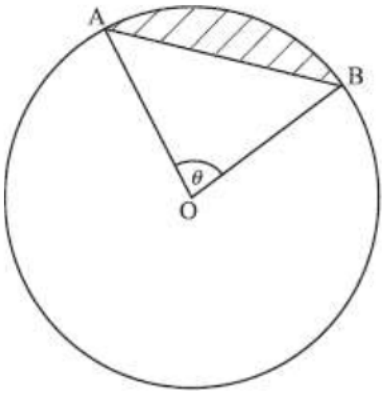
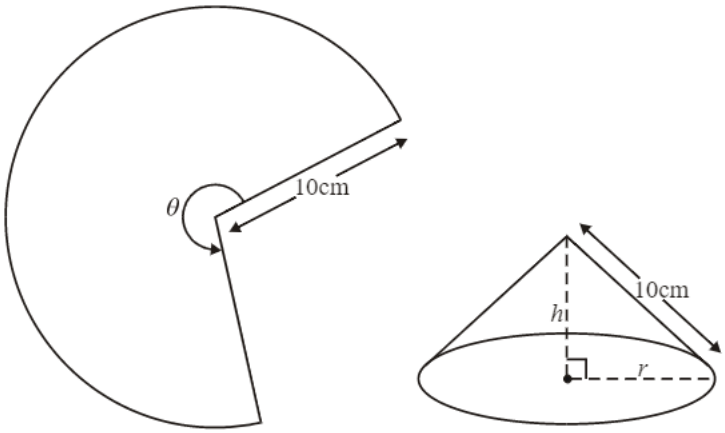
Sectors, Segments, Arcs – Extra Practice

1 The diagram shows a circle centre O and radius 1, with $\hat{AOB} = \theta$, $\theta \neq 0$. The area of ΔAOB is three times the shaded area.

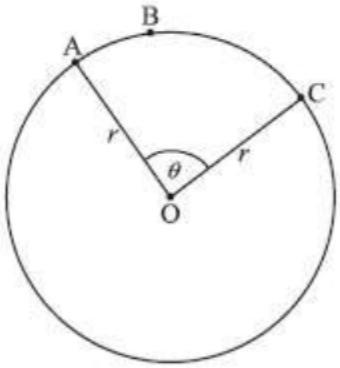


Find the value of θ.

2 The diagrams show a circular sector of radius 10 cm and angle θ radians which is formed into a cone of slant height 10 cm. The vertical height h of the cone is equal to the radius r of its base. Find the angle θ radians.



3 The following diagram shows a circle with radius r and centre O. The points A, B and C are on the circle and $\hat{AOC} = \theta$.



The area of sector OABC is $\frac{4}{3}\pi$ and the length of arc ABC is $\frac{2}{3}\pi$.

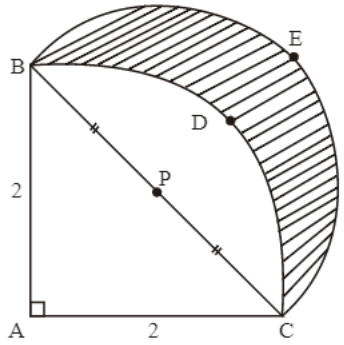
Find the value of r and of θ.

4. The diagram below shows a triangle and two arcs of circles.

The triangle ABC is a right-angled isosceles triangle, with AB = AC = 2. The point P is the midpoint of [BC].

The arc BDC is part of a circle with centre A.

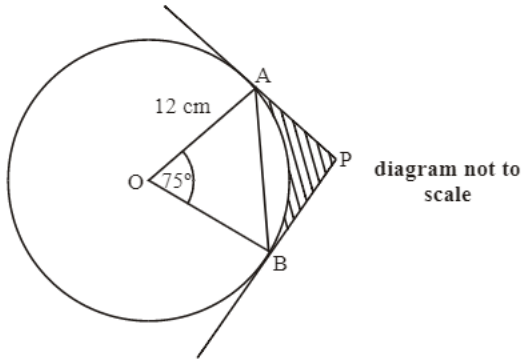
The arc BEC is part of a circle with centre P.



(a) Calculate the area of the segment BDCP.

(b) Calculate the area of the shaded region BECD.

5 The diagram below shows a circle, centre O, with a radius 12 cm. The chord AB subtends at an angle of 75° at the centre. The tangents to the circle at A and at B meet at P.



(a) Using the cosine rule, show that the length of AB is $12\sqrt{2(1-\cos 75°)}$.

(2)

(b) Find the length of BP.

(3)

(c) Hence find

(i) the area of triangle OBP;

(ii) the area of triangle ABP.

(4)

(d) Find the area of **sector** OAB.

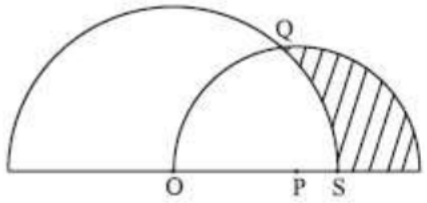
(2)

(e) Find the area of the shaded region.

(2)

(Total 13 marks)

6 The following diagram shows two semi-circles. The larger one has centre O and radius 4 cm. The smaller one has centre P, radius 3 cm, and passes through O. The line (OP) meets the larger semi-circle at S. The semi-circles intersect at Q.



(a) (i) Explain why OPQ is an isosceles triangle.

(ii) Use the cosine rule to show that $\cos \hat{OPQ} = \frac{1}{9}$.

(iii) Hence show that $\sin \hat{OPQ} = \frac{\sqrt{80}}{9}$.

(iv) Find the area of the triangle OPQ.

(7)

(b) Consider the smaller semi-circle, with centre P.

(i) Write down the size of $\hat{OPQ}$.

(ii) Calculate the area of the sector OPQ.

(3)

(c) Consider the larger semi-circle, with centre O. Calculate the area of the sector QOS.

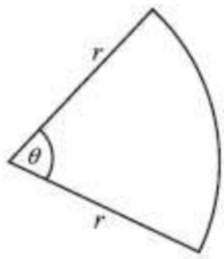
(3)

(d) Hence calculate the area of the shaded region.

(4)

(Total 17 marks)

7 The following diagram shows a sector of a circle of radius r cm, and angle θ at the centre. The perimeter of the sector is 20 cm.



(a) Show that $\theta = \frac{20-2r}{r}$.

(b) The area of the sector is 25 cm^2. Find the value of r.