Topic 2 Part 1 *[373 marks]*

1a. *[2 marks]*

Markscheme

EITHER

$f(-x) = f(x)$ **M1**

$\Rightarrow ax^2 - bx + c = ax^2 + bx + c \Rightarrow 2bx = 0, \ (\forall x \in \mathbb{R})$ **A1**

OR

y-axis is eqn of symmetry   **M1**

so $\frac{-b}{2a} = 0$ **A1**

THEN

$\Rightarrow b = 0$ **AG**

[2 marks]

Examiners report

Sometimes backwards working but many correct approaches.

1b. *[2 marks]*

Markscheme

$g(-x) = -g(x) \Rightarrow p\sin(-x) - qx + r = -p\sin x - qx - r$
$\Rightarrow -p\sin x - qx + r = -p\sin x - qx - r$ **M1**

Note: **M1** is for knowing properties of sin.

$\Rightarrow 2r = 0 \Rightarrow r = 0$ **A1**

Note: In (a) and (b) allow substitution of a particular value of x

[2 marks]

Examiners report

Some candidates did not know what odd and even functions were. Correct solutions from those who applied the definition.

1c. *[2 marks]*

Markscheme

$h(-x) = h(x) = -h(x) \Rightarrow 2h(x) = 0 \Rightarrow h(x) = 0, \ (\forall x)$ **M1A1**

Note: Accept geometrical explanations.

[2 marks]

Total [6 marks]

2a.   *[4 marks]*

Markscheme

$f : x \to y = \frac{3x-2}{2x-1}$ $f^{-1} : y \to x$

$y = \frac{3x-2}{2x-1} \Rightarrow 3x - 2 = 2xy - y$ **M1**

$\Rightarrow 3x - 2xy = -y + 2$ **M1**

$x(3 - 2y) = 2 - y$

$x = \frac{2-y}{3-2y}$ **A1**

$\left(f^{-1}(y) = \frac{2-y}{3-2y} \right)$

$f^{-1}(x) = \frac{2-x}{3-2x}$ $\left(x \neq \frac{3}{2} \right)$ **A1**

Note: x and y might be interchanged earlier.

Note: First **M1** is for interchange of variables second **M1** for manipulation

Note: Final answer must be a function of x

[4 marks]

2b.   *[2 marks]*

Markscheme

$\frac{3x-2}{2x-1} = A + \frac{B}{2x-1} \Rightarrow 3x - 2 = A(2x - 1) + B$

equating coefficients $3 = 2A$ and $-2 = -A + B$ *(M1)*

$A = \frac{3}{2}$ and $B = -\frac{1}{2}$ **A1**

Note: Could also be done by division or substitution of values.

[2 marks]

2c.

Markscheme

$\int f(x)\mathrm{d}x = \frac{3}{2}x - \frac{1}{4}\ln|2x-1| + c$ **A1**

Note: accept equivalent e.g.
$\ln|4x-2|$

[1 mark]

Total [7 marks]

Examiners report

Reasonable if they had an answer to (b) (follow through was given) usual mistakes with not allowing for the derivative of the bracket.

3a.

Markscheme

(i) $\left(-\frac{a_{n-1}}{a_n} = \right) - \frac{1}{2}$ **A1**

(ii) $\left((-1)^n \frac{a_0}{a_n} = \right) - \frac{36}{2} = (-18)$ **A1A1**

Note: First **A1** is for the negative sign.

[3 marks]

Examiners report

Both parts fine if they used the formula, some tried to use the quadratic equivalent formula. Surprisingly some even found all the roots.

3b.

Markscheme

METHOD 1

if λ satisfies $p(\lambda) = 0$ then $q(\lambda - 4) = 0$

so the roots of $q(x)$ are each 4 less than the roots of $p(x)$ **(R1)**

so sum of roots is $-\frac{1}{2} - 4 \times 5 = -20.5$ **A1**

METHOD 2

$p(x+4) = 2x^5 + 2 \times 5 \times 4x^4 \ldots + x^4 \ldots = 2x^5 + 41x^4 \ldots$ **(M1)**

so sum of roots is $-\frac{41}{2} = -20.5$ **A1**

[2 marks]

Tofal [5 marks]

Examiners report

Some notation problems for weaker candidates. Good candidates used either of the methods shown in the Markscheme.

4a.

Markscheme

$g \circ f(x) = g\left(f(x)\right)$ **M1**

$= g\left(2x + \frac{\pi}{5}\right)$

$= 3\sin\left(2x + \frac{\pi}{5}\right) + 4$ **AG**

[1 mark]

Examiners report

Well done.

4b.

Markscheme

since $-1 \le sin\theta \le +1$, range is $[1,\ 7]$ **(R1)A1**

[2 marks]

Examiners report

Generally well done, some used more complicated methods rather than considering the range of sine.

4c.

Markscheme

$3\sin\left(2x + \frac{\pi}{5}\right) + 4 = 7 \Rightarrow 2x + \frac{\pi}{5} = \frac{\pi}{2} + 2n\pi \Rightarrow x = \frac{3\pi}{20} + n\pi$ **(M1)**

so next biggest value is $\frac{23\pi}{20}$ **A1**

Note: Allow use of period.

[2 marks]

Examiners report

Fine if they realised the period was π, not if they thought it was 2π.

4d.

Markscheme

Note: Transformations can be in any order but see notes below.

stretch scale factor 3 parallel to
y axis (vertically) **A1**

vertical translation of 4 up **A1**

Note: Vertical translation is $\frac{4}{3}$ up if it occurs before stretch parallel to
y axis.

stretch scale factor $\frac{1}{2}$ parallel to
x axis (horizontally) **A1**

horizontal translation of $\frac{\pi}{10}$ to the left **A1**

Note: Horizontal translation is $\frac{\pi}{5}$ to the left if it occurs before stretch parallel to
x axis.

Note: Award **A1** for magnitude and direction in each case.

Accept any correct terminology provided that the meaning is clear *eg* shift for translation.

[4 marks]

Total [9 marks]

Examiners report

Typically 3 marks were gained. It was the shift in the axis χ of $\frac{\pi}{10}$ that caused the problem.

5a.

Markscheme

$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 \times \mathrm{e}^{3x} + x \times 3\mathrm{e}^{3x} = (\mathrm{e}^{3x} + 3x\mathrm{e}^{3x})$ **M1A1**

[2 marks]

Examiners report

Well done.

5b. [7 marks]

Markscheme

let $P(n)$ be the statement $\frac{\mathrm{d}^n y}{\mathrm{d}x^n} = n3^{n-1}\mathrm{e}^{3x} + x3^n\mathrm{e}^{3x}$

prove for $n = 1$ **M1**

LHS of $P(1)$ is $\frac{\mathrm{d}y}{\mathrm{d}x}$ which is $1 \times \mathrm{e}^{3x} + x \times 3\mathrm{e}^{3x}$ and
RHS is $3^0\mathrm{e}^{3x} + x3^1\mathrm{e}^{3x}$ **R1**

as $\mathrm{LHS} = \mathrm{RHS}$, $P(1)$ is true

assume $P(k)$ is true and attempt to prove $P(k+1)$ is true   **M1**

assuming $\frac{\mathrm{d}^k y}{\mathrm{d}x^k} = k3^{k-1}\mathrm{e}^{3x} + x3^k\mathrm{e}^{3x}$

$\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}^k y}{\mathrm{d}x^k}\right)$ **(M1)**

$= k3^{k-1} \times 3\mathrm{e}^{3x} + 1 \times 3^k\mathrm{e}^{3x} + x3^k \times 3\mathrm{e}^{3x}$ **A1**

$= (k+1)3^k\mathrm{e}^{3x} + x3^{k+1}\mathrm{e}^{3x}$ (as required)   **A1**

Note: Can award the **A** marks independent of the **M** marks

since $P(1)$ is true and $P(k)$ is true $\Rightarrow P(k+1)$ is true

then (by
PMI), $P(n)$ is true $(\forall n \in \mathbb{Z}^+)$ **R1**

Note: To gain last **R1** at least four of the above marks must have been gained.

[7 marks]

Examiners report

The logic of an induction proof was not known well enough. Many candidates used what they had to prove rather than differentiating what they had assumed. They did not have enough experience in doing Induction proofs.

5c. [5 marks]

Markscheme

$\mathrm{e}^{3x} + x \times 3\mathrm{e}^{3x} = 0 \Rightarrow 1 + 3x = 0 \Rightarrow x = -\frac{1}{3}$ **M1A1**

point is $\left(-\frac{1}{3}, -\frac{1}{3\mathrm{e}}\right)$ **A1**

EITHER

$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2 \times 3\mathrm{e}^{3x} + x \times 3^2\mathrm{e}^{3x}$

when $x = -\frac{1}{3}$, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} > 0$ therefore the point is a minimum   **M1A1**

OR

x	$-\dfrac{1}{3}$		
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	$-ve$	0	$+ve$

nature table shows point is a minimum   **M1A1**

[5 marks]

Examiners report

Good, some forgot to test for min/max, some forgot to give the y value.

5d. [5 marks]

Markscheme

$\frac{d^2y}{dx^2} = 2 \times 3e^{3x} + x \times 3^2 e^{3x}$ **A1**

$2 \times 3e^{3x} + x \times 3^2 e^{3x} = 0 \Rightarrow 2 + 3x = 0 \Rightarrow x = -\frac{2}{3}$ **M1A1**

point is $\left(-\frac{2}{3},\ -\frac{2}{3e^2}\right)$ **A1**

x	$-\dfrac{2}{3}$		
$\dfrac{d^2y}{dx^2}$	$-ve$	0	$+ve$

since the curvature does change (concave down to concave up) it is a point of inflection   **R1**

Note: Allow 3rd derivative is not zero at $-\frac{2}{3}$

[5 marks]

Examiners report

Again quite good, some forgot to check for change in curvature and some forgot the y value.

6a. [2 marks]

Markscheme

$a > 0$ **A1**

$a \neq 0$ **A1**

[2 marks]

Examiners report

[N/A]

6b.

Markscheme

METHOD 1

$\log_x y = \frac{\ln y}{\ln x}$ and $\log_y x = \frac{\ln x}{\ln y}$ **M1A1**

Note: Use of any base is permissible here, not just "e".

$\left(\frac{\ln y}{\ln x}\right)^2 = 4$ **A1**

$\ln y = \pm 2 \ln x$ **A1**

$y = x^2$ or $\frac{1}{x^2}$ **A1A1**

METHOD 2

$\log_y x = \frac{\log_x x}{\log_x y} = \frac{1}{\log_x y}$ **M1A1**

$(\log_x y)^2 = 4$ **A1**

$\log_x y = \pm 2$ **A1**

$y = x^2$ or $y = \frac{1}{x^2}$ **A1A1**

Note: The final two **A** marks are independent of the one coming before.

[6 marks]

Total [8 marks]

Examiners report

[N/A]

7a.

Markscheme
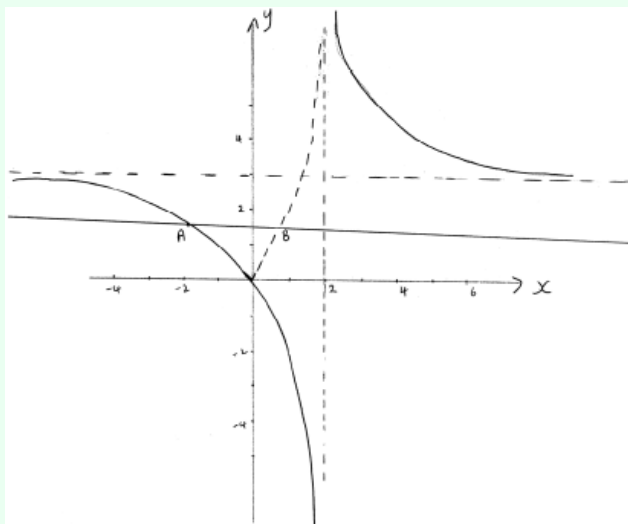
Note: In the diagram, points marked
A and
B refer to part (d) and do not need to be seen in part (a).

shape of curve   ***A1***

Note: This mark can only be awarded if there appear to be both horizontal and vertical asymptotes.

intersection at $(0,\ 0)$ ***A1***

horizontal asymptote at
$y = 3$ ***A1***

vertical asymptote at $x = 2$ ***A1***

[4 marks]

Examiners report

[N/A]

7b.

Markscheme

$y = \frac{3x}{x-2}$

$xy - 2y = 3x$ ***M1A1***

$xy - 3x = 2y$

$x = \frac{2y}{y-3}$

$\left(f^{-1}(x)\right) = \frac{2x}{x-3}$ ***M1A1***

Note: Final M1 is for interchanging of x and y, which may be seen at any stage.

[4 marks]

Examiners report

[N/A]

7c.

Markscheme

METHOD 1

attempt to solve $\frac{2x}{x-3} = \frac{3x}{x-2}$ **(M1)**

$2x(x-2) = 3x(x-3)$

$x\left[2(x-2) - 3(x-3)\right] = 0$

$x(5-x) = 0$

$x = 0$ or $x = 5$ **A1A1**

METHOD 2

$x = \frac{3x}{x-2}$ or $x = \frac{2x}{x-3}$ **(M1)**

$x = 0$ or $x = 5$ **A1A1**

[3 marks]

Examiners report

[N/A]

7d.

Markscheme

METHOD 1

at A : $\frac{3x}{x-2} = \frac{3}{2}$ AND at B : $\frac{3x}{x-2} = -\frac{3}{2}$ **M1**

$6x = 3x - 6$

$x = -2$ **A1**

$6x = 6 - 3x$

$x = \frac{2}{3}$ **A1**

solution is $-2 < x < \frac{2}{3}$ **A1**

METHOD 2

$\left(\frac{3x}{x-2}\right)^2 < \left(\frac{3}{2}\right)^2$ **M1**

$9x^2 < \frac{9}{4}(x-2)^2$

$3x^2 + 4x - 4 < 0$

$(3x-2)(x+2) < 0$

$x = -2$ **(A1)**

$x = \frac{2}{3}$ **(A1)**

solution is $-2 < x < \frac{2}{3}$ **A1**

[4 marks]

Examiners report

[N/A]

7e. *[2 marks]*

Markscheme

$-2 < x < 2$ ***A1A1***

Note: ***A1*** for correct end points, ***A1*** for correct inequalities.

Note: If working is shown, then ***A*** marks may only be awarded following correct working.

[2 marks]

Total [17 marks]

Examiners report

[N/A]

8a. *[2 marks]*

Markscheme

$g \circ f(x) = \frac{\tan x + 1}{\tan x - 1}$ ***A1***

$x \neq \frac{\pi}{4}, \; 0 \leq x < \frac{\pi}{2}$ ***A1***

[2 marks]

Examiners report

[N/A]

8b. *[2 marks]*

Markscheme

$\frac{\tan x + 1}{\tan x - 1} = \frac{\frac{\sin x}{\cos x} + 1}{\frac{\sin x}{\cos x} - 1}$ ***M1A1***

$= \frac{\sin x + \cos x}{\sin x - \cos x}$ ***AG***

[2 marks]

Examiners report

[N/A]

Markscheme

METHOD 1

$\frac{dy}{dx} = \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$ **M1(A1)**

$\frac{dy}{dx} = \frac{(2\sin x \cos x - \cos^2 x - \sin^2 x) - (2\sin x \cos x + \cos^2 x + \sin^2 x)}{\cos^2 x + \sin^2 x - 2\sin x \cos x}$

$= \frac{-2}{1 - \sin 2x}$

Substitute $\frac{\pi}{6}$ into any formula for $\frac{dy}{dx}$ **M1**

$\frac{-2}{1 - \sin \frac{\pi}{3}}$

$= \frac{-2}{1 - \frac{\sqrt{3}}{2}}$ **A1**

$= \frac{-4}{2 - \sqrt{3}}$

$= \frac{-4}{2 - \sqrt{3}}\left(\frac{2 + \sqrt{3}}{2 + \sqrt{3}}\right)$ **M1**

$= \frac{-8 - 4\sqrt{3}}{1} = -8 - 4\sqrt{3}$ **A1**

METHOD 2

$\frac{dy}{dx} = \frac{(\tan x - 1)\sec^2 x - (\tan x + 1)\sec^2 x}{(\tan x - 1)^2}$ **M1A1**

$= \frac{-2\sec^2 x}{(\tan x - 1)^2}$ **A1**

$= \frac{-2\sec^2 \frac{\pi}{6}}{\left(\tan \frac{\pi}{6} - 1\right)^2} = \frac{-2\left(\frac{4}{3}\right)}{\left(\frac{1}{\sqrt{3}} - 1\right)^2} = \frac{-8}{\left(1 - \sqrt{3}\right)^2}$ **M1**

Note: Award **M1** for substitution $\frac{\pi}{6}$.

$\frac{-8}{\left(1 - \sqrt{3}\right)^2} = \frac{-8}{\left(4 - 2\sqrt{3}\right)}\frac{\left(4 + 2\sqrt{3}\right)}{\left(4 + 2\sqrt{3}\right)} = -8 - 4\sqrt{3}$ **M1A1**

[6 marks]

Examiners report

[N/A]

Markscheme

Area $\left| \int_0^{\frac{\pi}{6}} \frac{\sin x + \cos x}{\sin x - \cos x} \, dx \right|$ **M1**

$= \left| \left[\ln|\sin x - \cos x| \right]_0^{\frac{\pi}{6}} \right|$ **A1**

Note: Condone absence of limits and absence of modulus signs at this stage.

$= \left| \ln|\sin \frac{\pi}{6} - \cos \frac{\pi}{6}| - \ln|\sin 0 - \cos 0| \right|$ **M1**

$= \left| \ln\left| \frac{1}{2} - \frac{\sqrt{3}}{2} \right| - 0 \right|$

$= \left| \ln\left(\frac{\sqrt{3}-1}{2} \right) \right|$ **A1**

$= -\ln\left(\frac{\sqrt{3}-1}{2} \right) = \ln\left(\frac{2}{\sqrt{3}-1} \right)$ **A1**

$= \ln\left(\frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \right)$ **M1**

$= \ln\left(\sqrt{3} + 1 \right)$ **AG**

[6 marks]

Total [16 marks]

Examiners report

[N/A]

Markscheme

(i)-(iii) given the three roots α, β, γ, we have

$x^3 + px^2 + qx + c = (x - \alpha)(x - \beta)(x - \gamma)$ **M1**

$= \left(x^2 - (\alpha + \beta)x + \alpha\beta \right)(x - \gamma)$ **A1**

$= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$ **A1**

comparing coefficients:

$p = -(\alpha + \beta + \gamma)$ **AG**

$q = (\alpha\beta + \beta\gamma + \gamma\alpha)$ **AG**

$c = -\alpha\beta\gamma$ **AG**

[3 marks]

Examiners report

[N/A]

9b.

Markscheme

METHOD 1

(i)   Given $-\alpha - \beta - \gamma = -6$

And $\alpha\beta + \beta\gamma + \gamma\alpha = 18$

Let the three roots be $\alpha,\ \beta,\ \gamma$

So $\beta - \alpha = \gamma - \beta$ ***M1***

or $2\beta = \alpha + \gamma$

Attempt to solve simultaneous equations: ***M1***

$\beta + 2\beta = 6$ ***A1***

$\beta = 2$ ***AG***

(ii) $\alpha + \gamma = 4$

$2\alpha + 2\gamma + \alpha\gamma = 18$

$\Rightarrow \gamma^2 - 4\gamma + 10 = 0$

$\Rightarrow \gamma = \frac{4 \pm i\sqrt{24}}{2}$ ***(A1)***

Therefore $c = -\alpha\beta\gamma = -\left(\frac{4+i\sqrt{24}}{2}\right)\left(\frac{4-i\sqrt{24}}{2}\right)2 = -20$ ***A1***

METHOD 2

(i)   let the three roots be $\alpha,\ \alpha - d,\ \alpha + d$ ***M1***

adding roots   ***M1***

to give $3\alpha = 6$ ***A1***

$\alpha = 2$ ***AG***

(ii) α is a root, so $2^3 - 6 \times 2^2 + 18 \times 2 + c = 0$ ***M1***

$8 - 24 + 36 + c = 0$

$c = -20$ ***A1***

METHOD 3

(i)   let the three roots be $\alpha,\ \alpha - d,\ \alpha + d$ ***M1***

adding roots   ***M1***

to give $3\alpha = 6$ ***A1***

$\alpha = 2$ ***AG***

(ii) $q = 18 = 2(2 - d) + (2 - d)(2 + d) + 2(2 + d)$ ***M1***

$d^2 = -6 \Rightarrow d = \sqrt{6}i$

$\Rightarrow c = -20$ ***A1***

[5 marks]

Examiners report

[N/A]

9c. [6 marks]

Markscheme

METHOD 1

Given $-\alpha - \beta - \gamma = -6$

And $\alpha\beta + \beta\gamma + \gamma\alpha = 18$

Let the three roots be $\alpha, \ \beta, \ \gamma$.

So $\frac{\beta}{\alpha} = \frac{\gamma}{\beta}$ *M1*

or $\beta^2 = \alpha\gamma$

Attempt to solve simultaneous equations: *M1*

$\alpha\beta + \gamma\beta + \beta^2 = 18$

$\beta(\alpha + \beta + \gamma) = 18$

$6\beta = 18$

$\beta = 3$ *A1*

$\alpha + \gamma = 3, \ \alpha = \frac{9}{\gamma}$

$\Rightarrow \gamma^2 - 3\gamma + 9 = 0$

$\Rightarrow \gamma = \frac{3 \pm i\sqrt{27}}{2}$ *(A1)(A1)*

Therefore $c = -\alpha\beta\gamma = -\left(\frac{3+i\sqrt{27}}{2}\right)\left(\frac{3-i\sqrt{27}}{2}\right)3 = -27$ *A1*

METHOD 2

let the three roots be $a, \ ar, \ ar^2$ *M1*

attempt at substitution of $a, \ ar, \ ar^2$ and p and q into equations from (a)   *M1*

$6 = a + ar + ar^2 \ \left(= a(1 + r + r^2)\right)$ *A1*

$18 = a^2r + a^2r^3 + a^2r^2 \ \left(= a^2r(1 + r + r^2)\right)$ *A1*

therefore $3 = ar$ *A1*

therefore $c = -a^3r^3 = -3^3 = -27$ *A1*

[6 marks]

Total [14 marks]

Examiners report

[N/A]

10a. [2 marks]

Markscheme

$\frac{1}{\sqrt{n}+\sqrt{n+1}} = \frac{1}{\sqrt{n}+\sqrt{n+1}} \times \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}-\sqrt{n}}$ *M1*

$= \frac{\sqrt{n+1}-\sqrt{n}}{(n+1)-n}$ *A1*

$= \sqrt{n+1} - \sqrt{n}$ *AG*

[2 marks]

Examiners report

[N/A]

10b.

Markscheme

$\sqrt{2} - 1 = \frac{1}{\sqrt{2}+\sqrt{1}}$ **A2**

$< \frac{1}{\sqrt{2}}$ **AG**

[2 marks]

Examiners report

[N/A]

10c.

Markscheme

consider the case $n = 2$: required to prove that $1 + \frac{1}{\sqrt{2}} > \sqrt{2}$ **M1**

from part (b) $\frac{1}{\sqrt{2}} > \sqrt{2} - 1$

hence $1 + \frac{1}{\sqrt{2}} > \sqrt{2}$ is true for $n = 2$ **A1**

now assume true for $n = k : \sum_{r=1}^{r=k} \frac{1}{\sqrt{r}} > \sqrt{k}$ **M1**

$\frac{1}{\sqrt{1}} + \ldots + \frac{\sqrt{1}}{\sqrt{k}} > \sqrt{k}$

attempt to prove true for $n = k + 1 : \frac{1}{\sqrt{1}} + \ldots + \frac{\sqrt{1}}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ **(M1)**

from assumption, we have that $\frac{1}{\sqrt{1}} + \ldots + \frac{\sqrt{1}}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$ **M1**

so attempt to show that $\sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$ **(M1)**

EITHER

$\frac{1}{\sqrt{k+1}} > \sqrt{k+1} - \sqrt{k}$ **A1**

$\frac{1}{\sqrt{k+1}} > \frac{1}{\sqrt{k}+\sqrt{k+1}}$, (from part a), which is true   **A1**

OR

$\sqrt{k} + \frac{1}{\sqrt{k+1}} = \frac{\sqrt{k+1}\sqrt{k}+1}{\sqrt{k+1}}$ **A1**

$> \frac{\sqrt{k}\sqrt{k}+1}{\sqrt{k+1}} = \sqrt{k+1}$ **A1**

THEN

so true for $n = 2$ and $n = k$ true $\Rightarrow n = k + 1$ true. Hence true for all $n \geq 2$ **R1**

Note: Award **R1** only if all previous **M** marks have been awarded.

[9 marks]

Total [13 marks]

Examiners report

[N/A]

11.

Markscheme

$f'(x) = 3x^2 + e^x$ **A1**

Note: Accept labelled diagram showing the graph $y = f'(x)$ above the *x*-axis;

do not accept unlabelled graphs nor graph of $y = f(x)$.

EITHER

this is always > 0 **R1**

so the function is (strictly) increasing   **R1**

and thus $1 - 1$ **A1**

OR

this is always > 0 $(\text{accept} \neq 0)$ **R1**

so there are no turning points   **R1**

and thus $1 - 1$ **A1**

Note: **A1** is dependent on the first **R1**.

[4 marks]

Examiners report

The differentiation was normally completed correctly, but then a large number did not realise what was required to determine the type of the original function. Most candidates scored 1/4 and wrote explanations that showed little or no understanding of the relation between first derivative and the given function. For example, it was common to see comments about horizontal and vertical line tests but applied to the incorrect function.In term of mathematical language, it was noted that candidates used many terms incorrectly showing no knowledge of the meaning of terms like 'parabola', 'even' or 'odd' (or no idea about these concepts).

12a.

Markscheme
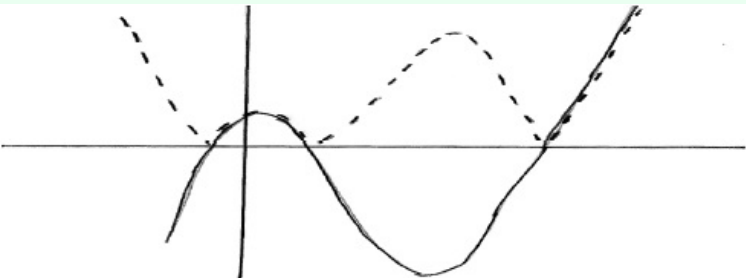
as roots of $f(x) = 0$ are $-1,\ 1,\ 5$ **(M1)**

solution is $]-\infty,\ -1[\cup]1,\ 5[$ $(x < -1 \ \ \text{or} \ \ 1 < x < 5)$ **A1A1**

Note: Award **A1A0** for closed intervals.

[3 marks]

Examiners report

In general part (a) was performed correctly, with the vast majority of candidates stating the correct open intervals as required.
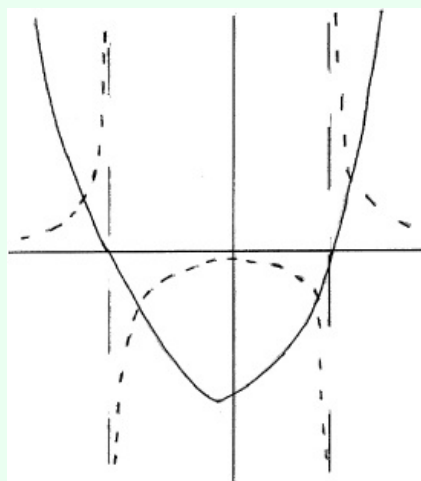
Markscheme

METHOD 1

$$\left(\text{graphs of } g(x) \text{ and } \tfrac{1}{g(x)}\right)$$

roots of $g(x) = 0$ are -3 and 2 **(M1)(A1)**

Notes: Award **M1** if quadratic graph is drawn or two roots obtained.

Roots may be indicated anywhere eg asymptotes on graph or in inequalities below.

the intersections of the graphs $g(x)$ and of $1/g(x)$

are $-3.19, \ -2.79, \ 1.79, \ 2.19$ **(M1)(A1)**

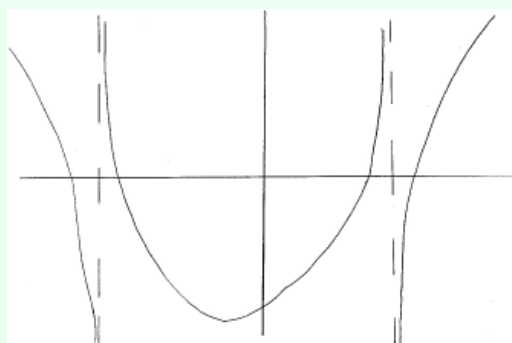**Note:**   Award **A1** for at least one of the values above seen anywhere.

solution is $]{-3.19}, \ -3[\,\cup\,]{-2.79}, \ 1.79[\,\cup\,]2, \ 2.19[$

$(-3.19 < x < -3 \quad \text{or} \quad -2.79 < x < 1.79 \quad \text{or} \quad 2 < x < 2.19)$ **A1A1A1**

Note: Award **A1A1A0** for closed intervals.

METHOD 2

$$\left(\text{graph of } g(x) - \tfrac{1}{g(x)}\right)$$

asymptotes at $x = -3$ and $x = 2$ **(M1)(A1)**

Note: May be indicated on the graph.

roots of graph are $-3.19, \ -2.79, \ 1.79, \ 2.19$ **(M1)(A1)**

solution is (when graph is negative)

$]-3.19, \ -3[\ \cup\ ]-2.79, \ 1.79[\ \cup\ ]2, \ 2.19[$

$(-3.19 < x < -3 \quad \text{or} \quad -2.79 < x < 1.79 \quad \text{or} \quad 2 < x < 2.19) \quad$ **A1A1A1**

Note: Award **A1A1A0** for closed intervals.

[7 marks]

Total [10 marks]

Examiners report

In part (b) many candidates scored a few marks by just finding intersection points and equations of asymptotes; many other candidates showed difficulties in manipulating inequalities and ignored the fact that the quantities could be negative. Candidates that used the graph well managed to achieve full marks. Unfortunately many sketches were very crudely drawn hence they were of limited value for assessment purposes.

---

13.   *[5 marks]*

Markscheme

$P(2) = 24 + 2a + b = 2, \ P(-1) = -3 - a + b = 5 \quad$ **M1A1A1**

$(2a + b = -22, \ -a + b = 8)$

Note: Award **M1** for substitution of 2 or −1 and equating to remainder, **A1** for each correct equation.

attempt to solve simultaneously   **M1**

$a = -10, \ b = -2 \quad$ **A1**

[5 marks]

Examiners report

[N/A]

---

14.   *[6 marks]*

Markscheme

$r_1 + r_2 + r_3 = \frac{-48}{5} \quad$ **(M1)(A1)**

$r_1 r_2 r_3 = \frac{a-2}{5} \quad$ **(M1)(A1)**

$\frac{-48}{5} + \frac{a-2}{5} = 0 \quad$ **M1**

$a = 50 \quad$ **A1**

Note: Award **M1A0M1A0M1A1** if answer of 50 is found using

$\frac{48}{5}$ and

$\frac{2-a}{5}$.

[6 marks]

Examiners report

[N/A]

15.

Markscheme

METHOD 1

substituting

$-5 + 12i + a(2 + 3i) + b = 0$ *(A1)*

equating real or imaginary parts   *(M1)*

$12 + 3a = 0 \Rightarrow a = -4$ *A1*

$-5 + 2a + b = 0 \Rightarrow b = 13$ *A1*

METHOD 2

other root is

$2 - 3i$ *(A1)*

considering either the sum or product of roots or multiplying factors   *(M1)*

$4 = -a$ (sum of roots) so

$a = -4$ *A1*

$13 = b$ (product of roots)   *A1*

[4 marks]

Examiners report

[N/A]

Markscheme

METHOD 1

sketch showing where the lines cross or zeros of

$y = x(x+2)^6 - x$ *(M1)*

$x = 0$ *(A1)*

$x = -1$ and

$x = -3$ *(A1)*

the solution is

$-3 < x < -1$ or

$x > 0$ *A1A1*

Note: Do not award either final *A1* mark if strict inequalities are not given.

METHOD 2

separating into two cases

$x > 0$ and

$x < 0$ *(M1)*

if

$x > 0$ then

$(x+2)^6 > 1 \Rightarrow$ always true   *(M1)*

if

$x < 0$ then

$(x+2)^6 < 1 \Rightarrow -3 < x < -1$ *(M1)*

so the solution is

$-3 < x < -1$ or

$x > 0$ *A1A1*

Note: Do not award either final *A1* mark if strict inequalities are not given.

METHOD 3

$f(x) = x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x$ *(A1)*

solutions to

$x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 63x = 0$ are   *(M1)*

$x = 0$, $x = -1$ and

$x = -3$ *(A1)*

so the solution is

$-3 < x < -1$ or

$x > 0$ *A1A1*

Note: Do not award either final *A1* mark if strict inequalities are not given.

METHOD 4

$f(x) = x$ when

$x(x+2)^6 = x$

either

$x = 0$ or

$(x+2)^6 = 1$ *(A1)*

if

$(x+2)^6 = 1$ then

$x + 2 = \pm 1$ so

$x = -1$ or

$x = -3$ *(M1)(A1)*

the solution is

$-3 < x < -1$ or

$x > 0$ *A1A1*

Note: Do not award either final *A1* mark if strict inequalities are not given.

[5 marks]

Examiners report

16b.   *[5 marks]*

Markscheme

METHOD 1 (by substitution)

substituting

$u = x + 2$ *(M1)*

$\mathrm{d}u = \mathrm{d}x$

$\int (u - 2) u^6 \, \mathrm{d}u$ *M1A1*

$= \frac{1}{8} u^8 - \frac{2}{7} u^7 (+c)$ *(A1)*

$= \frac{1}{8} (x+2)^8 - \frac{2}{7} (x+2)^7 (+c)$ *A1*

METHOD 2 (by parts)

$u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1,\ \frac{\mathrm{d}v}{\mathrm{d}x} = (x+2)^6 \Rightarrow v = \frac{1}{7}(x+2)^7$ *(M1)(A1)*

$\int x(x+2)^6 \, \mathrm{d}x = \frac{1}{7} x(x+2)^7 - \frac{1}{7} \int (x+2)^7 \, \mathrm{d}x$ *M1*

$= \frac{1}{7} x(x+2)^7 - \frac{1}{56}(x+2)^8 (+c)$ *A1A1*

METHOD 3 (by expansion)

$\int f(x) \, \mathrm{d}x = \int \left(x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x \right) \, \mathrm{d}x$ *M1A1*

$= \frac{1}{8} x^8 + \frac{12}{7} x^7 + 10x^6 + 32x^5 + 60x^4 + 64x^3 + 32x^2 (+c)$ *M1A2*

Note: Award *M1A1* if at least four terms are correct.

[5 marks]

Examiners report

17a.   *[4 marks]*

Markscheme

$x \to -\infty \Rightarrow y \to -\frac{1}{2}$ so

$y = -\frac{1}{2}$ is an asymptote   *(M1)A1*

$e^x - 2 = 0 \Rightarrow x = \ln 2$ so

$x = \ln 2 \ (= 0.693)$ is an asymptote   *(M1)A1*

[4 marks]

Examiners report

Markscheme

(i)

$f'(x) = \frac{2(e^x-2)e^{2x} - (e^{2x}+1)e^x}{(e^x-2)^2}$ *M1A1*

$= \frac{e^{3x} - 4e^{2x} - e^x}{(e^x-2)^2}$

(ii)

$f'(x) = 0$ when

$e^{3x} - 4e^{2x} - e^x = 0$ *M1*

$e^x \left(e^{2x} - 4e^x - 1\right) = 0$

$e^x = 0$, $e^x = -0.236$, $e^x = 4.24$ (or $e^x = 2 \pm \sqrt{5}$)   *A1A1*

Note: Award *A1* for zero, *A1* for other two solutions.

Accept any answers which show a zero, a negative and a positive.

as

$e^x > 0$ exactly one solution   *R1*

Note: Do not award marks for purely graphical solution.

(iii)   (1.44, 8.47)   *A1A1*

[8 marks]

Examiners report

[N/A]

Markscheme

$f'(0) = -4$ *(A1)*

so gradient of normal is

$\frac{1}{4}$ *(M1)*

$f(0) = -2$ *(A1)*

so equation of

L_1 is

$y = \frac{1}{4}x - 2$ *A1*

[4 marks]

Examiners report

[N/A]

Markscheme

$f'(x) = \frac{1}{4}$ *M1*

so

$x = 1.46$ *(M1)A1*

$f(1.46) = 8.47$ *(A1)*

equation of

L_2 is

$y - 8.47 = \frac{1}{4}(x - 1.46)$ *A1*

(or

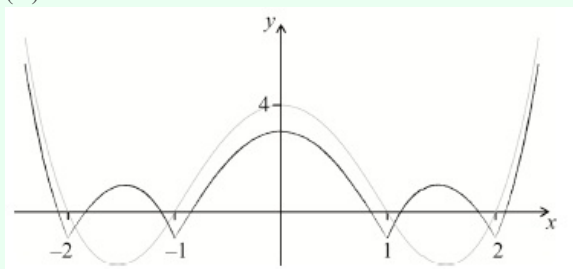$y = \frac{1}{4}x + 8.11$)

[5 marks]

Examiners report

[N/A]

$f'(x) = \frac{1}{4}$ *M1*

so

$x = 1.46$ *(M1)A1*

$f(1.46) = 8.47$ *(A1)*

equation of

L_2 is

$y - 8.47 = \frac{1}{4}(x - 1.46)$ *A1*

18.

Markscheme

(a)　(i)

$f(0) = -1$　　*(M1)A1*

(ii)

$(f \circ g)(0) = f(4) = 3$　　*A1*

(iii)

(M1)A1

Note:　Award *M1* for evidence that the lower part of the graph has been reflected and *A1* correct shape with *y*-intercept below 4.

[5 marks]

(b)　(i)
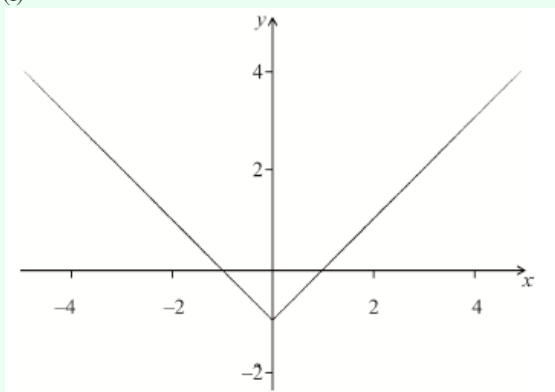
(M1)A1

Note:　Award *M1* for any translation of
$y = |x|$.

(ii)

± 1　　*A1*

Note:　Do not award the *A1* if coordinates given, but do not penalise in the rest of the question

[3 marks]

(c)　(i)
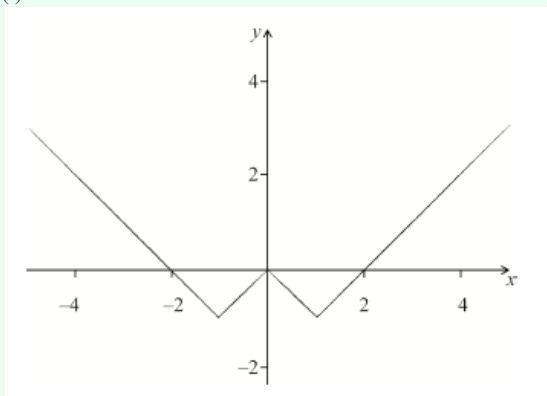
(M1)A1

Note: Award *M1* for evidence that lower part of (b) has been reflected in the *x*-axis and translated.

     (ii)

$0, \pm 2$ *A1*

[3 marks]

(d)   (i)

$\pm 1, \pm 3$ *A1*

     (ii)

$0, \pm 2, \pm 4$ *A1*

     (iii)

$0, \pm 2, \pm 4, \pm 6, \pm 8$ *A1*

[3 marks]

(e)   (i)

$(1, 3), (2, 5), \ldots$ *(M1)*

$N = 2n + 1$ *A1*

     (ii)   Using the formula of the sum of an arithmetic series   *(M1)*

     EITHER

$4(1 + 2 + 3 + \ldots + n) = \frac{4}{2}n(n + 1)$

$= 2n(n + 1)$ *A1*

     OR

$2(2 + 4 + 6 + \ldots + 2n) = \frac{2}{2}n(2n + 2)$

$= 2n(n + 1)$ *A1*

[4 marks]

Total [18 marks]

Examiners report

[N/A]

19. [6 marks]

Markscheme

(a) using the formulae for the sum and product of roots:

$\alpha + \beta = -2$ **A1**

$\alpha\beta = -\frac{1}{2}$ **A1**

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ **M1**

$= (-2)^2 - 2\left(-\frac{1}{2}\right)$

$= 5$ **A1**

Note: Award **M0** for attempt to solve quadratic equation.

[4 marks]

(b)

$(x - \alpha^2)(x - \beta^2) = x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2$ **M1**

$x^2 - 5x + \left(-\frac{1}{2}\right)^2 = 0$ **A1**

$x^2 - 5x + \frac{1}{4} = 0$

Note: Final answer must be an equation. Accept alternative correct forms.

[2 marks]

Total [6 marks]

Examiners report

[N/A]

20a. [2 marks]

Markscheme

A1A1
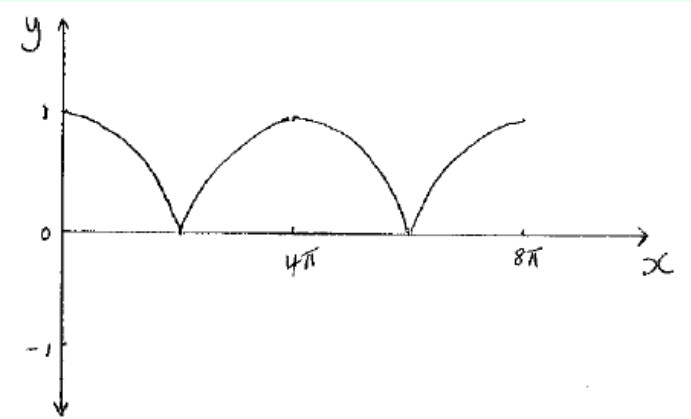
Note: Award **A1** for correct shape and **A1** for correct domain and range.

[2 marks]

Examiners report

[N/A]

20b.   *[3 marks]*

Markscheme

$\left|\cos\left(\frac{x}{4}\right)\right| = \frac{1}{2}$

$x = \frac{4\pi}{3}$ *A1*

attempting to find any other solutions   *M1*

Note: Award *(M1)* if at least one of the other solutions is correct (in radians or degrees) or clear use of symmetry is seen.

$x = 8\pi - \frac{4\pi}{3} = \frac{20\pi}{3}$
$x = 4\pi - \frac{4\pi}{3} = \frac{8\pi}{3}$
$x = 4\pi + \frac{4\pi}{3} = \frac{16\pi}{3}$ *A1*

Note: Award *A1* for all other three solutions correct and no extra solutions.

Note: If working in degrees, then max *A0M1A0*.

[3 marks]

Examiners report

[N/A]

21a.   *[2 marks]*

Markscheme

$1 - 2(2) = -3$ and
$\frac{3}{4}(2-2)^2 - 3 = -3$ *A1*

both answers are the same, hence f is continuous (at
$x = 2$)   *R1*

Note: *R1* may be awarded for justification using a graph or referring to limits. Do not award *A0R1*.

[2 marks]

Examiners report

[N/A]

21b.   [4 marks]
Markscheme

reflection in the y-axis

$$f(-x) = \begin{cases} 1 + 2x, & x \geq -2 \\ \frac{3}{4}(x+2)^2 - 3, & x < -2 \end{cases} \quad \textbf{(M1)}$$

Note: Award **M1** for evidence of reflecting a graph in y-axis.

translation

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$g(x) = \begin{cases} 2x - 3, & x \geq 0 \\ \frac{3}{4}x^2 - 3, & x < 0 \end{cases} \quad \textbf{(M1)A1A1}$$

Note: Award **(M1)** for attempting to substitute
$(x - 2)$ for x, or translating a graph along positive x-axis.

Award **A1** for the correct domains (this mark can be awarded independent of the **M1**).

Award **A1** for the correct expressions.

[4 marks]

Examiners report

[N/A]

22a.   [2 marks]
Markscheme
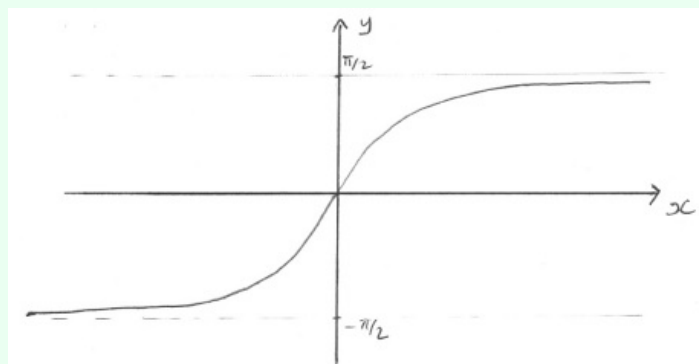
A1A1

Note: **A1** for correct shape, **A1** for asymptotic behaviour at
$y = \pm\frac{\pi}{2}$.

[2 marks]

Examiners report

[N/A]

22b.

Markscheme

$h \circ g(x) = \arctan\left(\frac{1}{x}\right)$ ***A1***

domain of

$h \circ g$ is equal to the domain of

$g : x \in \circ, \ x \neq 0$ ***A1***

[2 marks]

Examiners report

[N/A]

22c.

Markscheme

(i)

$f(x) = \arctan(x) + \arctan\left(\frac{1}{x}\right)$

$f'(x) = \frac{1}{1+x^2} + \frac{1}{1+\frac{1}{x^2}} \times -\frac{1}{x^2}$ ***M1A1***

$f'(x) = \frac{1}{1+x^2} + \frac{-\frac{1}{x^2}}{\frac{x^2+1}{x^2}}$ *(A1)*

$= \frac{1}{1+x^2} - \frac{1}{1+x^2}$

$= 0$ ***A1***

(ii)   **METHOD 1**

f is a constant   ***R1***

when

$x > 0$

$f(1) = \frac{\pi}{4} + \frac{\pi}{4}$ ***M1A1***

$= \frac{\pi}{2}$ ***AG***

METHOD 2

from diagram

$\theta = \arctan\frac{1}{x}$ ***A1***

$\alpha = \arctan x$ ***A1***

$\theta + \alpha = \frac{\pi}{2}$ ***R1***
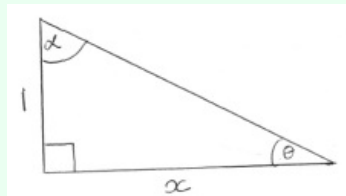
hence

$f(x) = \frac{\pi}{2}$ ***AG***

METHOD 3

$\tan(f(x)) = \tan\left(\arctan(x) + \arctan\left(\frac{1}{x}\right)\right)$ ***M1***

$= \frac{x + \frac{1}{x}}{1 - x\left(\frac{1}{x}\right)}$ ***A1***

denominator = 0, so

$f(x) = \frac{\pi}{2}$ (for $x > 0$)   ***R1***

[7 marks]

Examiners report

[N/A]

22d. *[3 marks]*

23a. *[2 marks]*

23b. *[3 marks]*

24a. *[2 marks]*

24b. [2 marks]

Markscheme

$x = -3$ **A1**

$y = 1$ **A1**

[2 marks]

Total [4 marks]

Examiners report

This question was generally well done. A few candidates made a sign error for the horizontal translation. A few candidates expressed the required equations for the asymptotes as 'inequalities', which received no marks.

25a. [2 marks]

Markscheme

using the formulae for the sum and product of roots:

(i) $\alpha + \beta = 4$ **A1**

(ii) $\alpha\beta = \frac{1}{2}$ **A1**

Note: Award **A0A0** if the above results are obtained by solving the original equation (except for the purpose of checking).

[2 marks]

Examiners report

Most candidates obtained full marks.

Markscheme

METHOD 1

required quadratic is of the form $x^2 - \left(\frac{2}{\alpha} + \frac{2}{\beta}\right)x + \left(\frac{2}{\alpha}\right)\left(\frac{2}{\beta}\right)$ **(M1)**

$q = \frac{4}{\alpha\beta}$

$q = 8$ **A1**

$p = -\left(\frac{2}{\alpha} + \frac{2}{\beta}\right)$

$= -\frac{2(\alpha+\beta)}{\alpha\beta}$ **M1**

$= -\frac{2\times 4}{\frac{1}{2}}$

$p = -16$ **A1**

Note: Accept the use of exact roots

METHOD 2

replacing x with $\frac{2}{x}$ **M1**

$2\left(\frac{2}{x}\right)^2 - 8\left(\frac{2}{x}\right) + 1 = 0$

$\frac{8}{x^2} - \frac{16}{x} + 1 = 0$ **(A1)**

$x^2 - 16x + 8 = 0$

$p = -16$ and $q = 8$ **A1A1**

Note: Award **A1A0** for $x^2 - 16x + 8 = 0$ *ie*, if $p = -16$ and $q = 8$ are not explicitly stated.

[4 marks]

Total [6 marks]

Examiners report

Many candidates obtained full marks, but some responses were inefficiently expressed. A very small minority attempted to use the exact roots, usually unsuccessfully.

Markscheme

(i) $x = e^{3y+1}$ **M1**

Note: The **M1** is for switching variables and can be awarded at any stage.

Further marks do not rely on this mark being awarded.

taking the natural logarithm of both sides and attempting to transpose   **M1**

$\left(f^{-1}(x)\right) = \frac{1}{3}(\ln x - 1)$ **A1**

(ii) $x \in \mathbb{R}^+$ or equivalent, for example $x > 0$.   **A1**

[4 marks]

Examiners report

26b. [5 marks]

Markscheme

$\ln x = \frac{1}{3}(\ln x - 1) \Rightarrow \ln x - \frac{1}{3}\ln x = -\frac{1}{3}$ (or equivalent)   **M1A1**

$\ln x = -\frac{1}{2}$ (or equivalent)   **A1**

$x = \mathrm{e}^{-\frac{1}{2}}$ **A1**

coordinates of P are $\left(\mathrm{e}^{-\frac{1}{2}}, \ -\frac{1}{2}\right)$ **A1**

[5 marks]

Examiners report

Generally very well done, even by candidates who had shown considerable weaknesses elsewhere on the paper.

26c. [3 marks]

Markscheme

coordinates of Q are $(1, 0)$ seen anywhere   **A1**

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$ **M1**

at Q, $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ **A1**

$y = x - 1$ **AG**

[3 marks]

Examiners report

Generally very well done, even by candidates who had shown considerable weaknesses elsewhere on the paper.

Markscheme

let the required area be A

$A = \int_1^e x - 1 \mathrm{d}x - \int_1^e \ln x \mathrm{d}x$ **M1**

Note: The **M1** is for a difference of integrals. Condone absence of limits here.

attempting to use integration by parts to find $\int \ln x \mathrm{d}x$ **(M1)**

$= \left[\frac{x^2}{2} - x \right]_1^e - [x \ln x - x]_1^e$ **A1A1**

Note: Award **A1** for $\frac{x^2}{2} - x$ and **A1** for $x \ln x - x$.

Note: The second **M1** and second **A1** are independent of the first **M1** and the first **A1**.

$= \frac{e^2}{2} - e - \frac{1}{2} \left(= \frac{e^2 - 2e - 1}{2} \right)$ **A1**

[5 marks]

Examiners report

A productive question for many candidates, but some didn't realise that a difference of areas/integrals was required.

Markscheme

(i) **METHOD 1**

consider for example $h(x) = x - 1 - \ln x$

$h(1) = 0$ and $h'(x) = 1 - \frac{1}{x}$ *(A1)*

as $h'(x) \geq 0$ for $x \geq 1$, then $h(x) \geq 0$ for $x \geq 1$ *R1*

as $h'(x) \leq 0$ for $0 < x \leq 1$, then $h(x) \geq 0$ for $0 < x \leq 1$ *R1*

so $g(x) \leq x - 1$, $x \in \mathbb{R}^+$ *AG*

METHOD 2

$g''(x) = -\frac{1}{x^2}$ *A1*
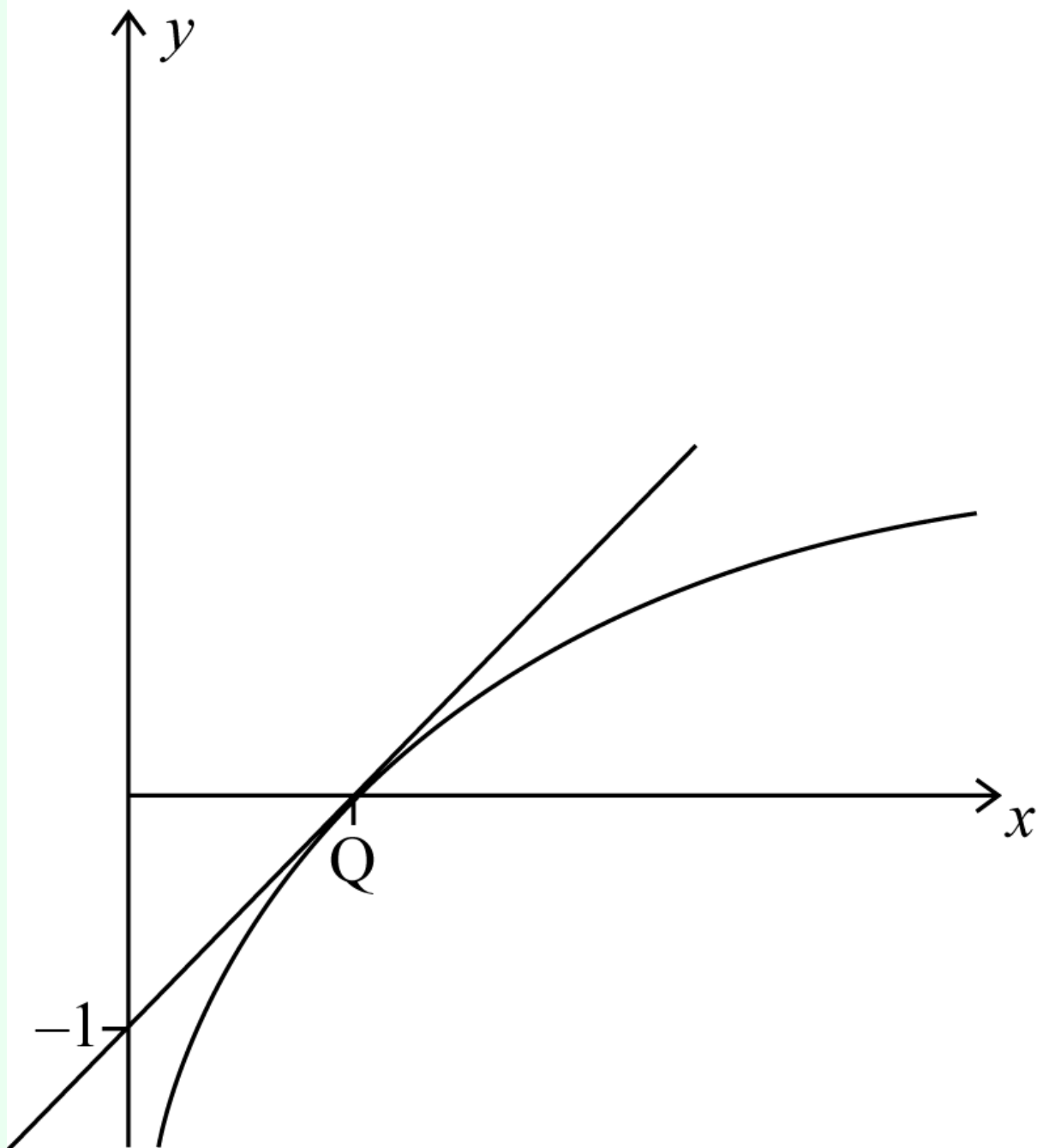
$g''(x) < 0$ (concave down) for $x \in \mathbb{R}^+$ *R1*

the graph of $y = g(x)$ is below its tangent $(y = x - 1$ at $x = 1)$ *R1*

so $g(x) \leq x - 1$, $x \in \mathbb{R}^+$ *AG*

Note: The reasoning may be supported by drawn graphical arguments.

METHOD 3

clear correct graphs of $y = x - 1$ and $\ln x$ for $x > 0$ **A1A1**

statement to the effect that the graph of $\ln x$ is below the graph of its tangent at $x = 1$ **R1AG**

(ii) replacing x by $\frac{1}{x}$ to obtain $\ln\left(\frac{1}{x}\right) \leq \frac{1}{x} - 1 \left(= \frac{1-x}{x}\right)$ **M1**

$-\ln x \leq \frac{1}{x} - 1 \left(= \frac{1-x}{x}\right)$ **(A1)**

$\ln x \geq 1 - \frac{1}{x} \left(= \frac{x-1}{x}\right)$ **A1**

so $\frac{x-1}{x} \leq g(x), \ x \in \mathbb{R}^+$ **AG**

[6 marks]

Total [23 marks]

Examiners report

(i) Many candidates adopted a graphical approach, but sometimes with unconvincing reasoning.

(ii) Poorly answered. Many candidates applied the suggested substitution only to one side of the inequality, and then had to fudge the answer.

Markscheme

using $p(a) = -7$ to obtain $3a^3 + a^2 + 5a + 7 = 0$ **M1A1**

$(a + 1)(3a^3 - 2a + 7) = 0$ **(M1)(A1)**

Note: Award **M1** for a cubic graph with correct shape and **A1** for clearly showing that the above cubic crosses the horizontal axis at $(-1, 0)$ only.

$a = -1$ **A1**

EITHER

showing that $3a^2 - 2a + 7 = 0$ has no real (two complex) solutions for a **R1**

OR

showing that $3a^3 + a^2 + 5a + 7 = 0$ has one real (and two complex) solutions for a **R1**

Note: Award **R1** for solutions that make specific reference to an appropriate graph.

[6 marks]

Examiners report

A large number of candidates, either by graphical (mostly) or algebraic or via use of a GDC solver, were able to readily obtain $a = -1$. Most candidates who were awarded full marks however, made specific reference to an appropriate graph. Only a small percentage of candidates used the discriminant to justify that only one value of a satisfied the required condition. A number of candidates erroneously obtained $3a^3 + a^2 + 5a - 7 = 0$ or equivalent rather than $3a^3 + a^2 + 5a + 7 = 0$.

Markscheme

using $r = \frac{u_2}{u_1} = \frac{u_3}{u_2}$ to form $\frac{a+2d}{a+6d} = \frac{a}{a+2d}$ **(M1)**

$a(a + 6d) = (a + 2d)^2$ **A1**

$2d(2d - a) = 0$ (or equivalent) **A1**

since $d \neq 0 \Rightarrow d = \frac{a}{2}$ **AG**

[3 marks]

Examiners report

Part (a) was reasonably well done. A number of candidates used $r = \frac{u_1}{u_2} = \frac{u_2}{u_3}$ rather than $r = \frac{u_2}{u_1} = \frac{u_3}{u_2}$. This invariably led to candidates obtaining $r = 2$ in part (b).

28b. [6 marks]

Markscheme

substituting $d = \frac{a}{2}$ into $a + 6d = 3$ and solving for a and d *(M1)*

$a = \frac{3}{4}$ and $d = \frac{3}{8}$ *(A1)*

$r = \frac{1}{2}$ *A1*

$$\frac{n}{2}\left(2 \times \frac{3}{4} + (n-1)\frac{3}{8}\right) - \frac{3\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}} \geq 200 \quad \textit{(A1)}$$

attempting to solve for n *(M1)*

$n \geq 31.68\ldots$

so the least value of n is 32 *A1*

[6 marks]

Total [9 marks]

Examiners report

In part (b), most candidates were able to correctly find the first term and the common difference for the arithmetic sequence. However a number of candidates either obtained $r = 2$ via means described in part (a) or confused the two sequences and used $u_1 = \frac{3}{4}$ for the geometric sequence.

29a. [3 marks]

Markscheme

each triangle has area $\frac{1}{8}x^2 \sin \frac{2\pi}{n}$ (use of $\frac{1}{2}ab \sin C$) *(M1)*

there are
n triangles so $A = \frac{1}{8}nx^2 \sin \frac{2\pi}{n}$ *A1*

$C = \dfrac{4\left(\frac{1}{8}nx^2 \sin \frac{2\pi}{n}\right)}{\pi n^2}$ *A1*

so $C = \frac{n}{2\pi}\sin \frac{2\pi}{n}$ *AG*

[3 marks]

Examiners report

Most candidates found this a difficult question with a large number of candidates either not attempting it or making little to no progress. In part (a), a number of candidates attempted to show the desired result using specific regular polygons. Some candidates attempted to fudge the result.

Markscheme

attempting to find the least value of
n such that $\frac{n}{2\pi}\sin\frac{2\pi}{n} > 0.99$ **(M1)**

$n = 26$ **A1**

attempting to find the least value of
n such that $\frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)} > 0.99$ **(M1)**

$n = 21$ (and so a regular polygon with 21 sides)   **A1**

Note: Award **(M0)A0(M1)A1** if $\frac{n}{2\pi}\sin\frac{2\pi}{n} > 0.99$ is not considered and $\frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)} > 0.99$ is correctly considered.

Award **(M1)A1(M0)A0** for $n = 26$.

[4 marks]

Examiners report

In part (b), the overwhelming majority of candidates that obtained either $n = 21$ or $n = 26$ or both used either a GDC numerical solve feature or a graphical approach rather than a tabular approach which is more appropriate for a discrete variable such as the number of sides of a regular polygon. Some candidates wasted valuable time by showing that $C = \frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)}$ (a given result).

Markscheme

attempting to use $V = \pi \int_a^b x^2 \, dy$ **(M1)**

attempting to express x^2 in terms of
y ie $x^2 = 4(y + 16)$ **(M1)**

for $y = h$, $V = 4\pi \int_0^h y + 16 \, dy$ **A1**

$V = 4\pi\left(\frac{h^2}{2} + 16h\right)$ **AG**

[3 marks]

Examiners report

This question was done reasonably well by a large proportion of candidates. Many candidates however were unable to show the required result in part (a). A number of candidates seemingly did not realize how the container was formed while other candidates attempted to fudge the result.

Markscheme

EITHER

the depth stabilizes when $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ *ie* $8.5 - \frac{250\sqrt{h}}{\pi(h+16)} = 0$ **R1**

attempting to solve $8.5 - \frac{250\sqrt{h}}{\pi(h+16)} = 0$ for h **(M1)**

OR

the depth stabilizes when $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$ *ie* $\frac{1}{4\pi(h+16)}\left(8.5 - \frac{250\sqrt{h}}{\pi(h+16)}\right) = 0$ **R1**

attempting to solve $\frac{1}{4\pi(h+16)}\left(8.5 - \frac{250\sqrt{h}}{\pi(h+16)}\right) = 0$ for h **(M1)**

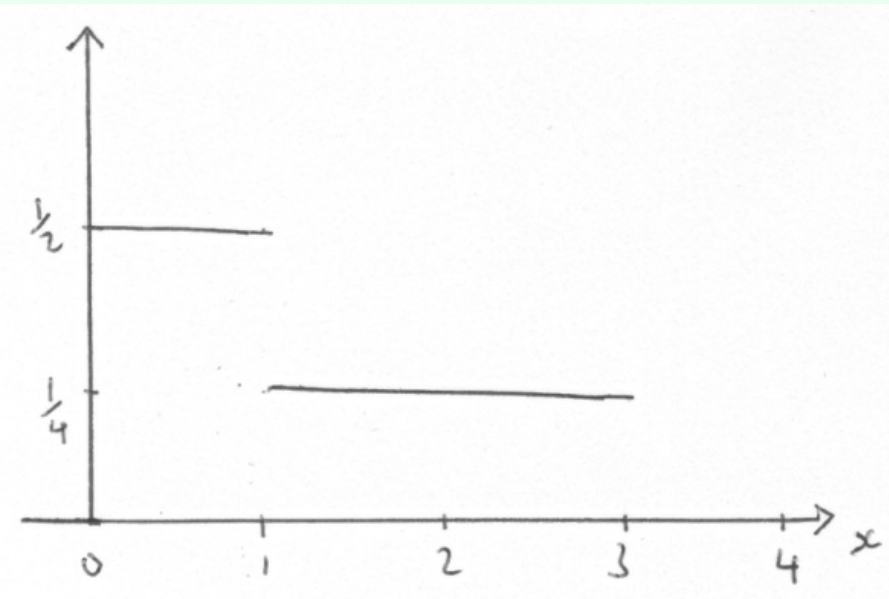**THEN**

$h = 5.06$ (cm) **A1**

[3 marks]

Total [16 marks]

Examiners report

In part (c), a pleasing number of candidates realized that the water depth stabilized when either $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$, sketched an appropriate graph and found the correct value of h. Some candidates misinterpreted the situation and attempted to find the coordinates of the local minimum of their graph.

Markscheme

A1

Note: Ignore open / closed endpoints and vertical lines.

Note: Award **A1** for a correct graph with scales on both axes and a clear indication of the relevant values.

[1 mark]

Examiners report

Part (a) was correctly answered by most candidates. Some graphs were difficult to mark because candidates drew their lines on top of the ruled lines in the answer book. Candidates should be advised not to do this. Candidates should also be aware that the command term 'sketch' requires relevant values to be indicated.

31b. *[5 marks]*

Markscheme

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \le x < 1 \\ \frac{x}{4} + \frac{1}{4} & 1 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

considering the areas in their sketch or using integration   *(M1)*

$F(x) = 0, \ x < 0, \ F(x) = 1, \ x \ge 3$ *A1*

$F(x) = \frac{x}{2}, \ 0 \le x < 1$ *A1*

$F(x) = \frac{x}{4} + \frac{1}{4}, \ 1 \le x < 3$ *A1A1*

Note: Accept $<$ for $\le$ in all places and also $>$ for $\ge$ first *A1*.

[5 marks]

Examiners report

In (b), most candidates realised that the cumulative distribution function had to be found by integration but the limits were sometimes incorrect.

31c. *[3 marks]*

Markscheme

$Q_3 = 2, \ Q_1 = 0.5$ *A1A1*

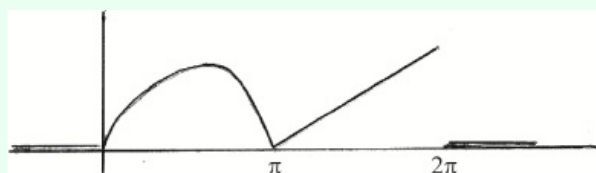IQR is $2 - 0.5 = 1.5$ *A1*

[3 marks]

Total [9 marks]

Examiners report

In (c), candidates who found the upper and lower quartiles correctly sometimes gave the interquartile range as $[0.5, \ 2]$. It is important for candidates to realise that that the word range has a different meaning in statistics compared with other branches of mathematics.

32a. *[2 marks]*

Markscheme

Award *A1* for sine curve from 0 to π, award *A1* for straight line from π to 2π *A1A1*

[2 marks]

Examiners report

Most candidates sketched the graph correctly. In a few cases candidates did not seem familiar with the shape of the graphs and ignored the fact that the graph represented a pdf. The correct sketch assisted greatly in the rest of the question.

32b.   *[2 marks]*

Markscheme

$\int_0^\pi \frac{\sin x}{4} \mathrm{d}x = \frac{1}{2}$ *(M1)A1*

[2 marks]

Examiners report

Most candidates answered this question correctly.

32c.   *[3 marks]*

Markscheme

METHOD 1

require $\frac{1}{2} + \int_\pi^{2\pi} a(x-\pi)\mathrm{d}x = 1$ *(M1)*

$\Rightarrow \frac{1}{2} + a\left[\frac{(x-\pi)^2}{2}\right]_\pi^{2\pi} = 1$ $\left(\text{or } \frac{1}{2} + a\left[\frac{x^2}{2} - \pi x\right]_\pi^{2\pi} = 1\right)$ *A1*

$\Rightarrow a\frac{\pi^2}{2} = \frac{1}{2}$ *A1*

$\Rightarrow a = \frac{1}{\pi^2}$ *AG*

Note: Must obtain the exact value. Do not accept answers obtained with calculator.

METHOD 2

$0.5 + \text{ area of triangle } = 1$ *R1*

area of triangle $= \frac{1}{2}\pi \times a\pi = 0.5$ *M1A1*

Note: Award *M1* for correct use of area formula $= 0.5$, *A1* for $a\pi$.

$a = \frac{1}{\pi^2}$ *AG*

[3 marks]

Examiners report

A few good proofs were seen but also many poor answers where the candidates assumed what you were trying to prove and verified numerically the result.

32d.   *[1 mark]*

Markscheme

median is π *A1*

[1 mark]

Examiners report

Most candidates stated the value correctly but many others showed no understanding of the concept.

32e. [3 marks]

Markscheme

$\mu = \int_0^\pi x \cdot \frac{\sin x}{4} \mathrm{d}x + \int_\pi^{2\pi} x \cdot \frac{x-\pi}{\pi^2} \mathrm{d}x$ *(M1)(A1)*

$= 3.40339\ldots = 3.40 \quad \left(\text{or } \frac{\pi}{4} + \frac{5\pi}{6} = \frac{13}{12}\pi\right)$ *A1*

[3 marks]

Examiners report

Many candidates scored full marks in this question; many others could not apply the formula due to difficulties in dealing with the piecewise function. For example, a number of candidates divided the final answer by two.

32f. [3 marks]

Markscheme

For $\mu = 3.40339\ldots$

EITHER

$\sigma^2 = \int_0^\pi x^2 \cdot \frac{\sin x}{4} \mathrm{d}x + \int_\pi^{2\pi} x^2 \cdot \frac{x-\pi}{\pi^2} \mathrm{d}x - \mu^2$ *(M1)(A1)*

OR

$\sigma^2 = \int_0^\pi (x-\mu)^2 \cdot \frac{\sin x}{4} \mathrm{d}x + \int_\pi^{2\pi} (x-\mu)^2 \cdot \frac{x-\pi}{\pi^2} \mathrm{d}x$ *(M1)(A1)*

THEN

$= 3.866277\ldots = 3.87$ *A1*

[3 marks]

Examiners report

Many misconceptions were identified: use of incorrect formula (e.g. formula for discrete distributions), use of both expressions as integrand and division of the result by 2 at the end.

32g. [2 marks]

Markscheme

$\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{4} \mathrm{d}x + \int_\pi^{\frac{3\pi}{2}} \frac{x-\pi}{\pi^2} \mathrm{d}x = 0.375 \quad \left(\text{or } \frac{1}{4} + \frac{1}{8} = \frac{3}{8}\right)$ *(M1)A1*

[2 marks]

Examiners report

This part was fairly well done with many candidates achieving full marks.

32h.

Markscheme

$P\left(\pi \le X \le 2\pi \,\middle|\, \frac{\pi}{2} \le X \le \frac{3\pi}{2}\right) = \dfrac{P\left(\pi \le X \le \frac{3\pi}{2}\right)}{P\left(\frac{\pi}{2} \le X \le \frac{3\pi}{2}\right)}$ **(M1)(A1)**

$= \dfrac{\int_{\pi}^{\frac{3\pi}{2}} \frac{(x-\pi)}{\pi^2}\,\mathrm{d}x}{0.375} = \dfrac{0.125}{0.375}$ $\left(\text{or } = \dfrac{\frac{1}{8}}{\frac{3}{8}} \text{ from diagram areas}\right)$ **(M1)**

$= \frac{1}{3}$ (0.333) **A1**

[4 marks]

Total [20 marks]

Examiners report

Many candidates had difficulties with this part showing that the concept of conditional probability was poorly understood. The best candidates did it correctly from the sketch.

33a.

Markscheme

(i) $(\cos\theta + \mathrm{i}\sin\theta)^5$

$= \cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta + 10\mathrm{i}^2\cos^3\theta\sin^2\theta +$

$10\mathrm{i}^3\cos^2\theta\sin^3\theta + 5\mathrm{i}^4\cos\theta\sin^4\theta + \mathrm{i}^5\sin^5\theta$ **A1A1**

$(= \cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta -$

$10\mathrm{i}\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + \mathrm{i}\sin^5\theta)$

Note: Award first **A1** for correct binomial coefficients.

(ii) $(\mathrm{cis}\theta)^5 = \mathrm{cis}5\theta = \cos 5\theta + \mathrm{i}\sin 5\theta$ **M1**

$= \cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10\mathrm{i}\cos^2\theta\sin^3\theta +$

$5\cos\theta\sin^4\theta + \mathrm{i}\sin^5\theta$ **A1**

Note: Previous line may be seen in (i)

equating imaginary terms   **M1**

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ **AG**

(iii)   equating real terms

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ **A1**

[6 marks]

Examiners report

In part (i) many candidates tried to multiply it out the binomials rather than using the binomial theorem. In parts (ii) and (iii) many candidates showed poor understanding of complex numbers and made no attempt to equate real and imaginary parts. In a some cases the correct answer to part (iii) was seen although it was unclear how it was obtained.

Markscheme

$(r\text{cis}\alpha)^5 = 1 \Rightarrow r^5\text{cis}5\alpha = 1\text{cis}0$ **M1**

$r^5 = 1 \Rightarrow r = 1$ **A1**

$5\alpha = 0 \pm 360k,\ k \in \mathbb{Z} \Rightarrow a = 72k$ **(M1)**

$\alpha = 72°$ **A1**

Note: Award **M1A0** if final answer is given in radians.

[4 marks]

Examiners report

This question was poorly done. Very few candidates made a good attempt to apply De Moivre's theorem and most of them could not even equate the moduli to obtain r.

Markscheme

use of $\sin(5 \times 72) = 0$ **OR** the imaginary part of 1 is 0 **(M1)**

$0 = 5\cos^4\alpha \sin\alpha - 10\cos^2\alpha\sin^3\alpha + \sin^5\alpha$ **A1**

$\sin\alpha \neq 0 \Rightarrow 0 = 5(1 - \sin^2\alpha)^2 - 10(1 - \sin^2\alpha)\sin^2\alpha + \sin^4\alpha$ **M1**

Note: Award **M1** for replacing $\cos^2\alpha$.

$0 = 5(1 - 2\sin^2\alpha + \sin^4\alpha) - 10\sin^2\alpha + 10\sin^4\alpha + \sin^4\alpha$ **A1**

Note: Award **A1** for any correct simplification.

so $16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$ **AG**

[4 marks]

Examiners report

This question was poorly done. From the few candidates that attempted it, many candidates started by writing down what they were trying to prove and made no progress.

Markscheme

$\sin^2\alpha = \frac{20\pm\sqrt{400-320}}{32}$ **M1A1**

$\sin\alpha = \pm\sqrt{\frac{20\pm\sqrt{80}}{32}}$

$\sin\alpha = \frac{\pm\sqrt{10\pm2\sqrt{5}}}{4}$ **A1**

Note: Award **A1** regardless of signs. Accept equivalent forms with integral denominator, simplification may be seen later.

as $72 > 60$, $\sin 72 > \frac{\sqrt{3}}{2} = 0.866\ldots$ we have to take both positive signs (or equivalent argument)   **R1**

Note: Allow verification of correct signs with calculator if clearly stated

$\sin 72 = \frac{\sqrt{10+2\sqrt{5}}}{4}$ **A1**

[5 marks]
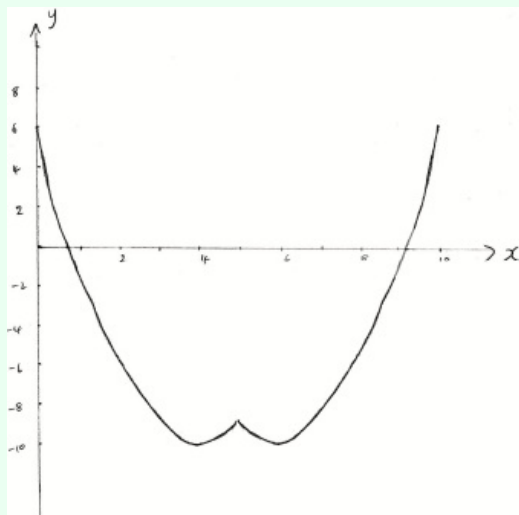
Total [19 marks]

Examiners report

Very few made a serious attempt to answer this question. Also very few realised that they could use the answers given in part (c) to attempt this part.

Markscheme

general shape including \[(\) minimums, cusp   **A1A1**

correct domain and symmetrical about the middle $(x = 5)$ **A1**

[3 marks]

Examiners report

[N/A]

34b. [2 marks]

Markscheme

$x = 9.16$ or $x = 0.838$ **A1A1**

[2 marks]

Total [5 marks]

Examiners report

[N/A]

35a. [4 marks]

Markscheme

EITHER

$y = \ln(x - a) + b = \ln(5x + 10)$ **(M1)**

$y = \ln(x - a) + \ln c = \ln(5x + 10)$

$y = \ln(c(x - a)) = \ln(5x + 10)$ **(M1)**

OR

$y = \ln(5x + 10) = \ln(5(x + 2))$ **(M1)**

$y = \ln(5) + \ln(x + 2)$ **(M1)**

THEN

$a = -2,\ b = \ln 5$ **A1A1**

Note: Accept graphical approaches.

Note: Accept $a = 2,\ b = 1.61$

[4 marks]

Examiners report

[N/A]

35b. [2 marks]

Markscheme

$V = \pi \int_e^{2e} \left[\ln(5x + 10)\right]^2 \mathrm{d}x$ **(M1)**
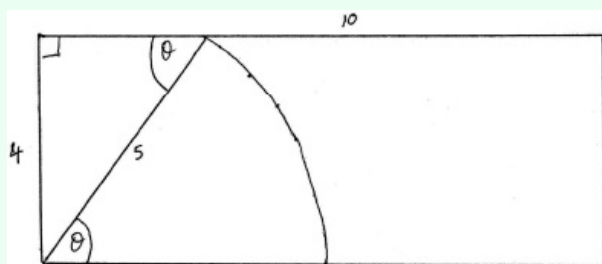
$= 99.2$ **A1**

[2 marks]

Total [6 marks]

Examiners report

[N/A]

Markscheme

EITHER

area of triangle $= \frac{1}{2} \times 3 \times 4 \quad (= 6) \quad$ ***A1***

area of sector $= \frac{1}{2}\arcsin\left(\frac{4}{5}\right) \times 5^2 \quad (= 11.5911\ldots) \quad$ ***A1***

OR

$\int_0^4 \sqrt{25 - x^2}\, dx \quad$ ***M1A1***

THEN

total area $= 17.5911\ldots \ \mathrm{m}^2 \quad$ ***(A1)***

percentage $= \frac{17.5911\ldots}{40} \times 100 = 44\% \quad$ ***A1***

[4 marks]

Examiners report

[N/A]

Markscheme
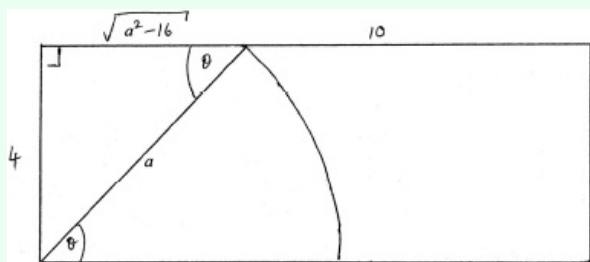
METHOD 1

area of triangle $= \frac{1}{2} \times 4 \times \sqrt{a^2 - 16}$ **A1**

$\theta = \arcsin\left(\frac{4}{a}\right)$ **(A1)**

area of sector $= \frac{1}{2} r^2 \theta = \frac{1}{2} a^2 \arcsin\left(\frac{4}{a}\right)$ **A1**

therefore total area $= 2\sqrt{a^2 - 16} + \frac{1}{2} a^2 \arcsin\left(\frac{4}{a}\right) = 20$ **A1**

rearrange to give: $a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40$ **AG**

METHOD 2

$\int_0^4 \sqrt{a^2 - x^2}\, \mathrm{d}x = 20$ **M1**

use substitution $x = a \sin\theta, \ \frac{\mathrm{d}x}{\mathrm{d}\theta} = a \cos\theta$

$\int_0^{\arcsin\left(\frac{4}{a}\right)} a^2 \cos^2\theta \mathrm{d}\theta = 20$

$\frac{a^2}{2} \int_0^{\arcsin\left(\frac{4}{a}\right)} (\cos 2\theta + 1)\mathrm{d}\theta = 20$ **M1**

$a^2 \left[\left(\frac{\sin 2\theta}{2} + \theta\right)\right]_0^{\arcsin\left(\frac{4}{a}\right)} = 40$ **A1**

$a^2 \left[(\sin\theta \cos\theta + \theta\right]_0^{\arcsin\left(\frac{4}{a}\right)} = 40$

$a^2 \arcsin\left(\frac{4}{a}\right) + a^2 \left(\frac{4}{a}\right) \sqrt{\left(1 - \left(\frac{4}{a}\right)^2\right)} = 40$ **A1**

$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40$ **AG**

[4 marks]

Examiners report

[N/A]

Markscheme

solving using $\mathrm{GDC} \Rightarrow a = 5.53$ cm   **A2**

[2 marks]

Total [10 marks]

Examiners report

[N/A]

37a.

Markscheme

METHOD 1

$s = \int (9t - 3t^2)\mathrm{d}t = \frac{9}{2}t^2 - t^3(+c)$ **(M1)**

$t = 0, \ s = 3 \Rightarrow c = 3$ **(A1)**

$t = 4 \Rightarrow s = 11$ **A1**

METHOD 2

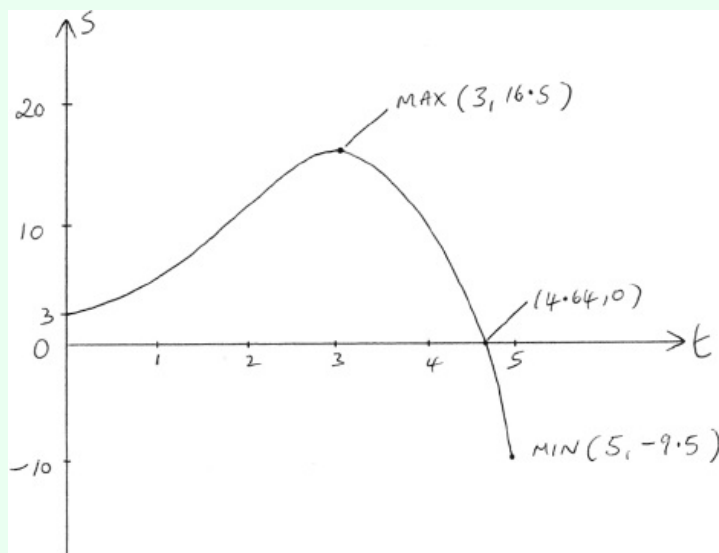$s = 3 + \int_0^4 (9t - 3t^2)\mathrm{d}t$ **(M1)(A1)**

$s = 11$ **A1**

[3 marks]

Examiners report

[N/A]

37b.

Markscheme

correct shape over correct domain **A1**

maximum at $(3, \ 16.5)$ **A1**

t intercept at 4.64, s intercept at 3 **A1**

minimum at $(5, \ -9.5)$ **A1**

[5 marks]

Examiners report

[N/A]

Markscheme

$-9.5 = a + b\cos 2\pi$

$16.5 = a + b\cos 3\pi$ *(M1)*

Note: Only award *M1* if two simultaneous equations are formed over the correct domain.

$a = \frac{7}{2}$ *A1*

$b = -13$ *A1*

[3 marks]

Examiners report

[N/A]

Markscheme

at t_1:

$3 + \frac{9}{2}t^2 - t^3 = 3$ *(M1)*

$t^2\left(\frac{9}{2} - t\right) = 0$

$t_1 = \frac{9}{2}$ *A1*

solving $\frac{7}{2} - 13\cos\frac{2\pi t}{5} = 3$ *(M1)*

$\text{GDC} \Rightarrow t_2 = 6.22$ *A1*

Note: Accept graphical approaches.

[4 marks]

Total [15 marks]

Examiners report

[N/A]