# Proof by Mathematical Induction

■ worked solutions attached ■

**1.** Use the method of mathematical induction to prove that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$, $n \in \mathbb{Z}^+$. [8 marks]

**2.** Prove by mathematical induction that $4^n + 2$ is a multiple of 3 for all positive integers. [8 marks]

**worked solutions**

**1.** Use the method of mathematical induction to prove that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$, $n \in \mathbb{Z}^+$. [8 marks]

**Solution**:

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}, \ n \in \mathbb{Z}^+$$

Show statement is true for $n = 1$:

$\text{RHS} = \sum_{r=1}^{1} \frac{1}{r(r+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$; $\text{LHS} = \frac{1}{1+1} = \frac{1}{2}$; thus, statement is true for $n = 1$

Assume the statement is true for *k*, a specific value of *n*. That is, assume that $\sum_{r=1}^{k} \frac{1}{r(r+1)} = \frac{k}{k+1}$.

Show it must follow that the statement is true foe $n = k+1$. That is, show that

$$\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k+1}{k+1+1} = \frac{k+1}{k+2}$$

$$\sum_{r=1}^{k} \frac{1}{r(r+1)} + \frac{1}{(k+1)(k+1+1)} = \text{RHS}$$

$$\frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \text{RHS}$$

$$\frac{k(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} = \text{RHS}$$

$$\frac{k^2+2k+1}{(k+1)(k+2)} = \text{RHS}$$

$$\frac{(k+1)(k+1)}{(k+1)(k+2)} = \text{RHS}$$

$$\frac{k+1}{k+2} = \frac{k+1}{k+2}$$ ***Q.E.D.***

Hence, by the principle of mathematical induction, the statement $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$ is true for all positive integer values of *n*.

**2.** Prove by mathematical induction that $4^n + 2$ is a multiple of 3 for all positive integers. [8 marks]

**Solution**:

Show statement is true for $n = 1$:

$4^1 + 2 = 6 = 2(3)$; thus, statement is true for $n = 1$

Assume that the statement is true for a specific value of *n*, call it *k*. That is, assume

$4^k + 2 = 3M$ where *M* is a positive integer

Show that it must follow that the statement is true for $n = k + 1$. That is, show that $4^{k+1} + 2$ must be a multiple of 3.

$4^{k+1} + 2 = 4 \cdot 4^k + 2$

From the assumption, it follows that $4^k = 3M - 2$. Substituting this, gives

$$4^{k+1} + 2 = 4(3M - 2) + 2$$

$$= 12M - 6$$

$$= 3(4M - 2)$$

Since *M* is a positive integer then $4M - 2$ must be a positive integer. And, since it was shown that $4^{k+1} + 2 = 3(4M - 2)$ then $4^{k+1} + 2$ is a multiple of 3.

Hence, by the principle of mathematical induction, the expression $4^n + 2$ must be a multiple of 3 for all positive integer values of *n*.