SL Paper 1

a. Find $\log_2 32$. [1]

b. Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px + qy$, find the value of p and of q. [4]

Markscheme

a. 5 ***A1*** ***N1***

[1 mark]

b. **METHOD 1**

$\log_2\left(\frac{32^x}{8^y}\right) = \log_2 32^x - \log_2 8^y$ ***(A1)***

$= x\log_2 32 - y\log_2 8$ ***(A1)***

$\log_2 8 = 3$ ***(A1)***

$p = 5$, $q = -3$ (accept $5x - 3y$) ***A1*** ***N3***

METHOD 2

$\frac{32^x}{8^y} = \frac{(2^5)^x}{(2^3)^y}$ ***(A1)***

$= \frac{2^{5x}}{2^{3y}}$ ***(A1)***

$= 2^{5x-3y}$ ***(A1)***

$\log_2(2^{5x-3y}) = 5x - 3y$

$p = 5$, $q = -3$ (accept $5x - 3y$) ***A1*** ***N3***

[4 marks]

Examiners report

a. Part (a) proved very accessible.

b. Although many found part (b) accessible as well, a good number of candidates could not complete their way to a final result. Many gave q as a positive value.

Find the value of each of the following, giving your answer as an integer.

a. $\log_6 36$ [2]

b. $\log_6 4 + \log_6 9$ [2]

c. $\log_6 2 - \log_6 12$ [3]

Markscheme

a. correct approach ***(A1)***

eg $6^x = 36,\ 6^2$

2 ***A1 N2***

[2 marks]

b. correct simplification ***(A1)***

eg $\log_6 36,\ \log(4 \times 9)$

2 ***A1 N2***

[2 marks]

c. correct simplification ***(A1)***

eg $\log_6 \frac{2}{12},\ \log(2 \div 12)$

correct working ***(A1)***

eg $\log_6 \frac{1}{6},\ 6^{-1} = \frac{1}{6}, 6^x = \frac{1}{6}$

-1 ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

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a. Expand $(2+x)^4$ and simplify your result. [3]

b. Hence, find the term in x^2 in $(2+x)^4\left(1+\frac{1}{x^2}\right)$. [3]

Markscheme

a. evidence of expanding ***M1***

e.g. $2^4 + 4(2^3)x + 6(2^2)x^2 + 4(2)x^3 + x^4$, $(4+4x+x^2)(4+4x+x^2)$

$(2+x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ ***A2 N2***

[3 marks]

b. finding coefficients 24 and 1 ***(A1)(A1)***

term is $25x^2$ ***A1 N3***

[3 marks]

Examiners report

a. Surprisingly few candidates employed the binomial theorem, choosing instead to expand by repeated use of the distributive property. This earned full marks if done correctly, but often proved prone to error.

b. Candidates often expanded the entire expression in part (b). Few recognized that only two distributions are required to answer the question. Some gave the coefficient as the final answer.

An arithmetic sequence has the first term $\ln a$ and a common difference $\ln 3$.

The 13th term in the sequence is $8\ln 9$. Find the value of a.

Markscheme

Note: There are many approaches to this question, and the steps may be done in any order. There are 3 relationships they may need to apply at some stage, for the 3rd, 4th and 5th marks. These are

equating bases *eg* recognising 9 is 3^2

log rules: $\ln b + \ln c = \ln(bc),\ \ln b - \ln c = \ln\left(\frac{b}{c}\right),$

exponent rule: $\ln b^n = n\ln b$.

The exception to the ***FT*** rule applies here, so that if they demonstrate correct application of the 3 relationships, they may be awarded the ***A*** marks, even if they have made a previous error. However all applications of a relationship need to be correct. Once an error has been made, do not award ***A1FT*** for their final answer, even if it follows from their working.

Please check working and award marks in line with the markscheme.

correct substitution into u_{13} formula ***(A1)***

eg $\ln a + (13-1)\ln 3$

set up equation for u_{13} in any form (seen anywhere) ***(M1)***

eg $\ln a + 12\ln 3 = 8\ln 9$

correct application of relationships ***(A1)(A1)(A1)***

$a = 81$ ***A1*** ***N3***

[6 marks]

Examples of application of relationships

Example 1

correct application of exponent rule for logs ***(A1)***

eg $\ln a + \ln 3^{12} = \ln 9^8$

correct application of addition rule for logs ***(A1)***

eg $\ln(a3^{12}) = \ln 9^8$

substituting for 9 or 3 in ln expression in equation ***(A1)***

eg $\ln(a3^{12}) = \ln 3^{16},\ \ln(a9^6) = \ln 9^8$

Example 2

recognising $9 = 3^2$ ***(A1)***

eg $\ln a + 12\ln 3 = 8\ln 3^2,\ \ln a + 12\ln 9^{\frac{1}{2}} = 8\ln 9$

one correct application of exponent rule for logs relating $\ln 9$ to $\ln 3$ ***(A1)***

eg $\ln a + 12 \ln 3 = 16 \ln 3,\ \ln a + 6 \ln 9 = 8 \ln 9$

another correct application of exponent rule for logs ***(A1)***

eg $\ln a = \ln 3^4,\ \ln a = \ln 9^2$

Examiners report

[N/A]

Consider the following sequence of figures.

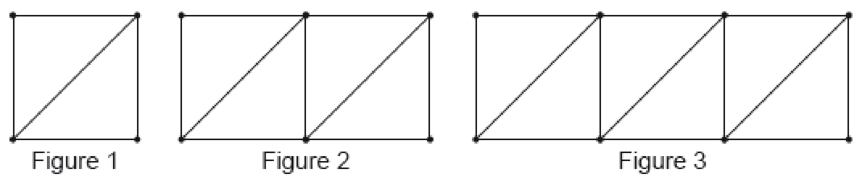

Figure 1 contains 5 line segments.

a. Given that Figure n contains 801 line segments, show that $n = 200$. [3]

b. Find the total number of line segments in the first 200 figures. [3]

Markscheme

a. recognizing that it is an arithmetic sequence ***(M1)***

eg $5,\ 5+4,\ 5+4+4,\ \ldots,\ d=4,\ u_n = u_1 + (n-1)d,\ 4n+1$

correct equation ***A1***

eg $5 + 4(n-1) = 801$

correct working (do not accept substituting $n = 200$) ***A1***

eg $4n - 4 = 796,\ n - 1 = \frac{796}{4}$

$n = 200$ ***AG*** ***N0***

[3 marks]

b. recognition of sum ***(M1)***

eg $S_{200},\ u_1 + u_2 + \ldots + u_{200},\ 5 + 9 + 13 + \ldots + 801$

correct working for AP ***(A1)***

eg $\frac{200}{2}(5 + 801),\ \frac{200}{2}\ (2(5) + 199(4))$

$80\,600$ ***A1*** ***N2***

[3 marks]

Examiners report

a. Most candidates recognized that the series was arithmetic but many worked backwards using $n = 200$ rather than creating and solving an equation of their own to produce the given answer.

b. Almost all students answered (b) correctly.

Let $f(x) = k\log_2 x$.

a. Given that $f^{-1}(1) = 8$, find the value of k . [3]

b. Find $f^{-1}\left(\frac{2}{3}\right)$. [4]

Markscheme

a. **METHOD 1**

recognizing that $f(8) = 1$ ***(M1)***

e.g. $1 = k\log_2 8$

recognizing that $\log_2 8 = 3$ ***(A1)***

e.g. $1 = 3k$

$k = \frac{1}{3}$ ***A1 N2***

METHOD 2

attempt to find the inverse of $f(x) = k\log_2 x$ ***(M1)***

e.g. $x = k\log_2 y$, $y = 2^{\frac{x}{k}}$

substituting 1 and 8 ***(M1)***

e.g. $1 = k\log_2 8$, $2^{\frac{1}{k}} = 8$

$k = \frac{1}{\log_2 8}$ $\left(k = \frac{1}{3}\right)$ ***A1 N2***

[3 marks]

b. **METHOD 1**

recognizing that $f(x) = \frac{2}{3}$ ***(M1)***

e.g. $\frac{2}{3} = \frac{1}{3}\log_2 x$

$\log_2 x = 2$ ***(A1)***

$f^{-1}\left(\frac{2}{3}\right) = 4$ (accept $x = 4$) ***A2 N3***

METHOD 2

attempt to find inverse of $f(x) = \frac{1}{3}\log_2 x$ ***(M1)***

e.g. interchanging x and y , substituting $k = \frac{1}{3}$ into $y = 2^{\frac{x}{k}}$

correct inverse ***(A1)***

e.g. $f^{-1}(x) = 2^{3x}$, 2^{3x}

$f^{-1}\left(\frac{2}{3}\right) = 4$ ***A2 N3***

[4 marks]

Examiners report

a. A very poorly done question. Most candidates attempted to find the inverse function for f and used that to answer parts (a) and (b). Few recognized that the explicit inverse function was not necessary to answer the question.

Although many candidates seem to know that they can find an inverse function by interchanging x and y, very few were able to actually get the correct inverse. Almost none recognized that if $f^{-1}(1) = 8$, then $f(8) = 1$. Many thought that the letters "log" could be simply "cancelled out", leaving the 2 and the 8.

b. A very poorly done question. Most candidates attempted to find the inverse function for f and used that to answer parts (a) and (b). Few recognized that the explicit inverse function was not necessary to answer the question.

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In an arithmetic sequence, the first term is 3 and the second term is 7.

a. Find the common difference. [2]

b. Find the tenth term. [2]

c. Find the sum of the first ten terms of the sequence. [2]

Markscheme

a. attempt to subtract terms ***(M1)***

eg $d = u_2 - u_1,\ 7 - 3$

$d = 4$ ***A1*** ***N2***

[2 marks]

b. correct approach ***(A1)***

eg $u_{10} = 3 + 9(4)$

$u_{10} = 39$ ***A1*** ***N2***

[2 marks]

c. correct substitution into sum ***(A1)***

eg $S_{10} = 5(3 + 39),\ S_{10} = \frac{10}{2}(2 \times 3 + 9 \times 4)$

$S_{10} = 210$ ***A1*** ***N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
[N/A]

c.

The first three terms of a infinite geometric sequence are $m-1,\ 6,\ m+4$, where $m \in \mathbb{Z}$.

a(i).Write down an expression for the common ratio, r. [2]

a(ii)Hence, show that m satisfies the equation $m^2+3m-40=0$. [2]

b(i).Find the two possible values of m. [3]

b(ii)Find the possible values of r. [3]

c(i).The sequence has a finite sum. [3]

State which value of r leads to this sum **and** justify your answer.

c(ii)The sequence has a finite sum. [3]

Calculate the sum of the sequence.

Markscheme

a(i).correct expression for r ***A1*** ***N1***

eg $r = \frac{6}{m-1},\ \frac{m+4}{6}$

[2 marks]

a(ii)correct equation ***A1***

eg $\frac{6}{m-1} = \frac{m+4}{6},\ \frac{6}{m+4} = \frac{m-1}{6}$

correct working ***(A1)***

eg $(m+4)(m-1) = 36$

correct working ***A1***

eg $m^2 - m + 4m - 4 = 36,\ m^2 + 3m - 4 = 36$

$m^2 + 3m - 40 = 0$ ***AG*** ***N0***

[2 marks]

b(i).valid attempt to solve ***(M1)***

eg $(m+8)(m-5) = 0,\ m = \frac{-3 \pm \sqrt{9+4\times 40}}{2}$

$m = -8,\ m = 5$ ***A1A1*** ***N3***

[3 marks]

b(ii)attempt to substitute **any** value of m to find r ***(M1)***

eg $\frac{6}{-8-1},\ \frac{5+4}{6}$

$r = \frac{3}{2},\ r = -\frac{2}{3}$ ***A1A1*** ***N3***

[3 marks]

c(i).$r = -\frac{2}{3}$ (may be seen in justification) ***A1***

valid reason ***R1*** ***N0***

eg $|r| < 1,\ -1 < \frac{-2}{3} < 1$

Notes: Award ***R1*** for $|r| < 1$ only if ***A1*** awarded.

[2 marks]

c(ii)finding the first term of the sequence which has $|r| < 1$ ***(A1)***

eg $-8-1,\ 6 \div \frac{-2}{3}$

$u_1 = -9$ (may be seen in formula) ***(A1)***

correct substitution of u_1 and their r into $\frac{u_1}{1-r}$, as long as $|r| < 1$ ***A1***

eg $S_\infty = \frac{-9}{1-\left(-\frac{2}{3}\right)}, \frac{-9}{\frac{5}{3}}$

$S_\infty = -\frac{27}{5}$ $(= -5.4)$ ***A1 N3***

[4 marks]

Examiners report

a(i).[N/A]
a(ii).[N/A]
b(i).[N/A]
b(ii).[N/A]
c(i).[N/A]
c(ii).[N/A]

In an arithmetic sequence, the first term is 2 and the second term is 5.

a. Find the common difference. [2]

b. Find the eighth term. [2]

c. Find the sum of the first eight terms of the sequence. [2]

Markscheme

a. correct approach ***(A1)***

eg $d = u_2 - u_1,\ 5 - 2$

$d = 3$ ***A1 N2***

[2 marks]

b. correct approach ***(A1)***

eg $u_8 = 2 + 7 \times 3$, listing terms

$u_8 = 23$ ***A1 N2***

[2 marks]

c. correct approach ***(A1)***

eg $S_8 = \frac{8}{2}(2 + 23)$, listing terms, $\frac{8}{2}(2(2) + 7(3))$

$S_8 = 100$ ***A1 N2***

[2 marks]

Total [6 marks]

Examiners report

a. All three parts of this question were very well done by the candidates. The occasional mistakes that were seen tended to be arithmetic errors which happened after the candidates had substituted correctly into the formulas given in the formula booklet.

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a. Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. [3]

b. Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$. [3]

Markscheme

a. correct application of $\ln a^b = b\ln a$ (seen anywhere) ***(A1)***

eg $\ln 4 = 2\ln 2,\ 3\ln 2 = \ln 2^3,\ 3\log 2 = \log 8$

correct working ***(A1)***

eg $3\ln 2 - 2\ln 2,\ \ln 8 - \ln 4$

$\ln 2 \quad (\text{accept } k = 2)$ ***A1 N2***

[3 marks]

b. **METHOD 1**

attempt to substitute **their** answer into the equation ***(M1)***

eg $\ln 2 = -\ln x$

correct application of a log rule ***(A1)***

eg $\ln \frac{1}{x},\ \ln \frac{1}{2} = \ln x,\ \ln 2 + \ln x = \ln 2x \quad (= 0)$

$x = \frac{1}{2}$ ***A1 N2***

METHOD 2

attempt to rearrange equation, with $3\ln 2$ written as $\ln 2^3$ or $\ln 8$ ***(M1)***

eg $\ln x = \ln 4 - \ln 2^3,\ \ln 8 + \ln x = \ln 4,\ \ln 2^3 = \ln 4 - \ln x$

correct working applying $\ln a \pm \ln b$ ***(A1)***

eg $\frac{4}{8},\ 8x = 4,\ \ln 2^3 = \ln \frac{4}{x}$

$x = \frac{1}{2}$ ***A1 N2***

[3 marks]

Total [6 marks]

Examiners report

a. Part (a) was answered correctly by a large number of candidates, though there were quite a few who applied the rules of logarithms in the wrong order.

b. In part (b), many candidates knew to set their answer from part (a) equal to $-\ln x$, but then a good number incorrectly said that $\ln 2 = -\ln x$ led to $2 = -x$.

The sums of the terms of a sequence follow the pattern

$S_1 = 1 + k,\ S_2 = 5 + 3k,\ S_3 = 12 + 7k,\ S_4 = 22 + 15k,\ \ldots,$ where $k \in \mathbb{Z}$.

a. Given that $u_1 = 1 + k$, find $u_2,\ u_3$ and u_4. [4]

b. Find a general expression for u_n. [4]

Markscheme

a. valid method ***(M1)***

eg $u_2 = S_2 - S_1,\ 1 + k + u_2 = 5 + 3k$

$u_2 = 4 + 2k,\ u_3 = 7 + 4k,\ u_4 = 10 + 8k$ ***A1A1A1 N4***

[4 marks]

b. correct AP **or** GP ***(A1)***

eg finding common difference is 3, common ratio is 2

valid approach using arithmetic **and** geometric formulas ***(M1)***

eg $1 + 3(n-1)$ **and** $r^{n-1}k$

$u_n = 3n - 2 + 2^{n-1}k$ ***A1A1 N4***

Note: Award ***A1*** for $3n - 2$, ***A1*** for $2^{n-1}k$.

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

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a. Consider the arithmetic sequence $2,\ 5,\ 8,\ 11,\ \ldots$. [3]

Find u_{101} .

b. Consider the arithmetic sequence $2,\ 5,\ 8,\ 11,\ \ldots$. [3]

Find the value of *n* so that $u_n = 152$.

Markscheme

a. $d = 3$ ***(A1)***

evidence of substitution into $u_n = a + (n-1)d$ ***(M1)***

e.g. $u_{101} = 2 + 100 \times 3$

$u_{101} = 302$ ***A1 N3***

[3 marks]

b. correct approach ***(M1)***

e.g. $152 = 2 + (n-1) \times 3$

correct simplification ***(A1)***

e.g. $150 = (n-1) \times 3$, $50 = n - 1$, $152 = -1 + 3n$

$n = 51$ ***A1 N2***

[3 marks]

Examiners report

a. Candidates probably had the most success with this question with many good solutions which were written with the working clearly shown.

Many used the alternate approach of $u_n = 3n - 1$.

b. Candidates probably had the most success with this question with many good solutions which were written with the working clearly shown.

Many used the alternate approach of $u_n = 3n - 1$.

The first two terms of an infinite geometric sequence, in order, are

$$2\log_2 x,\ \log_2 x, \text{ where } x > 0.$$

The first three terms of an arithmetic sequence, in order, are

$$\log_2 x,\ \log_2\left(\frac{x}{2}\right),\ \log_2\left(\frac{x}{4}\right), \text{ where } x > 0.$$

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

a. Find r. [2]

b. Show that the sum of the infinite sequence is $4\log_2 x$. [2]

c. Find d, giving your answer as an integer. [4]

d. Show that $S_{12} = 12\log_2 x - 66$. [2]

e. Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$. [3]

Markscheme

a. evidence of dividing terms (in any order) ***(M1)***

eg $\frac{\mu_2}{\mu_1}, \frac{2\log_2 x}{\log_2 x}$

$r = \frac{1}{2}$ ***A1 N2***

[2 marks]

b. correct substitution ***(A1)***

eg $\frac{2\log_2 x}{1-\frac{1}{2}}$

correct working ***A1***

eg $\frac{2\log_2 x}{\frac{1}{2}}$

$S_\infty = 4\log_2 x$ ***AG N0***

[2 marks]

c. evidence of subtracting two terms (in any order) ***(M1)***

eg $u_3 - u_2,\ \log_2 x - \log_2 \frac{x}{2}$

correct application of the properties of logs ***(A1)***

eg $\log_2\left(\frac{\frac{x}{2}}{x}\right),\ \log_2\left(\frac{x}{2} \times \frac{1}{x}\right),\ (\log_2 x - \log_2 2) - \log_2 x$

correct working ***(A1)***

eg $\log_2 \frac{1}{2},\ -\log_2 2$

$d = -1$ ***A1 N3***

[4 marks]

d. correct substitution into the formula for the sum of an arithmetic sequence ***(A1)***

eg $\frac{12}{2}(2\log_2 x + (12-1)(-1))$

correct working ***A1***

eg $6(2\log_2 x - 11),\ \frac{12}{2}(2\log_2 x - 11)$

$12\log_2 x - 66$ ***AG N0***

[2 marks]

e. correct equation ***(A1)***

eg $12\log_2 x - 66 = 2\log_2 x$

correct working ***(A1)***

eg $10\log_2 x = 66,\ \log_2 x = 6.6,\ 2^{66} = x^{10},\ \log_2\left(\frac{x^{12}}{x^2}\right) = 66$

$x = 2^{6.6}$ (accept $p = \frac{66}{10}$) ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
[N/A]

e.

The fifth term in the expansion of the binomial $(a+b)^n$ is given by $\binom{10}{4} p^6(2q)^4$.

a. Write down the value of n. [1]

b. Write down a and b, in terms of p and/or q. [2]

c. Write down an expression for the sixth term in the expansion. [3]

Markscheme

a. $n=10$ ***A1 N1***

[1 mark]

b. $a=p$, $b=2q$ (or $a=2q$, $b=p$) ***A1A1 N1N1***

[2 marks]

c. $\binom{10}{5} p^5(2q)^5$ ***A1A1A1 N3***

[3 marks]

Examiners report

a. [N/A]

b. [N/A]

c. The most common error was in (c) where many candidates interpreted the "sixth" term as using $\binom{10}{6}$, with accompanying powers of 4 and 6 in the expression.

An arithmetic sequence has $u_1=\log_c(p)$ and $u_2=\log_c(pq)$, where $c>1$ and $p,\ q>0$.

a. Show that $d=\log_c(q)$. [2]

b. Let $p=c^2$ and $q=c^3$. Find the value of $\sum_{n=1}^{20} u_n$. [6]

Markscheme

a. valid approach involving addition or subtraction ***M1***

eg $u_2=\log_c p+d,\ u_1-u_2$

correct application of log law ***A1***

eg $\log_c(pq)=\log_c p+\log_c q,\ \log_c\left(\frac{pq}{p}\right)$

$d = \log_c q$ ***AG N0***

[2 marks]

b. **METHOD 1** (finding u_1 and *d*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

attempt to find u_1 or *d* using $\log_c c^k = k$ ***(M1)***
eg $\log_c c$, $3\log_c c$, correct value of u_1 or *d*

u_1 = 2, *d* = 3 (seen anywhere) ***(A1)(A1)***

correct working ***(A1)***
eg $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)\,,\ S_{20} = \frac{20}{2}(2 + 59)\,,\ 10\,(61)$

$\sum\limits_{n=1}^{20} u_n$ = 610 ***A1 N2***

METHOD 2 (expressing *S* in terms of *c*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for *S* in terms of *c* ***(A1)***
eg $10\left(2\log_c c^2 + 19\log_c c^3\right)$

$\log_c c^2 = 2,\ \ \log_c c^3 = 3$ (seen anywhere) ***(A1)(A1)***

correct working ***(A1)***

eg $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)\,,\ S_{20} = \frac{20}{2}(2 + 59)\,,\ 10\,(61)$

$\sum\limits_{n=1}^{20} u_n$ = 610 ***A1 N2***

METHOD 3 (expressing *S* in terms of *c*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for *S* in terms of *c* ***(A1)***
eg $10\left(2\log_c c^2 + 19\log_c c^3\right)$

correct application of log law ***(A1)***
eg $2\log_c c^2 = \ \log_c c^4,\ 19\log_c c^3 = \ \log_c c^{57},\ 10\ \left(\log_c \left(c^2\right)^2 + \ \log_c \left(c^3\right)^{19}\right),\ 10\ \left(\log_c c^4 + \log_c c^{57}\right),\ 10\left(\log_c c^{61}\right)$

correct application of definition of log ***(A1)***
eg $\log_c c^{61} = 61,\ \log_c c^4 = 4,\ \log_c c^{57} = 57$

correct working ***(A1)***
eg $S_{20} = \frac{20}{2}(4 + 57)\,,\ 10\,(61)$

$\sum\limits_{n=1}^{20} u_n$ = 610 ***A1 N2***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]

In an arithmetic sequence, the third term is 10 and the fifth term is 16.

a. Find the common difference. [2]

b. Find the first term. [2]

c. Find the sum of the first 20 terms of the sequence. [3]

Markscheme

a. attempt to find d ***(M1)***

eg $\frac{16-10}{2}$, $10-2d=16-4d$, $2d=6$, $d=6$

$d=3$ ***A1*** ***N2***

[2 marks]

b. correct approach ***(A1)***

eg $10=u_1+2\times3$, $10-3-3$

$u_1=4$ ***A1*** ***N2***

[2 marks]

c. correct substitution into sum or term formula ***(A1)***

eg $\frac{20}{2}(2\times4+19\times3)$, $u_{20}=4+19\times3$

correct simplification ***(A1)***

eg $8+57$, $4+61$

$S_{20}=650$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

---

a. Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \ldots$. [1]

Write down the 10th term of the sequence. Do not simplify your answer.

b. Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \ldots$. [4]

Find the sum of the infinite sequence.

Markscheme

a. $u_{10}=3(0.9)^9$ ***A1*** ***N1***

[1 mark]

b. recognizing $r=0.9$ ***(A1)***

correct substitution ***A1***

e.g. $S=\frac{3}{1-0.9}$

$S = \frac{3}{0.1}$ ***(A1)***

$S = 30$ ***A1*** ***N3***

[4 marks]

Examiners report

a. This question was well done by most candidates. There were a surprising number of candidates who lost a mark for not simplifying $\frac{3}{0.1}$ to 30 , and there were a few candidates who used the formula for the finite sum unsuccessfully.

b. This question was well done by most candidates. There were a surprising number of candidates who lost a mark for not simplifying $\frac{3}{0.1}$ to 30, and there were a few candidates who used the formula for the finite sum unsuccessfully.

Let $f(x) = log_3\sqrt{x}$, for $x > 0$.

a. Show that $f^{-1}(x) = 3^{2x}$. [2]

b. Write down the range of f^{-1} . [1]

c. Let $g(x) = \log_3 x$, for $x > 0$. [4]

Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

Markscheme

a. interchanging *x* and *y* (seen anywhere) ***(M1)***

e.g. $x = \log\sqrt{y}$ (accept any base)

evidence of correct manipulation ***A1***

e.g. $3^x = \sqrt{y}$, $3^y = x^{\frac{1}{2}}$, $x = \frac{1}{2}\log_3 y$, $2y = \log_3 x$

$f^{-1}(x) = 3^{2x}$ ***AG*** ***N0***

[2 marks]

b. $y > 0$, $f^{-1}(x) > 0$ ***A1*** ***N1***

[1 mark]

c. **METHOD 1**

finding $g(2) = log_3 2$ (seen anywhere) ***A1***

attempt to substitute ***(M1)***

e.g. $(f^{-1} \circ g)(2) = 3^{2\log_3 2}$

evidence of using log or index rule ***(A1)***

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 4}$, $3^{\log_3 2^2}$

$(f^{-1} \circ g)(2) = 4$ ***A1*** ***N1***

METHOD 2

attempt to form composite (in any order) ***(M1)***

e.g. $(f^{-1} \circ g)(x) = 3^{2\log_3 x}$

evidence of using log or index rule ***(A1)***

e.g. $(f^{-1} \circ g)(x) = 3^{\log_3 x^2}$, $3^{\log_3 x^2}$

$(f^{-1} \circ g)(x) = x^2$ ***A1***

$(f^{-1} \circ g)(2) = 4$ ***A1*** ***N1***

[4 marks]

Examiners report

a. Candidates were generally skilled at finding the inverse of a logarithmic function.

b. Few correctly gave the range of this function, often stating “all real numbers” or “ $y \geq 0$ ”, missing the idea that the range of an inverse is the domain of the original function.

c. Some candidates answered part (c) correctly, although many did not get beyond $3^{2\log_3 2}$. Some attempted to form the composite in the incorrect order. Others interpreted $(f^{-1} \circ g)(2)$ as multiplication by 2.

The first three terms of an infinite geometric sequence are 32, 16 and 8.

a. Write down the value of r . [1]

b. Find u_6 . [2]

c. Find the sum to infinity of this sequence. [2]

Markscheme

a. $r = \frac{16}{32} \left(= \frac{1}{2}\right)$ ***A1*** ***N1***

[1 mark]

b. correct calculation or listing terms ***(A1)***

e.g. $32 \times \left(\frac{1}{2}\right)^{6-1}$, $8 \times \left(\frac{1}{2}\right)^{3}$, $32, \ldots 4, 2, 1$

$u_6 = 1$ ***A1*** ***N2***

[2 marks]

c. evidence of correct substitution in S_∞ ***A1***

e.g. $\frac{32}{1-\frac{1}{2}}$, $\frac{32}{\frac{1}{2}}$

$S_\infty = 64$ ***A1*** ***N1***

[2 marks]

Examiners report

a. This question was very well done by the majority of candidates. There were some who used a value of r greater than one, with the most common error being $r = 2$.

b. This question was very well done by the majority of candidates. There were some who used a value of r greater than one, with the most common error being $r = 2$.

c. A handful of candidates struggled with the basic computation involved in part (c).

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2 \theta$, where $0 < \theta < 2\pi$, and $\theta \ne \pi$.

a.i. Find an expression for r in terms of θ. [2]

a.ii. Find the possible values of r. [3]

b. Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. [4]

c. Find the values of θ which give the greatest value of the sum. [6]

Markscheme

a.i. valid approach ***(M1)***

eg $\frac{u_2}{u_1}, \frac{u_1}{u_2}$

$r = \frac{12\sin^2\theta}{18}\left(= \frac{2\sin^2\theta}{3}\right)$ ***A1 N2***

[2 marks]

a.ii. recognizing that $\sin\theta$ is bounded ***(M1)***

eg $0 \le \sin^2\theta \le 1$, $-1 \le \sin\theta \le 1$, $-1 < \sin\theta < 1$

$0 < r \le \frac{2}{3}$ ***A2 N3***

Note: If working shown, award ***M1A1*** for correct values with incorrect inequality sign(s).
If no working shown, award ***N1*** for correct values with incorrect inequality sign(s).

[3 marks]

b. correct substitution into formula for infinite sum ***A1***

eg $\frac{18}{1-\frac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for $\cos 2\theta$ (seen anywhere) ***(M1)***

eg $\cos 2\theta = 1 - 2\sin^2\theta$

correct substitution of identity/working (seen anywhere) ***(A1)***

eg $\frac{18}{1-\frac{2}{3}\left(\frac{1-\cos 2\theta}{2}\right)}, \frac{54}{3-2\left(\frac{1-\cos 2\theta}{2}\right)}, \frac{18}{\frac{3-2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer ***A1*** *eg* $\frac{18\times3}{2+(1-2\sin^2\theta)}$, $\frac{54}{3-(1-\cos 2\theta)}$

$\frac{54}{2+\cos(2\theta)}$ ***AG N0***

[4 marks]

c.

METHOD 1 (using differentiation)

recognizing $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}=0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}$ ***(A1)***

eg $\frac{0-54\times(-2\sin 2\theta)}{(2+\cos 2\theta)^2}$, $-54(2+\cos 2\theta)^{-2}\ (-2\sin 2\theta)$

correct working ***(A1)***

eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***

eg $0, \pi, \ldots$, sketch of sine curve with *x*-intercept(s) marked both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta = \pi, 3\pi$ (accept values in degrees)

both correct answers $\theta=\frac{\pi}{2}, \frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

METHOD 2 (using denominator)

recognizing when S_∞ is greatest ***(M1)***

eg $2+\cos 2\theta$ is a minimum, $1-r$ is smallest
correct working ***(A1)***

eg minimum value of $2+\cos 2\theta$ is 1, minimum $r=\frac{2}{3}$

correct working ***(A1)***

eg $\cos 2\theta=-1$, $\frac{2}{3}\sin^2\theta=\frac{2}{3}$, $\sin^2\theta=1$

EITHER (using $\cos 2\theta$)

any correct value for $\cos^{-1}(-1)$ (seen anywhere) ***(A1)***

eg $\pi, 3\pi, \ldots$ (accept values in degrees), sketch of cosine curve with *x*-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta = \pi, 3\pi$ (accept values in degrees)

OR (using $\sin\theta$)

$\sin\theta = \pm1$ (A1)

$\sin^{-1}(1)=\frac{\pi}{2}$ (accept values in degrees) (seen anywhere) ***A1***

THEN

both correct answers $\theta=\frac{\pi}{2}, \frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

[6 marks]

Examiners report

[N/A]

a.ii. [N/A]
b. [N/A]
c. [N/A]

Three consecutive terms of a geometric sequence are $x-3$, 6 and $x+2$.

Find the possible values of x.

Markscheme

METHOD 1

valid approach ***(M1)***

eg $r=\frac{6}{x-3},\ (x-3)\times r=6,\ (x-3)r^2=x+2$

correct equation in terms of x only ***A1***

eg $\frac{6}{x-3}=\frac{x+2}{6},\ (x-3)(x+2)=6^2,\ 36=x^2-x-6$

correct working ***(A1)***

eg $x^2-x-42,\ x^2-x=42$

valid attempt to solve **their** quadratic equation ***(M1)***

eg factorizing, formula, completing the square

evidence of correct working ***(A1)***

eg $(x-7)(x+6),\ \frac{1\pm\sqrt{169}}{2}$

$x=7,\ x=-6$ ***A1*** ***N4***

METHOD 2 (finding *r* first)

valid approach ***(M1)***

eg $r=\frac{6}{x-3},\ 6r=x+2,\ (x-3)r^2=x+2$

correct equation in terms of r only ***A1***

eg $\frac{6}{r}+3=6r-2,\ 6+3r=6r^2-2r,\ 6r^2-5r-6=0$

evidence of correct working ***(A1)***

eg $(3r+2)(2r-3),\ \frac{5\pm\sqrt{25+144}}{12}$

$r=-\frac{2}{3},\ r=\frac{3}{2}$ ***A1***

substituting their values of r to find x ***(M1)***

eg $(x-3)\left(\frac{2}{3}\right)=6,\ x=6\left(\frac{3}{2}\right)-2$

$x=7,\ x=-6$ ***A1*** ***N4***

[6 marks]

Examiners report

Nearly all candidates attempted to set up an expression, or pair of expressions, for the common ratio of the geometric sequence. When done correctly, these expressions led to a quadratic equation which was solved correctly by many candidates.

The values in the fourth row of Pascal's triangle are shown in the following table.

1	4	6	4	1

a. Write down the values in the fifth row of Pascal's triangle. [2]

b. Hence or otherwise, find the term in x^3 in the expansion of $(2x+3)^5$. [5]

Markscheme

a. 1, 5, 10, 10, 5, 1 ***A2*** ***N2***

[2 marks]

b. evidence of binomial expansion with binomial coefficient ***(M1)***

eg $\binom{n}{r} a^{n-r}b^r$, selecting correct term, $(2x)^5(3)^0 + 5(2x)^4(3)^1 + 10(2x)^3(3)^2 + \dots$

correct substitution into correct term ***(A1)(A1)(A1)***

eg $10(2)^3(3)^2$, $\binom{5}{3}(2x)^3(3)^2$

Note: Award ***A1*** for each factor.

$720x^3$ ***A1*** ***N2***

Notes: Do not award any marks if there is clear evidence of adding instead of multiplying.

Do not award final ***A1*** for a final answer of 720, even if $720x^3$ is seen previously.

[5 marks]

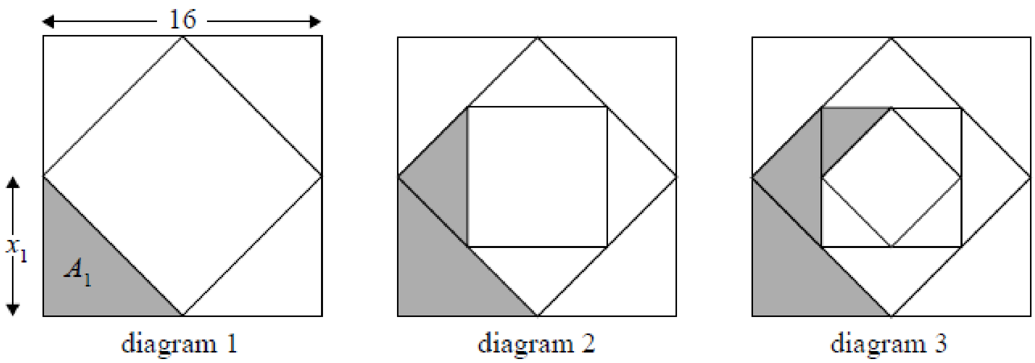
Examiners report

a. [N/A]

b. [N/A]

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1).

The process is repeated twice, as shown in diagrams 2 and 3.



Let x_n denote the length of one of the equal sides of each new triangle.

Let A_n denote the area of each new triangle.

a. The following table gives the values of x_n and A_n, for $1 \leqslant n \leqslant 3$. **Copy** and complete the table. *(Do **not** write on this page.)* [4]

n	1	2	3
x_n	8		4
A_n	32	16	

b. The process described above is repeated. Find A_6. [4]

c. Consider an initial square of side length k cm. The process described above is repeated indefinitely. The total area of the shaded regions is k cm^2. Find the value of k. [7]

Markscheme

a. valid method for finding side length ***(M1)***

eg $8^2 + 8^2 = c^2,\ 45 - 45 - 90$ side ratios, $8\sqrt{2},\ \frac{1}{2}s^2 = 16,\ x^2 + x^2 = 8^2$

correct working for area ***(A1)***

eg $\frac{1}{2} \times 4 \times 4$

n	1	2	3
x_n	8	$\sqrt{32}$	4
A_n	32	16	8

A1A1 N2N2

[4 marks]

b. **METHOD 1**

recognize geometric progression for A_n ***(R1)***

eg $u_n = u_1 r^{n-1}$

$r = \frac{1}{2}$ ***(A1)***

correct working ***(A1)***

eg $32\left(\frac{1}{2}\right)^5;\ 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

$A_6 = 1$ ***A1 N3***

METHOD 2

attempt to find x_6 ***(M1)***

eg $8\left(\frac{1}{\sqrt{2}}\right)^5,\ 2\sqrt{2},\ 2,\ \sqrt{2},\ 1,\ \ldots$

$x_6 = \sqrt{2}$ ***(A1)***

correct working ***(A1)***

eg $\frac{1}{2}\left(\sqrt{2}\right)^2$

$A_6 = 1$ ***A1*** ***N3***

[4 marks]

c. **METHOD 1**

recognize infinite geometric series ***(R1)***

eg $S_n = \frac{a}{1-r},\ |r| < 1$

area of first triangle in terms of k ***(A1)***

eg $\frac{1}{2}\left(\frac{k}{2}\right)^2$

attempt to substitute into sum of infinite geometric series (must have k) ***(M1)***

eg $\frac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}},\ \frac{k}{1-\frac{1}{2}}$

correct equation ***A1***

eg $\frac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}} = k,\ k = \frac{\frac{k^2}{8}}{\frac{1}{2}}$

correct working ***(A1)***

eg $k^2 = 4k$

valid attempt to solve **their** quadratic ***(M1)***

eg $k(k-4),\ k = 4$ or $k = 0$

$k = 4$ ***A1*** ***N2***

METHOD 2

recognizing that there are four sets of infinitely shaded regions with equal area ***R1***

area of original square is k^2 ***(A1)***

so total shaded area is $\frac{k^2}{4}$ ***(A1)***

correct equation $\frac{k^2}{4} = k$ ***A1***

$k^2 = 4k$ ***(A1)***

valid attempt to solve their quadratic ***(M1)***

eg $k(k-4),\ k = 4$ or $k = 0$

$k = 4$ ***A1*** ***N2***

[7 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

---

a. Find the value of $\log_2 40 - \log_2 5$. [3]

b. Find the value of $8^{\log_2 5}$. [4]

Markscheme

a. evidence of correct formula ***(M1)***

eg $\log a - \log b = \log \frac{a}{b}$, $\log\left(\frac{40}{5}\right)$, $\log 8 + \log 5 - \log 5$

Note: Ignore missing or incorrect base.

correct working ***(A1)***

eg $\log_2 8$, $2^3 = 8$

$\log_2 40 - \log_2 5 = 3$ ***A1 N2***

[3 marks]

b. attempt to write 8 as a power of 2 (seen anywhere) ***(M1)***

eg $(2^3)^{\log_2 5}$, $2^3 = 8$, 2^a

multiplying powers ***(M1)***

eg $2^{3\log_2 5}$, $a\log_2 5$

correct working ***(A1)***

eg $2^{\log_2 125}$, $\log_2 5^3$, $\left(2^{\log_2 5}\right)^3$

$8^{\log_2 5} = 125$ ***A1 N3***

[4 marks]

Examiners report

a. Many candidates readily earned marks in part (a). Some interpreted $\log_2 40 - \log_2 5$ to mean $\frac{\log_2 40}{\log_2 5}$, an error which led to no further marks. Others left the answer as $\log_2 5$ where an integer answer is expected.

b. Part (b) proved challenging for most candidates, with few recognizing that changing 8 to base 2 is a helpful move. Some made it as far as $2^{3\log_2 5}$ yet could not make that final leap to an integer.

---

a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. [2]

b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4]

Markscheme

a. $m = 3,\ n = 4$ ***A1A1 N2***

[2 marks]

b. attempt to apply $(2^a)^b = 2^{ab}$ ***(M1)***

eg $6x + 3,\ 4(2x - 3)$

equating **their** powers of 2 (seen anywhere) ***M1*** *eg* $3(2x+1) = 8x - 12$

correct working ***A1***

eg $8x - 12 = 6x + 3,\ 2x = 15$

$x = \frac{15}{2} \quad (7.5)$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

Examiners report

a. Indices laws were well understood with many candidates solving the equation correctly. Some candidates used logs, which took longer, and errors crept in.

b. Indices laws were well understood with many candidates solving the equation correctly. Some candidates used logs, which took longer, and errors crept in.

The first three terms of a geometric sequence are $\ln x^{16}$, $\ln x^8$, $\ln x^4$, for $x > 0$.

a. Find the common ratio. [3]

b. Solve $\sum\limits_{k=1}^{\infty} 2^{5-k} \ln x = 64$. [5]

Markscheme

a. correct use $\log x^n = n \log x$ ***A1***

eg $16 \ln x$

valid approach to find r ***(M1)***

eg $\frac{u_{n+1}}{u_n},\ \frac{\ln x^8}{\ln x^{16}},\ \frac{4 \ln x}{8 \ln x},\ \ln x^4 = \ln x^{16} \times r^2$

$r = \frac{1}{2}$ ***A1*** ***N2***

[3 marks]

b. recognizing a sum (finite or infinite) ***(M1)***

eg $2^4 \ln x + 2^3 \ln x,\ \frac{a}{1-r},\ S_\infty,\ 16 \ln x + \ldots$

valid approach (seen anywhere) ***(M1)***

eg recognizing GP is the same as part (a), using **their** r value from part (a), $r = \frac{1}{2}$

correct substitution into infinite sum (only if $|r|$ is a constant and less than 1) ***A1***

eg $\frac{2^4 \ln x}{1-\frac{1}{2}},\ \frac{\ln x^{16}}{\frac{1}{2}},\ 32 \ln x$

correct working ***(A1)***

eg $\ln x = 2$

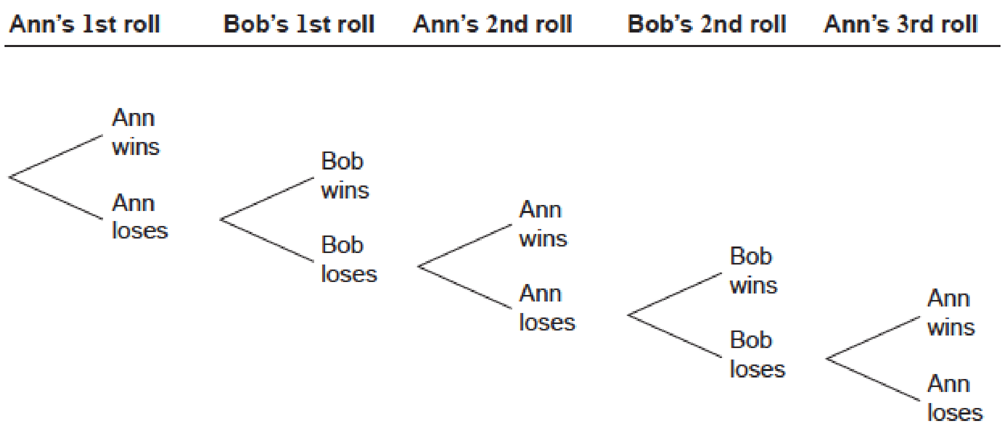
$x = \mathrm{e}^2$ ***A1*** ***N3***

[5 marks]

Examiners report

a. [N/A]
b. [N/A]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.



a. Find the probability that Ann wins on her first roll. [2]

b. (i) The probability that Ann wins on her third roll is $\frac{5}{8} \times \frac{4}{8} \times p \times q \times \frac{3}{8}$. [6]

Write down the value of p and of q.

(ii) The probability that Ann wins on her tenth roll is $\frac{3}{8}r^k$ where $r \in \mathbb{Q},\ k \in \mathbb{Z}$.

Find the value of r and of k.

c. Find the probability that Ann wins the game. [7]

Markscheme

a. recognizing Ann rolls green ***(M1)***

eg $\mathrm{P(G)}$

$\frac{3}{8}$ ***A1 N2***

[2 marks]

b. (i) $p = \frac{4}{8},\ q = \frac{5}{8}$ or $q = \frac{4}{8},\ p = \frac{5}{8}$ ***A1A1 N2***

(ii) recognizes Ann and Bob lose 9 times ***(M1)***

eg\(\;\;\;\(\overbrace {{A_L}{B_\(\overbrace {{A_L}{B_ \ldots \(\overbrace {{A_L}{B_{\text{ 9 times, }}\underbrace {\left({\frac{5}{8} \times \frac{4}{8}} \right) \times \ldots \times \left({\frac{5}{8} \times \frac{4}{8}} \right)}_{{\text{9 times}}}\)

$k = 9$ (seen anywhere) ***A1 N2***

correct working ***(A1)***

eg $\left(\frac{5}{8}\times\frac{4}{8}\right)^9\times\frac{3}{8},\ \left(\frac{5}{8}\times\frac{4}{8}\right)\times\ldots\times\left(\frac{5}{8}\times\frac{4}{8}\right)\times\frac{3}{8}$

$r=\frac{20}{64}\ \left(=\frac{5}{16}\right)$ ***A1*** ***N2***

[6 marks]

c. recognize the probability is an infinite sum ***(M1)***

eg Ann wins on her 1^{st} roll or 2^{nd} roll or 3^{rd} roll…, S_∞

recognizing GP ***(M1)***

$u_1=\frac{3}{8}$ (seen anywhere) ***A1***

$r=\frac{20}{64}$ (seen anywhere) ***A1***

correct substitution into infinite sum of GP ***A1***

eg $\frac{\frac{3}{8}}{1-\frac{5}{16}},\ \frac{3}{8}\left(\frac{1}{1-\left(\frac{5}{8}\times\frac{4}{8}\right)}\right),\ \frac{1}{1-\frac{5}{16}}$

correct working ***(A1)***

eg $\frac{\frac{3}{8}}{\frac{11}{16}},\ \frac{3}{8}\times\frac{16}{11}$

$\mathrm{P}\ (\text{Ann wins})=\frac{48}{88}\ \left(=\frac{6}{11}\right)$ ***A1*** ***N1***

[7 marks]

Total [15 marks]

Examiners report

a. Some teachers' comments suggested that the word 'loses' in the diagram was misleading. But candidate scripts did not indicate any adverse effect.

a) Very well answered.

b) i) Probabilities p and q were typically found correctly. ii) Fewer candidates identified the common ratio and number of rolls correctly.

Few candidates recognized that this was an infinite geometric sum although some did see that a geometric progression was involved.

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a) Very well answered.

b) i) Probabilities p and q were typically found correctly. ii) Fewer candidates identified the common ratio and number of rolls correctly.

Few candidates recognized that this was an infinite geometric sum although some did see that a geometric progression was involved.

Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

Markscheme

correct application of $\log a + \log b = \log ab$ ***(A1)***

eg $\log_2(2\sin x\cos x),\ \log 2 + \log(\sin x) + \log(\cos x)$

correct equation without logs ***A1***

eg $2\sin x\cos x = 2^{-1},\ \sin x\cos x = \frac{1}{4},\ \sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) ***A1***

eg $\log(\sin 2x),\ 2\sin x\cos x = \sin 2x,\ \sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\ (30°)$ ***(A1)***

correct working ***A1***

eg $x = \frac{\pi}{12} + 2\pi,\ 2x = \frac{25\pi}{6},\ \frac{29\pi}{6},\ 750°,\ 870°,\ x = \frac{\pi}{12}$ **and** $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12},\ \frac{29\pi}{12}$ (do not accept additional values) ***A2*** ***N0***

[7 marks]

Examiners report

[N/A]

Write down the value of

a(i).(i) $\log_3 27$; [1]

a(ii).(ii) $\log_8 \frac{1}{8}$; [1]

a(iii).(iii) $\log_{16} 4$. [1]

b. Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. [3]

Markscheme

a(i).(i) $\log_3 27 = 3$ ***A1*** ***N1***

[1 mark]

a(ii).(ii) $\log_8 \frac{1}{8} = -1$ ***A1*** ***N1***

[1 mark]

a(iii).(iii) $\log_{16} 4 = \frac{1}{2}$ ***A1*** ***N1***

[1 mark]

b. correct equation with **their** three values *(A1)*

eg $\frac{3}{2} = \log_4 x,\ 3 + (-1) - \frac{1}{2} = \log_4 x$

correct working involving powers *(A1)*

eg $x = 4^{\frac{3}{2}}, 4^{\frac{3}{2}} = 4^{\log_4 x}$

$x = 8$ ***A1*** ***N2***

[3 marks]

Examiners report

a(i). [N/A]
a(ii). [N/A]
a(iii). [N/A]
b. [N/A]

Solve $\log_2 x + \log_2 (x - 2) = 3$, for $x > 2$.

Markscheme

recognizing $\log a + \log b = \log ab$ (seen anywhere) ***(A1)***

e.g. $\log_2 (x(x - 2))$, $x^2 - 2x$

recognizing $\log_a b = x \Leftrightarrow a^x = b$ ***(A1)***

e.g. $2^3 = 8$

correct simplification ***A1***

e.g. $x(x - 2) = 2^3$, $x^2 - 2x - 8$

evidence of correct approach to solve ***(M1)***

e.g. factorizing, quadratic formula

correct working ***A1***

e.g. $(x - 4)(x + 2)$, $\frac{2 \pm \sqrt{36}}{2}$

$x = 4$ ***A2*** ***N3***

[7 marks]

Examiners report

Candidates secure in their understanding of logarithm properties usually had success with this problem, solving the resulting quadratic either by factoring or using the quadratic formula. The majority of successful candidates correctly rejected the solution that was not in the domain. A number of candidates, however, were unclear on logarithm properties. Some unsuccessful candidates were able to demonstrate understanding of one property but without both were not able to make much progress. A few candidates employed a “guess and check” strategy, but this did not earn full marks.

In the expansion of $(3x + 1)^n$, the coefficient of the term in x^2 is $135n$, where $n \in \mathbb{Z}^+$. Find n.

Markscheme

Note: Accept sloppy notation (such as missing brackets, or binomial coefficient which includes x^2).

evidence of valid binomial expansion with binomial coefficients ***(M1)***

eg $\binom{n}{r}(3x)^r(1)^{n-r},\ (3x)^n + n(3x)^{n-1} + \binom{n}{2}(3x)^{n-2} + \ldots,\ \binom{n}{r}(1)^{n-r}(3x)^r$

attempt to identify correct term ***(M1)***

eg $\binom{n}{n-2},\ (3x)^2,\ n-r=2$

setting **correct** coefficient or term equal to $135n$ (may be seen later) ***A1***

eg $9\binom{n}{2} = 135n,\ \binom{n}{n-2}(3x)^2 = 135n,\ \frac{9n(n-1)}{2} = 135nx^2$

correct working for binomial coefficient (using ${}_nC_r$ formula) ***(A1)***

eg $\frac{n(n-1)(n-2)(n-3)\ldots}{2\times1\times(n-2)(n-3)(n-4)\ldots},\ \frac{n(n-1)}{2}$

EITHER

evidence of correct working (with linear equation in n) ***(A1)***

eg $\frac{9(n-1)}{2} = 135,\ \frac{9(n-1)}{2}x^2 = 135x^2$

correct simplification ***(A1)***

eg $n-1 = \frac{135\times2}{9},\ \frac{(n-1)}{2} = 15$

$n = 31$ ***A1 N2***

OR

evidence of correct working (with quadratic equation in n) ***(A1)***

eg $9n^2 - 279n = 0,\ n^2 - n = 30n,\ (9\mathrm{n}^2 - 9n)x^2 = 270nx^2$

evidence of solving ***(A1)***

eg $9n(n-31) = 0,\ 9n^2 = 279n$

$n = 31$ ***A1 N2***

Note: Award ***A0*** for additional answers.

[7 marks]

Examiners report

[N/A]

Let $f(x) = 3\ln x$ and $g(x) = \ln 5x^3$.

a. Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. [4]

b. The graph of g is a transformation of the graph of f . Give a full geometric description of this transformation. [3]

Markscheme

a. attempt to apply rules of logarithms *(M1)*

e.g. $\ln a^b = b \ln a$, $\ln ab = \ln a + \ln b$

correct application of $\ln a^b = b \ln a$ (seen anywhere) ***A1***

e.g. $3 \ln x = \ln x^3$

correct application of $\ln ab = \ln a + \ln b$ (seen anywhere) ***A1***

e.g. $\ln 5x^3 = \ln 5 + \ln x^3$

so $\ln 5x^3 = \ln 5 + 3 \ln x$

$g(x) = f(x) + \ln 5$ (accept $g(x) = 3 \ln x + \ln 5$) ***A1*** ***N1***

[4 marks]

b. transformation with correct name, direction, and value ***A3***

e.g. translation by $\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by $\ln 5$, vertical translation of $\ln 5$

[3 marks]

Examiners report

a. This question was very poorly done by the majority of candidates. While candidates seemed to have a vague idea of how to apply the rules of logarithms in part (a), very few did so successfully. The most common error in part (a) was to begin incorrectly with $\ln 5x^3 = 3 \ln 5x$. This error was often followed by other errors.

b. In part (b), very few candidates were able to describe the transformation as a vertical translation (or shift). Many candidates attempted to describe numerous incorrect transformations, and some left part (b) entirely blank.

In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.

a. Find d . [2]

b. Find u_{20} . [2]

c. Find S_{20} . [2]

Markscheme

a. attempt to find d *(M1)*

e.g. $\frac{u_3 - u_1}{2}$, $8 = 2 + 2d$

$d = 3$ ***A1*** ***N2***

[2 marks]

b. correct substitution *(A1)*

e.g. $u_{20} = 2 + (20 - 1)3$, $u_{20} = 3 \times 20 - 1$

$u_{20} = 59$ ***A1*** ***N2***

[2 marks]

c. correct substitution *(A1)*

e.g. $S_{20} = \frac{20}{2}(2 + 59)$, $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$

$S_{20} = 610$ ***A1*** ***N2***

[2 marks]

Examiners report

a. This question was answered correctly by the large majority of candidates. The few mistakes seen were due to either incorrect substitution into the formula or simple arithmetic errors. Even where candidates made mistakes, they were usually able to earn full follow-through marks in the subsequent parts of the question.

b. This question was answered correctly by the large majority of candidates. The few mistakes seen were due to either incorrect substitution into the formula or simple arithmetic errors. Even where candidates made mistakes, they were usually able to earn full follow-through marks in the subsequent parts of the question.

c. This question was answered correctly by the large majority of candidates. The few mistakes seen were due to either incorrect substitution into the formula or simple arithmetic errors. Even where candidates made mistakes, they were usually able to earn full follow-through marks in the subsequent parts of the question.

Given that \({\left({1 + \frac{2}{3}x} \right)^n}{(3 + nx)^2} = 9 + 84x + \ldots \) , find the value of *n* .

Markscheme

attempt to expand $\left(1 + \frac{2}{3}x\right)^n$ ***(M1)***

e.g. Pascal's triangle, $\left(1 + \frac{2}{3}x\right)^n = 1 + \frac{2}{3}nx + \ldots$

correct first two terms of $\left(1 + \frac{2}{3}x\right)^n$ (seen anywhere) ***(A1)***

e.g. $1 + \frac{2}{3}nx$

correct first two terms of quadratic (seen anywhere) ***(A1)***

e.g. 9 , $6nx$, $(9 + 6nx + n^2x^2)$

correct calculation for the *x*-term ***A2***

e.g. $\frac{2}{3}nx \times 9 + 6nx$, $6n + 6n$, $12n$

correct equation ***A1***

e.g. $6n + 6n = 84$, $12nx = 84x$

$n = 7$ ***A1*** ***N1***

[7 marks]

Examiners report

This question proved quite challenging for the majority of candidates, although there were a small number who were able to find the correct value of *n* using algebraic and investigative methods. While most candidates recognized the need to apply the binomial theorem, the majority seemed to have no idea how to do so when the exponent was a variable, *n*, rather than a known integer. Most candidates who attempted this question did expand the quadratic correctly, but many went no further, or simply set the *x*-term of the quadratic equal to $84x$, ignoring the expansion of the first binomial altogether.

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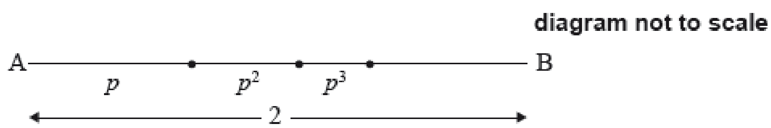
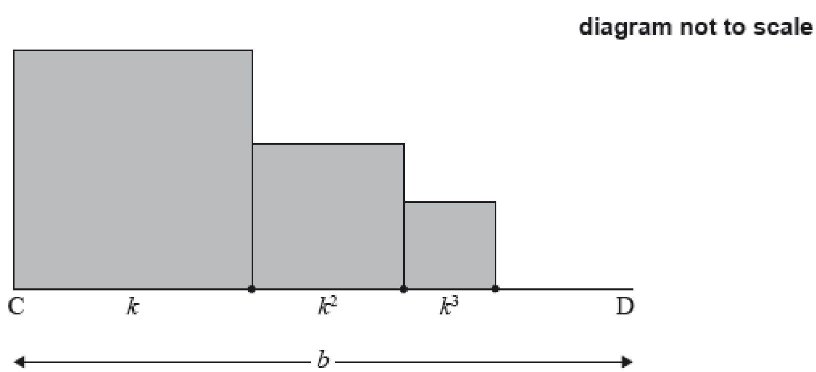
a. The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments. [5]



The length of the line segments are $p \text{ cm}, \ p^2 \text{ cm}, \ p^3 \text{ cm}, \ \ldots$, where $0 < p < 1$.

Show that $p = \frac{2}{3}$.

b. The following diagram shows [CD], with length $b \text{ cm}$, where $b > 1$. Squares with side lengths $k \text{ cm}, \ k^2 \text{ cm}, \ k^3 \text{ cm}, \ \ldots$, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares. [9]



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.

Markscheme

a. infinite sum of segments is 2 (seen anywhere) ***(A1)***

eg $p + p^2 + p^3 + \ldots = 2, \ \frac{u_1}{1-r} = 2$

recognizing GP ***(M1)***

eg ratio is $p, \ \frac{u_1}{1-r}, \ u_n = u_1 \times r^{n-1}, \ \frac{u_1(r^n-1)}{r-1}$

correct substitution into S_∞ formula (may be seen in equation) ***A1***

eg $\frac{p}{1-p}$

correct equation *(A1)*

eg $\frac{p}{1-p}=2,\ p=2-2p$

correct working leading to answer ***A1***

eg $3p=2,\ 2-3p=0$

$p=\frac{2}{3}\ (\mathrm{cm})$ ***AG*** ***N0***

[5 marks]

b. recognizing infinite geometric series with squares ***(M1)***

eg $k^2+k^4+k^6+\ldots,\ \frac{k^2}{1-k^2}$

correct substitution into $S_\infty=\frac{9}{16}$ (must substitute into formula) ***(A2)***

eg $\frac{k^2}{1-k^2}=\frac{9}{16}$

correct working ***(A1)***

eg $16k^2=9-9k^2,\ 25k^2=9,\ k^2=\frac{9}{25}$

$k=\frac{3}{5}$ (seen anywhere) ***A1***

valid approach with segments and CD (may be seen earlier) ***(M1)***

eg $r=k,\ S_\infty=b$

correct expression for b in terms of k (may be seen earlier) ***(A1)***

eg $b=\frac{k}{1-k},\ b=\sum_{n=1}^{\infty}k^n,\ b=k+k^2+k^3+\ldots$

substituting **their** value of k into **their** formula for b ***(M1)***

eg $\frac{\frac{3}{5}}{1-\frac{3}{5}},\ \frac{\left(\frac{3}{5}\right)}{\left(\frac{2}{5}\right)}$

$b=\frac{3}{2}$ ***A1*** ***N3***

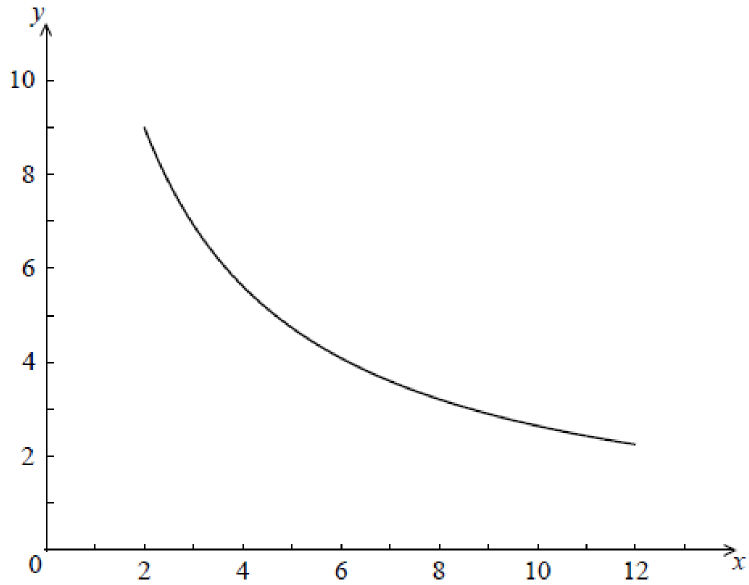
[9 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x)=\frac{1}{4}x^2+2$. The line L is the tangent to the curve of f at (4, 6) .

Let $g(x)=\frac{90}{3x+4}$, for $2\le x\le 12$. The following diagram shows the graph of g .



a. Find the equation of L . [4]

b. Find the area of the region enclosed by the curve of g , the x-axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. [6]

c. The graph of g is reflected in the x-axis to give the graph of h . The area of the region enclosed by the lines L , $x = 2$, $x = 12$ and the x-axis is 120 120 cm^2 . [3]

Find the area enclosed by the lines L , $x = 2$, $x = 12$ and the graph of h .

Markscheme

a. finding $f'(x) = \frac{1}{2}x$ ***A1***

attempt to find $f'(4)$ ***(M1)***

correct value $f'(4) = 2$ ***A1***

correct equation in any form ***A1*** ***N2***

e.g. $y - 6 = 2(x - 4)$, $y = 2x - 2$

[4 marks]

b. area $= \int_2^{12} \frac{90}{3x+4} \mathrm{d}x$

correct integral ***A1A1***

e.g. $30 \ln(3x + 4)$

substituting limits and subtracting ***(M1)***

e.g. $30 \ln(3 \times 12 + 4) - 30 \ln(3 \times 2 + 4)$, $30 \ln 40 - 30 \ln 10$

correct working ***(A1)***

e.g. $30(\ln 40 - \ln 10)$

correct application of $\ln b - \ln a$ ***(A1)***

e.g. $30 \ln \frac{40}{10}$

area $= 30 \ln 4$ ***A1*** ***N4***

[6 marks]

c. valid approach ***(M1)***

e.g. sketch, area h = area g , 120 + **their** answer from (b)

$\text{area} = 120 + 30\ln 4$ ***A2*** ***N3***

[3 marks]

Examiners report

a. While most candidates answered part (a) correctly, finding the equation of the tangent, there were some who did not consider the value of their derivative when $x = 4$.

b. In part (b), most candidates knew that they needed to integrate to find the area, but errors in integration, and misapplication of the rules of logarithms kept many from finding the correct area.

c. In part (c), it was clear that a significant number of candidates understood the idea of the reflected function, and some recognized that the integral was the negative of the integral from part (b), but only a few recognized the relationship between the areas. Many thought the area between h and the x-axis was 120.

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

a. $\ln\left(\frac{5}{3}\right)$. [2]

b. $\ln 45$. [4]

Markscheme

a. correct approach ***(A1)***

eg $\ln 5 - \ln 3$

$\ln\left(\frac{5}{3}\right) = y - x$ ***A1*** ***N2***

[2 marks]

b. recognizing factors of 45 (may be seen in log expansion) ***(M1)***

eg $\ln(9 \times 5),\ 3 \times 3 \times 5,\ \log 3^2 \times \log 5$

correct application of $\log(ab) = \log a + \log b$ ***(A1)***

eg $\ln 9 + \ln 5,\ \ln 3 + \ln 3 + \ln 5,\ \ln 3^2 + \ln 5$

correct working ***(A1)***

eg $2\ln 3 + \ln 5,\ x + x + y$

$\ln 45 = 2x + y$ ***A1*** ***N3***

[4 marks]

Examiners report

a. Most candidates were able to earn some or all the marks on this question. Part (a) was answered correctly by nearly all candidates.

b. Most candidates were able to earn some or all the marks on this question. In part (b), the majority of candidates knew they needed to factor 45, though some did not apply the log rules correctly to earn all the available marks here.

Let $\log_3 p = 6$ and $\log_3 q = 7$.

a. Find $\log_3 p^2$. [2]

b. Find $\log_3 \left(\frac{p}{q}\right)$. [2]

c. Find $\log_3 (9p)$. [3]

Markscheme

a. **METHOD 1**

evidence of correct formula ***(M1)***

eg $\log u^n = n \log u$, $2\log_3 p$

$\log_3 (p^2) = 12$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ ***(M1)***

eg $\log_3 (3^6)^2$, $\log 3^{12}$, $12\log_3 3$

$\log_3 (p^2) = 12$ ***A1*** ***N2***

[2 marks]

b. **METHOD 1**

evidence of correct formula ***(M1)***

eg $\log\left(\frac{p}{q}\right) = \log p - \log q$, $6 - 7$

$\log_3 \left(\frac{p}{q}\right) = -1$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ and $q = 3^7$ ***(M1)***

eg $\log_3 \left(\frac{3^6}{3^7}\right)$, $\log 3^{-1}$, $-\log_3 3$

$\log_3 \left(\frac{p}{q}\right) = -1$ ***A1*** ***N2***

[2 marks]

c. **METHOD 1**

evidence of correct formula ***(M1)***

eg $\log_3 uv = \log_3 u + \log_3 v$, $\log 9 + \log p$

$\log_3 9 = 2$ (may be seen in expression) ***A1***

eg $2 + \log p$

$\log_3 (9p) = 8$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ ***(M1)***

eg $\log_3 (9 \times 3^6)$, $\log_3 (3^2 \times 3^6)$

correct working ***A1***

eg $\log_3 9 + \log_3 3^6$, $\log_3 3^8$

$\log_3 (9p) = 8$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Examiners report

a. This question proved to be surprisingly challenging for many candidates. A common misunderstanding was to set p equal to 6 and q equal to A large number of candidates had trouble applying the rules of logarithms, and made multiple errors in each part of the question. Common types of errors included incorrect working such as $\log_3 p^2 = 36$ in part (a), $\log_3 \left(\frac{p}{q}\right) = \frac{\log_3 6}{\log_3 7}$ or $\log_3 \left(\frac{p}{q}\right) = \log_3 6 - \log_3 7$ in part (b), and $\log_3 (9p) = 54$ in part (c).

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Let $f(x) = \mathrm{e}^{x+3}$.

a. (i) Show that $f^{-1}(x) = \ln x - 3$. [3]

(ii) Write down the domain of f^{-1} .

b. Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$. [4]

Markscheme

a. (i) interchanging x and y (seen anywhere) ***M1***

e.g. $x = \mathrm{e}^{y+3}$

correct manipulation ***A1***

e.g. $\ln x = y + 3$, $\ln y = x + 3$

$f^{-1}(x) = \ln x - 3$ ***AG*** ***N0***

(ii) $x > 0$ ***A1 N1***

[3 marks]

b. collecting like terms; using laws of logs ***(A1)(A1)***

e.g. $\ln x - \ln\left(\frac{1}{x}\right) = 3$, $\ln x + \ln x = 3$, $\ln\left(\frac{x}{\frac{1}{x}}\right) = 3$, $\ln x^2 = 3$

simplify ***(A1)***

e.g. $\ln x = \frac{3}{2}$, $x^2 = \mathrm{e}^3$

$x = \mathrm{e}^{\frac{3}{2}} \left(= \sqrt{\mathrm{e}^3}\right)$ ***A1 N2***

[4 marks]

Examiners report

a. Many candidates interchanged the x and y to find the inverse function, but very few could write down the correct domain of the inverse, often giving $x \ge 0$, $x > 3$ and "all real numbers" as responses.

b. Where students attempted to solve the equation in (b), most treated $\ln x - 3$ as $\ln(x - 3)$ and created an incorrect equation from the outset. The few who applied laws of logarithms often carried the algebra through to completion.

In an arithmetic sequence, the first term is 8 and the second term is 5.

a. Find the common difference. [2]

b. Find the tenth term. [2]

c. Find the sum of the first ten terms. [2]

Markscheme

a. subtracting terms ***(M1)***

eg $5 - 8,\ u_2 - u_1$

$d = -3$ ***A1 N2***

[2 marks]

b. correct substitution into formula ***(A1)***

eg $u_{10} = 8 + (10 - 1)(-3),\ 8 - 27,\ -3(10) + 11$

$u_{10} = -19$ ***A1 N2***

[2 marks]

c. correct substitution into formula for sum ***(A1)***

eg $S_{10} = \frac{10}{2}(8 - 19),\ 5\,(2(8) + (10 - 1)(-3))$

$S_{10} = -55$ ***A1 N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

The y-coordinate of P is $\ln 27$.

a. Find the x-coordinate of P. [3]

b. Find $f(x)$, expressing your answer as a single logarithm. [8]

c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$. [[N/A]

Find the value of a and of b, where $a,\ b \in \mathbb{N}$.

Markscheme

a. recognizing $f'(x) = 0$ ***(M1)***

correct working ***(A1)***

eg $6 - 2x = 0$

$x = 3$ ***A1*** ***N2***

[3 marks]

b. evidence of integration ***(M1)***

eg $\int f',\ \int \frac{6-2x}{6x-x^2}\mathrm{d}x$

using substitution ***(A1)***

eg $\int \frac{1}{u}\mathrm{d}u$ where $u = 6x - x^2$

correct integral ***A1***

eg $\ln(u) + c,\ \ln(6x - x^2)$

substituting $(3,\ \ln 27)$ into **their** integrated expression (must have c) ***(M1)***

eg $\ln(6 \times 3 - 3^2) + c = \ln 27,\ \ln(18 - 9) + \ln k = \ln 27$

correct working ***(A1)***

eg $c = \ln 27 - \ln 9$

EITHER

$c = \ln 3$ ***(A1)***

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 3$ ***A1*** ***N4***

OR

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 27 - \ln 9$

correct use of a log law ***(A1)***

eg $f(x) = \ln(6x - x^2) + \ln\left(\frac{27}{9}\right),\ f(x) = \ln\big(27(6x - x^2)\big) - \ln 9$

$f(x) = \ln\big(3(6x - x^2)\big)$ ***A1 N4***

[8 marks]

c. $a = 3$ ***A1 N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

correct use of log law ***(A1)***

eg $\frac{3 \ln 3}{\ln 3},\ \log_3 27$

$b = 3$ ***A1 N2***

[4 marks]

Examiners report

a. Part a) was well answered.

b. In part b) most candidates realised that integration was required but fewer recognised the need to use integration by substitution. Quite a number of candidates who integrated correctly omitted finding the constant of integration.

c. In part c) many candidates showed good understanding of transformations and could apply them correctly, however, correct use of the laws of logarithms was challenging for many. In particular, a common error was $\frac{\ln 27}{\ln 3} = \ln 9$.