Relativistic Momentum Practice Problems

1 Momentum Calculation

1. An electron in a linear accelerator attains the speed v. Find the ratio of its momentum to its non-relativistic momentum mv if
   (a) $v = c/10$
   (b) $v = c/2$
   (c) $v = c(1 - 10^{-5})$
   (d) $v = c(1 - 10^{-10})$

2. A ship suddenly finds itself approaching a space-iceberg at velocity $(3/5)c$, so immediately fires a 100kg missile at speed $(4/5)c$ relative to the ship, in hopes of breaking up the ice before they crash. Find all components of the missile's momentum four-vector.
   (a) in the starship frame, assuming all motion is in the x direction.
   (b) in the iceberg frame.
   (c) The iceberg will break up if the momentum impacting it is at least $1.0 \times 10^{10}\text{kgms}^{-1}$. Does the iceberg break up?
   (d) What is the velocity of the missile in the iceberg frame?

3. What is the maximum relative speed v/c a particle can have so there is no more than a 1% error in using the non-relativistic momentum formula $P = mv$

4. (a) Find the momentum of a distant galaxy moving away from us at speed v = c/2 in our frame of reference. The galaxy contains 1011 stars of average mass 2×10^{30}kg, the mass of our Sun. This is 10% of the galaxy's mass; the other 90% is in some kind o f unseen "dark matter." (b) How fast would a single proton (mass 1.67 x 10-27 kg) have to move to have the same momentum? Express in the form $v/c = 1 - \epsilon$.

2 Conservation of Momentum and Collisions

1. A spaceship, at rest in some inertial frame in space, suddenly needs to accelerate. The ship forcibly expels 103kg of fuel from its rocket engine, almost instantaneously, at velocity $(3/5)c$ in the original inertial frame; afterwards the ship has a mass of 106 kg. How fast will the ship then be moving, valid to three significant figures?

2. An incident ball A of mass m and velocity v_0 moves to the right, striking a target ball B of mass 3m initially at rest in the lab frame S. Afterward, ball A bounces back with velocity $-v_0/2$ and ball B is pushed ahead with velocity $+v_0/2$.
   (a) Show that the sum of the classical momenta$p = mv$ is conserved in the lab frame.
   (b) Using the relativistic velocity transformation find the velocity o f each ball before and after the collision in an S$'$ frame moving to the right at velocity v_0 relative to the lab.
   (c) Using the results of part (b), show that the total classical momentum is not conserved in S'.

3. A photon of momentum P_y strikes an atomic nucleus at rest and is absorbed. If the mass of the final (excited) nucleus is M, calculate its speed.

4. Two particles make a head-on collision, stick together, and stop dead. The first particle has mass m and speed $(3/5)c$, and the second has mass M and speed $(4/5)c$. Find $]rmM$ in terms of m.

3 Lorentz Transformation

1. Rederive the velocity transformation for momentum using the Lorentz transformation for the velocity four-vector.

2. A ship suddenly finds itself approaching a space-iceberg at velocity (3/5)c, so immediately fires a 100 kg missile at speed (4/5)c relative to the ship, in hopes of breaking up the ice before they crash. Find all components of the missile's momentum four-vector

   (a) in the spaceship frame, assuming all motion is in the x direction;

   (b) in the iceberg frame.

   (c) The iceberg will break up if the momentum impacting it is at least $1.0 \times 10^{11} kgm/s$. Does the iceberg break up?

   (d) What is the velocity of the missile in the iceberg frame?

Problems are collected from Helliwell and Helliwell (2010). For more practicing problems, please see Science Knowledge or go to the next url: `https://scienceknowledge.webador.com/`. The solution can be found here.

References

T.M Helliwell and T.M Helliwell. page 108–111. University Science Books, 2010.