Finance Apps *[65 marks]*

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5 %, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

1a. Find the value of r, giving your answer to four significant figures. *[3 marks]*

Markscheme

$\left(1+\frac{5.5}{4\times100}\right)^4$ ***(M1)(A1)***

1.056 ***A1***

[3 marks]

1b. Laurie makes no further deposits to or withdrawals from the account. *[3 marks]*

Find the year in which the amount of money in Laurie's account will become double the amount she invested.

Markscheme

EITHER

$2P = P \times \left(1 + \frac{5.5}{100 \times 4}\right)^{4n}$ **OR** $2P = P \times (\text{their } (a))^{m}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

OR

PV = ±1
FV = ∓1
I% = 5.5
P/Y = 4
C/Y = 4
n = 50.756... ***(M1)(A1)***

OR

PV = ±1
FV = ∓2
I% = 100(their (*a*) − 1)
P/Y = 1
C/Y = 1 ***(M1)(A1)***

THEN

⇒ 12.7 years

Laurie will have double the amount she invested during 2032 ***A1***

[3 marks]

Give your answers in this question correct to the nearest whole number.

Imon invested $25\ 000$ Singapore dollars (SGD) in a fixed deposit account with a nominal annual interest rate of 3.6%, compounded **monthly**.

2a. Calculate the value of Imon's investment after 5 years. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.

$(FV =)\ 25\ 000 \times \left(1 + \frac{3.6}{100 \times 12}\right)^{12 \times 5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted compound interest formula, ***(A1)*** for correct substitutions.

OR

$N = 5$
$I\% = 3.6$
$PV = \mp 25\ 000$
$P/Y = 1$
$C/Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 12$ seen, ***(M1)*** for **all** other correct entries.

OR

$N = 60$
$I\% = 3.6$
$PV = \mp 25\ 000$
$P/Y = 12$
$C/Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 12$ seen, ***(M1)*** for **all** other correct entries.

$(FV =)\ 29\ 922\ (\text{SGD})$ ***(A1)*** ***(C3)***

Note: Do not award the final ***(A1)*** if answer is not given correct to the nearest integer.

[3 marks]

2b. At the end of the 5 years, Imon withdrew x SGD from the fixed deposit account and reinvested this into a super-savings account with a nominal annual interest rate of 5.7%, compounded **half-yearly**. *[3 marks]*

The value of the super-savings account increased to $20\ 000$ SGD after 18 months.

Find the value of x.

Markscheme

$20\,000 = PV \times \left(1 + \frac{5.7}{100 \times 2}\right)^{2 \times 1.5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted compound interest equated to $20\,000$. Award ***(A1)*** for correct substitutions.

OR

$N = 1.5$
$I\% = 5.7$
$FV = \pm 20\,000$
$P/Y = 1$
$C/Y = 2$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 2$ seen, ***(M1)*** for **all** other correct entries.

OR

$N = 3$
$I\% = 5.7$
$FV = \pm 20\,000$
$P/Y = 2$
$C/Y = 2$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 2$ seen, ***(M1)*** for **all** other correct entries.

$(x =)\ 18\,383\ (\text{SGD})$ ***(A1)*** ***(C3)***

Note: Do not award the final ***(A1)*** if answer is not given correct to the nearest integer (unless already penalized in part(a)).

[3 marks]

Tommaso plans to compete in a regional bicycle race after he graduates, however he needs to buy a racing bicycle. He finds a bicycle that costs 1100 euro (EUR). Tommaso has 950 EUR and invests this money in an account that pays 5 % interest per year, **compounded monthly**.

3a. Determine the amount that he will have in his account after 3 years. *[3 marks]*
Give your answer correct to two decimal places.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$950 \times \left(1 + \frac{5}{12 \times 100}\right)^{12 \times 3}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in the compound interest formula: ***(A1)*** for correct substitution.

OR

N = 3
I% = 5
PV = 950
P/Y = 1
C/Y = 12 ***(A1)(M1)***

Note: Award ***(A1)*** for C/Y = 12 seen, ***(M1)*** for other correct entries.

OR

N = 36
I% = 5
PV = 950
P/Y = 12
C/Y = 12 ***(A1)(M1)***

Note: Award ***(A1)*** for C/Y = 12 seen, ***(M1)*** for other correct entries.

1103.40 (EUR) ***(A1)(G3)***

Note: Answer must be given to 2 decimal places.

[3 marks]

The cost of the bicycle, C, can be modelled by $C = 20x + 1100$, where x is the number of years since Tommaso invested his money.

3b. Find the difference between the cost of the bicycle and the amount of money in Tommaso's account after 3 years. Give your answer correct to two decimal places. *[3 marks]*

Markscheme

(20 × 3 + 1100) − 1103.40 ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into cost of bike function, ***(M1)*** for subtracting their answer to part (a). This subtraction may be implied by their final answer (follow through from their part (a) for this implied subtraction).

55.60 (EUR) ***(A1)*****(ft)*****(G3)***

Note: Follow through from part (a). The answer must be two decimal places.

[3 marks]

3c. After m complete **months** Tommaso will, for the first time, have enough money in his account to buy the bicycle. *[5 marks]*

Find the value of m.

Markscheme

METHOD 1

$950 \times \left(1 + \frac{5}{12 \times 100}\right)^{12x} = 20x + 1100$ ***(M1)(M1)***

Note: Award ***(M1)*** for their correct substitution in the compound interest formula with a variable in the exponent; ***(M1)*** for comparing their expressions provided variables are the same (not an expression with x for years and another with x representing months). Award at most ***(M0)(M1)(A0)(M1)(A0)*** for substitution of an integer in both expressions and comparison of the results. Accept inequality.

(x =) 4.52157... (years) ***(A1)*****(ft)**

4.52157... × 12 (= 54.2588...) ***(M1)***

Note: Award ***(M1)*** for multiplying **their** value for x by 12. This may be implied.

m = 55 (months) ***(A1)*****(ft)*****(G4)***

METHOD 2

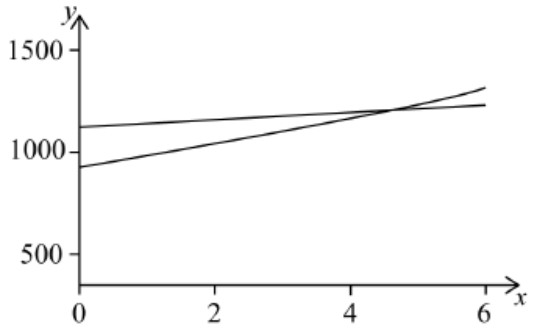
$950 \times \left(1 + \frac{5}{12 \times 100}\right)^{m} = 20 \times \frac{m}{12} + 1100$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for their correct substitution in the compound interest formula with a variable in the exponent to solve; **(M1)** for comparing their expressions provided variables are the same; ***(M1)*** for converting years to months in these expressions. Award at most ***(M0)(M1)(A0)(M1)(A0)*** for substitution of an integer in both expressions and comparison of the results. Accept inequality.

m = 54.2588... (months) ***(A1)*****(ft)**

m = 55 (months) ***(A1)*****(ft)*****(G4)***

METHOD 3



(M1)(M1)

Note: Award ***(M1)*** for each graph drawn.

(x =) 4.52157... (years) ***(A1)*****(ft)**

4.52157... × 12 (= 54.2588...) ***(M1)***

Note: Award ***(M1)*** for multiplying **their** value for x by 12. This may be implied.

If the graphs drawn are in terms of months, leading to a value of 54.2588..., award ***(M1)(M1)(M1)(A1)***, consistent with METHOD 2.

m = 55 (months) ***(A1)*****(ft)*****(G4)***

Note: Follow through for a compound interest formula consistent with their part (a). The final ***(A1)*****(ft)** can only be awarded for correct answer, or their correct answer following through from previous parts and only if value is rounded up. For example, do not award ***(M0)(M0)(A0)(M1)(A1)*****(ft)** for an unsupported "5 years × 12 = 60" or similar.

[5 marks]

In this question, give all answers to two decimal places.

Karl invests 1000 US dollars (USD) in an account that pays a nominal annual interest of 3.5%, **compounded quarterly**. He leaves the money in the account for 5 years.

4a. Calculate the amount of money he has in the account after 5 years. *[3 marks]*

Markscheme

$1000\left(1+\frac{3.5}{4\times100}\right)^{4\times5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitution.

OR

$N = 5$

$I = 3.5$

$PV = 1000$

$P/Y = 1$

$C/Y = 4$

Note: Award ***(A1)*** for $C/Y = 4$ seen, ***(M1)*** for other correct entries.

OR

$N = 5 \times 4$

$I = 3.5$

$PV = 1000$

$P/Y = 1$

$C/Y = 4$

Note: Award ***(A1)*** for $C/Y = 4$ seen, ***(M1)*** for other correct entries.

= 1190.34 (USD) ***(A1)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitution.

[3 marks]

4b. Write down the amount of **interest** he earned after 5 years. *[1 mark]*

Markscheme

190.34 (USD) ***(A1)*****(ft)** ***(C4)***

Note: Award ***(A1)*****(ft)** for subtraction of 1000 from their part (a)(i). Follow through from (a)(i).

[1 mark]

4c. Karl decides to donate this **interest** to a charity in France. The charity receives 170 euros (EUR). The exchange rate is 1 USD = t EUR. *[2 marks]*

Calculate the value of t.

Markscheme

$\frac{170}{190.34}$ ***(M1)***

Note: Award ***(M1)*** for division of 170 by their part (a)(ii).

= 0.89 ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their part (a)(ii).

[2 marks]

Juan buys a bicycle in a sale. He gets a discount of 30% off the original price and pays 560 US dollars (USD).

5a. Calculate the original price of the bicycle. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{560}{70} \times 100$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for dividing 560 by 0.7 or equivalent.

$= 800\ (\text{USD})$ ***(A1)*** ***(C2)***

[2 marks]

To buy the bicycle, Juan takes a loan of 560 USD for 6 months at a nominal annual interest rate of 75%, **compounded monthly**. Juan believes that the total amount he will pay will be less than the original price of the bicycle.

5b. Calculate the difference between the original price of the bicycle and the total amount Juan will pay. *[4 marks]*

Markscheme

$560\left(1+\frac{75}{12\times100}\right)^{12\times\frac{1}{2}}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into interest formula, ***(A1)*** for their correct substitution.

OR

$\mathrm{N}=\frac{1}{2}$

$\mathrm{I}\% \ =75$

$\mathrm{PV}=(\pm)560$

$\mathrm{P/Y}=1$

$\mathrm{C/Y}=12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $\mathrm{C/Y}=12$ seen, ***(M1)*** for all other entries correct.

OR

$\mathrm{N}=6$

$\mathrm{I}\% \ =75$

$\mathrm{PV}=(\pm)560$

$\mathrm{P/Y}=12$

$\mathrm{C/Y}=12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $\mathrm{C/Y}=12$ seen, ***(M1)*** for all other entries correct.

$=805.678\ldots\ (\mathrm{USD})$ ***(A1)***

Note: Award ***(A3)*** for 805.678... (806) seen without working.

(Juan spends) 5.68 (USD) (5.67828... USD) (more than the original price) ***(A1)*****(ft)** ***(C4)***

[4 marks]

Phil takes out a bank loan of $150 000 to buy a house, at an annual interest rate of 3.5%. The interest is calculated at the end of each year and added to the amount outstanding.

6a. Find the amount Phil would owe the bank after 20 years. Give your answer to the nearest dollar. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

150000×1.035^{20} ***(M1)(A1)***

$= \$298468$ ***A1***

Note: Only accept answers to the nearest dollar. Accept $298469.

[3 marks]

To pay off the loan, Phil makes annual deposits of $\$P$ at the end of every year in a savings account, paying an annual interest rate of 2% . He makes his first deposit at the end of the first year after taking out the loan.

6b. Show that the total value of Phil's savings after 20 years is $\frac{(1.02^{20}-1)P}{(1.02-1)}$. *[3 marks]*

Markscheme

attempt to look for a pattern by considering 1 year, 2 years *etc* ***(M1)***

recognising a geometric series with first term P and common ratio 1.02 ***(M1)***

EITHER

$P + 1.02P + \ldots + 1.02^{19}P \ \left(= P(1 + 1.02 + \ldots + 1.02^{19})\right)$ ***A1***

OR

explicitly identify $u_1 = P,\ r = 1.02$ and $n = 20$ (may be seen as S_{20}). ***A1***

THEN

$s_{20} = \frac{(1.02^{20}-1)P}{(1.02-1)}$ ***AG***

[3 marks]

6c. Given that Phil's aim is to own the house after 20 years, find the value for P to the nearest dollar. *[3 marks]*

Markscheme

$24.297\ldots P = 298468$ ***(M1)(A1)***

$P = 12284$ ***A1***

Note: Accept answers which round to 12284.

[3 marks]

David visits a different bank and makes a single deposit of $\$Q$, the annual interest rate being 2.8%.

6d. David wishes to withdraw \$5000 at the end of each year for a period of n *[3 marks]* years. Show that an expression for the minimum value of Q is

$\frac{5000}{1.028} + \frac{5000}{1.028^2} + \ldots + \frac{5000}{1.028^n}$.

Markscheme

METHOD 1

$Q(1.028^n) = 5000(1 + 1.028 + 1.028^2 + 1.028^3 + \ldots + 1.028^{n-1})$ ***M1A1***

$Q = \frac{5000(1+1.028+1.028^2+1.028^3+\ldots+1.028^{n-1})}{1.028^n}$ ***A1***

$= \frac{5000}{1.028} + \frac{5000}{1.028^2} + \ldots + \frac{5000}{1.028^n}$ ***AG***

METHOD 2

the initial value of the first withdrawal is $\frac{5000}{1.028}$ ***A1***

the initial value of the second withdrawal is $\frac{5000}{1.028^2}$ ***R1***

the investment required for these two withdrawals is $\frac{5000}{1.028} + \frac{5000}{1.028^2}$ ***R1***

$Q = \frac{5000}{1.028} + \frac{5000}{1.028^2} + \ldots + \frac{5000}{1.028^n}$ ***AG***

[3 Marks]

6e. Hence or otherwise, find the minimum value of Q that would permit David to withdraw annual amounts of \$5000 indefinitely. Give your answer to the nearest dollar. *[3 marks]*

Markscheme

sum to infinity is $\frac{\frac{5000}{1.028}}{1-\frac{1}{1.028}}$ ***(M1)(A1)***

$= 178571.428\ldots$

so minimum amount is \$178572 ***A1***

Note: Accept answers which round to \$178571 or \$178572.

[3 Marks]

Jashanti is saving money to buy a car. The price of the car, in US Dollars (USD), can be modelled by the equation

$$P = 8500(0.95)^t.$$

Jashanti's savings, in USD, can be modelled by the equation

$$S = 400t + 2000.$$

In both equations t is the time in months since Jashanti started saving for the car.

7a. Write down the amount of money Jashanti saves per month. *[1 mark]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

400 (USD) ***(A1)*** ***(C1)***

[1 mark]

7b. Use your graphic display calculator to find how long it will take for Jashanti to have saved enough money to buy the car. *[2 marks]*

Markscheme

$8500\ (0.95)^t = 400 \times t + 2000$ ***(M1)***

Note: Award ***(M1)*** for equating $8500(0.95)^t$ to $400 \times t + 2000$ or for comparing the difference between the two expressions to zero or for showing a sketch of both functions.

$(t =)\ 8.64\ (\text{months})\ (8.6414\ldots\ (\text{months}))$ ***(A1)*** ***(C2)***

Note: Accept 9 months.

[2 marks]

Jashanti does not want to wait too long and wants to buy the car two months after she started saving. She decides to ask her parents for the extra money that she needs.

7c. Calculate how much extra money Jashanti needs. *[3 marks]*

Markscheme

$8500(0.95)^2 - (400 \times 2 + 2000)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution of $t = 2$ into equation for P, ***(M1)*** for finding the difference between a value/expression for P and a value/expression for S. The first ***(M1)*** is implied if 7671.25 seen.

4870 (USD) (4871.25) ***(A1)*** ***(C3)***

Note: Accept 4871.3.

[3 marks]

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8. Carmen also deposited ARS in a bank account. Her account pays a nominal annual interest rate of 17%, **compounded yearly**. After three years, the total amount in Carmen's account is $10\,000\,\text{ARS}$. *[3 marks]*

Find the amount that Carmen deposited in the bank account.

Markscheme

$N = 3$

$I\% = 17$

$(PV = \pm 6243.705564)$

$PMT = 0$

$(FV = \mp 10\,000)$

$P/Y = 1$

$C/Y = 1$ ***(A1)(M1)***

Note: Award ***(A1)*** for $FV = \mp 10\,000$, ***(M1)*** for **all** other correct entries. $PV = \pm 6243.705564$ need not be seen.

OR

$10\,000 = PV \times \left(1 + \frac{17}{100}\right)^3$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting into compound interest formula, ***(A1)*** for equating $10\,000$ to the correctly substituted compounded interest formula.

$(PV =)\ 6243.71$ ***(A1)*** ***(C3)***

Note: Answer must be to two decimal places.

Pierre invests 5000 euros in a fixed deposit that pays a nominal annual interest rate of 4.5%, compounded **monthly**, for seven years.

9. Carla has 7000 dollars to invest in a fixed deposit which is compounded **annually**. *[3 marks]*

   She aims to double her money after 10 years.

   Calculate the minimum annual interest rate needed for Carla to achieve her aim.

Markscheme

$14000 = 7000\left(1 + \frac{r}{100}\right)^{10}$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substitution into compound interest formula equated to 14000 or equivalent.

Award ***(A1)*** for correct substitutions.

OR

$N = 10$

$PV = \pm 7000$

$FV \mp 14000$

$P/Y = 1$

$C/Y = 1$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 1$ seen, ***(M1)*** for other correct entries. PV and FV must have opposite signs.

$r = 7.18\%$ $(7.17734\ldots\%,\ 0.0718)$ ***(A1)*** ***(C3)***

Note: Do not penalize if $\%$ sign is missing. Do not accept 0.0718%.

Minta deposits 1000 euros in a bank account. The bank pays a nominal annual interest rate of 5%, **compounded quarterly**.

10. Minta will withdraw the money from her bank account when the interest earned is 300 euros. *[3 marks]*

    Find the time, in years, until Minta withdraws the money from her bank account.

Markscheme

$1000\left(1+\frac{5}{4\times 100}\right)^{4\times t}=1300$ ***(M1)(A1)***

Note: Award ***(M1)*** for using the compound interest formula with a variable for time, ***(A1)*** for substituting correct values and equating to 1300.

OR

$\mathrm{I}\%=5$

$\mathrm{PV}=\pm 1000$

$\mathrm{FV}=\mp 1300$

$\mathrm{P/Y}=1$

$\mathrm{C/Y}=4$ ***(A1)(M1)***

Note: Award ***(A1)*** for 1300 seen, ***(M1)*** for the other correct entries.

OR

$\mathrm{I}\%=5$

$\mathrm{PV}=\pm 1000$

$\mathrm{FV}=\mp 1300$

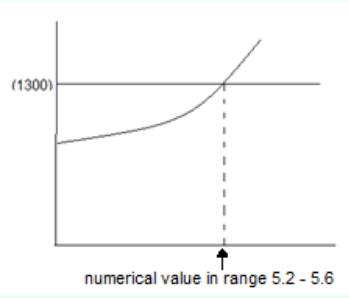
$\mathrm{P/Y}=4$

$\mathrm{C/Y}=4$ ***(A1)(M1)***

Note: Award ***(A1)*** for 1300 seen, ***(M1)*** for the other correct entries.

OR

Sketch drawn of two appropriate lines which intersect at a point



Note: Award ***(M1)*** for a sketch with a straight line intercepted by appropriate curve, ***(A1)*** for a numerical answer in the range $5.2-5.6$.

$t=5.28\ (\text{years})\quad (5.28001\ldots)$ ***(A1) (C3)***