## Markscheme

(i) theory suggests that no object can travel faster than light; the $1.7 c$ is not the speed of a physical object;
so is not in violation of the theory;
(ii) recognition that $v$ is negative relative to $u x$;
use of $\backslash\left(\right.$ ffrac $\left.\left.\left.\left.\left.\left.\{\{0.85 \mathrm{c}+0.85 \mathrm{c}\}\}\left\{\{1+\backslash \text { frac\{\{\{\{left( }\{0.85 \mathrm{c}\} \backslash \text { right) }\}^{\wedge} 2\right\}\right\}\right\}\left\{\left\{\left\{\mathrm{c}^{\wedge} 2\right\}\right\}\right\}\right\}\right\}\right\}\right)$;
(0.9869c $\approx$ ) 0.99c;

Accept first marking point implied in the second marking point.

## Examiners report

[N/A]

1b.

## Markscheme

(i) <br>(Igamma <br>)=1.89;
interval on Earth $=\backslash($ Igamma $\backslash) \times$ interval on spaceship;
(interval on Earth $1.90 \times 8$ years=) 15 years;
Award [3] for a bald correct answer.
(ii) time interval measured by observer on Earth is not proper because the time interval between the two events is not measured at same place/not the shortest time;
(iii) observer on Earth thinks spaceship has travelled for 15 years;
so distance is $0.85 \mathrm{c} \times 15=12.8 \approx 13 \mathrm{ly}$;
Award [2] for a bald correct answer.
or
the spaceship observer observes the distance moved by the Earth $=0.85 \mathrm{c} \times 8.0 \mathrm{yr}$; proper distance $=1.90 \times 0.85 \mathrm{c} \times 8.0 \mathrm{yr}=12.9 \approx 13 \mathrm{ly}$;
Award [2] for a bald correct answer.
(iv) Earth is at a distance of $4 \times 0.85 c=3.41 \mathrm{l}$ when signal is emitted; signal reaches Earth in time T where $c \mathrm{~T}+3.4=0.85 c \mathrm{~T}$;
$\mathrm{T}=22.7 \approx 23$ years;

## Examiners report

[N/A]
$2 a$.

## Markscheme

$E=\backslash($ gamma $\backslash) \mathrm{mc}^{2}=5 \mathrm{mc}^{2}$ or $\backslash($ ggamma $\backslash)=5$;
so <br>(\frac\{\{\{v^2\}\}\}\{\{\{c^2\}\}\} = 1-0.04));
$v=0.98 c$;

## Examiners report

[N/A]

## Markscheme

close to Newton for small $v$ and below Newton for large $V$; asymptotic to $c$;


## Examiners report

[N/A]

## Markscheme

$\backslash\left(\backslash \operatorname{left}\left(\left\{R=\backslash\right.\right.\right.$ frac $\left.\left.\{\{2 G M\}\}\left\{\left\{\left\{c^{\wedge} 2\right\}\right\}\right\}\right\} \backslash \operatorname{right}\right) \backslash$ frac $\{\{2$ \times $6.7 \backslash$ times $\{\{10\} \wedge\{-11\}\} \backslash$ times $5.0 \backslash$ times \{ \{10\}^\{30\}\}\}\} \{ \{9 \times $\{\{10\} \wedge\{16\}\}\}\} \backslash) ;$
$=7.4 \times 10^{3}(\mathrm{~m})$;

## Examiners report

[N/A]

## Markscheme

closest approach much larger than $R$;
so Newtonian mechanics will apply;
spacetime will not be significantly warped;

## Examiners report

[N/A]

## Markscheme

theory predicts that warping of spacetime will affect light; the massive star is warping spacetime;
causing the light to bend around it;
Award [2 max] to an answer that addresses the last two marking points.
Accept answer to the last two marking points in a form of labelled diagram.

## Examiners report

[N/A]

## Markscheme

(i) $\backslash\left(\backslash \operatorname{left}\left(\{\backslash \operatorname{frac}\{\{12\{\backslash \operatorname{rm}\{\operatorname{ly}\}\}\}\}\{\{0.60\{\backslash \operatorname{rm}\{\mathrm{c}\}\}\}\}=\} \backslash\right.\right.$ right)20\left( $\{\{\backslash \mathrm{rm}\{\mathrm{yr}\}\}\} \backslash$ right) $\backslash$ ) or $6.3 \times 10^{8}(\mathrm{~s})$;
(ii) $\backslash\left(y=\backslash \operatorname{left}\left(\left\{\backslash f r a c\{1\}\left\{\left\{\backslash\right.\right.\right.\right.\right.$ sqrt $\left.\left.\left.\left\{1-\left\{\{0.60\}^{\wedge} 2\right\}\right\}\right\}\right\}=\right\} \backslash$ right $) 1.25 \backslash$;
$\backslash\left(\backslash\right.$ Delta $\left\{t \_0\right\}=\operatorname{left}(\{\backslash f r a c\{\{\backslash D e l t a t\}\}\{y\}=\backslash$ frac $\{\{20\}\}\{\{1.25\}\}=\} \backslash$ right $) 16 \backslash \operatorname{left}(\{\{\backslash r m\{y r\}\}\} \backslash$ right $) \backslash$ or $5.0 \times 10^{8}$ (s); (allow ECF from (a)(i));

This question is worth [2], but it is easy to accidentally award [1].

## Examiners report

Part (a) was answered very well. This year almost nobody worked in seconds and so the answers were easily obtained. As usual there were candidates who got time dilation the wrong way round. The time interval for the Earth clocks is dilated (longer) but some candidates think that the time interval on the "moving" clock is dilated. It is best not to think of motion, but to realise that the single clock at both events records the shortest time interval.

## 4b.

## Markscheme

(i) the length of a body in the rest frame of the body;

Do not accept "event" instead of "object/body".
Do not accept "in the same frame" unless rest (OWTTE) is mentioned.
(ii) $\backslash(I=\backslash f r a c\{\{310\}\}\{\{1.25\}\} \backslash)$; (allow ECF from (a)(ii))
=250(m);
This question is worth [2], but it is easy to accidently award [1].

## Examiners report

In (b) a very common misconception with proper length is to just say that the object must be measured in the same frame of reference as the observer. Well this is always true of course, but only if the object is at rest in the observer's frame is it proper length. Everything is in everything else's frame. Gradually more and more candidates are answering simultaneity questions correctly. This year almost 3\% could correctly explain why light B emits waves before light F as perceived from the spacecraft frame. The other $97 \%$ thought that the question was asking about which light the spacecraft observer sees first.

## Markscheme

according to the spacecraft observer, the space station observer receives light from $B$ and $F$ at the same time; for the spacecraft observer the space station observer moves away from the waves from B/towards the waves from F; but the speed of light is constant;
according to the spacecraft observer light from B must be emitted first;
Do not award second marking point for answers that refer to light the spacecraft observer SEES or to distances to the spacecraft.

## Examiners report

[N/A]

## Markscheme

change in total energy/kinetic energy is 1.5 GeV ;
total energy is $1.5+0.938=2.4(\mathrm{GeV})$ or $3.8 \times 10^{-10}(\mathrm{~J})$ or $3.9 \times 10^{-10}(\mathrm{~J})$;
Award [2] for a bald correct answer.

## Examiners report

In part (a) the KE was usually easily identified and added to the proton rest energy.

## Markscheme

$\backslash\left(p c \backslash l e f t\left(\left\{=\backslash\right.\right.\right.$ sqrt $\left.\left\{\left\{E^{\wedge} 2\right\}-\left\{\left\{\backslash \operatorname{left}\left[\left\{m\left\{c^{\wedge} 2\right\}\right\} \backslash \text { right }\right]\right\}^{\wedge} 2\right\}\right\}\right\} \backslash$ right $)=\backslash$ sqrt $\left\{\left\{\{2.4\}^{\wedge} 2\right\}-\left\{\{\backslash \text { left }[\{0.938\} \backslash \text { right }]\}^{\wedge} 2\right\}\right\} \backslash$ ); (allow ECF from (a)) $\mathrm{p}=2.2$ or $2.3\left(\mathrm{GeVc}^{-1}\right)$ or $1.2 \times 10^{-18}\left(\mathrm{kgms}^{-1}\right)$;
Award [2] for a bald correct answer.

## Examiners report

Part (b): In any question with units expressed in terms of MeV and c there is enormous potential for confusion. However an increasing number of candidates are able use the relativistic energy - momentum equation $\left(E^{2}=\left(\mathrm{mc}^{2}\right)^{2}+p^{2} c^{2}\right)$ correctly as they realise that it becomes a Pythagorean $E^{2}=m^{2}+p^{2}$ when using the simpler units. The commonest mistake was to try to make use of the value of " $c$ " in the calculation instead of just sticking with the values given.

## Markscheme

$\backslash(y=\backslash \operatorname{left}(\{\backslash \operatorname{frac}\{\{2.44\{\backslash \operatorname{rm}\{\mathrm{GeV}\}\}\}\}\{\{0.938\{\backslash \operatorname{rm}\{\mathrm{GeV}\}\}\}\}\}$ \right) $=2.6 \backslash$ );
$\backslash\left(u=\backslash \operatorname{frac}\{p\}\left\{\left\{\backslash\right.\right.\right.$ gamma $\left.\left.\left.\left\{m \_0\right\}\right\}\right\}=\backslash \operatorname{frac}\{\{2.25\}\}\{\{2.6 \backslash \operatorname{times} 0.938\}\}=0.92\{\backslash r m\{c\}\} \backslash\right) ;($ allow ECF from (a) and (b)) Award [2] for a bald correct answer.
or
$\backslash\left(\backslash \operatorname{left}\left(\left\{p=\right.\right.\right.$ Igamma $\left\{m \_0\right\} u=$ Igamma $\left\{m \_0\right\}\left\{c^{\wedge} 2\right\} \backslash f r a c\{u\}\left\{\left\{\left\{c^{\wedge} 2\right\}\right\}\right\} \backslash$ Rightarrow $\} \backslash$ right $) u=\backslash$ frac $\left.\{\{p c\}\}\{E\} c \backslash\right) ;$
$\backslash(=\backslash$ left $(\{\backslash f r a c\{\{2.2\}\}\{\{2.4\}\} c=\} \backslash$ right $) 0.92 c \backslash$ ); (allow ECF from (a) and (b))
Award [2] for a bald correct answer.

## Examiners report

In (c) gamma was frequently found correctly and converted to a speed, but ECF was often necessary.

## Markscheme

there is a force of attraction between (the mass of) the Sun and (the mass of) the planet;
which acts as a central/centripetal force for the planet;

## Examiners report

Rather surprisingly candidates could explain the warping of spacetime and the shortest path followed by a planet but could not explain gravitational force providing centripetal motion.

## Markscheme

(mass of the) Sun curves/warps spacetime around it; planets move in paths of shortest length/geodesic (in this curved/warped spacetime);

## Examiners report

Rather surprisingly candidates could explain the warping of spacetime and the shortest path followed by a planet but could not explain gravitational force providing centripetal motion.
7.

## Markscheme

(i) any straight line with negative slope;
(ii) a downward curve (projectile like);

## Examiners report

In part (a) most candidates drew a projectile path for (ii) but thought that light would travel horizontally for the rocket observer in (i).

8 a.

## Markscheme

a co-ordinate system (in which measurements of distance and time can be made);
which is not accelerating;
in which Newton's laws are valid;

## Examiners report

There were a large variety of answers to (a). Many candidates stated that the frame of reference is not accelerated. Many candidates did not explain the term "frame of reference" in terms of a "co-ordinate system". It was a rare answer that earned more than one mark.

## Markscheme

(i) $\backslash(\backslash \operatorname{left}(\{\backslash \operatorname{frac}\{\{10\}\}\{\{0.90\{\backslash \mathrm{rm}\{\mathrm{c}\}\}\}\}=\}$ lright) $11\{\backslash \mathrm{rm}\{\mathrm{yr}\}\} \backslash)$;
<br>(\left( $\{=3.5$ \times $\{\{10\} \wedge 8\}\{\backslash \mathrm{rm}\{\mathrm{s}\}\}\} \backslash$ right) $)$;
This is a question testing units for this option. Do not award mark for an incorrect or missing unit.
(ii) distance according to spaceship observer $\backslash(=\backslash \operatorname{frac}\{\{10\}\}\{\{2.3\}\} \backslash$ left( $\{=4.3\{\backslash \mathrm{rm}\{\mathrm{ly}\}\}\} \backslash$ right) $)$ );
so time for spaceship $\backslash(=\backslash \operatorname{frac}\{\{4.3\}\}\{\{0.90\}\}=4.8 \backslash \operatorname{left}(\{\{\backslash r m\{y r\}\}\} \backslash r i g h t) \backslash$ );

## Examiners report

In (b)(i), the majority of candidates properly calculated the time. Some wrote the incorrect unit (ly) instead of $y$ or $s$. There is room for improvement in responses to (b)(ii). The vast majority of candidates used the formula for time dilation. They did not notice that it is not normal for the observer on the spaceship to know the time measured on the space station. The correct calculation, length and speed measured, appeared only very rarely.

## Markscheme

between two events occurring at the same point in space / shortest time measured;
so proper time interval measured by observer on spaceship;
Do not award second marking point unless a reason has been attempted.

## Examiners report

There was a good variety of answers to (c). Many candidates still do not know the term proper time interval, clearly defined in relativity. Many incorrectly referred to both events occurring in one frame of reference rather than one point in space in their answer. Most did attempt a reason.

## Markscheme

speed of light is the same for both observers $O$ and $S$ / events simultaneous in stationary reference frame are not (necessarily) simultaneous in moving reference frame;
$S$ is moving so PS will be longer than QS when light reaches S ;
so if light arrives simultaneously then light from $P$ will have been in transit for longer than $Q$;
therefore P emits a flash before Q;

## Examiners report

In (d) many candidates proved that they understood the concept of simultaneity. However, many did not respond to the command term "discuss". Many candidates were confused between object (in a specific frame of reference) and event.

## Markscheme

total energy of proton $=e V+$ rest mass;
$([2500+938] \mathrm{MeV}=) 3438$;
$\backslash\left(\left\{p^{\wedge} 2\right\}\left\{c^{\wedge} 2\right\}=\backslash \operatorname{left}\left(\left\{\left\{E_{-}\{\{\backslash \operatorname{rm}\{\operatorname{tot}\}\}\}\right\}^{\wedge} 2=\left\{m_{-} 0\right\}^{\wedge} 2\left\{c^{\wedge} 4\right\}=\right\} \backslash\right.\right.$ right $\left.)\left\{3438^{\wedge} 2\right\}-\left\{938^{\wedge} 2\right\} \backslash\right) ;$
$\backslash\left(p=3.3 \backslash \operatorname{left}\left(\left\{\{\backslash \mathrm{rm}\{\mathrm{GeV}\}\}\left\{\{\backslash \mathrm{rm}\{\mathrm{c}\}\}^{\wedge}\{-1\}\right\}\right\} \backslash \operatorname{right}\right) \backslash\right)$ or $1.76 \times 10^{-18}\left(\mathrm{kgms}^{-1}\right)$;
NOTE: The question paper stated the units of potential difference in GeV. Watch for answers stating that the unit of potential difference is V, not eV. For such answers without calculation, award [1].
Award [4] for correct use of potential difference (2.5GeV) divided by e.
ie $\mid\left(\mid f r a c\left\{\left\{2.5 \mid\right.\right.\right.$ times $\left\{\{10\}^{\wedge} 9\right\} \mid$ left ( $\{\mathrm{eV}\}$ |right) $\left.\}\right\}\{\{1.6$ |times $\{\{10\} \wedge\{19\}\} \mid / \mathrm{eft}(\{\mid r m\{C\}\} \mid$ right $)\}\}=1.56$ |times $\left\{10^{\wedge}\{28\}\right\} \mid$ left $\left.(V \mid r i g h t) \mid\right)$.

## Examiners report

The units of potential difference were incorrectly stated as GeV in this question. The markscheme was adjusted to ensure no candidate was disadvantaged and all examiners were asked to identify any candidates who appeared to have been thrown by the error. On the whole, candidates had interpreted the question with the correct unit of GV.

Correct answers were given by those who worked in logical manner and who clearly stated that total energy of the proton is the sum of kinetic energy and rest mass energy. The derivation of momentum from the formula was not easy for candidates. Candidates with basic arithmetic and algebra failed here. The more able candidates found the momentum in a clear, straightforward way. Looking through the formulas in data-booklet without understanding is not appropriate here. Many candidates confused kinetic energy with total energy.

## Markscheme

a frame of reference accelerating in outer space is equivalent to a frame of reference at rest in a gravitational field / an inertial frame of reference in outer space is equivalent to a freely falling frame of reference in a (uniform) gravitational field;
Award [0] for only "gravitational and inertial mass are equivalent".

## Examiners report

The principle of equivalence is generally well understood by the candidates. However, the majority of candidates wrote general statements (not wrong) but not in a sufficiently clear sequence. There were many vague statements about gravity and inertia in (a), which was not in response to the question of "state the principle...".

## Markscheme

(i) light source appears to be moving away from the observer;
so there is a red-shift (according to the Doppler effect);
or
spaceship (by equivalence) can be regarded as (at rest) in a gravitational field; photons lose energy in reaching observer (so frequency must be reduced);
(ii) the planet has a gravitational field;
so (by equivalence) the situation is as though light source is near a planet;
$f$ is still observed to be less than $f_{0} /$ period of the light can be taken as unit of time;
this can be interpreted as an increase in the time for emission of one wavelength / increase in the period (ie time is dilated);

## Examiners report

In (b)(ii), some candidates did not realize that the question remained focused on the principle of equivalence. Good answers to this question required a deep understanding of the principle.

## Markscheme

mention of gravitational lensing;
galaxy has a very large mass/gravitational field;
this field/mass bends the direction of light emitted by the quasar;
spacetime is distorted by this field/mass;
Award [1 max] for a diagram showing only a curve joining the quasar and the Earth.
Award [3] for an annotated diagram.

## Examiners report

Many well prepared candidates realized that the light is bent. However, "outline" requires a brief account or summary, so a more detailed answer was required here. Information such as the galaxy has very large mass should be mentioned, at least implicitly.

## Examiners report

[N/A]

## Markscheme

(i) speed of $X$ relative to $Y$ is $\backslash(\backslash \operatorname{left}(\{\backslash f r a c\{\{0.60\{\backslash r m\{c\}\}-\backslash \operatorname{left}(\{-0.60\{\backslash r m\{c\}\}\} \backslash r i g h t)\}\}\{\{1+\{\{0.60\} \wedge 2\}\}\}\}$ (right) $=0.882\{\backslash r m\{c\}\} \backslash$ );
gamma factor at this speed is $\backslash\left(\backslash\right.$ gamma $=\backslash$ frac $\{1\}\left\{\left\{\backslash\right.\right.$ sqrt $\left.\left.\left.\left\{1-\left\{\{0.882\}^{\wedge} 2\right\}\right\}\right\}\right\}=2.12 \backslash\right)$;
momentum is then $p=\gamma m v=2.12 \times 380 \times 0.882 c=710 \mathrm{MeVc}^{-1}$;
Award [3] for a bald correct answer between $700 \mathrm{MeVc}^{-1}$ and $713 \mathrm{MeVc}^{-1}$ due to rounding.
(ii) $\backslash\left(\mathrm{M}\left\{\mathrm{c}^{\wedge} 2\right\}=2 \backslash\right.$ times $\backslash$ gamma $m\left\{c^{\wedge} 2\right\}=2 \backslash$ times $\backslash$ frac $\{5\}\{4\} \backslash$ times $\left.380 \backslash\right)$;
$\Rightarrow M=950 \mathrm{MeVc}^{-2}$;
Award [2] for a bald correct answer.

## Examiners report

[N/A]

13a.

## Markscheme

inertial and gravitational effects are indistinguishable / a freely falling frame in a gravitational field is equivalent to an inertial frame far from all masses / an accelerating frame is equivalent to a frame at rest in a gravitational field;

## Examiners report

[N/A]

## Markscheme

(i) The question does not specifically state the location of the tower so allow any of the explanations below.
(the principle of equivalence predicts) photon energy decreases as it moves against a $g$ field;
this energy is given by $E=h f$;
hence as $E$ decreases, $f$ must also decrease;
or
the tower is equivalent to a frame accelerating upwards;
the top of the tower is moving away from the light emitted from the base;
and so by the Doppler effect/red-shift the frequency at the top will be less;
or
in freely falling tower the frequency at the top and bottom would be the same;
an outside observer sees the top moving towards the light emitted from the base and so (by the Doppler effect) expects a blue-shift;
for the frequency to be the same at the top the light moving upwards must suffer an equal red-shift;
(ii) $8.8 \times 10^{3} \mathrm{~Hz} / 8.9 \times 10^{3} \mathrm{~Hz} / 9.0 \times 10^{3} \mathrm{~Hz}$;
(iii) $\backslash(\backslash$ frac $\{\{\backslash$ Delta $f\}\}\{f\} \backslash \operatorname{left}(\{=\backslash$ frac $\{\{8.8 \backslash$ times $\{\{10\} \wedge 3\}\}\}\{\{3.5 \backslash$ times $\{\{10\} \wedge\{18\}\}\}\}\} \backslash$ right $) \backslash$ approx $\left.\left\{10^{\wedge}\{-15\}\right\} \backslash\right) /$ the shift is very small compared to the original frequency / the new frequency differs from the original in the 15 th decimal place;

## Examiners report

## 14a. <br> Markscheme which nothing can escape; <br> Examiners report <br> [N/A]

the distance from the black hole where the escape speed is the speed of light / the distance from the black hole inside

## 14b. <br> Markscheme <br> Examiners report <br> [N/A]

light travels along - geodesics/paths of shortest length; geodesics/light paths - follow the curvature of spacetime / OWTTE; spacetime is more curved nearer/less curved further from a black hole;
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