MAA HL

Test on Mathematical Induction

by Christos Nikolaidis

Date: 29 January 2020

Marks: /40

without GDC

Name of student: _____

1. [Maximum mark: 6]

Prove by mathematical induction that $3^{2n}+5^n+2$ is divisible by 4 for $n=0,1,2,3,...$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Turn over

2. [Maximum mark: 9]

(a) Prove by mathematical induction that [7]

$$\sum_{r=2}^{n} r \cdot 2^r = (n-1)2^{n+1} \quad \text{for any integer } n \geq 2.$$

(b) Confirm that the statement is true for $n = 3$. [2]

3. [Maximum mark: 7]

Prove by mathematical induction that

$$(n+1)! \geq 2^n n \qquad \text{for any integer } n \geq 3 .$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Turn over

4. [Maximum mark: 8]

A sequence is defined recursively as follows

$$u_1 = 1$$

$$u_{n+1} = 3u_n + 1$$

(a) Write down the first four terms of the sequence. [2]

(b) Prove by mathematical induction that

$$u_n = \frac{3^n - 1}{2} \qquad \text{for any } n = 1,2,3,\ldots . \qquad [6]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5. [Maximum mark: 10]

(a) Expand $(\sqrt{2}+\sqrt{5})^2$. [1]

(b) Prove by mathematical induction that $(\sqrt{2}+\sqrt{5})^{2n}$ has the form

$a+b\sqrt{10}$, where a is an odd integer and b is an even integer.

for any $n \in Z^+$. [7]

(c) **Hence**, show that $(\sqrt{2}+\sqrt{5})^{2n+1}$ has the form

$c\sqrt{2}+d\sqrt{5}$, where $c,d \in Z$ for any $n \in Z^+$

without using mathematical induction. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Turn over

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....