Relativistic Energy Practice Problems

1 Momentum and Energy

1. The total energy of a proton that has been accelerated in a synchrotron is 30 times its mass energy mc^2, In terms of m, find the proton's kinetic energy and the magnitude of its momentum. Also find its velocity expressed as a fraction of the speed of light.
2. The "Tevatron" at Fermilab near Chicago accelerates protons to energies of $1TeV = 10^{12}eV$ Find the speed of the protons. (Approximate $mc^2 \approx 1GeV = 10^9 eV$ for protons, and use both the form $v/c = 1 - \epsilon$ and the binomial approximation.)
3. A photon of energy $E = 5000MeV$ is absorbed by a nucleus of mass M_0 originally at rest. Afterward, the excited nucleus has mass M and is moving at (5/13c). Using MeV units, find
    (a) the momentum of mass M
    (b) the mass M
    (c) the mass M_0.

1.1 With Time Dilation and Length Contraction

1. Calculate the energy required to accelerate a 1.0×10^8kg spaceship to a speed such that in the Earth frame of reference a passenger inside ages only 10% as fast as we do on Earth. Compare this energy with the mass energy of the ship.
2. A Λ particle of mass energy 1116MeV has kinetic energy 1116MeV in the lab and travels 13.7cm before decaying. How long did it live in its own rest frame?
3. An unstable particle of mass m decays in time $\tau = 10^{-10}s$ in its own rest frame. If its energy is $E = 1000mc^2$? in the lab, how far (in meters) will it move before decaying?
4. A particular K^+ meson, with mass energy 494 MeV, decays in$1.2 \times 10^{-8}s$ in its rest frame. What is its energy in the lab frame, if it travels $10.0m$ before decaying?
5. The kinetic energy of a particular newly created particle in the laboratory happens to equal its mass energy. If it travels a distance d before decaying, find an expression for how long it lived in its own rest frame.

2 Conservation of Energy and Momentum

1. A free neutron at rest decays into a proton,an electron, and an antineutrino. The mass energies of the proton and electron are 938.3MeV and 0.5MeV, respectively, and the mass energy of the antineutrino (thought to be a small fraction of an eV) is negligible on the scale of MeV. Find the total kinetic energy of the decay products.
2. An antineutrino of energy 1.5MeV strikes a proton at rest. Is the reaction $\bar{v} + p \rightarrow n + e^+$ possible for this antineutrino?
3. A particle of mass $M_1 = 1440\text{MeV/c}^2$ is moving in the laboratory with speed (5/13). It decays into two particles with masses M_2 and M_3, where M_2 is at rest in the lab, and M3 moves at speed $(12c/13)$. Find M_2 and M_3 in units of MeV/c^2.
4. A particle of mass M is moving to the right at$V = (3/5)c$, when it suddenly disintegrates into two particles of mass$m = (2/5)M$, as shown. Find the vertical component of velocity v_y for each particle of mass m.

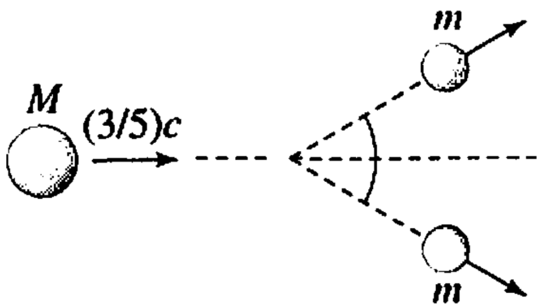

Figure 2.0.1: Collision

3 Energy and Momentum of a Photon

1. A gamma-ray photon emitted from an excited ^{7}Li nucleus has an energy of 0.478MeV. Calculate its momentum in MeV/c, and its frequency in inverse seconds.

4 Tips

- When solving problems related to collisions(problems involving momentum and energy problems in general), think about the conservation of momentum and energy and write out the equations even if you are confused. This will help you be more explicit about the problems and get helpful information.
- When a problem asks you to solve for speed or velocity, be aware that the Lorentz factor γ is expressed in terms of speed. In other words, you can know an object's speed by knowing the relativistic factor.
- Draw diagrams! Special relativity is about frames and changing frames. There are primed and unprimed frames, and observers in different frames view events. All of these need to be clarified, and you will get lost if you are unclear about the system you use.
- Calculate the relativistic factor γ first and substitute its value directly into the equation you use when solving problems. Try to avoid too much variables, such as $\frac{1}{\sqrt{1-v^2/c^2}}$ (just use γ), to avoid calculation mistakes.

Problems are collected from Helliwell and Helliwell (2010). For more practicing problems, please see Science Knowledge or go to the next url: `https://scienceknowledge.webador.com/`. The solution can be found here.

References

T.M Helliwell and T.M Helliwell. page 108–111. University Science Books, 2010.