Momentum and Impulse

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Learning Objectives

- Understand what momentum is
  - Defintion, SI unit, calculation
- Understand the connection between Newton's second law and momentum
  - Net force F=ma
  - Momentum p=mv
- Understand that change in momentum is equal to impulse
- Basic diagram
  - Momentum and force-time graph
- Understand conservation of momentum
  - Problem Solving
  - Collision and momentum

What is Momentum?

Momentum and Newton’s Law

Impulse

Example 1

- A ball is thrown rightward at a spped of v. After t seconds, the ball collides with a wall and bounces back with a final speed of v. What is the change in momentum?

Example 2

- You've been rowdy and obnoxious in a bar and now are in the process of being thrown out by the bouncer by the scruff of the neck. The bouncer has hold of you for 5.0 s and you are given a final velocity of 2.75 m/s. If your mass is 70.0 kg, what was your final momentum? What impulse and average force did the bouncer exert on you? Assume all motion is in a straight line.

https://www.kpu.ca/sites/default/files/Faculty%20of%20Science%20%26%20Horticulture/Physics/PHYS%201100%20Collision%20%26%20Momentum%20Solutions.pdf

Momentum is defined by $p = mv$. Taking the direction of motion as positive, your initial momentum was zero and your final momentum is

$$p = (70.0 \text{ kg})(2.75 \text{ m/s}) = 192.5 \text{ kg-m/s} .$$

Impulse is defined as the change in momentum

$$I = p_f - p_i = 192.5 \text{ kg-m/s} .$$

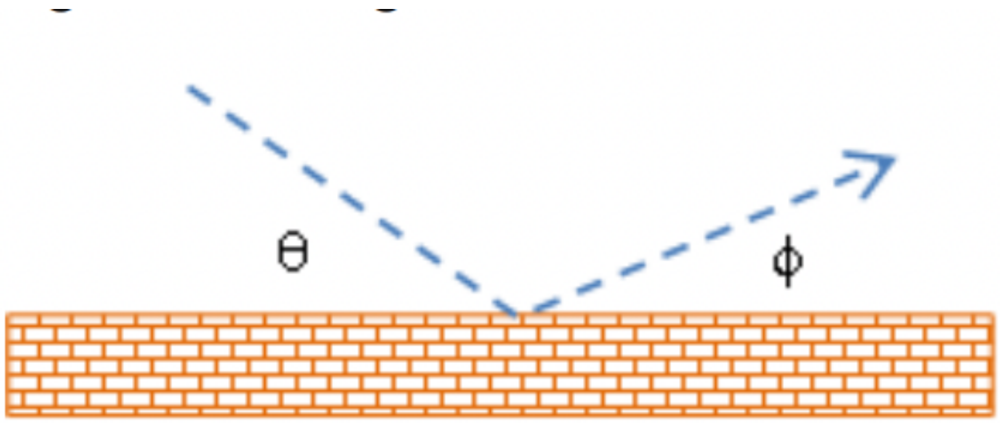
Average force is related to impulse by $I = F_{average}t$, so

$$F_{average} = I / t = 192.5 \text{ kg-m/s} / 5 \text{ s} = 38.5 \text{ N}.$$

This is the average force exerted on you and is in the same direction as your motion.

Example 3

- A ball of mass 0.25 kg glances of a wall as shown in the diagram. The ball approaches at 15 m/s at $\theta = 30°$ and leaves at 12 m/s at $\phi = 20°$. The collision lasts for 15 milliseconds. (a) What are the components of the impulse experienced by the ball? (b) What are the components of the average force acting on the ball?



a. We know Impulse is equal to the difference in momentum, $\mathbf{I} = m\mathbf{v}_f - \mathbf{v}_i$. This is a vector equation and to get components we consider the x and y components separately.

$I_x = mv_{fx} - mv_{ix} = (0.25) \times (12\cos 20° - 15\cos 30°) = -0.4285$ N-s

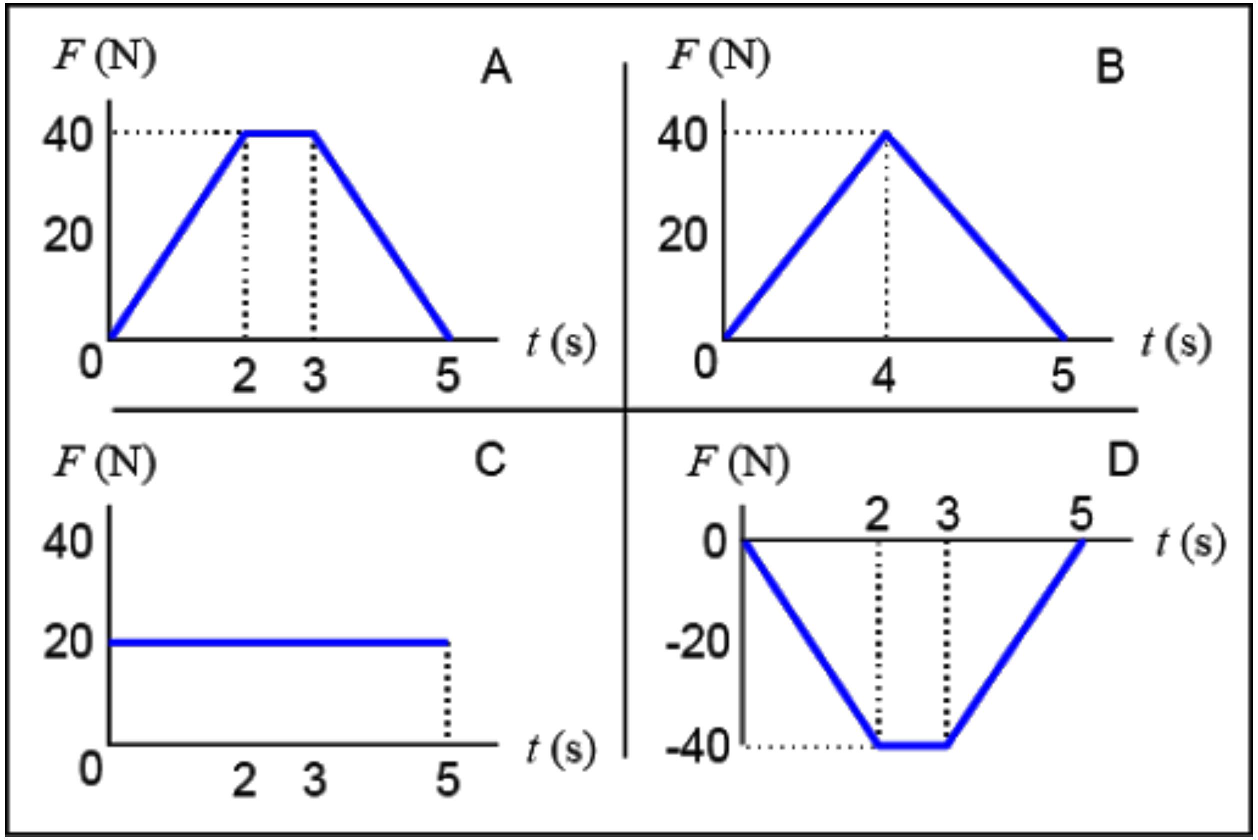
$I_y = mv_{fy} - mv_{iy} = (0.25) \times (12\sin 20° - (-15\sin 30°)) = +2.9011$ N-s

or $\mathbf{I} = -\boldsymbol{i}0.4285 + \boldsymbol{j}2.9011$ N-s.

b. We know $\mathbf{I} = \mathbf{F}_{ave}\Delta t$ where Δt is how long the collision lasts. We have already calculated I and we are given $\Delta t =$ 15 ms, so $\mathbf{F}_{ave} = \mathbf{I}/\Delta t$

so $\mathbf{F}_{ave} = (-\boldsymbol{i}0.4285 + \boldsymbol{j}2.9011 \text{ N-s}) / (15 \text{ ms}) = -\boldsymbol{i}28.6 + \boldsymbol{j}193.4$ N.

Force and Time Graph



Example 1

A 0.50 kg ball is dropped from rest above a hard floor. When it reaches the floor it has a velocity of 4.0 m s^{-1}. The ball then bounces vertically upwards. Figure **2.77** is the graph of velocity against time for the ball. The positive direction for velocity is upwards.

a Find the magnitude of the momentum change of the ball during the bounce.

b The ball stayed in contact with the floor for 0.15 s. What average force did the floor exert on the ball?

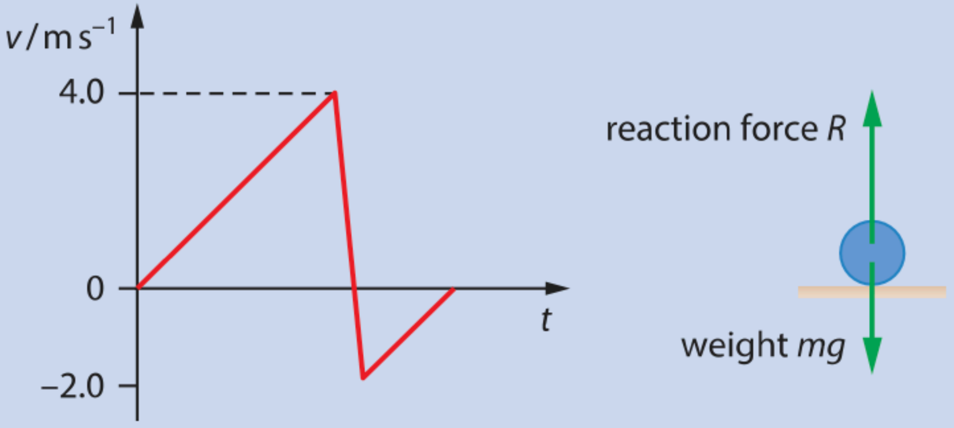


Figure 2.77

a The momentum when the ball hits the floor is: $0.50 \times 4.0 = 2.0\,\text{N s}$

The momentum when the ball rebounds from the floor is: $0.50 \times (-2.0) = -1.0\,\text{N s}$

The magnitude of the momentum change is therefore 3.0 N s.

b The forces on the ball are its weight and the reaction from the floor, *R*.

$F_{net} = R - mg$

This is also the force that produces the change in momentum:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

Substituting in this equation:

$$F_{net} = \frac{3.0}{0.15} = 20\,\text{N}$$

We need to find *R*, so:

$R = 20 + 5.0 = 25\,\text{N}$.

The average force exerted on the ball by the floor is 25 N.

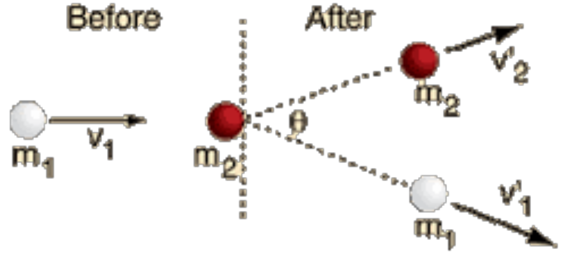
Exam tip

This is a very tricky problem with lots of possibilities for error. A lot of people forget to include the minus sign in the rebound velocity and also forget the weight, so they answer incorrectly that $R = 20\,\text{N}$.

Conservation of Momentum

Collision and Momentum

Elastic Collisions - Target Initially at Rest*



*This can actually apply to all elastic collisions since we can always choose a reference frame riding with m_2 before the collision.

General relationships:

a. Conservation of momentum: $m_1\vec{v}_1 = m_1\vec{v}'_1 + m_2\vec{v}'_2$

b. Conservation of kinetic energy: (elastic collision assumption) $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2$

c. For head-on collisions: $v'_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}v_1$; $v'_2 = \frac{2m_1}{(m_1 + m_2)}v_1$

d. For head-on collisions the velocity of approach is equal to the velocity of separation.

	Before	After
Momentum	$m_1 v_1$	$(m_1 + m_2)v_2$
Kinetic energy	$\frac{1}{2}m_1 v_1^2$	$\frac{1}{2}(m_1 + m_2)v_2^2$

From conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)v_2 \Rightarrow v_2 = \frac{m_1}{m_1 + m_2} v_1$$

Ratio of kinetic energies before and after collision:

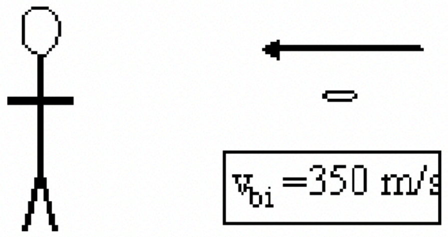
$$\frac{KE_f}{KE_i} = \frac{m_1}{m_1 + m_2}$$

Fraction of kinetic energy lost in the collision:

$$\frac{KE_i - KE_f}{KE_i} = \frac{m_2}{m_1 + m_2}$$

Example 1

- While chasing an armed suspect into and onto an ice rink, a police constable is shot. Fortunately, the constable is wearing a bulletproof vest which absorbs the bullet. If the muzzle velocity of the bullet is 350 m/s and the its mass is 100 g. Find the final velocity of the constable and bullet if her mass is 69.5 kg. Assume all motion is in a straight line and ignore friction. Assume that the constable is at rest.



We have a totally inelastic collision, so momentum is conserved. For this particular problem

$$(m_{police} + m_{bullet})v_{pf} = m_{police}v_{pi} + m_{bullet}v_{bi}.$$

Since we are told $v_{pi} = 0$,

$$v_{pf} = m_{bullet}v_{bi}/(m_{police} + m_{bullet}) = (0.100 \text{ kg})(-350 \text{ m/s})/(69.5 \text{ kg} + 0.1 \text{ kg}) = -0.503 \text{ m/s} .$$

So the constable is knocked backwards at 0.50 m/s.

Example 3

A 3000 kg truck travelling at 50 km/hr strikes a stationary 1000 kg car, locking the two vehicles together.
A) What is the final velocity of the two vehicles?

References

- http://hyperphysics.phy-astr.gsu.edu/hbase/inecol.html
- http://hyperphysics.phy-astr.gsu.edu/hbase/elacol2.html