5.2 Heating effect of electric currents

This section will introduce the main ideas behind electric circuits. We begin by discussing how the movement of electrons inside conductors (i.e. electric current) results in heating of the conductor.

Learning objectives

- Understand how current in a circuit component generates thermal energy.
- Find current, potential difference and power dissipated in circuit components.
- Define and understand electric resistance.
- Describe Ohm's law.
- Investigate factors that affect resistance.
- Apply Kirchhoff's laws to more complicated circuits.

Collisions of electrons with lattice atoms

The effect of an electric field within a conductor, for example in a metal wire, is to accelerate the free electrons. The electrons therefore gain kinetic energy as they move through the metal. The electrons suffer **inelastic** collisions with the metal atoms, which means they lose energy to the atoms of the wire. The electric field will again accelerate the electrons until the next collision, and this process repeats. In this way, the electrons keep providing energy to the atoms of the wire. The atoms in the wire vibrate about their equilibrium positions with increased kinetic energy. This shows up **macroscopically** as an increase in the temperature of the wire.

Electric resistance

In Subtopic **5.1** we stressed that whenever there is a potential difference there must also be an electric field. So when a potential difference is established at the ends of a conductor, an electric field is established within the conductor that forces electrons to move, i.e. creating an electric current (Figure **5.13a**). Now, when the same potential difference is established at the ends of different conductors, the size of the current is different in the different conductors. What determines how much current will flow for a given potential difference is a property of the conductor called its **electric resistance**.

The electric resistance R of a conductor is defined as the potential difference V across its ends divided by the current I passing through it:

$$R = \frac{V}{I}$$

The unit of electric resistance is the volt per ampere. This is defined to be the ohm, symbol Ω.

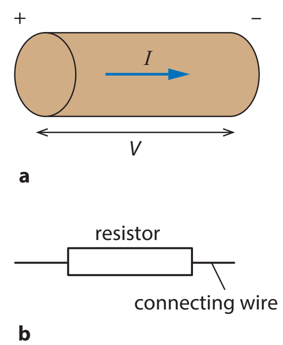


Figure 5.13 **a** The potential difference V across the ends of the conductor creates an electric field within the conductor that forces a current I through the conductor. **b** How we represent a resistor and connecting wires in a circuit diagram.

The electric resistance of conducting wires is very small so it is a good approximation to ignore this resistance. Conducting wires are represented by thin line segments in diagrams. Conductors whose resistance cannot be neglected are denoted by boxes; they are called **resistors** (Figure **5.13b**).

In 1826, the German scientist Georg Ohm (1789–1854) discovered that, when the temperature of most metallic conductors is kept constant, the current through the conductor is proportional to the potential difference across it:

$$I \propto V$$

This statement is known as **Ohm's law**.

Materials that obey Ohm's law have a constant resistance at constant temperature. For these ohmic materials, a graph of I versus V gives a straight line through the origin (Figure **5.14a**).

A filament light bulb will obey Ohm's law as long as the current through it is small. As the current is increased, the temperature of the filament increases and so does the resistance. Other devices, such as the **diode** or a **thermistor**, also deviate from Ohm's law. Graphs of current versus potential difference for these devices are shown in Figure **5.14**.

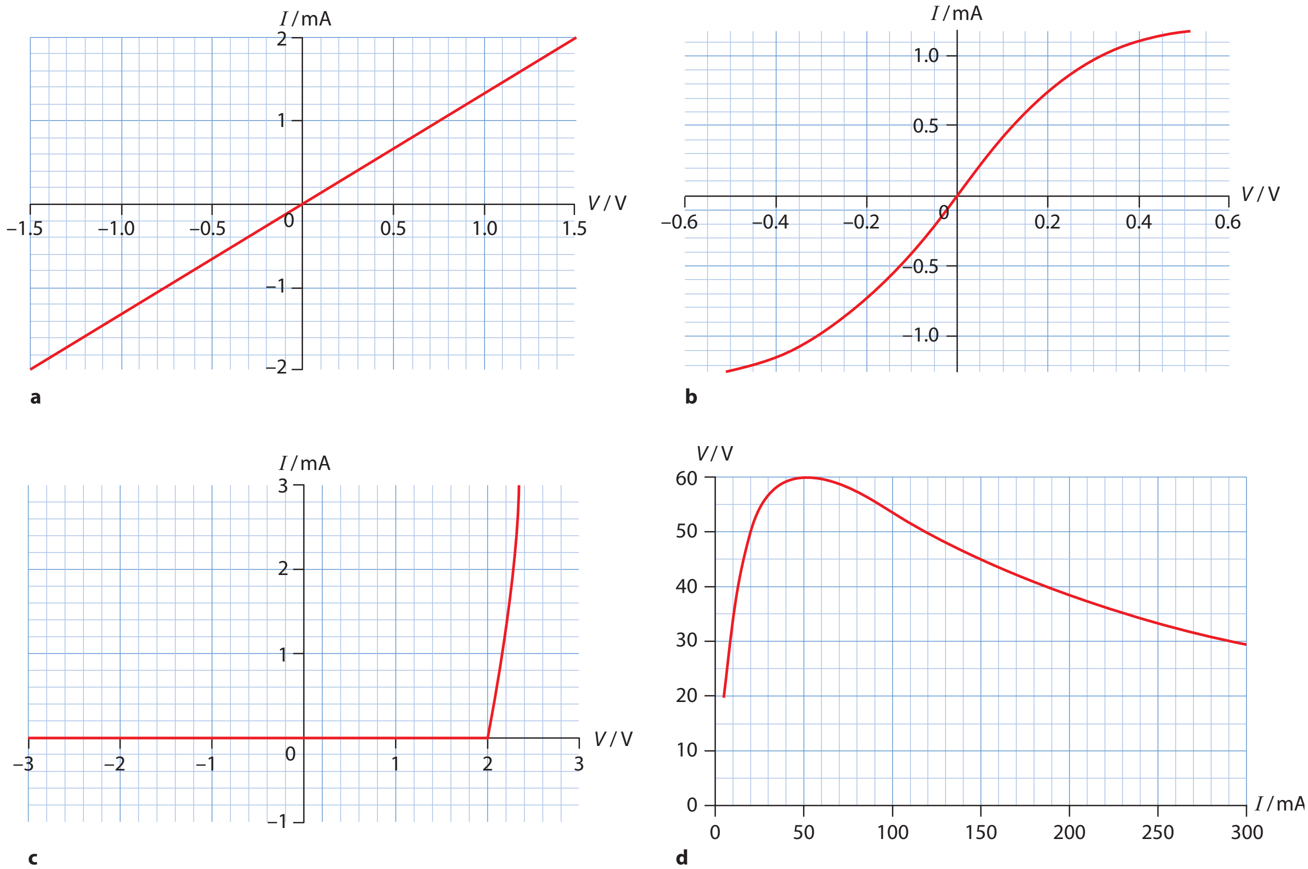


Figure 5.14 Graph **a** shows the current–potential difference graph for a material that obeys Ohm's law. The graphs for **b** a lamp filament, **c** a diode and **d** a thermistor show that these devices do not obey this law. (Notice that for the thermistor we plot voltage versus current.)

In the first graph for the ohmic material, no matter which point on the graph we choose (say the one with voltage 1.2 V and current 1.6 mA), the resistance is always the same:

$$R = \frac{1.2}{1.6 \times 10^{-3}} = 750\,\Omega$$

However, looking at the graph in Figure **5.14b** (the lamp filament), we see that at a voltage of 0.2 V the current is 0.8 mA and so the resistance is:

$$R = \frac{0.2}{0.8 \times 10^{-3}} = 250\,\Omega$$

At a voltage of 0.3 V the current is 1.0 mA and the resistance is:

$$R = \frac{0.3}{1.0 \times 10^{-3}} = 300\,\Omega$$

We see that as the current in the filament increases the resistance increases, so Ohm's law is not obeyed. This is a **non-ohmic** device.

Experiments show that three factors affect the resistance of a wire kept at constant temperature. They are:

- the nature of the material
- the length of the wire
- the cross-sectional area of the wire.

For most metallic materials, an increase in the temperature results in an increase in the resistance.

It is found from experiment that the electric resistance R of a wire (at fixed temperature) is proportional to its length L and inversely proportional to the cross-sectional area A:

$$R = \rho \frac{L}{A}$$

The constant ρ is called **resistivity** and depends on the material of the conductor and the temperature. The unit of resistivity is $\Omega\,\text{m}$.

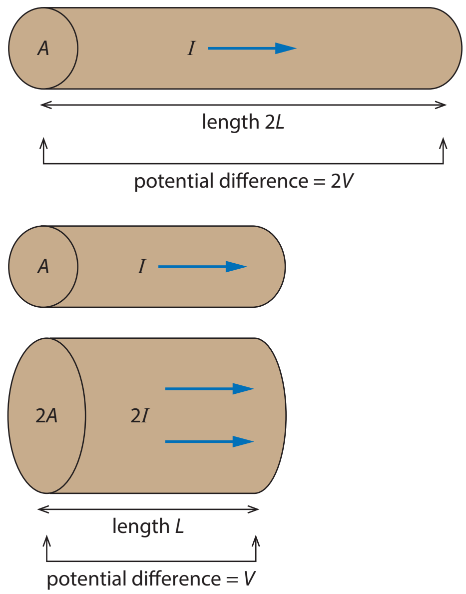


Figure 5.15 The effect of change in length L and cross-sectional area A on the current flowing in a wire.

The formula for resistance shows that if we double the cross-sectional area of the conductor the resistance halves; and if we double the length, the resistance doubles. How do we understand these results? Figure **5.15** shows that if we double the cross-sectional area A of a wire, the current in the metal for the same potential difference will double as well (recall that $I = nAvq$). Since $R = \frac{V}{I}$, the resistance R halves. What if we double the length L of the wire? The work done to move a charge q can be calculated two ways: one is through $W = qV$. The other is through $W = FL = qEL$. So, if L doubles the potential difference must also double. The current stays the same and so the resistance R doubles.

For most metallic conductors, increasing the temperature increases the resistance. With an increased temperature the atoms of the conductor vibrate more and this increases the number of collisions per second. This in turn means that the average distance travelled by the electrons between collisions is reduced, i.e. the drift speed is reduced. This means the current is reduced and so resistance increases.

Worked example

5.10 The resistivity of copper is $1.68 \times 10^{-8}\,\Omega\,\text{m}$. Calculate the length of a copper wire of diameter 4.00 mm that has a resistance of $5.00\,\Omega$.

We use $R = \rho\frac{L}{A}$ to get $L = \frac{RA}{\rho}$ and so:

$$L = \frac{5.00 \times \pi \times (2.00 \times 10^{-3})^2}{1.68 \times 10^{-8}}$$

$$L = 3739\,\text{m}$$

The length of copper wire is about 3.74 km.

Exam tip
Do not confuse diameter with radius.

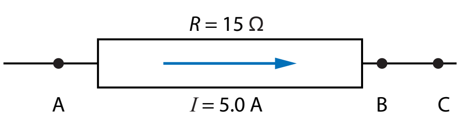


Figure 5.16 There is a voltage across points A and B and zero voltage across B and C.

Voltage

The defining equation for resistance, $R = \frac{V}{I}$, can be rearranged in terms of the potential difference V:

$$V = IR$$

This says that if there is a current through a conductor that has resistance, i.e. a resistor, then there must be a potential difference across the ends of that resistor. The term **voltage** is commonly used for the potential difference at the ends of a resistor.

Figure **5.16** shows part of a circuit. The current is 5.0 A and the resistance is $15\,\Omega$. The voltage across the resistor is given by $V = IR = 5.0 \times 15 = 75\,\text{V}$. The resistance between B and C is zero, so the voltage across B and C is zero.

Electric power

We saw earlier that whenever an electric charge q is moved from one point to another when there is a potential difference V between these points, work is done. This work is given by $W = qV$.

Consider a resistor with a potential difference V across its ends. Since power is the rate of doing work, the power P dissipated in the resistor in moving a charge q across it in time t is:

$$P = \frac{\text{work done}}{\text{time taken}}$$

$$P = \frac{qV}{t}$$

But $\frac{q}{t}$ is the current I in the resistor, so the **power** is given by:

$$P = IV$$

This power manifests itself in thermal energy and/or work performed by an electrical device (Figure 5.17). We can use $R = \frac{V}{I}$ to rewrite the formula for power in equivalent ways:

$$P = IV = RI^2 = \frac{V^2}{R}$$

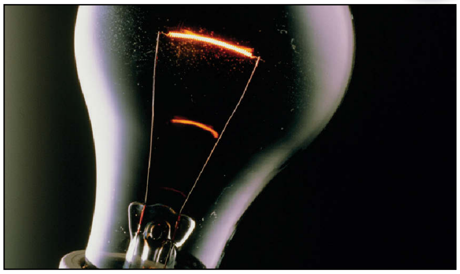


Figure 5.17 The metal filament in a light bulb glows as the current passes through it. It is also very hot. This shows that electrical energy is converted into both thermal energy and light.

Worked examples

5.11 A resistor of resistance 12 Ω has a current of 2.0 A flowing through it. How much energy is generated in the resistor in one minute?

The power generated in the resistor is:

$$P = RI^2$$

$$P = 12 \times 4 = 48\,\text{W}$$

Thus, in one minute (60 s) the energy E generated is:

$$E = 48 \times 60\,\text{J} = 2.9 \times 10^3\,\text{J}$$

Electrical devices are usually rated according to the power they use. A light bulb rated as 60 W at 220 V means that it will dissipate 60 W when a potential difference of 220 V is applied across its ends. If the potential difference across its ends is anything other than 220 V, the power dissipated will be different from 60 W.

5.12 A light bulb rated as 60 W at 220 V has a potential difference of 110 V across its ends. Find the power dissipated in this light bulb.

Exam tip
The power of the light bulb is 60 W only when the voltage across it is 220 V. If we change the voltage we will change the power.

Let R be the resistance of the light bulb and P the power we want to find. Assuming R stays constant (so that it is the same when 220 V and 110 V are applied to its ends), we have:

$$P = \frac{110^2}{R} \quad \text{and} \quad 60 = \frac{220^2}{R}$$

Dividing the first equation by the second, we find:

$$\frac{P}{60} = \frac{110^2}{220^2}$$

This gives:

$$P = 15\,\text{W}$$

> **Exam tip**
> You must understand the ideas that keep coming up in this topic: to make charges move in the same direction we need an electric field to exert forces on the charges. To have an electric field means there must be a potential difference. So something must provide that potential difference.

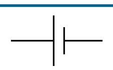
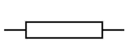
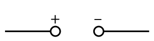
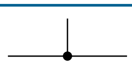
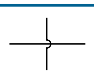
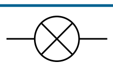
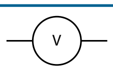
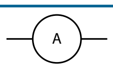
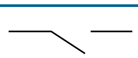
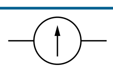
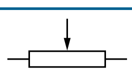
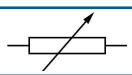
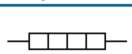
Symbol	Component name
	connection lead
	cell
	battery of cells
	resistor
	dc power supply
	ac power supply
	junction of conductors
	crossing conductors (no connection)
	lamp
	voltmeter
	ammeter
	switch
	galvanometer
	potentiometer
	variable resistor
	heating element

Table 5.1 Names of electrical components and their circuit symbols.

Electromotive force (emf)

The concept of emf will be discussed in detail in Subtopic **5.3**. Here we need a first look at emf in order to start discussing circuits. Charges need to be pushed in order to drift in the same direction inside a conductor. To do this we need an electric field. To have an electric field requires a source of potential difference. Cells use the energy from chemical reactions to provide potential difference. Figure **5.18** shows a simple circuit in which the potential difference is supplied by a battery – a battery is a collection of cells. The symbols for cells and batteries are shown in Table **5.1**.

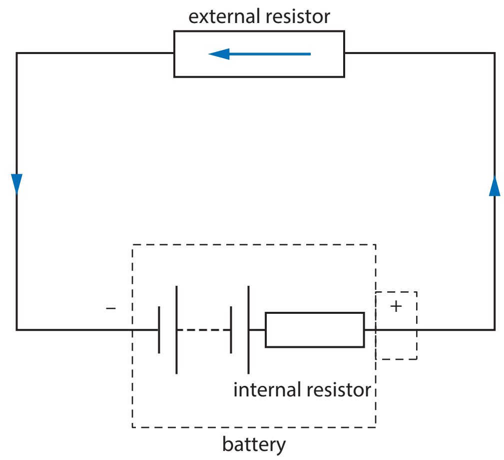


Figure 5.18 A simple circuit consisting of a battery, connecting wires and a resistor. Note that the battery has internal resistavnce. The current enters the circuit from the positive pole of the battery.

We define **emf** as the work done per unit charge in moving charge across the battery terminals. As we will see in Subtopic **5.3**, emf is the potential difference across the battery terminals when the battery has no internal resistance. Emf is measured in volts. Emf is also the power provided by the battery per unit current:

$$\varepsilon = \text{emf} = \frac{W}{q} = \frac{P}{I}$$

This definition is very useful when discussing circuits.

Simple circuits

We have so far defined emf, voltage, resistance, current and power dissipated in a resistor. This means that we are now ready to put all these ideas together to start discussing the main topic of this chapter, electric circuits. The circuits we will study at Standard Level will include cells and batteries, connecting wires, ammeters (to measure current) and voltmeters (to measure voltage). The symbols used for these circuit components are shown in Table **5.1**. In Topic **11** we will extend things so as to include another type of circuit element, the capacitor.

We start with the simplest type of circuit – a single-loop circuit, as shown in Figure 5.19. The current enters the circuit from the positive terminal of the cell. The direction of the current is shown by the blue arrow. The terminals of the cell are directly connected to the ends of the resistor (there is no intervening internal resistor). Therefore the potential difference at the ends of the resistor is 12 V. Using the definition of resistance we write $R = \frac{V}{I}$, i.e. $24 = \frac{12}{I}$, giving the current in the circuit to be $I = 0.5$ A.

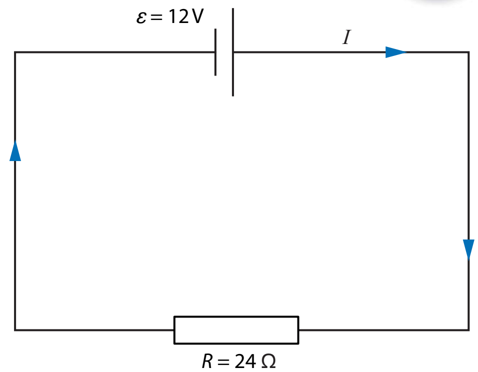


Figure 5.19 A simple one-loop circuit with one cell with negligible internal resistance and one resistor.

Resistors in series

Figure 5.20 shows part of a simple circuit, but now there are three resistors connected in series. Connecting resistors in **series** means that there are no junctions in the wire connecting any two resistors and so the current through all of them is the same. Let I be the common current in the three resistors.

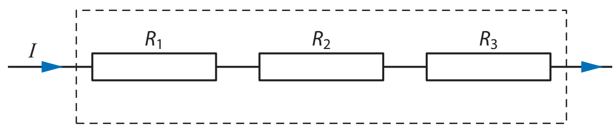


Figure 5.20 Three resistors in series.

The potential difference across each of the resistors is:

$$V_1 = IR_1, \quad V_2 = IR_2 \quad \text{and} \quad V_3 = IR_3$$

The sum of the potential differences is thus:

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

If we were to replace the three resistors by a single resistor of value $R_1 + R_2 + R_3$ (in other words, if we were to replace the contents of the dotted box in Figure 5.20 with a single resistor, as in the circuit shown in Figure 5.21), we would not be able to tell the difference. The same current comes into the dotted box and the same potential difference exists across its ends.

We thus define the equivalent or total resistance of the three resistors of Figure 5.21 by:

$$R_{total} = R_1 + R_2 + R_3$$

If more than three were present, we would simply add all of them. Adding resistors in series increases the total resistance.

In a circuit, a combination of resistors like those in Figure 5.21 is equivalent to the single total or equivalent resistor. Suppose we now connect the three resistors to a battery of negligible internal resistance and emf equal to 24 V. Suppose that $R_1 = 2.0\,\Omega$, $R_2 = 6.0\,\Omega$ and $R_3 = 4.0\,\Omega$. We replace the three resistors by the equivalent resistor of $R_{total} = 2.0 + 6.0 + 4.0 = 12\,\Omega$. We now observe that the potential difference

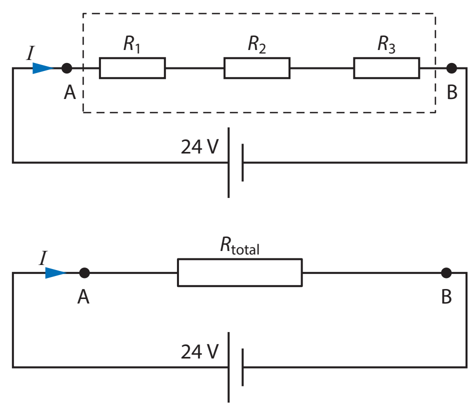


Figure 5.21 The top circuit is replaced by the equivalent circuit containing just one resistor.

across the equivalent resistor is known. It is simply 24 V and hence the current through the equivalent resistor is found as follows:

$$R=\frac{V}{I}$$

$$\Rightarrow \quad I=\frac{V}{R}=\frac{24}{12}=2.0\,\text{A}$$

This current, therefore, is also the current that enters the dotted box: that is, it is the current in each of the three resistors of the original circuit. We may thus deduce that the potential differences across the three resistors are:

$$V_1=IR_1=4.0\,\text{V}$$

$$V_2=IR_2=12\,\text{V}$$

$$V_3=IR_3=8.0\,\text{V}$$

Resistors in parallel

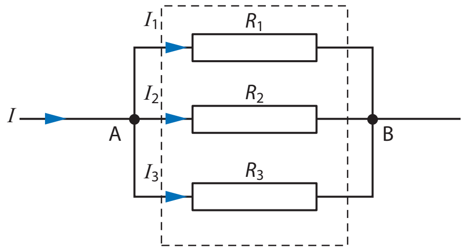


Figure 5.22 Three resistors connected in parallel.

Consider now part of another circuit, in which the current splits into three other currents that flow in three resistors, as shown in Figure **5.22**. The current that enters the junction at A must equal the current that leaves the junction at B, by the law of conservation of charge. The left ends of the three resistors are connected at the same point and the same is true for the right ends. This means that three resistors have the same potential difference across them. This is called a **parallel connection**.

We must then have that:

$$I=I_1+I_2+I_3$$

This is a consequence of charge conservation. The current entering the junction is I and the currents leaving the junction are I_1, I_2 and I_3 Whatever charge enters the junction must exit the junction and so the sum of the currents into a junction equals the sum of the currents leaving the junction. This is known as **Kirchhoff's current law**.

Kirchhoff's current law (Kirchhoff's first law) states that:

$$\Sigma I_{\text{in}}=\Sigma I_{\text{out}}$$

Let V be the common potential difference across the resistors. Then:

$$I_1=\frac{V}{R_1}, \quad I_2=\frac{V}{R_2} \quad \text{and} \quad I_3=\frac{V}{R_3}$$

and so:

$$I=\frac{V}{R_1}+\frac{V}{R_2}+\frac{V}{R_3}$$

$$I=V\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

If we replace the three resistors in the dotted box with a single resistor, the potential difference across it would be V and the current through it would be I. Thus:

$$I = \frac{V}{R_{total}}$$

Comparing with the last equation, we find:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The formula shows that the total resistance is ***smaller*** than any of the individual resistances being added.

We have thus learned how to replace resistors that are connected in series or parallel by a single resistor in each case, thus greatly simplifying the circuit.

Exam tip
Adding resistors in series increases the total resistance of a circuit (and so decreases the current leaving the battery). Adding resistors in parallel decreases the total resistance of the circuit (and so increases the current leaving the battery).

More complex circuits

A typical circuit will contain both parallel and **series connections**.

In Figure **5.23**, the two top resistors are in series. They are equivalent to a single resistor of 8.0 Ω. This resistor and the 24 Ω resistor are in parallel, so together they are equivalent to a single resistor of:

$$\frac{1}{R_{total}} = \frac{1}{8.0} + \frac{1}{24} = \frac{1}{6}$$

$$\Rightarrow R_{total} = 6.0\,\Omega$$

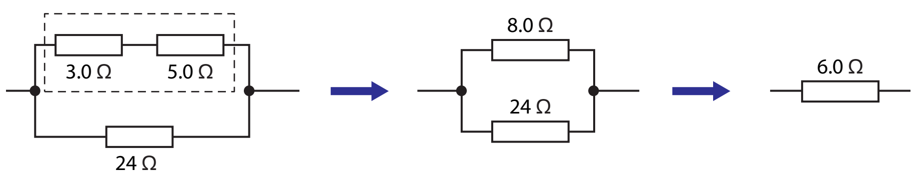


Figure 5.23 Part of a circuit with both series and parallel connections.

Consider now Figure **5.24**. The two top 6.0 Ω resistors are in series, so they are equivalent to a 12 Ω resistor. This, in turn, is in parallel with the other 6.0 Ω resistor, so the left block is equivalent to:

$$\frac{1}{R_{total}} = \frac{1}{12} + \frac{1}{6.0} = \frac{1}{4}$$

$$\Rightarrow R_{total} = 4.0\,\Omega$$

Let us go to the right block. The 12 Ω and the 24 Ω resistors are in series, so they are equivalent to 36 Ω. This is in parallel with the top 12 Ω, so the equivalent resistor of the right block is:

$$\frac{1}{R_{total}} = \frac{1}{36} + \frac{1}{12} = \frac{1}{9}$$

$$\Rightarrow R_{total} = 9.0\,\Omega$$

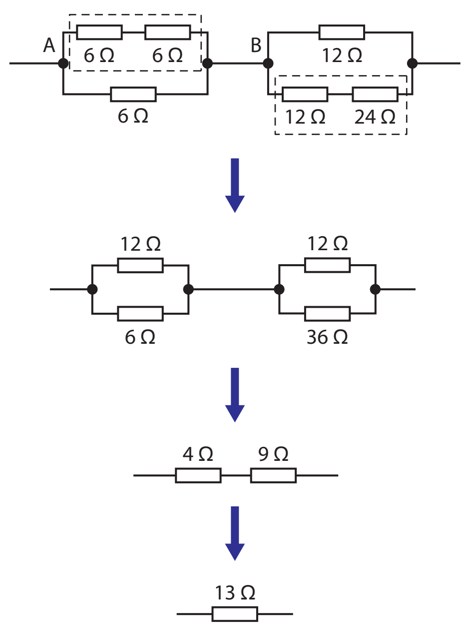


Figure 5.24 A complicated part of a circuit containing many parallel and series connections.

The overall resistance is thus:

$$4.0 + 9.0 = 13\,\Omega$$

Suppose now that this part of the circuit is connected to a source of emf 156 V (and negligible internal resistance). The current that leaves the source is $I = \frac{156}{13} = 12\,\text{A}$. When it arrives at point A, it will split into two parts. Let the current in the top part be I_1 and that in the bottom part I_2. We have $I_1 + I_2 = 12\,\text{A}$. We also have that $12I_1 = 6I_2$, since the top and bottom resistors of the block beginning at point A are in parallel and so have the same potential difference across them. Thus, $I_1 = 4.0\,\text{A}$ and $I_2 = 8.0\,\text{A}$. Similarly, in the block beginning at point B the top current is 9.0 A and the bottom current is 3.0 A.

Worked examples

5.13 **a** Determine the total resistance of the circuit shown in Figure **5.25**.

b Hence calculate the current and power dissipated in each of the resistors.

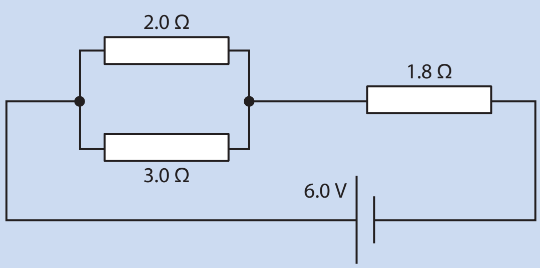


Figure 5.25

a The resistors of 2.0 Ω and 3.0 Ω are connected in parallel and are equivalent to a single resistor of resistance R that may be found from:

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$\Rightarrow\ R = \frac{6}{5} = 1.2\,\Omega$$

In turn, this is in series with the resistance of 1.8 Ω, so the total equivalent circuit resistance is $1.8 + 1.2 = 3.0\,\Omega$.

b The current that leaves the battery is thus:

$$I = \frac{6.0}{3.0} = 2.0\,\text{A}$$

The potential difference across the 1.8 Ω resistor is $V = 1.8 \times 2.0 = 3.6\,\text{V}$, leading to a potential difference across the two parallel resistors of $V = 6.0 - 3.6 = 2.4\,\text{V}$. Thus the current in the 2 Ω resistor is:

$$I = \frac{2.4}{2.0} = 1.2\,\text{A}$$

This leads to power dissipated of:

$$P = RI^2 = 2.0 \times 1.2^2 = 2.9\,\text{W}$$

or $P = \frac{V^2}{R} = \frac{2.4^2}{2.0} = 2.9\,\text{W}$

or $P = VI = 2.4 \times 1.2 = 2.9\,\text{W}$

For the 3 Ω resistor:

$$I = \frac{2.4}{3.0} = 0.80\,\text{A}$$

which leads to power dissipated of $P = RI^2 = 3.0 \times 0.80^2 = 1.9\,\text{W}$

The power in the 1.8 Ω resistor is $P = RI^2 = 1.8 \times 2.0^2 = 7.2\,\text{W}$

5.14 In the circuit of Figure **5.26** the three lamps are identical and may be assumed to have a constant resistance. Discuss what happens to the brightness of lamp A and lamp B when the switch is closed. (The cell is ideal, i.e. it has negligible internal resistance.)

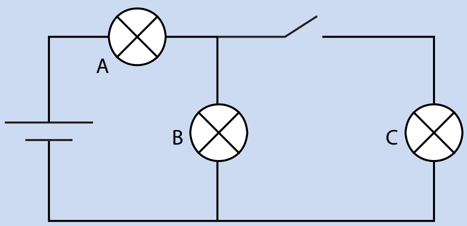


Figure 5.26

Method 1

A mathematical answer. Let the emf of the cell be ε and the resistance of each lamp be R: before the switch is closed A and B take equal current $\frac{\varepsilon}{(2R)}$ and so are equally bright (the total resistance is $2R$). When the switch is closed, the total resistance of the circuit changes and so the current changes as well. The new total resistance is $\frac{3R}{2}$ (lamps B and C in parallel and the result in series with A) so the total current is now $\frac{2\varepsilon}{(3R)}$, larger than before. The current in A is thus greater and so the power, i.e. the brightness, is greater than before. The current of $\frac{2\varepsilon}{(3R)}$ is divided equally between B and C. So B now takes a current $\frac{\varepsilon}{(3R)}$, which is smaller than before. So B is dimmer.

Method 2

The potential difference across A and B before the switch is closed is $\frac{\varepsilon}{2}$ and so A and B are equally bright. When the switch is closed the potential difference across A is double that across B since the resistance of A is double the parallel combination of resistance of B and C. This means that the potential difference across A is $2\frac{\varepsilon}{3}$ and across B it is $\frac{\varepsilon}{3}$. Hence A increases in brightness and B gets dimmer.

5.15 Look at Figure **5.27**. Determine the current in the 2.0 Ω resistor and the potential difference across the two marked points, A and B, when the switch is **a** open and **b** closed.

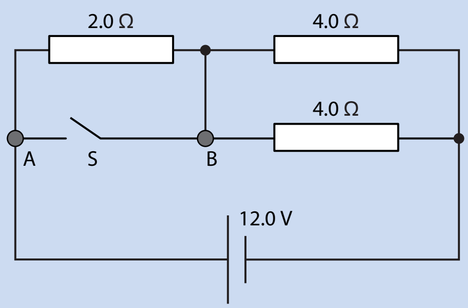


Figure 5.27

a When the switch is open, the total resistance is 4.0 Ω and thus the total current is 3.0 A.

The potential difference across the 2.0 Ω resistor is $2.0 \times 3.0 = 6.0$ V.

The potential difference across points A and B is thus 6.0 V.

b When the switch is closed, no current flows through the 2.0 Ω resistor, since all the current takes the path through the switch, which offers no resistance. (The 2.0 Ω resistor has been shorted out.)

The resistance of the circuit is then 2.0 Ω and the current leaving the battery is 6.0 A.

The potential difference across points A and B is now zero. (There is current flowing from A to B, but the resistance from A to B is zero, hence the potential difference is $6.0 \times 0 = 0$ V.)

5.16 Four lamps each of constant resistance 60 Ω are connected as shown in Figure **5.28**.

a Determine the power in each lamp.

b Lamp A burns out. Calculate the power in each lamp and the potential difference across the burnt-out lamp.

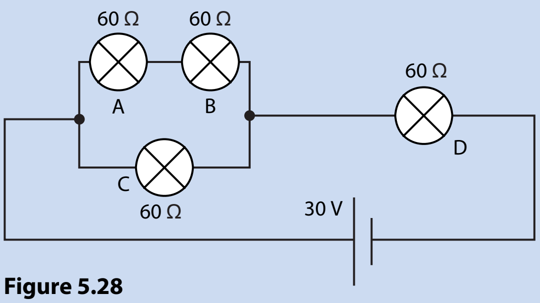


Figure 5.28

a We know the resistance of each lamp, so to find the power we need to find the current in each lamp.

Lamps A and B are connected in series so they are equivalent to one resistor of value $R_{AB} = 60 + 60 = 120$ Ω. This is connected in parallel to C, giving a total resistance of:

$$\frac{1}{R_{ABC}} = \frac{1}{120} + \frac{1}{60}$$

$$\frac{1}{R_{ABC}} = \frac{1}{40}$$

$$\Rightarrow R_{ABC} = 40\ \Omega$$

Finally, this is in series with D, giving a total circuit resistance of:

$$R_{total} = 40 + 60 = 100\,\Omega$$

The current leaving the battery is thus:

$$I = \frac{30}{100} = 0.30\,\text{A}$$

The current through A and B is 0.10 A and that through C is 0.20 A. The current through D is 0.30 A. Hence the power in each lamp is:

$P_A = P_B$

$P_A = 60 \times (0.10)^2 = 0.6\,\text{W}$

$P_C = 60 \times (0.20)^2 = 2.4\,\text{W}$

$P_D = 60 \times (0.30)^2 = 5.4\,\text{W}$

b With lamp A burnt out, the circuit is as shown in **Figure 5.29**.

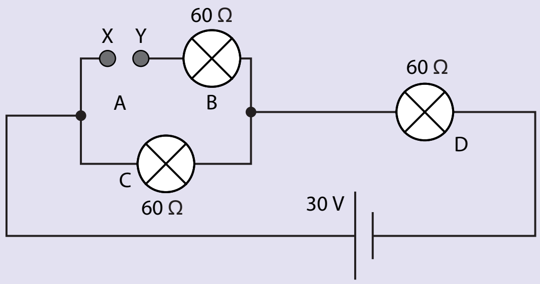


Figure 5.29

Lamp B gets no current, so we are left with only C and D connected in series, giving a total resistance of:

$$R_{total} = 60 + 60 = 120\,\Omega$$

The current is thus $I = \frac{30}{120} = 0.25\,\text{A}$. The power in C and D is thus:

$$P_C = P_D = 60 \times (0.25)^2 = 3.8\,\text{W}$$

We see that D becomes dimmer and C brighter. The potential difference across lamp C is:

$V = IR$

$V = 0.25 \times 60$

$V = 15\,\text{V}$

Lamp B takes no current, so the potential difference across it is zero. The potential difference across points X and Y is the same as that across lamp C, i.e. 15 V.

Multi-loop circuits

In the circuit shown earlier in Figure **5.19**, we found the current in the circuit quite easily. Let us find the current again using a different approach (Figure **5.30**). This approach will use **Kirchhoff's loop law**, which will be stated shortly. This method is best used for complicated multi-loop circuits, but once you master it, you can easily apply it in simple circuits as well, such as the circuit of Figure **5.30**.

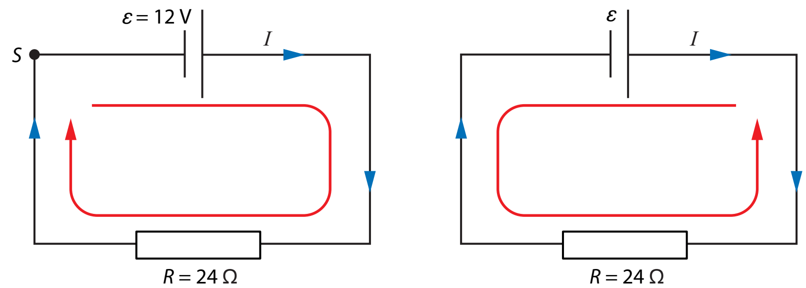


Figure 5.30 Solving a circuit using loops.

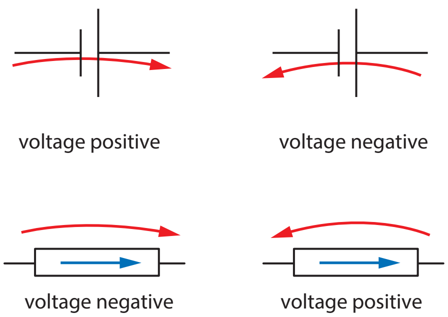


Figure 5.31 The rules for signs of voltages in Kirchhoff's loop law. The blue arrow shows the direction of the current through the resistor.

Draw a loop through the circuit and put an arrow on it (red loop). This indicates the direction in which we will go around the circuit. In the left-hand diagram we have chosen a clockwise direction. Now follow *the loop* starting anywhere; we will choose to start at point S. As we travel along the circuit we calculate the quantity $\sum V$, i.e. the sum of the voltages across each resistor or cell that the loop takes us through, according to the rules in Figure **5.31**.

Follow the clockwise loop. First we go through the cell whose emf is $\varepsilon = 12\,\text{V}$. The loop takes us through the cell from the negative to the positive terminal and so we count the voltage as $+\varepsilon$, i.e. as $+12\,\text{V}$.

Next we go through a resistor. The loop direction is the same as the direction of the current so we take the voltage across the resistor as negative, i.e. $-RI$, which gives $-24I$.

So the quantity $\sum V$ is $12 - 24I$.

> Kirchhoff's loop law (Kirchhoff's second law) states that:
> $\sum V = 0$

The loop law is a consequence of energy conservation: the power delivered into the circuit by the cell is εI. The power dissipated in the resistor is RI^2. Therefore $\varepsilon I = RI^2$. Cancelling one power of the current, this implies $\varepsilon = RI$ or $\varepsilon - RI = 0$ which is simply the Kirchhoff loop law for this circuit. So $12 - 24I = 0$, which allows us to solve for the current as 0.50 A.

Had we chosen a counter-clockwise loop (right-hand diagram in Figure **5.30**) we would find $\sum V = -12 + 24I = 0$, giving the same answer for the current. (This is because we go through the cell from positive to negative so we count the voltage as negative, and we go through resistors in a direction opposite to that of the current so we count the voltage as positive.)

Consider now the circuit with two cells, shown in Figure **5.32**. Again, choose a loop along which to travel through the circuit. We choose a clockwise loop. Draw the arrow for the current. With two cells it is not obvious what the correct direction for the current is. But it does not matter, as we will see. Let's calculate $\sum V$. The cells give $+12-9.0$ since we go through the lower cell from positive to negative. The resistors give $-4.0I-2.0I$ and so $12-9.0-4.0I-2.0I=0$ which gives $I=0.50$ A. The current has come out with a positive sign, so our original guess about its direction is correct. Had the current come out negative, the actual direction would be opposite to what we assumed.

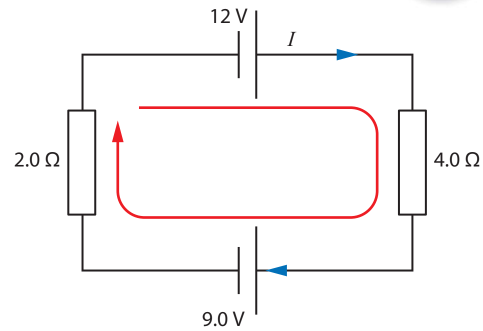


Figure 5.32 A single-loop circuit with two cells.

Figure **5.33** is another example of a circuit with two sources of emf. Each of the four resistors in the circuit of Figure **5.33** is 2.0 Ω. Let's determine the currents in the circuit.

First we assign directions to the currents. Again it does not matter which directions we choose. Call the currents I_1, I_2 and I_3. The loop law states that:

top loop: $\sum V = +6.0 - 2I_1 - 2I_2 - 2I_1 = 0$

bottom loop: $\sum V = +6.0 - 2I_2 - 2I_3 = 0$

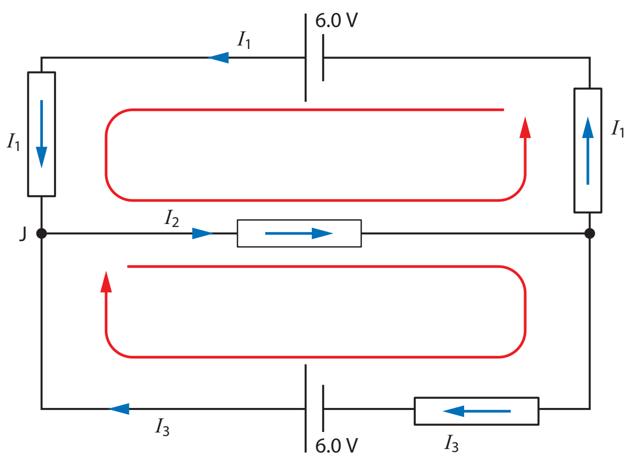


Figure 5.33 A circuit with more than one loop.

From Kirchhoff's current law at junction J (Figure **5.34**):

$$\underbrace{I_1 + I_3}_{\text{current in}} = \underbrace{I_2}_{\text{current out}}$$

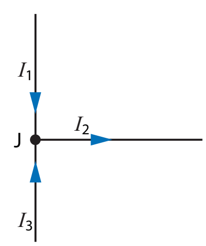


Figure 5.34 Currents at junction J.

So the first loop equation becomes:

$$+6.0 - 2I_1 - 2(I_1 + I_3) - 2I_1 = 0$$

$$\Rightarrow \; 6I_1 + 2I_3 = 6.0$$

$$\Rightarrow \; 3I_1 + I_3 = 3.0$$

and the second loop equation becomes:

$$6.0 - 2(I_1 + I_3) - 2I_3 = 0$$

$$\Rightarrow \; 2I_1 - 4I_3 = 6.0$$

Exam tip
Using the current law we eliminate one of the currents (I_2), making the algebra easier.

So we need to solve the system of equations:

$$3I_1 + I_3 = 3.0$$

$$I_1 - 2I_3 = 3.0$$

Exam tip

1 For each loop in the circuit, give a name to each current in each resistor in the loop and show its direction.
2 Indicate the direction in which the loop will be travelled.
3 Calculate $\sum V$ for every cell or battery and every resistor:
   - For a cell or battery V is counted positive if the cell or battery is travelled from the negative to the positive terminal; negative otherwise.
   - For resistors the value of V is negative $(-RI)$ if the resistor is travelled in the direction of the current; positive otherwise.
4 Set $\sum V = 0$.
5 Repeat for other loops.
6 Use Kirchhoff's current law to reduce the number of currents that need to be found.

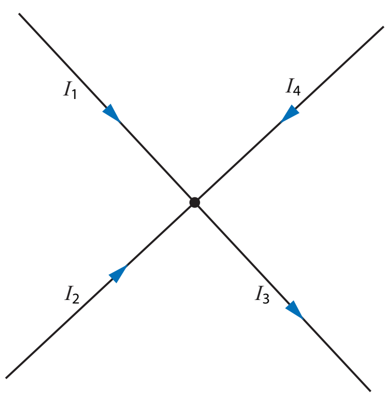


Figure 5.35 Currents entering and leaving a junction.

Solving, $I_1 = 0.60$A. Substituting this into the equations gives $I_3 = 1.2$ A and $I_2 = 1.8$

The IB data booklet writes the Kirchhoff current law as $\sum I = 0$. This is completely equivalent to the version $\sum I_{in} = \sum I_{out}$ used here. In using the booklet's formula you must include a plus sign for a current entering a junction and minus sign for currents leaving. So consider Figure **5.35**.

We would write $I_1 + I_2 + I_4 = I_3$. The booklet formula would write this as $I_1 + I_2 + I_4 - I_3 = 0$, two identical results.

Ammeters and voltmeters

The current through a resistor is measured by an instrument called an **ammeter**, which is connected in series to the resistor as shown in Figure **5.36**.

The ammeter itself has a small electric resistance. An **ideal ammeter**

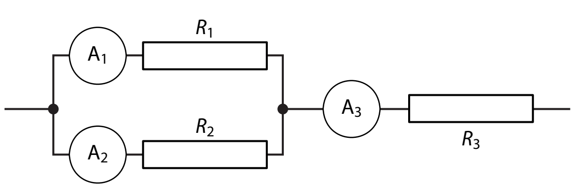


Figure 5.36 An ammeter measures the current in the resistor connected in series to it.

has zero resistance. The potential difference across a device is measured with a **voltmeter** connected in parallel to the device (Figure **5.37**).

An **ideal voltmeter** has infinite resistance, which means that it takes

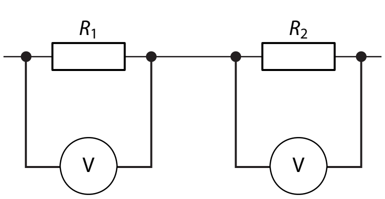


Figure 5.37 A voltmeter is connected in parallel to the device we want to measure the potential difference across.

no current when it is connected to a resistor. Real voltmeters have very high resistance. Unless otherwise stated, ammeters and voltmeters will be assumed to be ideal.

Thus, to measure the potential difference across and current through a resistor, the arrangement shown in Figure **5.38** is used.

Voltmeters and ammeters are both based on a current sensor called a galvanometer. An ammeter has a small resistance connected in parallel to the galvanometer and a voltmeter is a galvanometer connected to a large resistance in series.

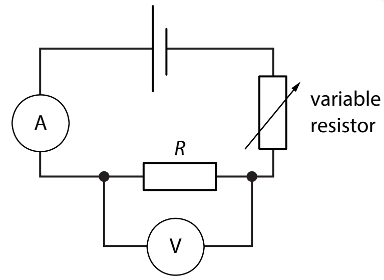


Figure 5.38 The correct arrangement for measuring the current through and potential difference across a resistor. The variable resistor allows the current in the resistor R to be varied so as to collect lots of data for current and voltage.

Worked example

5.17 In the circuit in Figure **5.39**, the emf of the cell is 9.00 V and the internal resistance is assumed negligible. A non-ideal voltmeter whose resistance is 500 kΩ is connected in parallel to a resistor of 500 kΩ.

a Determine the reading of the (ideal) ammeter.

b A student is shown the circuit and assumes, incorrectly, that the voltmeter is ideal. Estimate the resistance the student would calculate if he were to use the current found in **a**.

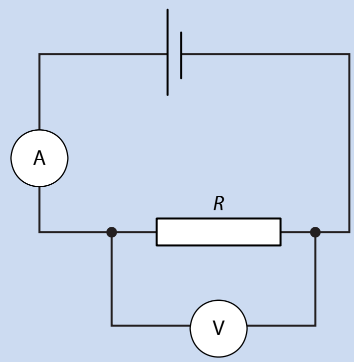


Figure 5.39

a Since the two 500 kΩ resistances are in parallel, the total resistance of the circuit is found from:

$$\frac{1}{R}=\frac{1}{500}+\frac{1}{500}=\frac{1}{250}$$

$$\Rightarrow \quad R=250\,\text{k}\Omega$$

Using $I=\frac{V}{R}$, the current that leaves the battery is:

$$I=\frac{9.0}{250\,000}=3.6\times10^{-5}\,\text{A}$$

$$I=36\,\mu\text{A}$$

This is the reading of the ammeter in the circuit.

b The reading of the voltmeter is 9.0 V. If the student assumes the voltmeter is ideal, he would conclude that the current in the resistor is 36 μA. He would then calculate that:

$R=\frac{V}{I}=\frac{9.0\,\text{V}}{36\,\mu\text{A}}=250\,\text{k}\Omega$ and would get the wrong answer for the resistance.

The potential divider

The circuit in Figure **5.40a** shows a potential divider. It can be used to investigate, for example, the current–voltage characteristic of some device denoted by resistance R. This complicated-looking circuit is simply equivalent to the circuit in Figure **5.40b**. In this circuit, the resistance R_1 is the resistance of the resistor XY from end X to the slider S, and R_2 is the resistance of the resistor from S to end Y. The current that leaves the cell splits at point M. Part of the current goes from M to N, and the rest goes into the device with resistance R. The right end of the resistance R can be connected to a point S on the resistor XY.

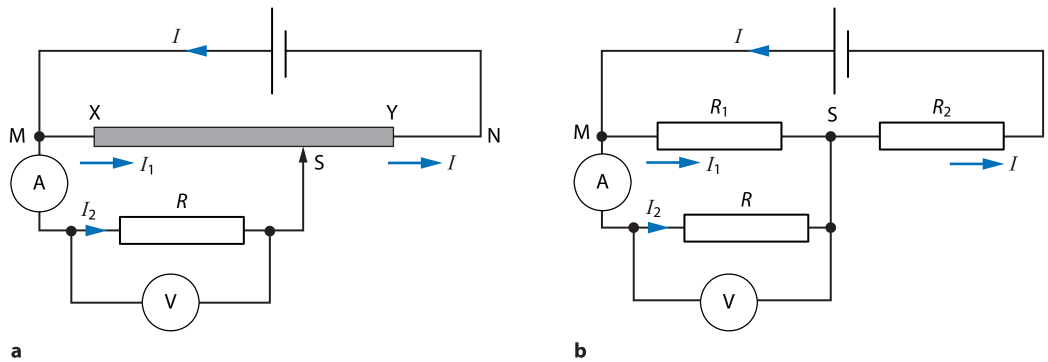


Figure 5.40 **a** This circuit uses a potential divider. The voltage and current in the device with resistance R can be varied by varying the point where the slider S is attached to the variable resistor. **b** The potential divider circuit is equivalent to this simpler-looking circuit.

By varying where the slider S connects to XY, different potential differences and currents are obtained for the device R. The resistor XY could also be just a wire of uniform diameter. One advantage of the potential divider over the conventional circuit arrangement (Figure **5.38**) is that now the potential difference across the resistor can be varied from a minimum of zero volts, when the slider S is placed at X, to a maximum of ε, the emf of the battery (assuming zero internal resistance), by connecting the slider S to point Y. In the conventional arrangement of Figure **5.38**, the voltage can be varied from zero volts up to some maximum value less than the emf.

Worked example

5.18 In the circuit in Figure **5.41**, the battery has emf ε and negligible internal resistance. Derive an expression for the voltage V_1 across resistor R_1 and the voltage V_2 across resistor R_2.

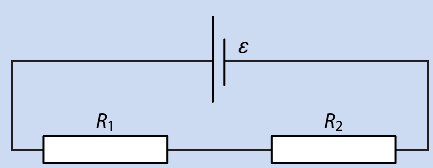


Figure 5.41

Since $I = \dfrac{\varepsilon}{R_1 + R_2}$ and $V = IR$, we have that:

$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)\varepsilon \quad \text{and} \quad V_2 = \left(\frac{R_2}{R_1 + R_2}\right)\varepsilon$$

Nature of science

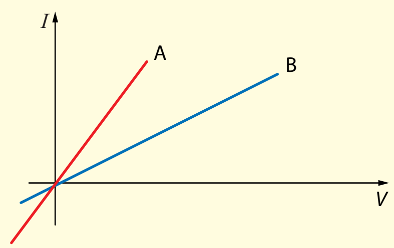
In 1825 in England Peter Barlow proposed a law explaining how wires conducted electricity. His careful experiments using a constant voltage showed good agreement, and his theory was accepted. At about the same time in Germany, Georg Ohm proposed a different law backed up by experimental evidence using a range of voltages. The experimental approach to science was not popular in Germany, and Ohm's findings were rejected. It was not until 1841 that the value of his work was recognised, first in England and later in Germany. In modern science, before research findings are published they are reviewed by other scientists working in the same area (peer review). This would have shown the errors in Barlow's work and given Ohm recognition sooner.

? Test yourself

15 Outline the mechanism by which electric current heats up the material through which it flows.

16 Explain why doubling the length of a wire, at constant temperature, will double its resistance.

17 The graphs show the current as a function of voltage across the same piece of metal wire which is kept at two different temperatures.
- **a** Discuss whether the wire obey Ohm's law.
- **b** Suggest which of the two lines on the graph corresponds to the higher temperature.

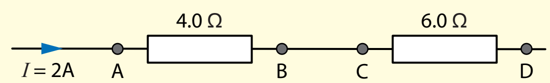


18 The current in a device obeying Ohm's law is 1.5 A when connected to a source of potential difference 6.0 V. What will the potential difference across the same device be when a current of 3.5 A flows in it?

19 A resistor obeying Ohm's law is measured to have a resistance of 12 Ω when a current of 3.0 A flows in it. Determine the resistance when the current is 4.0 A.

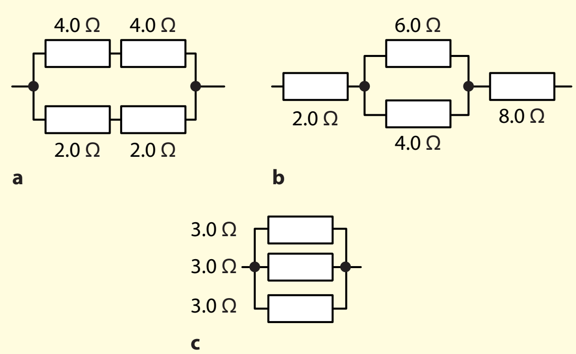
20 The heating element of an electric kettle has a current of 15 A when connected to a source of potential difference 220 V. Calculate the resistance of the heating element.

21 The diagram shows two resistors with a current of 2.0 A flowing in the wire.
- **a** Calculate the potential difference across each resistor.
- **b** State the potential between points B and C.

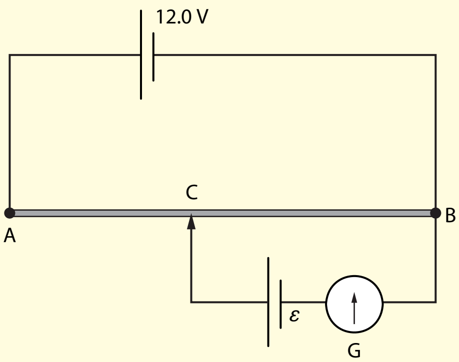


22 The filament of a lamp rated as 120 W at 220 V has resistivity $2.0 \times 10^{-6}\,\Omega\,\text{m}$.
- **a** Calculate the resistance of the lamp when it is connected to a source of 220 V.
- **b** The radius of the filament is 0.030 mm. Determine its length.

23 Determine the total resistance for each of the circuit parts in the diagram.



24 In the potentiometer in the diagram, wire AB is uniform and has a length of 1.00 m. When contact is made at C with BC = 54.0 cm, the galvanometer G shows zero current. Determine the emf of the second cell.

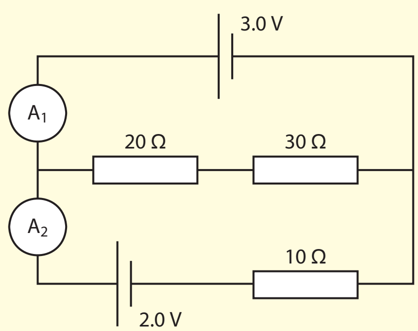


25 In the circuit shown the top cell has emf 3.0 V and the lower cell has emf 2.0 V. Both cells have negligible internal resistance.

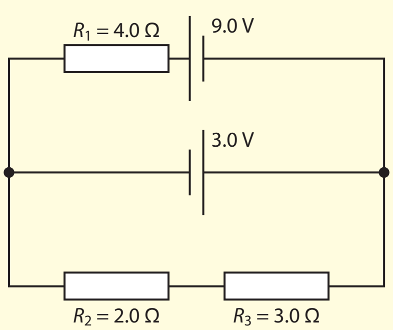
Calculate:

a the readings of the two ammeters

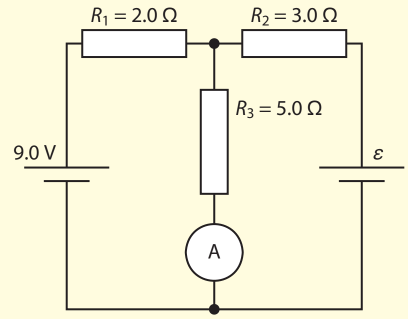
b the potential difference across each resistor.



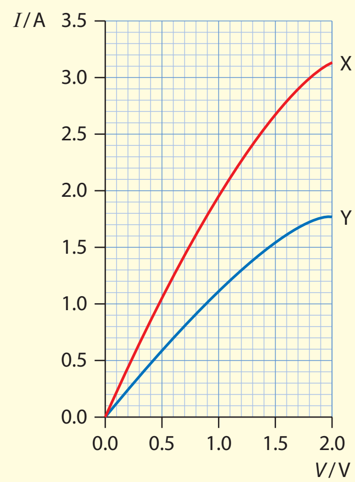
26 Calculate the current in each resistor in the circuit shown in the diagram.



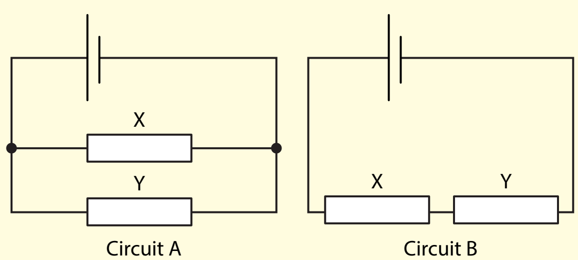
27 In the circuit in the diagram the ammeter reads 7.0 A. Determine the unknown emf ε.



28 Two resistors, X and Y, have I–V characteristics given by the graph.

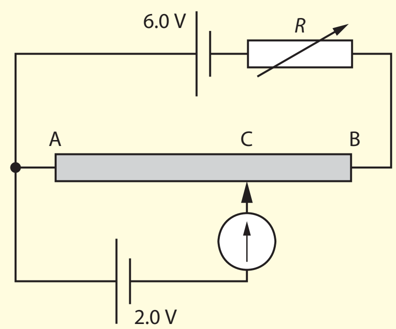


a Circuit A shows the resistors X and Y connected in parallel to a cell of emf 1.5 V and negligible internal resistance. Calculate the total current leaving the cell.



b In circuit B the resistors X and Y are connected in series to the same cell. Estimate the total current leaving the cell in this circuit.

29 The top cell in the circuit in the diagram has emf 6.0 V. The emf of the cell in the lower part of the circuit is 2.0 V. Both cells have negligible internal resistance. AB is a uniform wire of length 1.0 m and resistance 4.0 Ω.
When the variable resistor is set at 3.2 Ω the galvanometer shows zero current. Determine the length AC.



5.3 Electric cells

Batteries are now used to power watches, laptops, cars and entire submarines. Substantial advances in battery technology have resulted in batteries that store more energy, recharge faster and pose smaller environmental dangers.

Learning objectives

- Distinguish between primary and secondary cells.
- Understand the presence of an internal resistance.
- Distinguish between emf and terminal potential difference.

Emf

We have already discussed that electric charges will not drift in the same direction inside a conductor unless a potential difference is established at the ends of the conductor. In a circuit we therefore need a source of potential difference. The most common is the connection of a **battery** in the circuit. (Others include a generator, a thermocouple or a solar cell.) What these sources do is to convert various forms of energy into **electrical energy**.

To understand the function of the battery, we can compare a battery to a pump that forces water through pipes up to a certain height and down again (Figure **5.42**). The pump provides the gravitational potential energy mgh of the water that is raised. The water, descending, converts its gravitational potential energy into thermal energy (frictional losses) and mechanical work. Once the water reaches the pump, its gravitational potential energy has been exhausted and the pump must again perform work to raise the water so that the cycle repeats.

In an electric circuit a battery performs a role similar to the pump's. A battery connected to an outside circuit will force current in the circuit. Thus, the chemical energy of the battery is eventually converted into thermal energy (the current heats up the wires), into mechanical work (the circuit may contain a motor that may be used to raise a load) and into chemical energy again if it is used to charge another battery in the external circuit. Within the battery itself, negative ions are pushed from the negative to the positive terminal and positive ions in the opposite direction. This requires work that must be done on the ions (Figure **5.43**). This work is provided by the **chemical energy** stored in the battery and is released by chemical reactions taking place inside the battery.

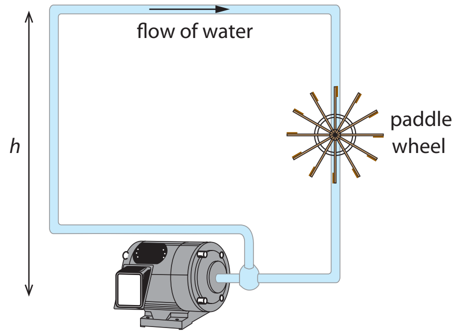


Figure 5.42 In the absence of the pump, the water flow would stop. The work done by the pump equals the work done to overcome frictional forces plus work done to operate devices, such as, for example, a paddle wheel.

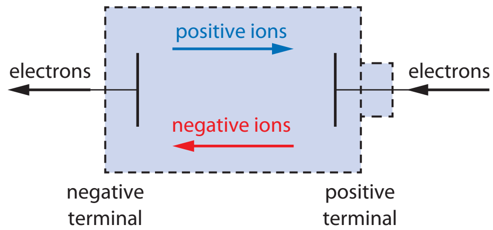


Figure 5.43 Inside the battery, negative ions move from the negative to the positive terminal of the battery. Positive ions move in the opposite direction. In the external circuit, electrons leave the negative battery terminal, travel through the circuit and return to the battery at the positive terminal.