ANSWERS

1. (1) 2

   (2) 0

   (3) $\frac{1}{2}$

   (4) -3

   (5) 0

   (6) 3

   (7) -1

   (8) -2

   (9) 2

   (10) $\frac{1}{3}$

   (11) $-\frac{3}{2}$

   (12) -2

2. (1) 0

   (2) 1

   (3) 3

   (4) 3

   (5) $\frac{3}{2}$

   (6) 64

3. (1) $2\log x + \log y$

   (2) $3\log x + 2\log y - \log z$

   (3) $\frac{1}{2}\log x + \frac{2}{3}\log y - 4\log z$

   (4) $\log x + \log y + \log z$

   (5) $\log x - \log y - \log z$

   (6) $2\log x - 2\log y$

   (7) $\frac{1}{3}\log x + \frac{1}{3}\log y$

   (8) $\log x + \frac{1}{2}\log z$

   (9) $\frac{1}{3}(\log x - \log y - \log z)$

   (10) $\frac{1}{4}\log x + \frac{1}{2}\log y - \log z$

   (11) $\frac{5}{4}\log x - \frac{1}{2}\log z$

   (12) $\frac{1}{2}\log x + \log y - 4\log z$

4. (1) $3^4 = 81$

   (2) $7^1 = 7$

   (3) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

   (4) $3^0 = 1$

   (5) $4^{-3} = \frac{1}{64}$

   (6) $6^{-2} = \frac{1}{36}$

   (7) $x^z = y$

   (8) $m^{\frac{1}{2}} = n$

5. (1) $\log_8 64 = 2$

   (2) $\log_{10} 10000 = 3$

   (3) $\log_4 \frac{1}{16} = -2$

   (4) $\log_3 \frac{1}{81} = -4$

   (5) $\log_{\frac{1}{2}} 32 = -5$

   (6) $\log_{\frac{1}{3}} 27 = -3$

   (7) $\log_x y = 2z$

   (8) $\log_x y = \frac{1}{2}$

6. (1) True

   (2) False

   (3) True

   (4) False

   (5) False

   (6) False

   (7) True

   (8) True

7. (1) $S = \{e^{-3}\}$

   (2) $S = \{34\}$

   (3) $S = \{2, 4\}$

   (4) $S = \{5\}$

   (5) $S = \{2\}$

   (6) $S = \{6\}$

   (7) $S = \{5\}$

   (8) $S = \{3\}$

8. (1)

$$\log_{\sqrt{b}} x = 2\log_b x$$

$$\begin{aligned}\log_{\sqrt{b}} x &= \frac{\log x}{\log \sqrt{b}} \\ &= \frac{\log x}{\frac{1}{2}\log b} \\ &= 2\frac{\log x}{\log b} \\ &= 2\log_b x \quad \square\end{aligned}$$

(2)

$$\log_{\frac{1}{\sqrt{b}}} \sqrt{x} = -\log_b x$$

$$\begin{aligned}\log_{\frac{1}{\sqrt{b}}} \sqrt{x} &= \frac{\log \sqrt{x}}{\log \frac{1}{\sqrt{b}}} \\ &= \frac{\frac{1}{2}\log x}{-\frac{1}{2}\log b} \\ &= -\frac{\log x}{\log b} \\ &= -\log_b x \quad \square\end{aligned}$$

(3)

$$\log_{b^4} x^2 = \log_b \sqrt{x}$$

$$\begin{aligned}\log_{b^4} x^2 &= \frac{\log x^2}{\log b^4} \\ &= \frac{2\log x}{4\log b} \\ &= \frac{1}{2}\frac{\log x}{\log b} \\ &= \frac{1}{2}\log_b x \\ &= \log_b \sqrt{x} \quad \square\end{aligned}$$

9. (1) $2x + y$

(2) $x + 2$

(3) $x - y + z$

(4) $y - 1$

(5) $y - x$

(6) $y + z - x$

(7) $1 - x + y$

(8) $x + y - z + 3$

10. (1) $S = \{2.402\}$

(2) $S = \{0.369\}$

(3) $S = \{-7.213\}$

(4) $S = \{0.438\}$

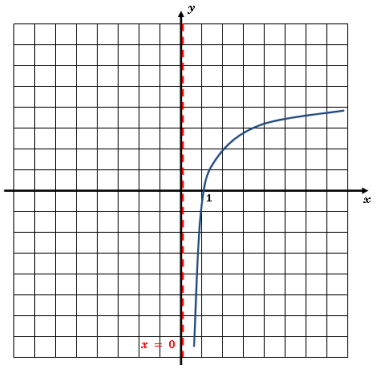
(5) $S = \{-1.652\}$

(6) $S = \{-\ln 4\}$

(7) $S = \{\log_5 4\}$

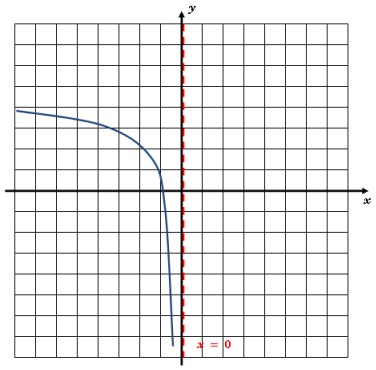
(8) $S = \{\ln 5\}$

11. (1)



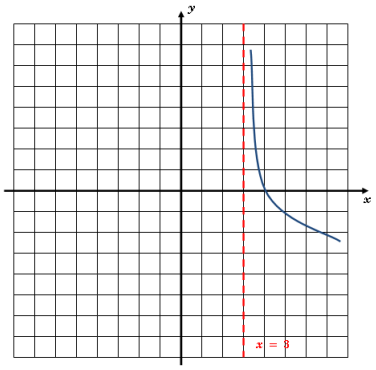
$\mathrm{Dom}(f) =]0, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 1
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in]0, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]0, 1]$
$f(x) \leq 0$ if $x \in [1, +\infty[$

(2)



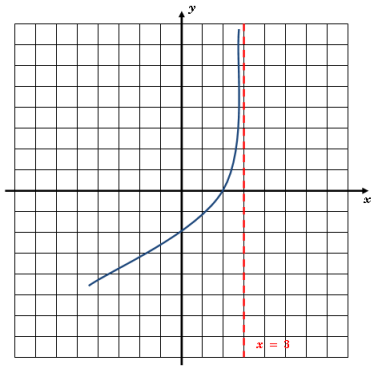
$\mathrm{Dom}(f) =]-\infty, 0[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]-\infty, 0[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]-\infty, -1]$
$f(x) \leq 0$ if $x \in [-1, 0[$

(3)



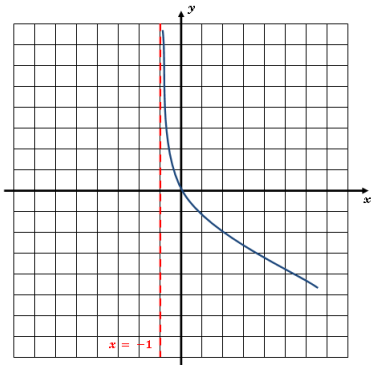
$\mathrm{Dom}(f) =]3, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 4
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]3, +\infty[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]3, 4]$
$f(x) \leq 0$ if $x \in [4, +\infty[$

(4)



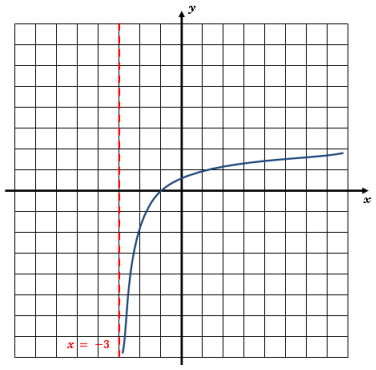
$\mathrm{Dom}(f) =]-\infty, 3[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 2
Y-intercept: -2
Variation:
$f(x) \nearrow$ if $x \in]-\infty, 3[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]2, 3[$
$f(x) \leq 0$ if $x \in]-\infty, 2[$

(5)



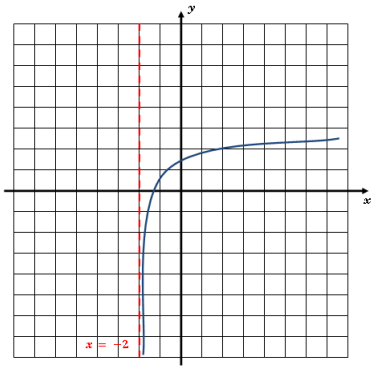
$\mathrm{Dom}(f) =]-1, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 0
Y-intercept: 0
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]-1, +\infty[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]-1, 0[$
$f(x) \leq 0$ if $x \in]0, +\infty[$

(6)



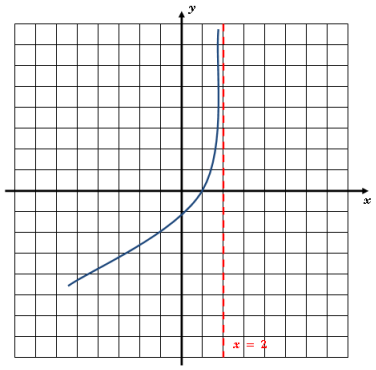
$\mathrm{Dom}(f) =]-3, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1
Y-intercept: $2 \ln \frac{3}{2}$
Variation:
$f(x) \nearrow$ if $x \in]-3, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [-1, +\infty[$
$f(x) \leq 0$ if $x \in]-3, -1]$

(7)



$\mathrm{Dom}(f) =]-2, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1.5
Y-intercept: $\ln 4$
Variation:
$f(x) \nearrow$ if $x \in]-2, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [-1.5, +\infty[$
$f(x) \leq 0$ if $x \in]-2, -1.5]$

(8)



$\mathrm{Dom}(f) =]-\infty, 2[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: $\dfrac{5}{3}$
Y-intercept: $-2\ln 6$
Variation:
$f(x) \nearrow$ if $x \in]-\infty, 2[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [\frac{5}{3}, 2[$
$f(x) \leq 0$ if $x \in]-\infty, \frac{5}{3}[$

12. (1) $f^{-1}(x) = 2^{x+5} + 3$

    (2) $f^{-1}(x) = 3^{\frac{x-1}{3}} - 3$

    (3) $f^{-1}(x) = \frac{1}{2}10^{\frac{2-x}{2}} + 1$

    (4) $f^{-1}(x) = -\frac{1}{2}e^{1-x} + \frac{1}{2}$

    (5) $f^{-1}(x) = \log_2(x+3)$

    (6) $f^{-1}(x) = \frac{1}{3}\log_3\left(\frac{x+1}{2}\right)$

    (7) $f^{-1}(x) = -\ln\left(\frac{2-x}{5}\right)$

    (8) $f^{-1}(x) = -\frac{1}{2}\ln\left(\frac{1-x}{2}\right)$

13. 37 years.

14. 9 years.

15. 53 years old.

16. (a) $f(t) = 10000 \cdot 2^{1.5t}$. Where t is the number of hours.

    (b) 28 284 bacteria.

    (c) 92.88 minutes.