

Note: For an algebraic solution award ***A1*** for **two** correct equations in two variables.

$$\begin{pmatrix} 2 & -1 & 3 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 3 & 2a+3 & 2b+2 \end{pmatrix} \to R3 - 3R2$$

$$\begin{pmatrix} 2 & -1 & 3 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 2a+6 & 2b+8 \end{pmatrix}$$

Note: Accept alternative correct row reductions.

recognition of the need for 4 zeroes ***M1***

so for multiple solutions $a = -3$ and $b = -4$ ***A1A1***

[5 marks]

OR

$$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 1 & 2 \\ -1 & 2 & a \end{vmatrix} = 0$$ ***M1***

$\Rightarrow 2(a-4) + (3a+2) + 3(6+1) = 0$

$\Rightarrow 5a + 15 = 0$

$\Rightarrow a = -3$ ***A1***

$$\begin{vmatrix} 2 & -1 & 2 \\ 3 & 1 & -2 \\ -1 & 2 & b \end{vmatrix} = 0$$ ***M1***

$\Rightarrow 2(b+4) + (3b-2) + 2(6+1) = 0$ ***A1***

$\Rightarrow 5b + 20 = 0$

$\Rightarrow b = -4$ ***A1***

[5 marks]

Examiners report

Many candidates attempted an algebraic approach that used excessive time but still allowed few to arrive at a solution. Of those that recognised the question should be done by matrices, some were unaware that for more than one solution a complete line of zeros is necessary.

10. *[7 marks]*

Markscheme

(a) $z^3 = 2\sqrt{2}e^{\frac{3\pi i}{4}}$ ***(M1)(A1)***

$z_1 = \sqrt{2}e^{\frac{\pi i}{4}}$ ***A1***

adding or subtracting $\frac{2\pi i}{3}$ ***M1***

$z_2 = \sqrt{2}e^{\frac{\pi i}{4}+\frac{2\pi i}{3}} = \sqrt{2}e^{\frac{11\pi i}{12}}$ ***A1***

$z_3 = \sqrt{2}e^{\frac{\pi i}{4}-\frac{2\pi i}{3}} = \sqrt{2}e^{-\frac{5\pi i}{12}}$ ***A1***

Notes: Accept equivalent solutions *e.g.* $z_3 = \sqrt{2}e^{\frac{19\pi i}{12}}$

Award marks as appropriate for solving $(a+bi)^3 = -2+2i$.

Accept answers in degrees.

(b) $\sqrt{2}e^{\frac{\pi i}{4}}$ $\left(= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\right)$ ***A1***

= 1 + i ***AG***

Note: Accept geometrical reasoning.

[7 marks]

Examiners report

Many students incorrectly found the argument of z^3 to be $\arctan\left(\frac{2}{-2}\right) = -\frac{\pi}{4}$. Of those students correctly finding one solution, many were unable to use symmetry around the origin, to find the other two. In part (b) many students found the cube of 1 + *i* which could not be awarded marks as it was not “hence”.

11. *[5 marks]*

Markscheme

$(100 + 101 + 102 + \ldots + 999) - (102 + 105 + \ldots + 999)$ ***(M1)***

$= \frac{900}{2}(100 + 999) - \frac{300}{2}(102 + 999)$ ***M1A1A1***

$= 329\,400$ ***A1*** ***N5***

Note: A variety of other acceptable methods may be seen including for example $\frac{300}{2}(201 + 1995)$ or $\frac{600}{2}(100 + 998)$.

[5 marks]

Examiners report

There were many good solutions seen by a variety of different methods.

12. *[10 marks]*

Markscheme

(a) (i) $1 \times 2 + 2 \times 3 + \ldots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$ ***R1***

(ii) LHS = 40; RHS = 40 ***A1***

[2 marks]

(b) the sequence of values are:

5, 7, 11, 19, 35 … or an example ***A1***

35 is not prime, so Bill's conjecture is false ***R1AG***

[2 marks]

(c) $\mathrm{P}(n) : 5 \times 7^n + 1$ is divisible by 6

$\mathrm{P}(1) : 36$ is divisible by $6 \Rightarrow \mathrm{P}(1)$ true ***A1***

assume $\mathrm{P}(k)$ is true $(5 \times 7^k + 1 = 6r)$ ***M1***

Note: Do **not** award ***M1*** for statement starting 'let $n = k$'.

Subsequent marks are independent of this ***M1***.

consider $5 \times 7^{k+1} + 1$ ***M1***

$= 7(6r - 1) + 1$ ***(A1)***

$= 6(7r - 1) \Rightarrow \mathrm{P}(k+1)$ is true ***A1***

P(1) true and $\mathrm{P}(k)$ true $\Rightarrow \mathrm{P}(k+1)$ true, so by MI $\mathrm{P}(n)$ is true for all $n \in \mathbb{Z}^+$ ***R1***

Note: Only award ***R1*** if there is consideration of P(1), $\mathrm{P}(k)$ and $\mathrm{P}(k+1)$ in the final statement.

Only award ***R1*** if at least one of the two preceding ***A*** marks has been awarded.

[6 marks]

Total [10 marks]

Examiners report

Although there were a good number of wholly correct solutions to this question, it was clear that a number of students had not been prepared for questions on conjectures. The proof by induction was relatively well done, but candidates often showed a lack of rigour in the proof. It was fairly common to see students who did not appreciate the idea that $\mathrm{P}(k)$ is assumed not given and this was penalised. Also it appeared that a number of students had been taught to write down the final reasoning for a proof by induction, even if no attempt of a proof had taken place. In these cases, the final reasoning mark was not awarded.

13. *[16 marks]*

Markscheme

(a) (i) $\omega^3 = \left(\cos\left(\frac{2\pi}{3}\right) + \mathrm{i}\sin\left(\frac{2\pi}{3}\right)\right)^3$

$= \cos\left(x \times \frac{2\pi}{3}\right) + \mathrm{i}\sin\left(3 \times \frac{2\pi}{3}\right)$ ***(M1)***

$= \cos 2\pi + \mathrm{i}\sin 2\pi$ ***A1***

$= 1$ ***AG***

(ii) $1 + \omega + \omega^2 = 1 + \cos\left(\frac{2\pi}{3}\right) + \mathrm{i}\sin\left(\frac{2\pi}{3}\right) + \cos\left(\frac{4\pi}{3}\right) + \mathrm{i}\sin\left(\frac{4\pi}{3}\right)$ ***M1A1***

$= 1 + -\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2} - \frac{1}{2} - \mathrm{i}\frac{\sqrt{3}}{2}$ ***A1***

$= 0$ ***AG***

[5 marks]

(b) (i) $\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{\mathrm{i}\left(\theta + \frac{2\pi}{3}\right)} + \mathrm{e}^{\mathrm{i}\left(\theta + \frac{4\pi}{3}\right)}$

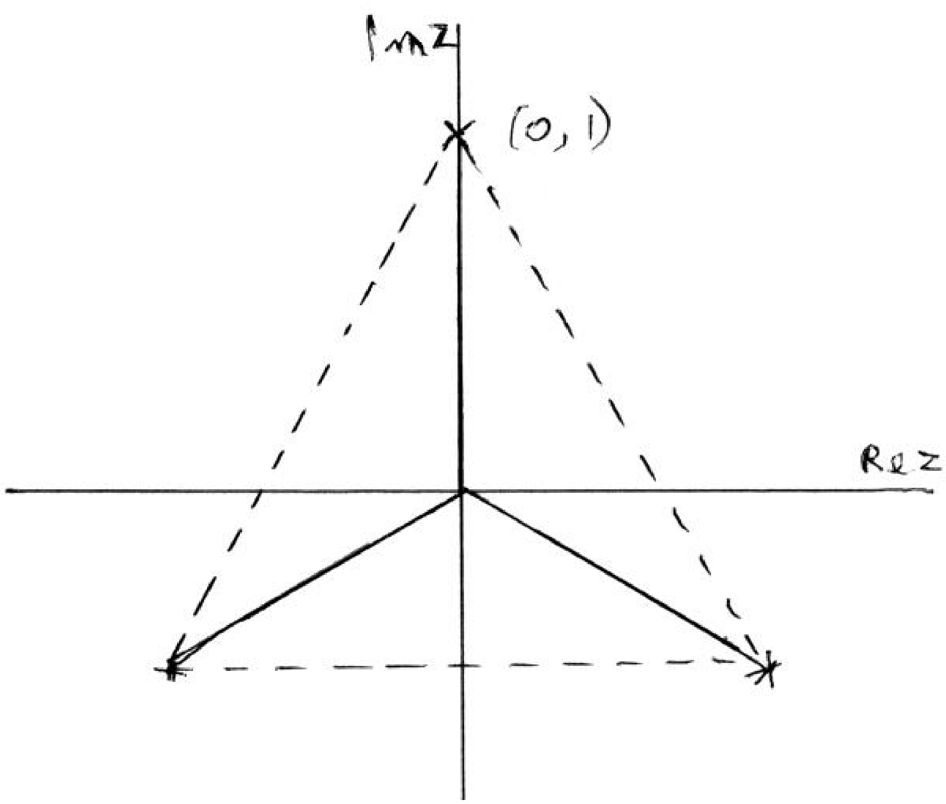
$= \mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\mathrm{i}\left(\frac{2\pi}{3}\right)} + \mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\mathrm{i}\left(\frac{4\pi}{3}\right)}$ ***(M1)***

$= \left(\mathrm{e}^{\mathrm{i}\theta}\left(1 + \mathrm{e}^{\mathrm{i}\left(\frac{2\pi}{3}\right)} + \mathrm{e}^{\mathrm{i}\left(\frac{4\pi}{3}\right)}\right)\right)$

$= \mathrm{e}^{\mathrm{i}\theta}(1 + \omega + \omega^2)$ ***A1***

$= 0$ ***AG***

(ii)



A1A1

Note: Award ***A1*** for one point on the imaginary axis and another point marked with approximately correct modulus and argument.

Award ***A1*** for third point marked to form an equilateral triangle centred on the origin.

[4 marks]

(c) (i) attempt at the expansion of at least two linear factors ***(M1)***

$(z-1)z^2 - z(\omega + \omega^2) + \omega^3$ or equivalent ***(A1)***

use of earlier result ***(M1)***

$F(z) = (z-1)(z^2+z+1) = z^3 - 1$ ***A1***

(ii) equation to solve is $z^3 = 8$ ***(M1)***

$z = 2,\ 2\omega,\ 2\omega^2$ ***A2***

Note: Award ***A1*** for 2 correct solutions.

[7 marks]

Total [16 marks]

Examiners report

Most candidates were able to make a meaningful start to part (a) with many fully correct answers seen. Part (b) was the exact opposite with the majority of candidates not knowing what was required and failing to spot the connection to part (a). Candidates made a reasonable start to part (c), but often did not recognise the need to use the result that $1 + \omega + \omega^2 = 0$. This meant that most candidates were unable to make any progress on part (c) (ii).

14. *[4 marks]*

Markscheme

$\left(x^2 - \frac{2}{x}\right)^4 = (x^2)^4 + 4(x^2)^3\left(-\frac{2}{x}\right) + 6(x^2)^2\left(-\frac{2}{x}\right)^2 + 4(x^2)\left(-\frac{2}{x}\right)^3 + \left(-\frac{2}{x}\right)^4$ ***(M1)***

$= x^8 - 8x^5 + 24x^2 - \frac{32}{x} + \frac{16}{x^4}$ ***A3***

Note: Deduct one ***A*** mark for each incorrect or omitted term.

[4 marks]

Examiners report

Most candidates solved this question correctly with most candidates who explained how they obtained their coefficients using Pascal’s triangle rather than the combination formula.

15. *[6 marks]*

Markscheme

METHOD 1

$5(2a + 9d) = 60$ (or $2a + 9d = 12$)   ***M1A1***

$10(2a + 19d) = 320$ (or $2a + 19d = 32$)   ***A1***

solve simultaneously to obtain   ***M1***

$a = -3,\ d = 2$ ***A1***

the 15^{th} term is $-3 + 14 \times 2 = 25$ ***A1***

Note: ***FT*** the final ***A1*** on the values found in the penultimate line.

METHOD 2

with an AP the mean of an even number of consecutive terms equals the mean of the middle terms   ***(M1)***

$\frac{a_{10}+a_{11}}{2} = 16 \quad (\text{or } a_{10} + a_{11} = 32)$ ***A1***

$\frac{a_5+a_6}{2} = 6 \quad (\text{or } a_5 + a_6 = 12)$ ***A1***

$a_{10} - a_5 + a_{11} - a_6 = 20$ ***M1***

$5d + 5d = 20$

$d = 2$ and $a = -3 \quad (\text{or } a_5 = 5 \text{ or } a_{10} = 15)$ ***A1***

the 15^{th} term is $-3 + 14 \times 2 = 25 \quad (\text{or } 5 + 10 \times 2 = 25 \text{ or } 15 + 5 \times 2 = 25)$ ***A1***

Note: ***FT*** the final ***A1*** on the values found in the penultimate line.

[6 marks]

Examiners report

Many candidates had difficulties with this question with the given information often translated into incorrect equations.

16. *[8 marks]*

Markscheme

METHOD 1

(a) $u_n = S_n - S_{n-1}$ ***(M1)***

$= \frac{7^n - a^n}{7^n} - \frac{7^{n-1} - a^{n-1}}{7^{n-1}}$ ***A1***

(b) **EITHER**

$u_1 = 1 - \frac{a}{7}$ ***A1***

$u_2 = 1 - \frac{a^2}{7^2} - \left(1 - \frac{a}{7}\right)$ ***M1***

$= \frac{a}{7}\left(1 - \frac{a}{7}\right)$ ***A1***

common ratio $= \frac{a}{7}$ ***A1***

OR

$u_n = 1 - \left(\frac{a}{7}\right)^n - 1 + \left(\frac{a}{7}\right)^{n-1}$ ***M1***

$= \left(\frac{a}{7}\right)^{n-1}\left(1 - \frac{a}{7}\right)$

$= \frac{7-a}{7}\left(\frac{a}{7}\right)^{n-1}$ ***A1***

$u_1 = \frac{7-a}{7}$, common ratio $= \frac{a}{7}$ ***A1A1***

(c) (i) $0 < a < 7$ (accep $a < 7$) ***A1***

(ii) 1 ***A1***

[8 marks]

METHOD 2

(a) $u_n = br^{n-1} = \left(\frac{7-a}{7}\right)\left(\frac{a}{7}\right)^{n-1}$ ***A1A1***

(b) for a GP with first term b and common ratio r

$S_n = \frac{b(1-r^n)}{1-r} = \left(\frac{b}{1-r}\right) - \left(\frac{b}{1-r}\right)r^n$ ***M1***

as $S_n = \frac{7^n - a^n}{7^n} = 1 - \left(\frac{a}{7}\right)^n$

comparing both expressions ***M1***

$\frac{b}{1-r} = 1$ and $r = \frac{a}{7}$

$b = 1 - \frac{a}{7} = \frac{7-a}{7}$

$u_1 = b = \frac{7-a}{7}$, common ratio $= r = \frac{a}{7}$ ***A1A1***

Note: Award method marks if the expressions for b and r are deduced in part (a).

(c) (i) $0 < a < 7$ (accept $a < 7$) ***A1***

(ii) 1 ***A1***

[8 marks]

Examiners report

Many candidates found this question difficult. In (a), few seemed to realise that $u_n = S_n - S_{n-1}$. In (b), few candidates realised that $u_1 = S_1$ and in (c) that S_n could be written as $1 - \left(\frac{a}{7}\right)^n$ from which it follows immediately that the sum to infinity exists when a < 7 and is equal to 1.

17. *[19 marks]*

Markscheme

(a) **METHOD 1**

$\frac{z+\mathrm{i}}{z+2}=\mathrm{i}$

$z+\mathrm{i}=\mathrm{i}z+2\mathrm{i}$ ***M1***

$(1-\mathrm{i})z=\mathrm{i}$ ***A1***

$z=\frac{\mathrm{i}}{1-\mathrm{i}}$ ***A1***

EITHER

$z=\frac{\operatorname{cis}\left(\frac{\pi}{2}\right)}{\sqrt{2}\operatorname{cis}\left(\frac{3\pi}{4}\right)}$ ***M1***

$z=\frac{\sqrt{2}}{2}\operatorname{cis}\left(\frac{3\pi}{4}\right)\ \left(\text{or } \frac{1}{\sqrt{2}}\operatorname{cis}\left(\frac{4\pi}{4}\right)\right)$ ***A1A1***

OR

$z=\frac{-1+\mathrm{i}}{2}\ \left(=-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$ ***M1***

$z=\frac{\sqrt{2}}{2}\operatorname{cis}\left(\frac{3\pi}{4}\right)\ \left(\text{or } \frac{1}{\sqrt{2}}\operatorname{cis}\left(\frac{3\pi}{4}\right)\right)$ ***A1A1***

[6 marks]

METHOD 2

$\mathrm{i}=\frac{x+\mathrm{i}(y+1)}{x+2+\mathrm{i}y}$ ***M1***

$x+\mathrm{i}(y+1)=-y+\mathrm{i}(x+2)$ ***A1***

$x=-y;\ x+2=y+1$ ***A1***

solving, $x=-\frac{1}{2};\ y=\frac{1}{2}$ ***A1***

$z=-\frac{1}{2}+\frac{1}{2}\mathrm{i}$

$z=\frac{\sqrt{2}}{2}\operatorname{cis}\left(\frac{3\pi}{4}\right)\ \left(\text{or } \frac{1}{\sqrt{2}}\operatorname{cis}\left(\frac{3\pi}{4}\right)\right)$ ***A1A1***

Note: Award ***A1*** fort the correct modulus and ***A1*** for the correct argument, but the final answer must be in the form r cis θ Accept 135° for the argument.

[6 marks]

(b) substituting $z=x+\mathrm{i}y$ to obtain $w=\frac{x+(y+1)\mathrm{i}}{(x+2)+y\mathrm{i}}$ ***(A1)***

use of $(x+2)-y\mathrm{i}$ to rationalize the denominator ***M1***

$\omega=\frac{x(x+2)+y(y+1)+\mathrm{i}(-xy+(y+1)(x+2))}{(x+2)^2+y^2}$ ***A1***

$=\frac{(x^2+2x+y^2+y)+\mathrm{i}(x+2y+2)}{(x+2)^2+y^2}$ ***AG***

[3 marks]

(c) $\operatorname{Re}\omega=\frac{x^2+2x+y^2+y}{(x+2)^2+y^2}=1$ ***M1***

$\Rightarrow x^2+2x+y^2+y=x^2+4x+4+y^2$ ***A1***

$\Rightarrow y=2x+4$ ***A1***

which has gradient $m=2$ ***A1***

[4 marks]

(d) **EITHER**

$\arg(z) = \frac{\pi}{4} \Rightarrow x = y\,(\text{and } x,\ y > 0)$ ***(A1)***

$\omega = \frac{2x^2+3x}{(x+2)^2+x^2} + \frac{\mathrm{i}(3x+2)}{(x+2)^2+x^2}$

if $\arg(\omega) = \theta \Rightarrow \tan\theta = \frac{3x+2}{2x^2+3x}$ ***(M1)***

$\frac{3x+2}{2x^2+3x} = 1$ ***M1A1***

OR

$\arg(z) = \frac{\pi}{4} \Rightarrow x = y\,(\text{and } x,\ y > 0)$ ***A1***

$\arg(w) = \frac{\pi}{4} \Rightarrow x^2 + 2x + y^2 + y = x + 2y + 2$ ***M1***

solve simultaneously ***M1***

$x^2 + 2x + x^2 + x = x + 2x + 2$ (or equivalent) ***A1***

THEN

$x^2 = 1$

$x = 1\ (\text{as } x > 0)$ ***A1***

Note: Award ***A0*** for $x = \pm 1$.

$|z| = \sqrt{2}$ ***A1***

Note: Allow ***FT*** from incorrect values of x.

[6 marks]

Total [19 marks]

Examiners report

Many candidates knew what had to be done in (a) but algebraic errors were fairly common. Parts (b) and (c) were well answered in general. Part (d), however, proved beyond many candidates who had no idea how to convert the given information into mathematical equations.

18. *[7 marks]*

Markscheme

METHOD 1

$1 + \mathrm{i}$ is a zero $\Rightarrow$ $1 - \mathrm{i}$ is a zero ***(A1)***

$1 - 2\mathrm{i}$ is a zero $\Rightarrow$ $1 + 2\mathrm{i}$ is a zero ***(A1)***

$(x - (1 - \mathrm{i}))\ (x - (1 + \mathrm{i})) = (x^2 - 2x + 2)$ ***(M1)A1***

$(x - (1 - 2\mathrm{i}))\ (x - (1 + 2\mathrm{i})) = (x^2 - 2x + 5)$ ***A1***

$p(x) = (x^2 - 2x + 2)(x^2 - 2x + 5)$ ***M1***

$= x^4 - 4x^3 + 11x^2 - 14x + 10$ ***A1***

$a = -4,\ b = 11,\ c = -14,\ d = 10$

[7 marks]

METHOD 2

$p(1 + \mathrm{i}) = -4 + (-2 + 2\mathrm{i})a + (2\mathrm{i})b + (1 + \mathrm{i})c + d$ ***M1***

$p(1 + \mathrm{i}) = 0 \Rightarrow \begin{cases} -4 - 2a + c + d = 0 \\ 2a + 2b + c = 0 \end{cases}$ ***M1A1A1***

$p(1 - 2\mathrm{i}) = -7 + 24\mathrm{i} + (-11 + 2\mathrm{i})a + (-3 - 4\mathrm{i})b + (1 - 2\mathrm{i})c + d$

$p(1 - 2\mathrm{i}) = 0 \Rightarrow \begin{cases} -7 - 11a - 3b + c + d = 0 \\ 24 + 2a - 4b - 2c = 0 \end{cases}$ ***A1***

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 \\ -11 & -3 & 1 & 1 \\ 2 & -4 & -2 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 0 \\ 7 \\ -24 \end{pmatrix} = \begin{pmatrix} -4 \\ 11 \\ -14 \\ 10 \end{pmatrix}$$ ***M1A1***

$a = -4,\ b = 11,\ c = -14,\ d = 10$

[7 marks]

Examiners report

Most candidates attempted this question, using different approaches. The most successful approach was the method of complex conjugates and the product of linear factors. Candidates who used this method were in general successful whereas candidates who attempted direct substitution and separation of real and imaginary parts to obtain four equations in four unknowns were less successful because either they left the work incomplete or made algebraic errors that led to incorrect answers.

19a. *[2 marks]*

Markscheme

$u_1 = 27$

$\frac{81}{2} = \frac{27}{1-r}$ ***M1***

$r = \frac{1}{3}$ ***A1***

[2 marks]

Examiners report

Part (a) was well done by most candidates. However (b) caused difficulty to most candidates. Although a number of different approaches were seen, just a small number of candidates obtained full marks for this question.

19b. *[5 marks]*

Markscheme

$v_2 = 9$
$v_4 = 1$
$2d = -8 \Rightarrow d = -4$ ***(A1)***
$v_1 = 13$ ***(A1)***

$\frac{N}{2}\left(2 \times 13 - 4\left(N-1\right)\right) > 0$ (accept equality) ***M1***

$\frac{N}{2}\left(30 - 4N\right) > 0$

$N\left(15 - 2N\right) > 0$

$N < 7.5$ ***(M1)***

$N = 7$ ***A1***

Note: $13 + 9 + 5 + 1 - 3 - 7 - 11 > 0 \Rightarrow N = 7$ or equivalent receives full marks.

[5 marks]

Examiners report

Part (a) was well done by most candidates. However (b) caused difficulty to most candidates. Although a number of different approaches were seen, just a small number of candidates obtained full marks for this question.

20a. *[2 marks]*

Markscheme

$\left(\cos\theta + \mathrm{i}\sin\theta\right)^3 = \cos^3\theta + 3\cos^2\theta\left(\mathrm{i}\sin\theta\right) + 3\cos\theta(\mathrm{i}\sin\theta)^2 + (\mathrm{i}\sin\theta)^3$ ***(M1)***

$= \cos^3\theta - 3\cos\theta\sin^2\theta + \mathrm{i}\left(3\cos^2\theta\sin\theta - \sin^3\theta\right)$ ***A1***

[2 marks]

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

20b. *[3 marks]*

Markscheme

from De Moivre's theorem

$(\cos\theta + \mathrm{i}\sin\theta)^3 = \cos 3\theta + \mathrm{i}\sin 3\theta$ ***(M1)***

$\cos 3\theta + \mathrm{i}\sin 3\theta = \left(\cos^3\theta - 3\cos\theta\sin^2\theta\right) + \mathrm{i}\left(3\cos^2\theta\sin\theta - \sin^3\theta\right)$

equating real parts ***M1***

$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

$= \cos^3\theta - 3\cos\theta\left(1 - \cos^2\theta\right)$ ***A1***

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$ ***AG***

Note: Do not award marks if part (a) is not used.

[3 marks]

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

20c. *[3 marks]*

Markscheme

$(\cos\theta + \mathrm{i}\sin\theta)^5 =$

$\cos^5\theta + 5\cos^4\theta\,(\mathrm{i}\sin\theta) + 10\cos^3\theta(\mathrm{i}\sin\theta)^2 + 10\cos^2\theta(\mathrm{i}\sin\theta)^3 + 5\cos\theta(\mathrm{i}\sin\theta)^4 + (\mathrm{i}\sin\theta)^5$ ***(A1)***

from De Moivre's theorem

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ ***M1***

$= \cos^5\theta - 10\cos^3\theta\left(1 - \cos^2\theta\right) + 5\cos\theta\left(1 - \cos^2\theta\right)^2$ ***A1***

$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$

$\therefore \cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ ***AG***

Note: If compound angles used in (b) and (c), then marks can be allocated in (c) only.

[3 marks]

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

20d. *[6 marks]*

Markscheme

$\cos 5\theta + \cos 3\theta + \cos \theta$

$= \left(16\cos^5\theta - 20\cos^3\theta + 5\cos\theta\right) + \left(4\cos^3\theta - 3\cos\theta\right) + \cos\theta = 0$ ***M1***

$16\cos^5\theta - 16\cos^3\theta + 3\cos\theta = 0$ ***A1***

$\cos\theta\left(16\cos^4\theta - 16\cos^2\theta + 3\right) = 0$

$\cos\theta\left(4\cos^2\theta - 3\right)\left(4\cos^2\theta - 1\right) = 0$ ***A1***

$\therefore \cos\theta = 0;\ \pm\frac{\sqrt{3}}{2};\ \pm\frac{1}{2}$ ***A1***

$\therefore \theta = \pm\frac{\pi}{6};\ \pm\frac{\pi}{3};\ \pm\frac{\pi}{2}$ ***A2***

[6 marks]

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

20e. *[8 marks]*

Markscheme

$\cos 5\theta = 0$

$5\theta = \ldots\frac{\pi}{2};\ \left(\frac{3\pi}{2};\ \frac{5\pi}{2}\right);\ \frac{7\pi}{2};\ \ldots$ ***(M1)***

$\theta = \ldots\frac{\pi}{10};\ \left(\frac{3\pi}{10};\ \frac{5\pi}{10}\right);\ \frac{7\pi}{10};\ \ldots$ ***(M1)***

Note: These marks can be awarded for verifications later in the question.

now consider $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$ ***M1***

$\cos\theta\left(16\cos^4\theta - 20\cos^2\theta + 5\right) = 0$

$\cos^2\theta = \frac{20\pm\sqrt{400-4(16)(5)}}{32};\ \cos\theta = 0$ ***A1***

$\cos\theta = \pm\sqrt{\frac{20\pm\sqrt{400-4(16)(5)}}{32}}$

$\cos\frac{\pi}{10} = \sqrt{\frac{20+\sqrt{400-4(16)(5)}}{32}}$ since max value of cosine $\Rightarrow$ angle closest to zero ***R1***

$\cos\frac{\pi}{10} = \sqrt{\frac{4.5+4\sqrt{25-4(5)}}{4.8}} = \sqrt{\frac{5+\sqrt{5}}{8}}$ ***A1***

$\cos\frac{7\pi}{10} = -\sqrt{\frac{5-\sqrt{5}}{8}}$ ***A1A1***

[8 marks]

Examiners report

This question proved to be very difficult for most candidates. Many had difficulties in following the instructions and attempted to use addition formulae rather than binomial expansions. A small number of candidates used the results given and made a good attempt to part (d) but very few answered part (e).

21a. *[1 mark]*

Markscheme

$2x^2 + x - 3 = (2x + 3)(x - 1)$ ***A1***

Note: Accept $2\left(x + \frac{3}{2}\right)(x - 1)$.

Note: Either of these may be seen in (b) and if so ***A1*** should be awarded.

[1 mark]

Examiners report

Many candidates struggled to find an efficient approach to this problem by applying the Binomial Theorem. A disappointing number of candidates attempted the whole expansion which was clearly an unrealistic approach when it is noted that the expansion is to the 8^{th} power. The fact that some candidates wrote down Pascal's Triangle suggested that they had not studied the Binomial Theorem in enough depth or in a sufficient variety of contexts.

21b. *[4 marks]*

Markscheme

EITHER

$\left(2x^2 + x - 3\right)^8 = (2x + 3)^8(x - 1)^8$ ***M1***

$= \left(3^8 + 8\left(3^7\right)(2x) + \ldots\right)\left((-1)^8 + 8(-1)^7(x) + \ldots\right)$ ***(A1)***

coefficient of $x = 3^8 \times 8 \times (-1)^7 + 3^7 \times 8 \times 2 \times (-1)^8$ ***M1***

$= -17\,496$ ***A1***

Note: Under ft, final ***A1*** can only be achieved for an integer answer.

OR

$\left(2x^2 + x - 3\right)^8 = \left(3 - \left(x - 2x^2\right)\right)^8$ ***M1***

$= 3^8 + 8\left(-\left(x - 2x^2\right)\left(3^7\right) + \ldots\right)$ ***(A1)***

coefficient of $x = 8 \times (-1) \times 3^7$ ***M1***

$= -17\,496$ ***A1***

Note: Under ft, final ***A1*** can only be achieved for an integer answer.

[4 marks]

Examiners report

Many candidates struggled to find an efficient approach to this problem by applying the Binomial Theorem. A disappointing number of candidates attempted the whole expansion which was clearly an unrealistic approach when it is noted that the expansion is to the 8^{th} power. The fact that some candidates wrote down Pascal's Triangle suggested that they had not studied the Binomial Theorem in enough depth or in a sufficient variety of contexts.

22. *[6 markets]*

Markscheme

$\log_{x+1} y = 2$

$\log_{y+1} x = \frac{1}{4}$

so $(x+1)^2 = y$ ***A1***

$(y+1)^{\frac{1}{4}} = x$ ***A1***

EITHER

$x^4 - 1 = (x+1)^2$ ***M1***

$x = -1$, not possible ***R1***

$x = 1.70$, $y = 7.27$ ***A1A1***

OR

1

$\left(x^2 + 2x + 2\right)^{\frac{1}{4}} - x = 0$ ***M1***

attempt to solve or graph of LHS ***M1***

$x = 1.70$, $y = 7.27$ ***A1A1***

[6 marks]

Examiners report

This question was well answered by a significant number of candidates. There was evidence of good understanding of logarithms. The algebra required to solve the problem did not intimidate candidates and the vast majority noticed the necessity of technology to solve the final equation. Not all candidates recognized the extraneous solution and there were situations where a rounded value of x was used to calculate the value of y leading to an incorrect solution.

23a. *[3 marks]*

Markscheme

$$\begin{pmatrix} 0 & 2 & 1 \\ -1 & 1 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ k \end{pmatrix}$$

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 & 3 \\ -2 & 1 & 2 \end{vmatrix} = 0 - 2(-2+6) + (-1+2) = -7$$ ***M1A1***

since determinant $\neq 0 \Rightarrow$ unique solution to the system ***R1***

planes intersect in a point ***AG***

Note: For any method, including row reduction, leading to the explicit solution $\left(\frac{6-5k}{7}, \frac{10+k}{7}, \frac{1-2k}{7}\right)$, award ***M1*** for an attempt at a correct method, ***A1*** for two correct coordinates and ***A1*** for a third correct coordinate.

[3 marks]

Examiners report

It was disappointing to see that a significant number of candidates did not appear to be well prepared for this question and made no progress at all. There were a number of schools where no candidate made any appreciable progress with the question. This was of concern since this is quite a standard problem in Mathematics HL exams. Parts (a) and (b) were intended to be answered by the use of determinants, but many candidates were not aware of this technique and used elimination. Whilst a valid method, elimination led to a long and cumbersome solution when a much more straightforward solution was available using determinants.

23b. *[5 marks]*

Markscheme

$$\begin{vmatrix} a & 2 & 1 \\ -1 & a+1 & 3 \\ -2 & 1 & a+2 \end{vmatrix} = a\left((a+1)(a+2)-3\right) - 2\left(-1(a+2)+6\right) + \left(-1+2(a+1)\right)$$ ***M1(A1)***

planes not meeting in a point $\Rightarrow$ no unique solution *i.e.* determinant $= 0$ ***(M1)***

$a\left(a^2+3a-1\right) + (2a-8) + (2a+1) = 0$

$a^3+3a^2+3a-7=0$ ***A1***

\(a =1\) ***A1***

[5 marks]

Examiners report

It was disappointing to see that a significant number of candidates did not appear to be well prepared for this question and made no progress at all. There were a number of schools where no candidate made any appreciable progress with the question. This was of concern since this is quite a standard problem in Mathematics HL exams. Parts (a) and (b) were intended to be answered by the use of determinants, but many candidates were not aware of this technique and used elimination. Whilst a valid method, elimination led to a long and cumbersome solution when a much more straightforward solution was available using determinants.

23c. *[6 marks]*

Markscheme

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 4 & 4 \\ -2 & 1 & 3 & k \end{pmatrix} r_1 + r_2$$ ***M1***

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 4 & 4 \\ 0 & 5 & 5 & 6+k \end{pmatrix} 2r_1 + r_3$$ ***(A1)***

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 4+4k \end{pmatrix} 4r_3 + 5r_2$$ ***(A1)***

for an infinite number of solutions to exist, $4 + 4k = 0 \Rightarrow k = -1$ ***A1***

$x + 2y + z = 3$

$y + z = 1$ ***M1***

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$ ***A1***

Note: Accept methods involving elimination.

Note: Accept any equivalent form *e.g.* $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

Award ***A0*** if $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ or $r =$ is absent.

[6 marks]

Examiners report

It was disappointing to see that a significant number of candidates did not appear to be well prepared for this question and made no progress at all. There were a number of schools where no candidate made any appreciable progress with the question. This was of concern since this is quite a standard problem in Mathematics HL exams. Parts (a) and (b) were intended to be answered by the use of determinants, but many candidates were not aware of this technique and used elimination. Whilst a valid method, elimination led to a long and cumbersome solution when a much more straightforward solution was available using determinants. Part (c) was also a standard question but more challenging. Very few candidates made progress on (c).

24. *[5 marks]*

Markscheme

METHOD 1

$r = 2,\ \theta = -\frac{\pi}{3}$ ***(A1)(A1)***

$\therefore (1 - \mathrm{i}\sqrt{3})^{-3} = 2^{-3}\left(\cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)^{-3}$ ***M1***

$= \frac{1}{8}(\cos\pi + \mathrm{i}\sin\pi)$ ***(M1)***

$= -\frac{1}{8}$ ***A1***

[5 marks]

METHOD 2

$(1 - \sqrt{3}\mathrm{i})(1 - \sqrt{3}\mathrm{i}) = 1 - 2\sqrt{3}\mathrm{i} - 3 \quad (= -2 - 2\sqrt{3}\mathrm{i})$ ***(M1)A1***

$(-2 - 2\sqrt{3}\mathrm{i})(1 - \sqrt{3}\mathrm{i}) = -8$ ***(M1)(A1)***

$\therefore \frac{1}{(1-\sqrt{3}\mathrm{i})^3} = -\frac{1}{8}$ ***A1***

[5 marks]

METHOD 3

Attempt at Binomial expansion ***M1***

$(1 - \sqrt{3}\mathrm{i})^3 = 1 + 3(-\sqrt{3}\mathrm{i}) + 3(-\sqrt{3}\mathrm{i})^2 + (-\sqrt{3}\mathrm{i})^3$ ***(A1)***

$= 1 - 3\sqrt{3}\mathrm{i} - 9 + 3\sqrt{3}\mathrm{i}$ ***(A1)***

$= -8$ ***A1***

$\therefore \frac{1}{(1-\sqrt{3}\mathrm{i})^3} = -\frac{1}{8}$ ***M1***

[5 marks]

Examiners report

Most candidates made a meaningful attempt at this question using a variety of different, but correct methods. Weaker candidates sometimes made errors with the manipulation of the square roots, but there were many fully correct solutions.

25. *[6 marks]*

Markscheme

(a) $0 < 2^x < 1$ *(M1)*

$x < 0$ ***A1*** ***N2***

(b) $\frac{35}{1-r} = 40$ ***M1***

$\Rightarrow 40 - 40 \times r = 35$

$\Rightarrow -40 \times r = -5$ *(A1)*

$\Rightarrow r = 2^x = \frac{1}{8}$ ***A1***

$\Rightarrow x = \log_2 \frac{1}{8} \ (= -3)$ ***A1***

Note: The substitution $r = 2^x$ may be seen at any stage in the solution.

[6 marks]

Examiners report

Part (a) was the first question that a significant majority of candidates struggled with. Only the best candidates were able to find the required set of values. However, it was pleasing to see that the majority of candidates made a meaningful start to part (b). Many candidates gained wholly correct answers to part (b).

26a. *[3 marks]*

Markscheme

$r = -\frac{1}{3}$ *(A1)*

$S_\infty = \frac{27}{1+\frac{1}{3}}$ ***M1***

$S_\infty = \frac{81}{4} \quad (= 20.25)$ ***A1*** ***N1***

[3 marks]

Examiners report

Part (a) was correctly answered by the majority of candidates, although a few found $r = -3$.

26b. *[7 marks]*

Markscheme

Attempting to show that the result is true for $n = 1$ ***M1***

LHS = a **and** RHS $= \frac{a(1-r)}{1-r} = a$ ***A1***

Hence the result is true for $n = 1$

Assume it is true for $n = k$

$a + ar + ar^2 + \ldots + ar^{k-1} = \frac{a(1-rk)}{1-r}$ ***M1***

Consider $n = k + 1$:

$a + ar + ar^2 + \ldots + ar^{k-1} + ar^k = \frac{a(1-r^k)}{1-r} + ar^k$ ***M1***

$= \frac{a(1-r^k)+ar^k(1-r)}{1-r}$

$= \frac{a-ar^k+ar^k-ar^{k+1}}{1-r}$ ***A1***

Note: Award ***A1*** for an equivalent correct intermediate step.

$= \frac{a-ar^{k+1}}{1-r}$

$= \frac{a(1-r^{k+1})}{1-r}$ ***A1***

Note: Illogical attempted proofs that use the result to be proved would gain ***M1A0A0*** for the last three above marks.

The result is true for $n = k \Rightarrow$ it is true for $n = k + 1$ **and** as it is true for $n = 1$, the result is proved by mathematical induction.

R1 N0

Note: To obtain the final ***R1*** mark a reasonable attempt must have been made to prove the $k + 1$ step.

[7 marks]

Examiners report

Part (b) was often started off well, but a number of candidates failed to initiate the $n = k + 1$ step in a satisfactory way. A number of candidates omitted the ‘P(1) is true’ part of the concluding statement.

27. *[12 marks]*

Markscheme

(a) **EITHER**

$w^5 = \left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)^5$ ***(M1)***

$= \cos 2\pi + \mathrm{i}\sin 2\pi$ ***A1***

$= 1$ ***A1***

Hence w is a root of $z^5 - 1 = 0$ ***AG***

OR

Solving $z^5 = 1$ ***(M1)***

$z = \cos\frac{2\pi}{5}n + \mathrm{i}\sin\frac{2\pi}{5}n\,,\quad n = 0, 1, 2, 3, 4$ ***A1***

$n = 1$ gives $\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}$ which is w ***A1***

[3 marks]

(b) $(w-1)(1+w+w^2+w^3+w^4) = w+w^2+w^3+w^4+w^5-1-w-w^2-w^3-w^4$ ***M1***

$= w^5 - 1$ ***A1***

Since $w^5 - 1 = 0$ **and** $w \neq 1$, $w^4+w^3+w^2+w+1=0$. ***R1***

[3 marks]

(c) $1+w+w^2+w^3+w^4 =$

$1+\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}+\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)^2+\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)^3+\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)^4$ ***(M1)***

$= 1+\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}+\cos\frac{4\pi}{5}+\mathrm{i}\sin\frac{4\pi}{5}+\cos\frac{6\pi}{5}+\mathrm{i}\sin\frac{6\pi}{5}+\cos\frac{8\pi}{5}+\mathrm{i}\sin\frac{8\pi}{5}$ ***M1***

$= 1+\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}+\cos\frac{4\pi}{5}+\mathrm{i}\sin\frac{4\pi}{5}+\cos\frac{4\pi}{5}-\mathrm{i}\sin\frac{4\pi}{5}+\cos\frac{2\pi}{5}-\mathrm{i}\sin\frac{2\pi}{5}$ ***M1A1A1***

Note: Award ***M1*** for attempting to replace 6π and 8π by 4π and 2π .

Award ***A1*** for correct cosine terms and ***A1*** for correct sine terms.

$= 1 + 2\cos\frac{4\pi}{5} + 2\cos\frac{2\pi}{5} = 0$ ***A1***

Note: Correct methods involving equating real parts, use of conjugates or reciprocals are also accepted.

$\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$ ***AG***

[6 marks]

Note: Use of cis notation is acceptable throughout this question.

Total [12 marks]

Examiners report

Parts (a) and (b) were generally well done, although very few stated that $w \neq 1$ in (b). Part (c), the last question on the paper was challenging. Those candidates who gained some credit correctly focussed on the real part of the identity and realise that different cosine were related.

28. *[4 marks]*

Markscheme

$81 = \frac{n}{2}(1.5 + 7.5)$ ***M1***

$\Rightarrow n = 18$ ***A1***

$1.5 + 17d = 7.5$ ***M1***

$\Rightarrow d = \frac{6}{17}$ ***A1*** ***N0***

[4 marks]

Examiners report

There were many totally correct solutions to this question, but a number of candidates found two simultaneous equations and then spent a lot of time and working trying, often unsuccessfully, to solve these equations.

29a. *[12 marks]*

Markscheme

(a) $z = (1 - \mathrm{i})^{\frac{1}{4}}$

Let $1 - \mathrm{i} = r(\cos\theta + \mathrm{i}\sin\theta)$

$\Rightarrow r = \sqrt{2}$ ***A1***

$\theta = -\frac{\pi}{4}$ ***A1***

$z = \left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)\right)^{\frac{1}{4}}$ ***M1***

$= \left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4} + 2n\pi\right) + \mathrm{i}\sin\left(-\frac{\pi}{4} + 2n\pi\right)\right)\right)^{\frac{1}{4}}$

$= 2^{\frac{1}{8}}\left(\cos\left(-\frac{\pi}{16} + \frac{n\pi}{2}\right) + \mathrm{i}\sin\left(-\frac{\pi}{16} + \frac{n\pi}{2}\right)\right)$ ***M1***

$= 2^{\frac{1}{8}}\left(\cos\left(-\frac{\pi}{16}\right) + \mathrm{i}\sin\left(-\frac{\pi}{16}\right)\right)$

Note: Award ***M1*** above for this line if the candidate has forgotten to add 2π and no other solution given.

$= 2^{\frac{1}{8}}\left(\cos\left(\frac{7\pi}{16}\right) + \mathrm{i}\sin\left(\frac{7\pi}{16}\right)\right)$

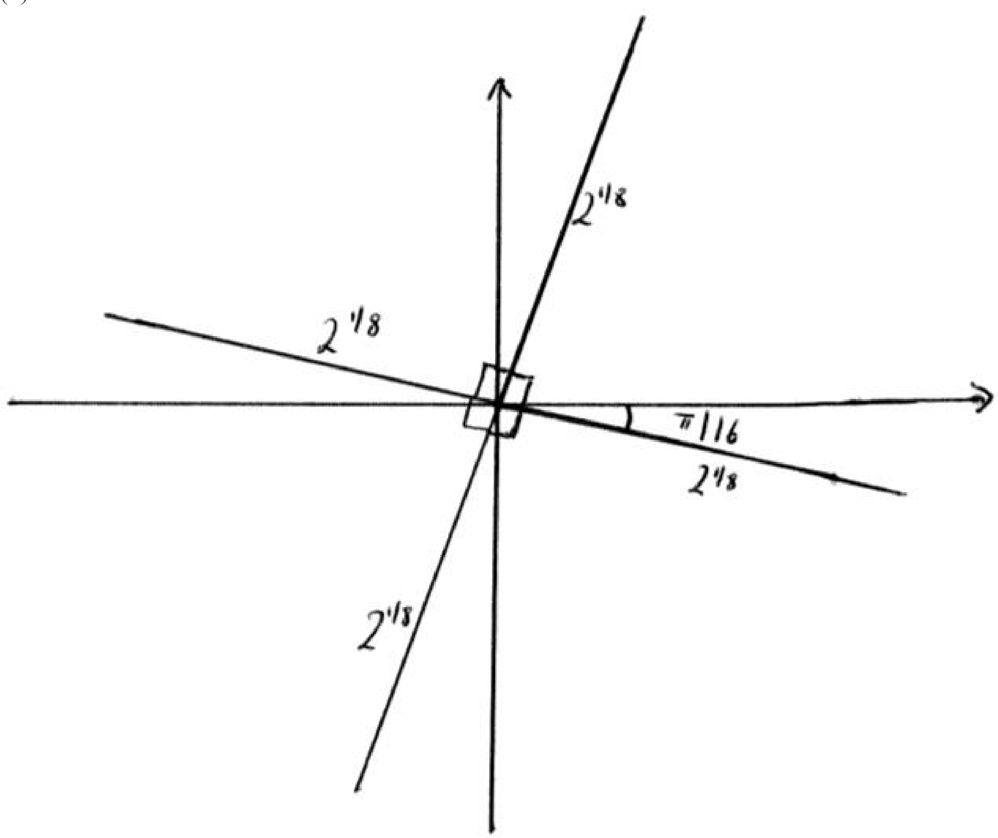
$= 2^{\frac{1}{8}}\left(\cos\left(\frac{15\pi}{16}\right) + \mathrm{i}\sin\left(\frac{15\pi}{16}\right)\right)$

$= 2^{\frac{1}{8}}\left(\cos\left(-\frac{9\pi}{16}\right) + \mathrm{i}\sin\left(-\frac{9\pi}{16}\right)\right)$ ***A2***

Note: Award ***A1*** for 2 correct answers. Accept any equivalent form.

[6 marks]

(b)



A2

Note: Award ***A1*** for roots being shown equidistant from the origin and one in each quadrant.

A1 for correct angular positions. It is not necessary to see written evidence of angle, but must agree with the diagram.

[2 marks]

(c) $\frac{z_2}{z_1} = \frac{2^{\frac{1}{8}}\left(\left(\cos\frac{15\pi}{16}\right)+\mathrm{i}\sin\left(\frac{15\pi}{16}\right)\right)}{2^{\frac{1}{8}}\left(\left(\cos\frac{7\pi}{16}\right)+\mathrm{i}\sin\left(\frac{7\pi}{16}\right)\right)}$ ***M1A1***

$= \cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}$ ***(A1)***

$= \mathrm{i}$ ***A1 N2***

$(\Rightarrow a = 0,\ b = 1)$

[4 marks]

Examiners report

The response to Part A was disappointing. Many candidates did not know that they had to apply de Moivre's theorem and did not appreciate that they needed to find four roots.

29b. *[13 marks]*

Markscheme

(a) $(x-1)(x^4+x^3+x^2+x+1)$

$= x^5+x^4+x^3+x^2+x-x^4-x^3-x^2-x-1$ ***(M1)***

$= x^5-1$ ***A1***

[2 marks]

(b) b is a root

$f(b) = 0$

$b^5 = 1$ ***M1***

$b^5 - 1 = 0$ ***A1***

$(b-1)(b^4+b^3+b^2+b+1) = 0$

$b \neq 1$ ***R1***

$1+b+b^2+b^3+b^4 = 0$ as shown. ***AG***

[3 marks]

(c) (i) $u+v = b^4+b^3+b^2+b = -1$ ***A1***

$uv = (b+b^4)(b^2+b^3) = b^3+b^4+b^6+b^7$ ***A1***

Now $b^5 = 1$ ***(A1)***

Hence $uv = b^3+b^4+b+b^2 = -1$ ***A1***

Hence $u+v = uv = -1$ ***AG***

(ii) $(u-v)^2 = (u^2+v^2) - 2uv$ ***(M1)***

$= \left((u+v)^2 - 2uv\right) - 2uv \quad \left(= (u+v)^2 - 4uv\right)$ ***(M1)A1***

Given $u - v > 0$

$u - v = \sqrt{(u+v)^2 - 4uv}$

$= \sqrt{(-1)^2 - 4(-1)}$

$= \sqrt{1+4}$ ***A1***

$= \sqrt{5}$ ***AG***

Note: Award ***A0*** unless an indicator is given that $u - v = -\sqrt{5}$ is invalid.

[8 marks]

Total [13 marks]

Examiners report

Part B started well for most candidates, but in part (b) many candidates did not appreciate the significance of b not lying on the real axis. A majority of candidates started (c) (i) and many fully correct answers were seen. Part (c) (ii) proved unsuccessful for all but the very best candidates.

30. *[5 marks]*

Markscheme

$2 \times 1.05^{n-1} > 500$ ***M1***

$n - 1 > \frac{\log\ 250}{\log\ 1.05}$ ***M1***

$n - 1 > 113.1675\ldots$ ***A1***

$n = 115$ ***(A1)***

$u_{115} = 521$ ***A1*** ***N5***

Note: Accept graphical solution with appropriate sketch.

[5 marks]

Examiners report

Many candidates misread the question and stopped at showing that the required term was the 115^{th}.

31. *[5 marks]*

Markscheme

(a)

$$\begin{pmatrix} 1 & 2 & -3 & k \\ 3 & 1 & 2 & 4 \\ 5 & 0 & 7 & 5 \end{pmatrix}$$ ***M1***

$R_1 - 2R_2$

$$\begin{pmatrix} -5 & 0 & -7 & k-8 \\ 3 & 1 & 2 & 4 \\ 5 & 0 & 7 & 5 \end{pmatrix}$$ ***(A1)***

$R_1 + R_3$

$$\begin{pmatrix} 0 & 0 & 0 & k-3 \\ 3 & 1 & 2 & 4 \\ 5 & 0 & 7 & 5 \end{pmatrix}$$ ***(A1)***

Hence no solutions if $k \in \mathbb{R}, k \neq 3$ ***A1***

(b) Two planes meet in a line and the third plane is parallel to that line.

[5 marks]

Examiners report

Most candidates realised that some form of row operations was appropriate here but arithmetic errors were fairly common. Many candidates whose arithmetic was correct gave their answer as $k = 3$ instead of $k \neq 3$. Very few candidates gave a correct answer to (b) with most failing to realise that stating that there was no common point was not enough to answer the question.

32. *[6 marks]*

Markscheme

(a) $|z| = \sqrt{5}$ and $|w| = \sqrt{4 + a^2}$

$|w| = 2|z|$

$\sqrt{4 + a^2} = 2\sqrt{5}$

attempt to solve equation ***M1***

Note: Award ***M0*** if modulus is not used.

$a = \pm 4$ ***A1A1 N0***

(b) $zw = (2 - 2a) + (4 + a)\mathrm{i}$ ***A1***

forming equation $2 - 2a = 2(4 + a)$ ***M1***

$a = -\frac{3}{2}$ ***A1 N0***

[6 marks]

Examiners report

Most candidates made good attempts to answer this question. Weaker candidates did not get full marks due to difficulties recognizing the notation and working with modulus of a complex number.

33a. *[9 marks]*

Markscheme

(a) $|z| = z$, $\arg(z) = 0$ ***A1A1***

so $L(z) = \ln z$ ***AG N0***

[2 marks]

(b) (i) $L(-1) = \ln 1 + \mathrm{i}\pi = \mathrm{i}\pi$ ***A1A1 N2***

(ii) $L(1 - \mathrm{i}) = \ln \sqrt{2} + \mathrm{i}\,\frac{7\pi}{4}$ ***A1A1 N2***

(iii) $L(-1 + \mathrm{i}) = \ln \sqrt{2} + \mathrm{i}\,\frac{3\pi}{4}$ ***A1 N1***

[5 marks]

(c) for comparing the product of two of the above results with the third ***M1***

for stating the result $-1 + \mathrm{i} = -1 \times (1 - \mathrm{i})$ and $L(-1 + \mathrm{i}) \neq L(-1) + L(1 - \mathrm{i})$ ***R1***

hence, the property $L(z_1 z_2) = L(z_1) + L(z_2)$

does not hold for all values of z_1 and z_2 ***AG N0***

[2 marks]

Total [9 marks]

Examiners report

Part A was answered well by a fair amount of candidates, with some making mistakes in calculating the arguments of complex numbers, as well as careless mistakes in finding the products of complex numbers.

33b. *[14 marks]*

Markscheme

(a) from $f(x+y)=f(x)f(y)$

for $x=y=0$ ***M1***

we have $f(0+0)=f(0)f(0) \Leftrightarrow f(0)=(f(0))^2$ ***A1***

as $f(0)\neq 0$, this implies that $f(0)=1$ ***R1AG*** ***N0***

[3 marks]

(b) **METHOD 1**

from $f(x+y)=f(x)f(y)$

for $y=-x$, we have $f(x-x)=f(x)f(-x) \Leftrightarrow f(0)=f(x)f(-x)$ ***M1A1***

as $f(0)\neq 0$ this implies that $f(x)\neq 0$ ***R1AG*** ***N0***

METHOD 2

suppose that, for a value of x, $f(x)=0$ ***M1***

from $f(x+y)=f(x)f(y)$

for $y=-x$, we have $f(x-x)=f(x)f(-x) \Leftrightarrow f(0)=f(x)f(-x)$ ***A1***

substituting $f(x)$ by 0 gives $f(0)=0$ which contradicts part (a) ***R1***

therefore $f(x)\neq 0$ for all x. ***AG*** ***N0***

[3 marks]

(c) by the definition of derivative

$f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ ***(M1)***

$=\lim_{h\to 0}\left(\frac{f(x)f(h)-f(x)f(0)}{h}\right)$ ***A1(A1)***

$=\lim_{h\to 0}\left(\frac{f(h)-f(0)}{h}\right)f(x)$ ***A1***

$=f'(0)f(x) \quad (=k\ f(x))$ ***AG*** ***N0***

[4 marks]

(d) $\int \frac{f'(x)}{f(x)}\,\mathrm{d}x=\int k\mathrm{d}x \Rightarrow \ln f(x)=kx+C$ ***M1A1***

$\ln f(0)=C \Rightarrow C=0$ ***A1***

$f(x)=\mathrm{e}^{kx}$ ***A1*** ***N1***

Note: Award ***M1A0A0A0*** if no arbitrary constant C.

[4 marks]

Total [14 marks]

Examiners report

Part B proved demanding for most candidates, particularly parts (c) and (d). A surprising number of candidates did not seem to know what was meant by the ‘definition of derivative’ in part (c) as they attempted to use quotient rule rather than first principles.

34. *[8 marks]*

Markscheme

(a) $1-\sqrt{3}\mathrm{i}$ ***A1***

(b) **EITHER**

$\left(z-(1+\sqrt{3}\mathrm{i})\right)\left(z-(1-\sqrt{3}\mathrm{i})\right)=z^2-2z+4$ ***(M1)A1***

$p(z)=(z-2)(z^2-2z+4)$ ***(M1)***

$=z^3-4z^2+8z-8$ ***A1***

therefore $b=-4,\ c=8,\ d=-8$

OR

relating coefficients of cubic equations to roots

$-b=2+1+\sqrt{3}\mathrm{i}+1-\sqrt{3}\mathrm{i}=4$ ***M1***

$c=2(1+\sqrt{3}\mathrm{i})+2(1-\sqrt{3}\mathrm{i})+(1+\sqrt{3}\mathrm{i})(1-\sqrt{3}\mathrm{i})=8$

$-d=2(1+\sqrt{3}\mathrm{i})(1-\sqrt{3}\mathrm{i})=8$

$b=-4,\ c=8,\ d=-8$ ***A1A1A1***

(c) $z_2=2\mathrm{e}^{\frac{\mathrm{i}\pi}{3}},\ z_3=2\mathrm{e}^{-\frac{\mathrm{i}\pi}{3}}$ ***A1A1A1***

Note: Award ***A1*** for modulus,

A1 for each argument.

[8 marks]

Examiners report

Parts a) and c) were done quite well by many but the method used in b) often lead to tedious and long algebraic manipulations in which students got lost and so did not get to the correct solution. Many did not give the principal argument in c).

35. *[8 marks]*

Markscheme

let $n = 1$

$\text{LHS} = 1 \times 1! = 1$

$\text{RHS} = (1 + 1)! - 1 = 2 - 1 = 1$

hence true for $n = 1$ ***R1***

assume true for $n = k$

$\sum_{r=1}^{k} r(r!) = (k + 1)! - 1$ ***M1***

$\sum_{r=1}^{k+1} r(r!) = (k + 1)! - 1 + (k + 1) \times (k + 1)!$ ***M1A1***

$= (k + 1)!(1 + k + 1) - 1$

$= (k + 1)!(k + 2) - 1$ ***A1***

$= (k + 2)! - 1$ ***A1***

hence if true for $n = k$, true for $n = k + 1$ ***R1***

since the result is true for $n = 1$ and $\mathrm{P}(k) \Rightarrow \mathrm{P}(k + 1)$ the result is proved by mathematical induction $\forall n \in \mathbb{Z}^{+}$ ***R1***

[8 marks]

Examiners report

This question was done poorly on a number of levels. Many students knew the structure of induction but did not show that they understood what they were doing. The general notation was poor for both the induction itself and the sigma notation.

In noting the case for $n = 1$ too many stated the equation rather than using the LHS and RHS separately and concluding with a statement. There were also too many who did not state the conclusion for this case.

Many did not state the assumption for $n = k$ as an assumption.

Most stated the equation for $n = k + 1$ and worked with the equation. Also common was the lack of sigma and inappropriate use of *n* and *k* in the statement. There were some very nice solutions however.

The final conclusion was often not complete or not considered which would lead to the conclusion that the student did not really understand what induction is about.

36. *[22 marks]*

Markscheme

(a) any appropriate form, *e.g.* $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos(n\theta) + \mathrm{i}\sin(n\theta)$ ***A1***

[1 mark]

(b) $z^n = \cos n\theta + \mathrm{i}\sin n\theta$ ***A1***

$\frac{1}{z^n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ ***(M1)***

$= \cos n\theta - \mathrm{i}\sin(n\theta)$ ***A1***

therefore $z^n - \frac{1}{z^n} = 2\mathrm{i}\sin(n\theta)$ ***AG***

[3 marks]

(c) $\left(z - \frac{1}{z}\right)^5 = z^5 + \binom{5}{1} z^4\left(-\frac{1}{z}\right) + \binom{5}{2} z^3\left(-\frac{1}{z}\right)^2 + \binom{5}{3} z^2\left(-\frac{1}{z}\right)^3 + \binom{5}{4} z\left(-\frac{1}{z}\right)^4 + \left(-\frac{1}{z}\right)^5$ ***(M1)(A1)***

$= z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$ ***A1***

[3 marks]

(d) $\left(z - \frac{1}{z}\right)^5 = z^5 - \frac{1}{z^5} - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$ ***M1A1***

$(2\mathrm{i}\sin\theta)^5 = 2\mathrm{i}\sin 5\theta - 10\mathrm{i}\sin 3\theta + 20\mathrm{i}\sin\theta$ ***M1A1***

$16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$ ***AG***

[4 marks]

(e) $16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$

$\text{LHS} = 16\left(\sin\frac{\pi}{4}\right)^5$

$= 16\left(\frac{\sqrt{2}}{2}\right)^5$

$= 2\sqrt{2} \quad \left(= \frac{4}{\sqrt{2}}\right)$ ***A1***

$\text{RHS} = \sin\left(\frac{5\pi}{4}\right) - 5\sin\left(\frac{3\pi}{4}\right) + 10\sin\left(\frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2} - 5\left(\frac{\sqrt{2}}{2}\right) + 10\left(\frac{\sqrt{2}}{2}\right)$ ***M1A1***

Note: Award ***M1*** for attempted substitution.

$= 2\sqrt{2} \quad \left(= \frac{4}{\sqrt{2}}\right)$ ***A1***

hence this is true for $\theta = \frac{\pi}{4}$ ***AG***

[4 marks]

(f) $\int_0^{\frac{\pi}{2}} \sin^5\theta \mathrm{d}\theta = \frac{1}{16}\int_0^{\frac{\pi}{2}} (\sin 5\theta - 5\sin 3\theta + 10\sin\theta)\mathrm{d}\theta$ ***M1***

$= \frac{1}{16}\left[-\frac{\cos 5\theta}{5} + \frac{5\cos 3\theta}{3} - 10\cos\theta\right]_0^{\frac{\pi}{2}}$ ***A1***

$= \frac{1}{16}\left[0 - \left(-\frac{1}{5} + \frac{5}{3} - 10\right)\right]$ ***A1***

$= \frac{8}{15}$ ***A1***

[4 marks]

(g) $\int_0^{\frac{\pi}{2}} \cos^5\theta \mathrm{d}\theta = \frac{8}{15}$, with appropriate reference to symmetry and graphs. ***A1R1R1***

Note: Award first ***R1*** for partially correct reasoning *e.g.* sketches of graphs of sin and cos.

Award second ***R1*** for fully correct reasoning involving $\sin^5$ and $\cos^5$.

[3 marks]

Total [22 marks]

Examiners report

Many students in b) substituted for the second term (again not making the connection to part a)) on the LHS and multiplied by the conjugate, which some managed well but it is inefficient. The binomial expansion was done well even if students did not do the earlier part. The connection between d) and f) was missed by many which lead to some creative attempts at the integral. Very few attempted the last part and of those many attempted another integral, ignoring the hence, while others related to the graph of sin and cos but not to the particular graphs here.

37. *[5 marks]*

Markscheme

EITHER

changing to modulus-argument form

$r = 2$

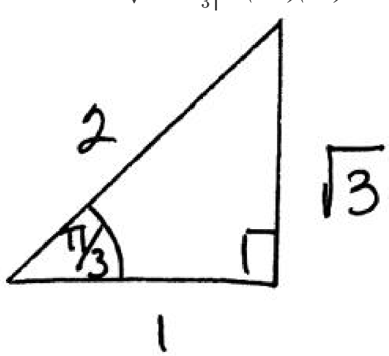
$\theta = \arctan\sqrt{3} = \frac{\pi}{3}$ ***(M1)A1***

$\Rightarrow 1 + \sqrt{3}^n = 2^n\left(\cos\frac{n\pi}{3} + \mathrm{i}\sin\frac{n\pi}{3}\right)$ ***M1***

if $\sin\frac{n\pi}{3} = 0 \Rightarrow n = \{0,\ \pm 3,\ \pm 6,\ \ldots\}$ ***(M1)A1*** ***N2***

OR

$\theta = \arctan\sqrt{3} = \frac{\pi}{3}$ ***(M1)(A1)***



M1

$n \in \mathbb{R} \Rightarrow \frac{n\pi}{3} = k\pi,\ k \in \mathbb{Z}$ ***M1***

$\Rightarrow n = 3k,\ k \in \mathbb{Z}$ ***A1*** ***N2***

[5 marks]

Examiners report

Some candidates did not consider changing the number to modulus-argument form. Among those that did this successfully, many considered individual values of n, or only positive values. Very few candidates considered negative multiples of 3.

38a. *[14 marks]*

Markscheme

(a) $S_6 = 81 \Rightarrow 81 = \frac{6}{2}(2a + 5d)$ ***M1A1***

$\Rightarrow 27 = 2a + 5d$

$S_{11} = 231 \Rightarrow 231 = \frac{11}{2}(2a + 10d)$ ***M1A1***

$\Rightarrow 21 = a + 5d$

solving simultaneously, $a = 6$, $d = 3$ ***A1A1***

[6 marks]

(b) $a + ar = 1$ ***A1***

$a + ar + ar^2 + ar^3 = 5$ ***A1***

$\Rightarrow (a + ar) + ar^2(1 + r) = 5$

$\Rightarrow 1 + ar^2 \times \frac{1}{a} = 5$

obtaining $r^2 - 4 = 0$ ***M1***

$\Rightarrow r = \pm 2$

$r = 2$ (since all terms are positive) ***A1***

$a = \frac{1}{3}$ ***A1***

[5 marks]

(c) AP r^{th} term is $3r + 3$ ***A1***

GP r^{th} term is $\frac{1}{3}2^{r-1}$ ***A1***

$3(r + 1) \times \frac{1}{3}2^{r-1} = (r + 1)2^{r-1}$ ***M1AG***

[3 marks]

Total [14 marks]

Examiners report

Parts (a), (b) and (c) were answered successfully by a large number of candidates. Some, however, had difficulty with the arithmetic.

38b. *[7 marks]*

Markscheme

prove: $P_n : \sum_{r=1}^{n}(r+1)2^{r-1} = n2^n,\ n \in \mathbb{Z}^+.$

show true for $n = 1$, *i.e.*

$\text{LHS} = 2 \times 2^0 = 2 = \text{RHS}$ ***A1***

assume true for $n = k$, *i.e.* ***M1***

$\sum_{r=1}^{k}(r+1)2^{r-1} = k2^k,\ k \in \mathbb{Z}^+$

consider $n = k + 1$

$\sum_{r=1}^{k+1}(r+1)2^{r-1} = k2^k + (k+2)2^k$ ***M1A1***

$= 2^k(k+k+2)$

$= 2(k+1)2^k$ ***A1***

$= (k+1)2^{k+1}$ ***A1***

hence true for $n = k + 1$

P_{k+1} is true whenever P_k is true, and P_1 is true, therefore P_n is true ***R1***

for $n \in \mathbb{Z}^+$

[7 marks]

Examiners report

In part (d) many candidates showed little understanding of sigma notation and proof by induction. There were cases of circular reasoning and using *n*, *k* and *r* randomly. A concluding sentence almost always appeared, even if the proof was done incorrectly, or not done at all.

39a. *[8 marks]*

Markscheme

(i) $\frac{1}{z} = \frac{1}{x+\mathrm{i}y} \times \frac{x-\mathrm{i}y}{x-\mathrm{i}y} = \frac{x}{x^2+y^2} - \mathrm{i}\frac{y}{x^2+y^2}$ ***(M1)A1***

(ii) $z + \frac{1}{z} = x + \frac{x}{x^2+y^2} + \mathrm{i}\left(y - \frac{y}{x^2+y^2}\right) = k$ ***(A1)***

for k to be real, $y - \frac{y}{x^2+y^2} = 0 \Rightarrow y(x^2+y^2-1) = 0$ ***M1A1***

hence, $y = 0$ or $x^2 + y^2 - 1 = 0 \Rightarrow x^2 + y^2 = 1$ ***AG***

(iii) when $x^2 + y^2 = 1,\ z + \frac{1}{z} = 2x$ ***(M1)A1***

$|x| \leqslant 1$ ***R1***

$\Rightarrow\ |k| \leqslant 2$ ***AG***

[8 marks]

Examiners report

A large number of candidates did not attempt part (a), or did so unsuccessfully.

39b. *[14 marks]*

Markscheme

(i) $w^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta) = \cos n\theta - \mathrm{i}\sin n\theta$ ***M1A1***

$\Rightarrow w^n + w^{-n} = (\cos n\theta + \mathrm{i}\sin n\theta) + (\cos n\theta - \mathrm{i}\sin n\theta) = 2\cos n\theta$ ***M1AG***

(ii) (rearranging)

$3(w^2 + w^{-2}) - (w + w^{-1}) + 2 = 0$ ***(M1)***

$\Rightarrow 3(2\cos 2\theta) - 2\cos\theta + 2 = 0$ ***A1***

$\Rightarrow 2(3\cos 2\theta - \cos\theta + 1) = 0$

$\Rightarrow 3(2\cos^2\theta - 1) - \cos\theta + 1 = 0$ ***M1***

$\Rightarrow 6\cos^2\theta - \cos\theta - 2 = 0$ ***A1***

$\Rightarrow (3\cos\theta - 2)(2\cos\theta + 1) = 0$ ***M1***

$\therefore \cos\theta = \frac{2}{3},\ \cos\theta = -\frac{1}{2}$ ***A1A1***

$\cos\theta = \frac{2}{3} \Rightarrow \sin\theta = \pm\frac{\sqrt{5}}{3}$ ***A1***

$\cos\theta = -\frac{1}{2} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$ ***A1***

$\therefore w = \frac{2}{3} \pm \frac{\mathrm{i}\sqrt{5}}{3}, -\frac{1}{2} \pm \frac{\mathrm{i}\sqrt{3}}{2}$ ***A1A1***

Note: Allow ***FT*** from incorrect $\cos\theta$ and/or $\sin\theta$.

[14 marks]

Examiners report

It was obvious that many candidates had been trained to answer questions of the type in part (b), and hence of those who attempted it, many did so successfully. Quite a few however failed to find all solutions.

40. *[7 marks]*

Markscheme

(a) There are 3! ways of arranging the Mathematics books, 5! ways of arranging the English books and 4! ways of arranging the Science books. ***(A1)***

Then we have 4 types of books which can be arranged in 4! ways. ***(A1)***

$3! \times 5! \times 4 \times 4! = 414\,720$ ***(M1)A1***

(b) There are 3! ways of arranging the subject books, and for each of these there are 2 ways of putting the dictionary next to the Mathematics books. ***(M1)(A1)***

$3! \times 5! \times 4! \times 3! \times 2 = 207\,360$ ***A1***

[7 marks]

Examiners report

Many students added instead of multiplying. There were, however, quite a few good answers to this question.

41. *[4 marks]*

Markscheme

(a) $18n - 10$ (or equivalent) ***A1***

(b) $\sum\limits_{1}^{n} (18r - 10)$ (or equivalent) ***A1***

(c) by use of GDC or algebraic summation or sum of an AP ***(M1)***

$\sum\limits_{1}^{15} (18r - 10) = 2010$ ***A1***

[4 marks]

Examiners report

An easy starter question, but few candidates seem to be familiar with the conventions of sigma notation.

42a. *[2 marks]*

Markscheme

(a) use GDC or manual method to find a, b and c ***(M1)***

obtain $a = 2,\ b = -1,\ c = 3$ (in any identifiable form) ***A1***

[2 marks]

Examiners report

Generally well done.

42b. *[4 marks]*

Markscheme

use GDC or manual method to solve second set of equations ***(M1)***

obtain $x = \frac{4-11t}{2}$; $y = \frac{-7t}{2}$; $z = t$ (or equivalent) ***(A1)***

$r = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5.5 \\ -3.5 \\ 1 \end{pmatrix}$ (accept equivalent vector forms) ***M1A1***

Note: Final ***A1*** requires $r =$ or equivalent.

[4 marks]

Examiners report

Moderate success here. Some forgot that an equation must have an = sign.

43. *[7 marks]*

Markscheme

(a) using de Moivre's theorem

$z^n + \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta \ (= 2\cos n\theta)$, imaginary part of which is 0 ***M1A1***

so $\mathrm{Im}\left(z^n + \frac{1}{z^n}\right) = 0$ ***AG***

(b) $\frac{z-1}{z+1} = \frac{\cos\theta + \mathrm{i}\sin\theta - 1}{\cos\theta + \mathrm{i}\sin\theta + 1}$

$= \frac{(\cos\theta - 1 + \mathrm{i}\sin\theta)(\cos\theta + 1 - \mathrm{i}\sin\theta)}{(\cos\theta + 1 + \mathrm{i}\sin\theta)(\cos\theta + 1 - \mathrm{i}\sin\theta)}$ ***M1A1***

Note: Award ***M1*** for an attempt to multiply numerator and denominator by the complex conjugate of their denominator.

$\Rightarrow \mathrm{Re}\left(\frac{z-1}{z+1}\right) = \frac{(\cos\theta - 1)(\cos\theta + 1) + \sin^2\theta}{\text{real denominator}}$ ***M1A1***

Note: Award ***M1*** for multiplying out the numerator.

$= \frac{\cos^2\theta + \sin^2\theta - 1}{\text{real denominator}}$ ***A1***

$= 0$ ***AG***

[7 marks]

Examiners report

Part(a) - The majority either obtained full marks or no marks here.

Part(b) - This question was algebraically complex and caused some candidates to waste their efforts for little credit.

44. *[12 marks]*

Markscheme

(a) the area of the first sector is $\frac{1}{2}2^2\theta$ ***(A1)***

the sequence of areas is $2\theta,\ 2k\theta,\ 2k^2\theta\ldots$ ***(A1)***

the sum of these areas is $2\theta(1+k+k^2+\ldots)$ ***(M1)***

$= \frac{2\theta}{1-k} = 4\pi$ ***M1A1***

hence $\theta = 2\pi(1-k)$ ***AG***

Note: Accept solutions where candidates deal with angles instead of area.

[5 marks]

(b) the perimeter of the first sector is $4+2\theta$ ***(A1)***

the perimeter of the third sector is $4+2k^2\theta$ ***(A1)***

the given condition is $4+2k^2\theta = 2+\theta$ ***M1***

which simplifies to $2 = \theta(1-2k^2)$ ***A1***

eliminating θ, obtain cubic in k: $\pi(1-k)(1-2k^2)-1=0$ ***A1***

or equivalent

solve for $k = 0.456$ and then $\theta = 3.42$ ***A1A1***

[7 marks]

Total [12 marks]

Examiners report

This was a disappointingly answered question.

Part(a) - Many candidates correctly assumed that the areas of the sectors were proportional to their angles, but did not actually state that fact.

Part(b) - Few candidates seem to know what the term ‘perimeter’ means.

45. *[6 marks]*

Markscheme

(a) coefficient of x^3 is $\binom{n}{3}\left(\frac{1}{2}\right)^3 = 70$ ***M1(A1)***

$\frac{n!}{3!(n-3)!} \times \frac{1}{8} = 70$ ***(A1)***

$\Rightarrow \frac{n(n-1)(n-2)}{48} = 70$ ***(M1)***

$n = 16$ ***A1***

(b) $\binom{16}{2}\left(\frac{1}{2}\right)^2 = 30$ ***A1***

[6 marks]

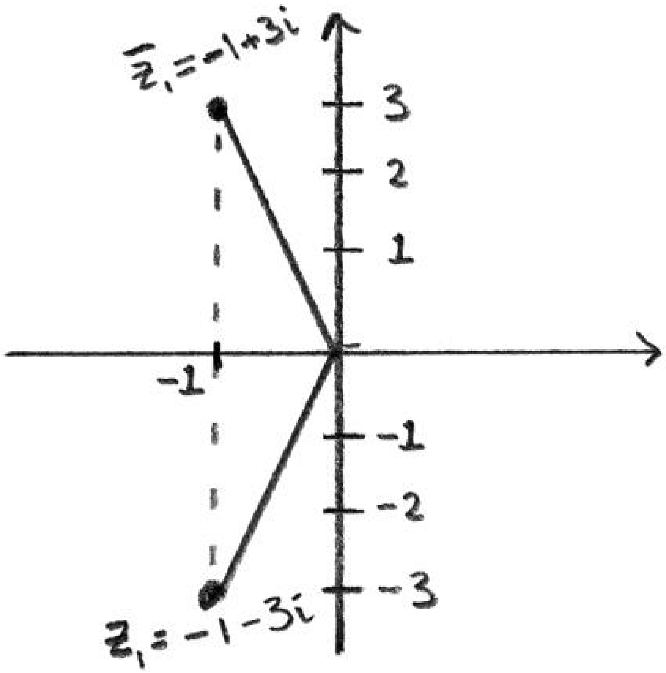
Examiners report

Most candidates were able to answer this question well.

46\. *[7 marks]*

Markscheme

(a) one root is $-1 - 3\mathrm{i}$ ***A1***



distance between roots is 6, implies height is 3 ***(M1)A1***

EITHER

$-1 + 3 = 2 \Rightarrow$ third root is 2 ***A1***

OR

$-1 - 3 = -4 \Rightarrow$ third root is 4 ***A1***

(b) **EITHER**

$z - \ (-1 + 3\mathrm{i}) \ z - \ (-1 - 3\mathrm{i}) \ (z - 2) = 0$ ***M1***

$\Rightarrow \ (z^2 + 2z + 10)\ (z - 2) = 0$ ***(A1)***

$z^2 + 6z - 20 = 0$ ***A1***

$a = 0$, $b = 6$ and $c = 20$

OR

$z - \ (-1 + 3\mathrm{i}) \ z - \ (-1 - 3\mathrm{i}) \ (z + 4) = 0$ ***M1***

$\Rightarrow \ (z^2 + 2z + 10)\ (z + 4) = 0$ ***(A1)***

$z^2 + 6z^2 + 18z + 40 = 0$ ***A1***

$a = 6$, $b = 18$ and $c = 40$

[7 marks]

Examiners report

Most students were able to state the conjugate root, but many were unable to take the question further. Of those that then recognised the method, the question was well answered.

47. *[6 marks]*

Markscheme

(a) $\mathrm{i}^4 - 5\mathrm{i}^3 + 7\mathrm{i}^2 - 5\mathrm{i} + 6 = 1 + 5\mathrm{i} - 7 - 5\mathrm{i} + 6$ ***M1A1***

$= 0$ ***AG N0***

(b) i root $\Rightarrow$ $-\mathrm{i}$ is second root ***(M1)A1***

moreover, $x^4 - 5x^3 + 7x^2 - 5x + 6 = (x - \mathrm{i})(x + \mathrm{i})\,q(x)$

where $q(x) = x^2 - 5x + 6$

finding roots of $q(x)$

the other two roots are 2 and 3 ***A1A1***

Note: Final ***A1A1*** is independent of previous work.

[6 marks]

Examiners report

A surprising number of candidates solved the question by dividing the expression by $1 - i$ rather than substituting i into the expression. Many students were not aware that complex roots occur in conjugate pairs, and many did not appreciate the difference between a factor and a root. Generally the question was well done.

48. *[5 marks]*

Markscheme

EITHER

with no restrictions six people can be seated in $5! = 120$ ways ***A1***

we now count the number of ways in which the two restricted people will be sitting next to each other

call the two restricted people p_1 and p_2

they sit next to each other in two ways ***A1***

the remaining people can then be seated in $4!$ ways ***A1***

the six may be seated p_1 and p_2 next to each other) in $2 \times 4! = 48$ ways ***M1***

$\therefore$ with p_1 and (p_2 not next to each other the number of ways $= 120 - 48 = 72$ ***A1 N3***

[5 marks]

OR

person p_1 seated at table in 1 way ***A1***

p_2 then sits in any of 3 seats (not next to p_1) ***M1A1***

the remaining 4 people can then be seated in $4!$ ways ***A1***

$\therefore$ number ways with p_1 not next to $p_2 = 3 \times 4! = 72$ ways ***A1 N3***

Note: If candidate starts with $6!$ instead of $5!$, potentially leading to an answer of 432, do not penalise.

[5 marks]

Examiners report

Very few candidates provided evidence of a clear strategy for solving such a question. The problem which was set in a circular scenario was no more difficult than an analogous linear one.

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