HL Paper 2

This question is about the thermodynamics of a car engine and the dynamics of the car.

A car engine consists of four cylinders. In each of the cylinders, a fuel-air mixture explodes to supply power at the appropriate moment in the cycle.

The diagram models the variation of pressure *P* with volume *V* for one cycle of the gas, ABCDA, in one of the cylinders of the engine. The gas in the cylinder has a fixed mass and can be assumed to be ideal.



The car is travelling at its maximum speed of 56 m s^{-1} . At this speed, the energy provided by the fuel injected into one cylinder in each cycle is 9200 J. One litre of fuel provides 56 MJ of energy.

A car is travelling along a straight horizontal road at its maximum speed of 56 m s^{-1} . The power output required at the wheels is 0.13 MW.

A driver moves a car in a horizontal circular path of radius 200 m. Each of the four tyres will not grip the road if the frictional force between a tyre and the road becomes less than 1500 N.

a.	At point A in the cycle, the fuel-air mixture is at 18 °C. During process AB, the gas is compressed to 0.046 of its original volume and the	[1]
	pressure increases by a factor of 40. Calculate the temperature of the gas at point B.	
b.	State the nature of the change in the gas that takes place during process BC in the cycle.	[1]
c.	Process CD is an adiabatic change. Discuss, with reference to the first law of thermodynamics, the change in temperature of the gas in the	[3]
	cylinder during process CD.	
d.	Explain how the diagram can be used to calculate the net work done during one cycle.	[2]
e.	(i) Calculate the volume of fuel injected into one cylinder during one cycle.	[3]
	(ii) Each of the four cylinders completes a cycle 18 times every second. Calculate the distance the car can travel on one litre of fuel at a speed of 56 m s^{-1} .	

f. A car accelerates uniformly along a straight horizontal road from an initial speed of 12 m s^{-1} to a final speed of 28 m s^{-1} in a distance of 250 [4]

m. The mass of the car is 1200 kg. Determine the rate at which the engine is supplying kinetic energy to the car as it accelerates.

g. (i) Calculate the total resistive force acting on the car when it is travelling at a constant speed of 56 m s^{-1} .

(ii) The mass of the car is 1200 kg. The resistive force *F* is related to the speed *v* by $F \propto v^2$. Using your answer to (g)(i), determine the maximum theoretical acceleration of the car at a speed of 28 m s^{-1} .

h. (i) Calculate the maximum speed of the car at which it can continue to move in the circular path. Assume that the radius of the path is the [6]

same for each tyre.

(ii) While the car is travelling around the circle, the people in the car have the sensation that they are being thrown outwards. Outline how Newton's first law of motion accounts for this sensation.

Markscheme

- a. 535 (K) / 262 (°C);
- b. constant volume change / isochoric / isovolumetric / OWTTE;
- c. Q/thermal energy transfer is zero;

$$\Delta U = -W;$$

as work is done by gas internal energy falls;

temperature falls as temperature is measure of average kinetic energy;

d. work done is estimated by evaluating area;

inside the loop / OWTTE;

e. (i) 1.6×10^{-4} (litre);

(ii) one litre =
$$\left(\frac{1}{4 \times 18 \times 1.64 \times 10^{-4}}\right)$$
 87 s of travel;

$$(87 \times 56) = 4.7 \text{ (km)};$$

Allow rounded 1.6 value to be used, giving 4.9 (km).

f. use of a kinematic equation to determine motion time (= 12.5 s);

change in kinetic energy $=\frac{1}{2} \times 1200 \times [28^2 - 12^2]$ (= 384 kJ); rate of change in kinetic energy $=\frac{384000}{12.5}$; } (allow ECF of 16² from (28 - 12)² for this mark) 31 (kW);

or

use of a kinematic equation to determine motion time (= 12.5 s);

use of a kinematic equation to determine acceleration (= $1.28~{
m m\,s^{-2}}$);

work done
$$\frac{F \times s}{\text{time}} = \frac{1536 \times 250}{12.5}$$
;

g. (i) force
$$= rac{ ext{power}}{ ext{speed}};$$

2300 *or* 2.3k (N);

Award [2] for a bald correct answer.

- (ii) resistive force $=\frac{2300}{4}$ or $\frac{2321}{4}$ (= 575); (allow ECF)
- so accelerating force $(2300-580=)\ 1725\ (N)$ or 1741 (N);

 $a = rac{1725}{1200} = 1.44 \ ({
m m \, s^{-2}})$ or $a = rac{1741}{1200} = 1.45 \ ({
m m \, s^{-2}});$

Award [2 max] for an answer of 0.49 (m s⁻² (omits 2300 N).

h. (i) centripetal force must be $< 6000~({
m N})$; (allow force 6000 N)

 $v^2 = F imes rac{r}{m};$

 $31.6 (m s^{-1});$

Allow [3] for a bald correct answer.

Allow **[2 max]** if $4 \times$ is omitted, giving 15.8 (m s⁻¹).

(ii) statement of Newton's first law;

(hence) without car wall/restraint/friction at seat, the people in the car would move in a straight line/at a tangent to circle;

(hence) seat/seat belt/door exerts centripetal force;

(in frame of reference of the people) straight ahead movement is interpreted as "outwards";

Examiners report

- a. This simple gas law calculation was surprisingly badly done. Certainly similar questions have attracted better scores in previous examinations.
 Common errors included the inevitable failure to work in kelvin, and simple arithmetic errors.
- b. Most candidates were able to describe the constant volume nature of the change in question.
- c. Many candidates scored full credit in a question that has been well rehearsed in previous examinations. The zero change in thermal energy transfer was common and many were able to deduce that ΔU is therefore equal to -W. This led immediately to a deduction of temperature decrease.
- d. Almost all recognised that the work done was related to some area under the graph. In a small minority of cases the exact specification of the area was too imprecise to gain the second mark.
- e. (i) It was common to see a correct value for the volume of fuel used though not a correct unit.

(ii) Many were able to arrive at a travel time for the fuel and therefore the distance travelled. However, routes were indirect and lengthy and few could see a direct way to the answer.

f. There were at least two routes to tackle this problem. Some solutions were so confused that it was difficult to decide which method had been used. Common errors included: forgetting that the initial speed was 12 m s^{-1} not zero, power of ten errors, and simple mistakes in the use of the kinematic equations, or failure to evaluate work done = force \times distance correctly. However, many candidates scored partial credit. Scores of two or three out of the maximum four were common showing that many are persevering to get as far as they can.

g. (i) Many correct solutions were seen. Candidates are clearly comfortable with the use of the equation force = power/speed.

(ii) The method to be used here was obvious to many. What was missing was a clear appreciation of what was happening in terms of resistive force in the system. Many scored two out of three because they indicated a sensible method but did not use the correct value for the force. Scoring two marks does require that the explanation of the method is at least competent. Those candidates who give limited explanations of their method leading to a wrong answer will generally accumulate little credit. A suggestion (never seen in answers) is that candidates should have begun from a free-body force diagram which would have revealed the relationship of all the forces.

h. (i) The major problem here was that most candidates did not recognise that 1500 N of force acting at each of four wheels will imply a total force

of 6 kN. Again, partial credit was available only if it was clear what the candidate was doing and what the error was.

(ii) Statements of Newton's first law were surprisingly poor. As in previous examinations, few candidates appear to have learnt this essential rule by heart and they produce a garbled and incomplete version under examination pressure. The first law was then only loosely connected to the particular context of the question. Candidates have apparently not learnt to relate the physics they learn to everyday contexts.

This question is about the energy of an orbiting satellite.

A space shuttle of mass *m* is launched in the direction of the Earth's South Pole.

The shuttle enters a circular orbit of radius R around the Earth.

a. The kinetic energy $E_{\rm K}$ given to the shuttle at its launch is given by the expression

$$E_{
m K}=rac{7GMm}{8R_{
m E}}$$

where G is the gravitational constant, M is mass of the Earth and $R_{\rm E}$ is the radius of the Earth. Deduce that the shuttle cannot escape the gravitational field of the Earth.

b.i. Show that the total energy of the shuttle in its orbit is given by $-\frac{GMm}{2R}$. Air resistance is negligible. [3]

b.ii.Using the expression for $E_{
m K}$ in (a) and your answer to (b)(i), determine R in terms of $R_{
m E}$.

c. In practice, the total energy of the shuttle decreases as it collides with air molecules in the upper atmosphere. Outline what happens to the [2] speed of the shuttle when this occurs.

Markscheme

a. KE needs to be \geq (magnitude of) GPE at surface $\left(-\frac{GMm}{R_{F}}\right)$;

But KE is
$$\frac{7GMm}{8R_{\rm E}} < \frac{GMm}{R_{\rm E}}$$
 / OWTTE;

or

shows that total energy at launch $=-rac{GMm}{8R_{
m E}}$; (appropriate working required)

this is < 0, so escape impossible;

or

states that escape velocity needed is $\sqrt{\frac{2GM}{R_{\rm E}}}$; shows launch velocity is only $\sqrt{\frac{7GM}{4R_{\rm E}}}$; (appropriate working required)

b.i.
$$E_{
m tot}={
m PE}+{
m KE};$$

<u>shows that</u> kinetic energy = $(\frac{1}{2}mv^2 =) \frac{GMm}{2R}$; (appropriate working required) adds PE $\left(-\frac{GMm}{R}\right)$ and KE to get given answer; (appropriate working required) b.ii. $-\frac{GMm}{R_{\rm E}} + \frac{7GMm}{8R_{\rm E}} = -\frac{GMm}{2R}$; (equating total energy at launch and in orbit) $\frac{1}{8R_{\rm E}} = \frac{1}{2R}$;

$$R=4R_{\mathrm{E}};$$

[2]

[3]

Award **[0]** for an answer such as $R=rac{4R_{
m E}}{7}.$

c. total energy decreases/becomes a greater negative value, so R decreases;

as R decreases kinetic energy increases;

speed increases;

Allow third marking point even if reasoning is incorrect.

Examiners report

- a. Candidates were asked to show that the shuttle could not escape the Earth's field. There are many ways of approaching this, but in general answers were good, with only the weakest candidates failing to know where to start.
- b.i. The determination of total energy of a mass in Earth orbit is a standard classroom derivation. Most were able to reproduce it, but not everyone explained how the formula for orbital KE was derived.
- b.iiDetermining the radius of the orbit proved difficult for most candidates with many obtaining negative values or orbits with a radius less than the radius of the Earth. This was due to carelessness with the symbols used for the two different radii.
- c. Few candidates equated a decrease in total energy with an <u>increasingly</u> negative value. The consequent fact that the radius of the orbit decreases and velocity increases was counter-intuitive for most candidates. Most incorrectly opted for a decrease in KE due to resistive forces.

This question is about escape speed and gravitational effects.

- a. Explain what is meant by escape speed.
- b. Titania is a moon that orbits the planet Uranus. The mass of Titania is 3.5×10²¹kg. The radius of Titania is 800 km.

(i) Use the data to calculate the gravitational potential at the surface of Titania.

(ii) Use your answer to (b)(i) to determine the escape speed for Titania.

c. An astronaut visiting Titania throws an object away from him with an initial horizontal velocity of 1.8 m s⁻¹. The object is 1.5 m above the moon's [3] surface when it is thrown. The gravitational field strength at the surface of Titania is 0.37 N kg⁻¹.

[2]

[5]

Calculate the distance from the astronaut at which the object first strikes the surface.

Markscheme

a. (minimum) speed of object to escape gravitational field of a planet/travel to infinity;

at surface of planet;

without (further) energy input;

b. (i) $-rac{6.67 imes 10^{-11} imes 3.5 imes 10^{21}}{8.0 imes 10^5}$;

 -2.9×10^{5} Jkg⁻¹; (allow Nmkg⁻¹)

Award [1 max] if negative sign omitted.

(ii) $\frac{1}{2}mv^2 = mV$; speed== $\sqrt{2 \times 2.9 \times 10^5}$; (allow ECF from (b)(i)) 7.6 ×10²ms⁻¹; Ignore sign. Award **[3]** for a bald correct answer.

c. time to hit surface
$$=\sqrt{rac{2.0 imes 1.5}{0.37}}\,(=2.85\mathrm{s});$$

distance to impact = 2.85×1.8 ;

5.1m;

Examiners report

a. ^[N/A] b. ^[N/A] c. ^[N/A]

This question is about a probe in orbit.

A probe of mass *m* is in a circular orbit of radius *r* around a spherical planet of mass *M*.



- a. State why the work done by the gravitational force during one full revolution of the probe is zero.
- b. Deduce for the probe in orbit that its

(i) speed is
$$v=\sqrt{rac{GM}{r}}.$$

(ii) total energy is $E = -\frac{GMm}{2r}$.

c. It is now required to place the probe in another circular orbit further away from the planet. To do this, the probe's engines will be fired for a very [2]

short time.

State and explain whether the work done on the probe by the engines is positive, negative or zero.

[4]

Markscheme

a. because the force is always at right angles to the velocity / motion/orbit is an equipotential surface;

Do not accept answers based on the displacement being zero for a full revolution.

b. (i) equating gravitational force $\frac{GMm}{r^2}$; to centripetal force $\frac{mv^2}{r}$ to get result;

(ii) kinetic energy is $\frac{GMm}{2r}$; addition to potential energy $-\frac{GMm}{r}$ to get result;

c. the total energy (at the new orbit) will be greater than before/is less negative;

hence probe engines must be fired to produce force in the direction of motion / positive work must be done (on the probe);

Award [1] for mention of only potential energy increasing.

Examiners report

a. ^[N/A] b. ^[N/A]

c. [N/A]

A non-uniform electric field, with field lines as shown, exists in a region where there is no gravitational field. X is a point in the electric field. The field

lines and X lie in the plane of the paper.



- a. Outline what is meant by electric field strength.
- b. An electron is placed at X and released from rest. Draw, on the diagram, the direction of the force acting on the electron due to the field. [1]

[2]

c. The electron is replaced by a proton which is also released from rest at X. Compare, without calculation, the motion of the electron with the [4] motion of the proton after release. You may assume that no frictional forces act on the electron or the proton.

Markscheme

a. force per unit charge

acting on a small/test positive charge

b. horizontally to the left

Arrow does not need to touch X

- c. proton moves to the right/they move in opposite directions
 - force on each is initially the same

proton accelerates less than electron initially «because mass is greater»

field is stronger on right than left «as lines closer»

proton acceleration increases «as it is moving into stronger field»

OR

electron acceleration decreases «as it is moving into weaker field»

Allow ECF from (b)

Accept converse argument for electron

Examiners report

a. ^[N/A]

- b. ^[N/A]
- c. [N/A]

This question is in two parts. Part 1 is about collisions. Part 2 is about the gravitational field of Mars.

Part 1 Collisions

The experiment is repeated with the clay block placed at the edge of the table so that it is fired away from the table. The initial speed of the clay block

is $4.3\ m\ s^{-1}$ horizontally. The table surface is 0.85 m above the ground.



The graph shows the variation with distance r from the centre of Mars of the gravitational potential V. R is the radius of Mars which is 3.3 Mm.

(Values of V for r < R are not shown.)



A rocket of mass $1.2 imes 10^4 \ {
m kg}$ lifts off from the surface of Mars. Use the graph to

Par(i).c. Ignoring air resistance, calculate the horizontal distance travelled by the clay block before it strikes the ground.

(ii) The diagram in (c) shows the path of the clay block neglecting air resistance. On the diagram, draw the approximate shape of the path that the clay block will take assuming that air resistance acts on the clay block.

[7]

[1]

[5]

PartDefane gravitational potential energy of a mass at a point.

Part b.calculate the change in gravitational potential energy of the rocket at a distance 4R from the centre of Mars.

(ii) show that the magnitude of the gravitational field strength at a distance 4R from the centre of Mars is 0.23 N kg^{-1} .

Markscheme

Par(i) .c. use of kinematic equation to yield time;

$$t=\sqrt{rac{2s}{g}}~(=0.42~{
m s});$$

s =horizontal speed \times time;

= 1.8 m;

- Accept $g = 10 \text{ m s}^{-2}$ equivalent answers 1.79 from 9.8, 1.77 from 10.
- (ii) initial drawn velocity horizontal; (judge by eye)

reasonable shape; (i.e. quasi-parabolic)

horizontal distance moved always decreasing when compared to given path / range less than original;



Part20ak done in moving mass from infinity to a point;

Parti2.b.read offs -12.6 and -3.2;

gain in gpe $1.2 \times 10^4 \times [12.6 - 3.2]$ or gain in g potential $[12.6 \times 10^6 - 3.2 \times 10^6]$; = $1.13 \pm 0.05 \times 10^5$ MJ or $1.13 \pm 0.05 \times 10^{11}$ J; (ii) use of gradient of graph to determine g; values substituted from drawn gradient $(\text{typically } \frac{6.7 \times 10^6}{7 \times 3.3 \times 10^6})$; = 0.23 N kg^{-1} (allow answers in the range of 0.20 to 0.26 N kg-1) Award [0] for solutions from $\frac{V}{r}$.

Examiners report

Par(i) .c. Candidates were required to determine the time taken to fall to the floor and then use this time to evaluate the distance travelled horizontally.

Many managed this with more or less success.

(ii) Many candidates produced poor attempts at the sketch. Initial trajectories were not horizontal and the general shapes of the curves were usually not quasi-parabolic.

Parsome candidates gave a definition of gravitational potential, i.e. they related the energy to that of a unit mass.

Part Pathoughout this part candidates were instructed to use the graph, those who used other non-graphical methods were penalised.

- (i) There were many good evaluations with complete and well presented solutions.
- (ii) Use of the Data Booklet equation for gravitational field strength without reference to the graph was common.

Hydrogen atoms in an ultraviolet (UV) lamp make transitions from the first excited state to the ground state. Photons are emitted and are incident on a photoelectric surface as shown.



The photons cause the emission of electrons from the photoelectric surface. The work function of the photoelectric surface is 5.1 eV.

The electric potential of the photoelectric surface is 0 V. The variable voltage is adjusted so that the collecting plate is at -1.2 V.



a. Show that the energy of photons from the UV lamp is about 10 eV.	[2]
b.i.Calculate, in J, the maximum kinetic energy of the emitted electrons.	[2]
b.iiSuggest, with reference to conservation of energy, how the variable voltage source can be used to stop all emitted electrons from reaching the	[2]

collecting plate.

b.iiThe variable voltage can be adjusted so that no electrons reach the collecting plate. Write down the minimum value of the voltage for which no [1]

electrons reach the collecting plate.

c.i. On the diagram, draw and label the equipotential lines at –0.4 V and –0.8 V. [2]

c.ii An electron is emitted from the photoelectric surface with kinetic energy 2.1 eV. Calculate the speed of the electron at the collecting plate. [2]

Markscheme

a. $E_1 = -13.6 \text{ «eV» } E_2 = -\frac{13.6}{4} = -3.4 \text{ «eV»}$

energy of photon is difference $E_2 - E_1 = 10.2 \ll 10 \text{ eV}$ »

Must see at least 10.2 eV.

[2 marks]

b.i.10 - 5.1 = 4.9 «eV»

 $4.9 \times 1.6 \times 10^{-19} = 7.8 \times 10^{-19} \text{ «J»}$

Allow 5.1 if 10.2 is used to give 8.2×10⁻¹⁹ «J».

b.iiEPE produced by battery

exceeds maximum KE of electrons / electrons don't have enough KE

For first mark, accept explanation in terms of electric potential energy difference of electrons between surface and plate.

[2 marks]

b.iii4.9 «V»

Allow 5.1 if 10.2 is used in (b)(i).

Ignore sign on answer.

[1 mark]

c.i. two equally spaced vertical lines (judge by eye) at approximately 1/3 and 2/3

labelled correctly





c.ii.kinetic energy at collecting plate = 0.9 «eV»

speed = "
$$\sqrt{\frac{2 \times 0.9 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}}$$
" = 5.6 × 10⁵ "ms⁻¹"

Allow ECF from MP1

[2 marks]

Examiners report

a. [N/A] b.i.[N/A] b.ii.[N/A] b.iii.[N/A] c.i. [N/A] c.ii.[N/A] This question is about motion in a magnetic field.

An electron, that has been accelerated from rest by a potential difference of 250 V, enters a region of magnetic field of strength 0.12 T that is directed into the plane of the page.



a. The electron's path while in the region of magnetic field is a quarter circle. Show that the

time the electron spends in the region of magnetic field is 7.5×10^{-11} s.

c. A square loop of conducting wire is placed near a straight wire carrying a constant current *I*. The wire is in the same plane as the loop. [4]

[1]



The loop is made to move with constant speed v towards the wire.

(i) Explain, by reference to Faraday's and Lenz's laws of electromagnetic induction, why work must be done on the loop.

(ii) Suggest what becomes of the work done on the loop.

Markscheme

a.
$$t = \frac{1}{4} \frac{2\pi \times 4.5 \times 10^{-4}}{9.4 \times 10^{6}};$$

=7.5×10⁻¹¹s

c. (i) the flux in the loop is changing and so (by Faraday's law) an emf will be induced in the loop;

(by Lenz's law) the induced current will be (counter-clockwise) and so there will be a magnetic force opposing the motion;

requiring work to be done on the loop;

(ii) it is dissipated as thermal energy (due to the resistance) in the loop / radiation;

Examiners report

a. ^[N/A]

c. [N/A]

Curling is a game played on a horizontal ice surface. A player pushes a large smooth stone across the ice for several seconds and then releases it. The stone moves until friction brings it to rest. The graph shows the variation of speed of the stone with time.



[3]

[3]

The total distance travelled by the stone in 17.5 s is 29.8 m.

- b. Determine the coefficient of dynamic friction between the stone and the ice during the last 14.0 s of the stone's motion.
- c. The diagram shows the stone during its motion after release.



Label the diagram to show the forces acting on the stone. Your answer should include the name, the direction **and** point of application of each force.

Markscheme

b. ALTERNATIVE 1

$$\begin{split} \text{ ``deceleration''} &= \frac{3.41}{14.0} \text{ ``= } 0.243 \, \mathrm{m \, s^{-2} \text{''}} \\ F &= 0.243 \times m \\ \mu &= \frac{0.243 \times m}{m \times 9.81} = 0.025 \end{split}$$

ALTERNATIVE 2

distance travelled after release = 23.85 smKE lost = 5.81m sJ

$$\mu_{
m d} = rac{{
m KE \ lost}}{mg imes {
m distance}} = rac{5.81m}{23.85mg} = 0.025$$

Award [3] for a bald correct answer.

Ignore sign in acceleration.

Allow ECF from (a) (note that $\mu=0.0073~{
m x}$ candidate answer to (a)).

Ignore any units in answer.

Condone omission of m in solution.

Allow $g = 10 \text{ N kg}^{-1}$ (gives 0.024).

c. normal force, upwards, ignore point of application

Force must be labeled for its mark to be awarded. Blob at poa not required. Allow OWTTE for normal force. Allow N, R, reaction. The vertical forces must lie within the middle third of the stone

weight/weight force/force of gravity, downwards, ignore point of application

Allow mg, W but not "gravity".

Penalise gross deviations from vertical/horizontal once only

friction/resistive force, to left, at bottom of stone, point of application must be on the interface between ice and stone

Allow F, μ R. Only allow arrows/lines that lie on the interface. Take the tail of the arrow as the definitive point of application and expect line to be drawn horizontal.

Award [2 max] if any force arrow does not touch the stone

Do not award MP3 if a "driving force" is shown acting to the right. This need not be labelled to disqualify the mark. Treat arrows labelled "air resistance" as neutral.



N.B: Diagram in MS is drawn with the vertical forces not direction of travel collinear for clarity

Examiners report

b. ^[N/A] c. ^[N/A] A company designs a spring system for loading ice blocks onto a truck. The ice block is placed in a holder H in front of the spring and an electric motor compresses the spring by pushing H to the left. When the spring is released the ice block is accelerated towards a ramp ABC. When the spring is fully decompressed, the ice block loses contact with the spring at A. The mass of the ice block is 55 kg.



Assume that the surface of the ramp is frictionless and that the masses of the spring and the holder are negligible compared to the mass of the ice block.

On a particular day, the ice blocks experience a frictional force because the section of the ramp from A to B is not cleaned properly. The coefficient of dynamic friction between the ice blocks and the ramp AB is 0.030. The length of AB is 2.0 m.

Determine whether the ice blocks will be able to reach C.

Markscheme

«energy dissipated in friction =» 0.03 × 55 × 9.8 × 2.0 «= 32.3»

hence use result to show that block cannot reach C

FOR EXAMPLE

total energy at C is 670 - 32.3 - 646.8 = -9.1 J

negative value of energy means cannot reach C

Allow ECF from (a)(ii).

Allow calculation of deceleration (a = -0.29 m s^{-2}) using coefficient of dynamic friction. Hence KE available at B = 628 J.

Examiners report

[N/A]

A test model of a two-stage rocket is fired vertically upwards from the surface of Earth. The sketch graph shows how the vertical speed of the rocket varies with time from take-off until the first stage of the rocket reaches its maximum height.



a. (i) Show that the maximum height reached by the first stage of the rocket is about 170 m.

[7]

(ii) On reaching its maximum height, the first stage of the rocket falls away and the second stage fires so that the rocket acquires a constant horizontal velocity of 56 m s⁻¹. Calculate the velocity at the instant when the second stage of the rocket returns to the surface of the Earth. Ignore air resistance.

b. A full-scale version of the rocket reaches a height of 260km when the first stage falls away. Using the data below, calculate the speed at which [3] the second stage of the rocket will orbit the Earth at a height of 260km.

Mass of Earth = 6.0×10^{24} kg

Radius of Earth = 6.4×10^6 m

Markscheme

a. (i) attempt at area under graph;

appropriate triangle 175 m;

a comment about missing area making answer a little less / OWTTE;

(ii)
$$t = \sqrt{\frac{2 \times 170}{9.81}} (= 5.89 \text{s});$$

 $u = 57.8 \text{(ms}^{-1}) \text{ or } u^2 = 3340 \text{m}^2 \text{s}^{-2};$
speed ($= \sqrt{(57.8^2 + 56^2)} = 80.4 \text{ms}^{-1};$
46° to horizontal;

b. $\frac{GMm}{r^2} = \frac{mv^2}{r};$ $v=\sqrt{rac{GM}{r}};$ 7.75×10³ms⁻¹;

Examiners report

a. [N/A]

b. [N/A]

Part 2 Projectile motion

A ball is projected horizontally at 5.0ms⁻¹ from a vertical cliff of height 110m. Assume that air resistance is negligible and use *g*=10ms⁻².



a. (i) State the magnitude of the horizontal component of acceleration of the ball after it leaves the cliff.

(ii) On the axes below, sketch graphs to show how the horizontal and vertical components of the velocity of the ball, v_x and v_y , change with time *t* until just before the ball hits the ground. It is not necessary to calculate any values.



[3]

- (ii) Calculate the horizontal distance travelled by the ball until just before it reaches the ground.
- c. Another projectile is launched at an angle to the ground. In the absence of air resistance it follows the parabolic path shown below.



On the diagram above, sketch the path that the projectile would follow if air resistance were not negligible.

Markscheme

a. (i) zero;

(ii) horizontal: any horizontal line not on *t*-axis (accept lines above or below *t*-axis); vertical: any diagonal line starting at origin (accept positive or negative gradients);



b. (i) $s_y=rac{1}{2}a_yt^2\Rightarrow 110=rac{1}{2} imes 10 imes t^2;$

t=4.690≈4.7s;

(ii) s_x=u_xt=5.0×4.690; s_x=23m;

c. lower maximum height;

lower horizontal range;

asymmetrical with horizontal range before maximum height more than horizontal

range after maximum height;

[3]



Examiners report

a. ^[N/A] b. ^[N/A] c. ^[N/A]