Topic 3 Part 2 *[221 marks]*

Consider the statement
$p \Rightarrow q$.

If I break my arm, then it will hurt.

1a. Write down in words, the inverse of $p \Rightarrow q$. *[2 marks]*

1b. Complete the following truth table. *[2 marks]*

p	q	$p \Rightarrow q$	Inverse of $p \Rightarrow q$	Converse of $p \Rightarrow q$
T	T	T		
T	F	F		
F	T	T		
F	F	T		

1c. State whether the converse and the inverse of an implication are logically equivalent. *[2 marks]*

Justify your answer.

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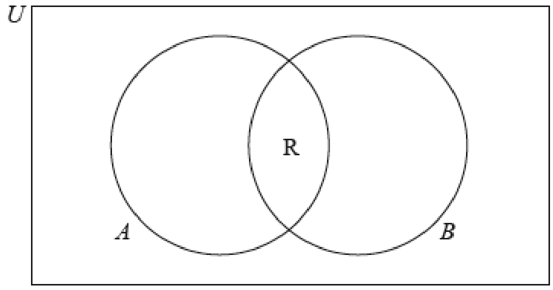
Tuti has the following polygons to classify: rectangle (R), rhombus (H), isosceles triangle (I), regular pentagon (P), and scalene triangle (T).

In the Venn diagram below, set A consists of the polygons that have at least one pair of parallel sides, and set B consists of the polygons that have at least one pair of equal sides.



2a. Complete the Venn diagram by placing the letter corresponding to each polygon in the appropriate region. For example, R has already been placed, and represents the rectangle. *[3 marks]*

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2b. State which polygons from Tuti’s list are elements of *[3 marks]*

(i) $A \cap B$;

(ii) $(A \cup B)'$.

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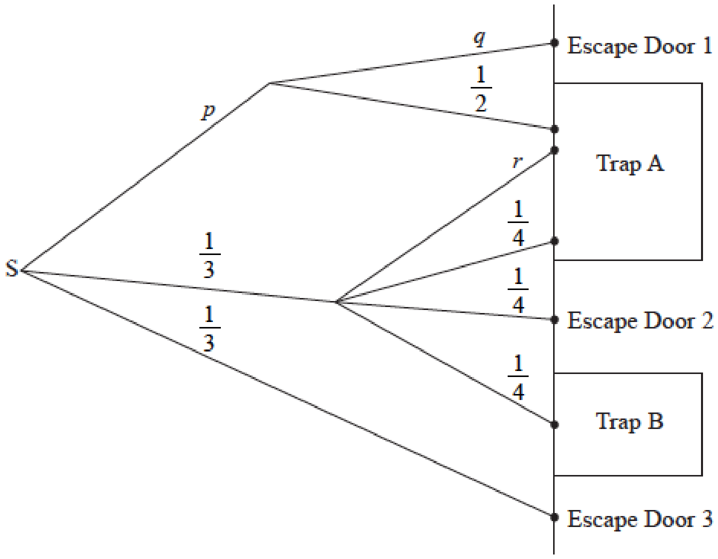
Mike, the laboratory mouse, is placed at the starting point, S, of a maze. Some paths in the maze lead to Trap A, some to Trap B, and others to escape doors. Some paths have one and some have two sections. If his path forks, Mike randomly chooses a path **forward**.

The following tree diagram represents the maze, showing all possible paths, and the probability that Mike chooses a certain section of a path through the maze.



3a. Write down the value of *[3 marks]*

(i)
p ;

(ii)
q ;

(iii)
r.

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3b. (i) Find the probability that Mike reaches Trap B. *[7 marks]*

(ii) Find the probability that Mike reaches Trap A.

(iii) Find the probability that Mike escapes from the maze.

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3c. Sonya, a lab assistant, counts the number of paths that lead to traps or escape doors. She believes that the probability that Mike will be trapped is greater than the probability that he will escape. *[2 marks]*

State whether Sonya is correct. Give a mathematical justification for your conclusion.

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3d. During the first trial Mike escapes. *[3 marks]*

Given that Mike escaped, find the probability that he went directly from S to Escape Door 3.

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The IB grades attained by a group of students are listed as follows.

$$6 \quad 4 \quad 5 \quad 3 \quad 7 \quad 3 \quad 5 \quad 4 \quad 2 \quad 5$$

4a. Find the median grade. *[2 marks]*

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4b. Calculate the interquartile range. *[2 marks]*

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4c. Find the probability that a student chosen at random from the group scored at least a grade 4. *[2 marks]*

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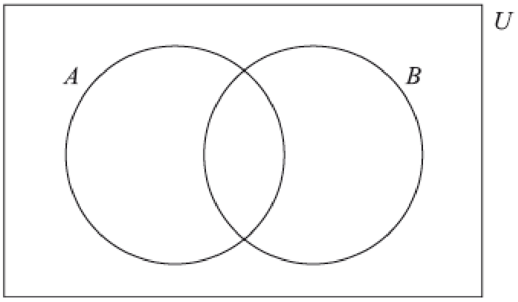
Aleph has an unbiased cubical (six faced) die on which are written the numbers

1 , 2 , 3 , 4 , 5 and 6.

Beth has an unbiased tetrahedral (four faced) die on which are written the numbers

2 , 3 , 5 and 7.

5a. Complete the Venn diagram with the numbers written on Aleph's die (A) and Beth's die (B). *[2 marks]*



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5b. Find $n(B \cap A')$. *[2 marks]*

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5c. Aleph and Beth are each going to roll their die once only. Shin says the probability that each die will show the same number is $\frac{1}{8}$. *[2 marks]*

Determine whether Shin is correct. Give a reason.

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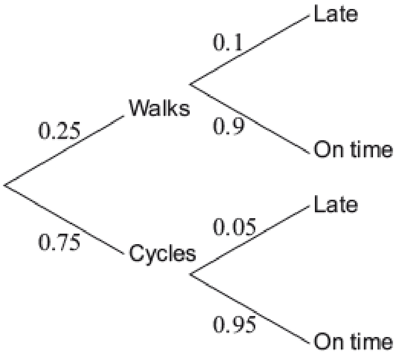
Peter either walks or cycles to work. The probability that he walks is 0.25. If Peter walks to work, the probability that he is late is 0.1. If he cycles to work, the probability that he is late is 0.05. The tree diagram for this information is shown.



6a. On a day chosen at random, Peter walked to work. *[1 mark]*

Write down the probability that he was on time.

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6b. For a different day, also chosen at random, *[2 marks]*

find the probability that Peter cycled to work and was late.

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6c. For a different day, also chosen at random, *[3 marks]*

find the probability that, given Peter was late, he cycled to work.

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Consider the propositions r, p and q.

7a. Complete the following truth table. *[4 marks]*

r	p	q	$r \wedge p$	$\neg q$	$(r \wedge p) \vee \neg q$	$\neg((r \wedge p) \vee \neg q)$	$\neg(r \wedge p)$	$\neg(r \wedge p) \wedge q$
T	T	T		F			F	
T	T	F		T			F	
T	F	T		F			T	
T	F	F		T			T	
F	T	T		F			T	
F	T	F		T			T	
F	F	T		F			T	
F	F	F		T			T	

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7b. Determine whether the compound proposition $\neg\left((r \wedge p) \vee \neg q\right) \Leftrightarrow \neg(r \wedge p) \wedge q$ is a tautology, a contradiction or neither. *[2 marks]*

Give a reason.

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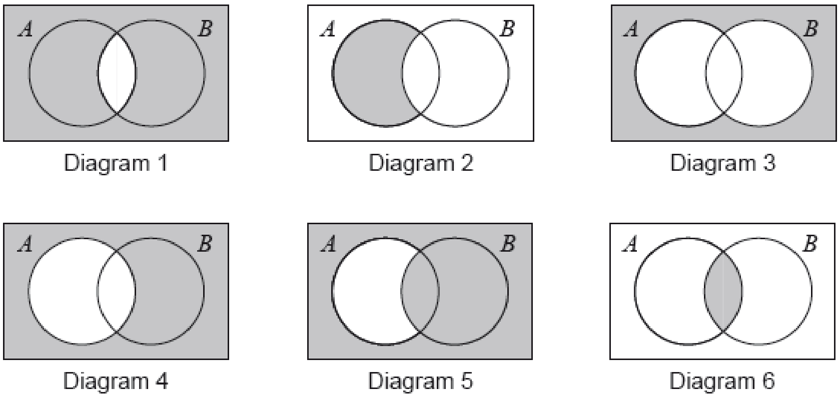
8. Consider the following Venn diagrams. Each diagram is shaded differently. *[6 marks]*



In the following table there are six sets. Each of these sets corresponds to the shaded region of one of the Venn diagrams. In the correct space, write the number of the diagram that corresponds to that set.

Set	**Diagram**
$(A \cup B)'$	. . . .
$A' \cup B'$	. . . .
$A \cap B'$	. . . .
$A \cap B$	. . . .
$A' \cup B$	. . . .
A'	. . . .

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p : x is a multiple of 12

q : x is a multiple of 6.

9a. Write down in words $\neg p$. *[1 mark]*

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9b. Write down in symbolic form the compound statement *[2 marks]*

r : If
x is a multiple of 12, then
x is a multiple of 6.

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9c. Consider the compound statement *[1 mark]*

s : If
x is a multiple of 6, then
x is a multiple of 12.

Identify whether s : is the inverse, the converse or the contrapositive of r.

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9d. Consider the compound statement *[2 marks]*

s : If
x is a multiple of 6, then
x is a multiple of 12.

Determine the validity of s. Justify your decision.

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Consider the following statements.

p: the land has been purchased

q: the building permit has been obtained

r: the land can be used for residential purposes

10a. Write the following argument in symbolic form. *[3 marks]*

"If the land has been purchased and the building permit has been obtained, then the land can be used for residential purposes."

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10b. **In your answer booklet**, copy and complete a truth table for the argument in part (a). *[2 marks]*

Begin your truth table as follows.

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

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10c. Use your truth table to determine whether the argument in part (a) is valid. *[2 marks]*

Give a reason for your decision.

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10d. Write down the inverse of the argument in part (a) *[4 marks]*

(i) in symbolic form;

(ii) in words.

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A group of 100 customers in a restaurant are asked which fruits they like from a choice of mangoes, bananas and kiwi fruits. The results are as follows.

15 like all three fruits

22 like mangoes and bananas

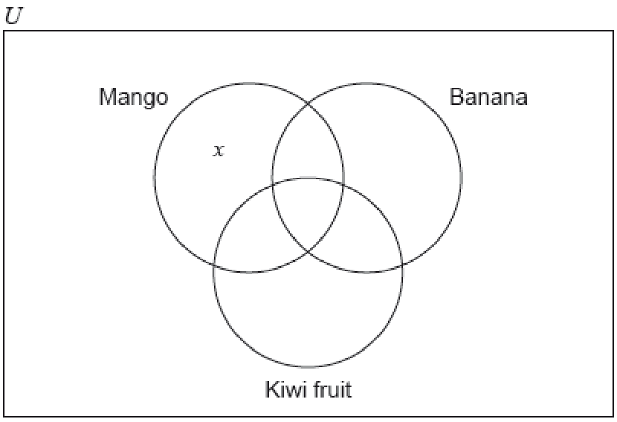
33 like mangoes and kiwi fruits

27 like bananas and kiwi fruits

8 like none of these three fruits

x like **only** mangoes

11a. **Copy** the following Venn diagram and correctly insert all values from the above information. *[3 marks]*



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11b. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. *[2 marks]*
This number is half of the number of customers that like **only** bananas.

Complete your Venn diagram from part (a) with this additional information **in terms of** x.

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11c. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. *[2 marks]*
This number is half of the number of customers that like **only** bananas.

Find the value of x.

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11d. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. *[2 marks]*
This number is half of the number of customers that like **only** bananas.

Write down the number of customers who like

(i) mangoes;

(ii) mangoes or bananas.

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11e. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. *[4 marks]*
This number is half of the number of customers that like **only** bananas.

A customer is chosen at random from the 100 customers. Find the probability that this customer

(i) likes none of the three fruits;

(ii) likes only two of the fruits;

(iii) likes all three fruits given that the customer likes mangoes and bananas.

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11f. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. *[3 marks]*
This number is half of the number of customers that like **only** bananas.

Two customers are chosen at random from the 100 customers. Find the probability that the two customers like none of the three fruits.

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Two propositions
p and
q are defined as follows

p: Eva is on a diet

q: Eva is losing weight.

12a. Write down the following statement **in words**. *[2 marks]*

$$q \Rightarrow p$$

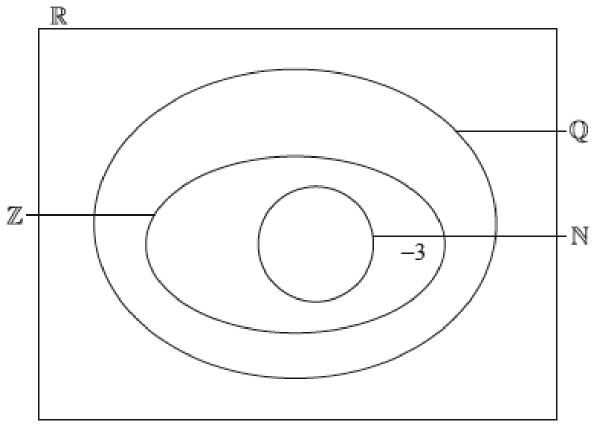
12b. Write down, in words, the contrapositive statement of *[2 marks]*
$q \Rightarrow p$.

12c. Determine whether your statement in part (a) is logically equivalent to your statement in part (b). Justify your answer. *[2 marks]*

The following Venn diagram shows the relationship between the sets of numbers

$$\mathbb{N},\ \mathbb{Z},\ \mathbb{Q} \text{ and } \mathbb{R}.$$

The number –3 belongs to the set of
$\mathbb{Z}$, $\mathbb{Q}$ and
$\mathbb{R}$, but not
$\mathbb{N}$, and is placed in the appropriate position on the Venn diagram as an example.



Write down the following numbers in the appropriate place in the Venn diagram.

13a. 4 *[1 mark]*

13b. $\frac{1}{3}$ *[1 mark]*

13c. π *[1 mark]*

13d. 0.38 *[1 mark]*

13e. $\sqrt{5}$ *[1 mark]*

13f. -0.25 *[1 mark]*

Tomek is attending a conference in Singapore. He has both trousers and shorts to wear. He also has the choice of wearing a tie or not.

The probability Tomek wears trousers is

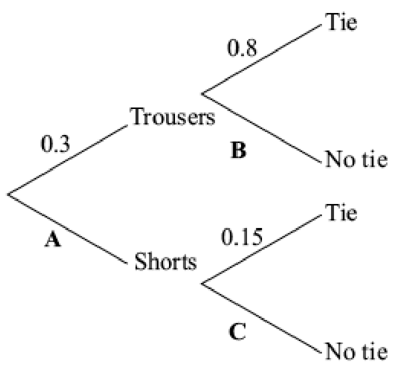
0.3. If he wears trousers, the probability that he wears a tie is

0.8.

If Tomek wears shorts, the probability that he wears a tie is

0.15.

The following tree diagram shows the probabilities for Tomek's clothing options at the conference.



14a. Find the value of *[3 marks]*

(i)

A;

(ii)

B;

(iii)

C.

14b. Calculate the probability that Tomek wears *[8 marks]*

(i) shorts and no tie;

(ii) no tie;

(iii) shorts given that he is not wearing a tie.

14c. The conference lasts for two days. *[2 marks]*

Calculate the probability that Tomek wears trousers on both days.

14d. The conference lasts for two days. *[3 marks]*

Calculate the probability that Tomek wears trousers on one of the days, and shorts on the other day.

U is the set of **positive** integers less than or equal to 10. A, B and C are subsets of U.

$A = \{\text{even integers}\}$

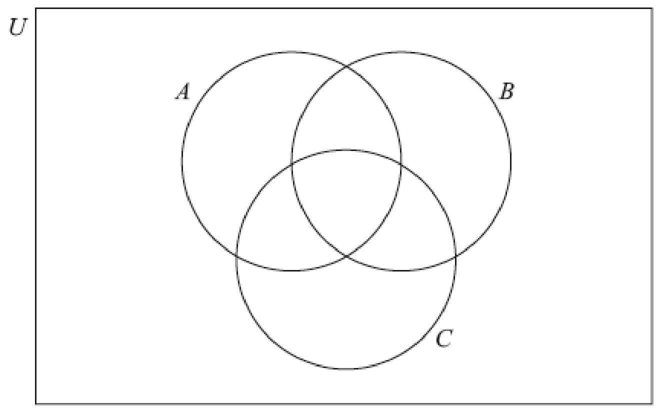
$B = \{\text{multiples of } 3\}$

$C = \{6,\ 7,\ 8,\ 9\}$

15a. List the elements of A. *[1 mark]*

15b. List the elements of B. *[1 mark]*

15c. Complete the Venn diagram with **all** the elements of U. *[4 marks]*



Consider the three propositions p, q and r.

p: *The food is well cooked*

q: *The drinks are chilled*

r: *Dinner is spoilt*

16a. Write the following compound proposition in words. *[3 marks]*

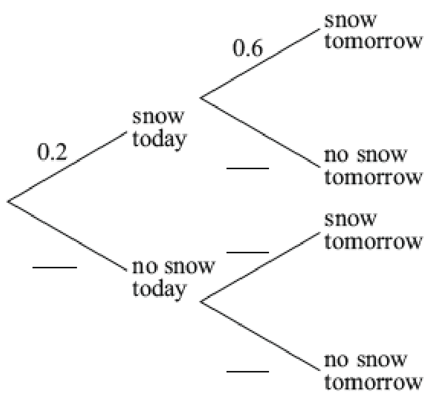
$$(p \wedge q) \Rightarrow \neg r$$

16b. Complete the following truth table. *[3 marks]*

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \Rightarrow \neg r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

The probability that it snows today is 0.2. If it does snow today, the probability that it will snow tomorrow is 0.6. If it does not snow today, the probability that it will not snow tomorrow is 0.9.

17a. Using the information given, complete the following tree diagram. *[3 marks]*



17b. Calculate the probability that it will snow tomorrow. *[3 marks]*

Two propositions are defined as follows:

p : *Quadrilateral ABCD has two diagonals that are equal in length.*

q : *Quadrilateral ABCD is a rectangle.*

18a. Express the following in symbolic form. *[2 marks]*

"A rectangle always has two diagonals that are equal in length."

18b. Write down in symbolic form the converse of the statement in (a). *[1 mark]*

18c. Determine, **without** using a truth table, whether the statements in (a) and (b) are logically equivalent. *[2 marks]*

18d. Write down the name of the statement that is logically equivalent to the converse. *[1 mark]*

A group of tourists went on safari to a game reserve. The game warden wanted to know how many of the tourists saw Leopard (
L), Cheetah (
C) or Rhino (
R). The results are given as follows.

5 of the tourists saw all three
7 saw Leopard and Rhino
1 saw Cheetah and Leopard **but not** Rhino
4 saw Leopard **only**
3 saw Cheetah **only**
9 saw Rhino **only**

19a. Draw a Venn diagram to show this information. *[4 marks]*

19b. There were 25 tourists in the group and every tourist saw at least one of the three types of animal. *[2 marks]*
Find the number of tourists that saw Cheetah and Rhino **but not** Leopard.

19c. There were 25 tourists in the group and every tourist saw at least one of the three types of animal. *[6 marks]*
Calculate the probability that a tourist chosen at random from the group
(i) saw Leopard;
(ii) saw **only one** of the three types of animal;
(iii) saw **only** Leopard, given that he saw only one of the three types of animal.

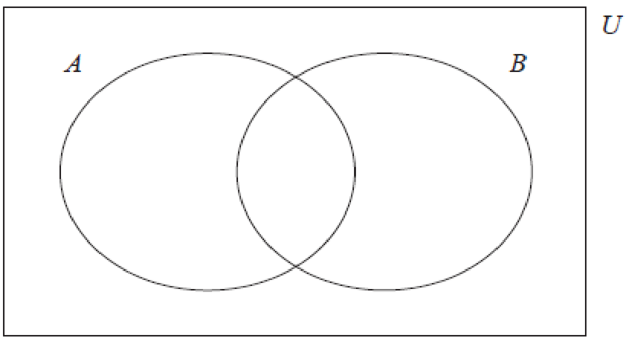
19d. There were 25 tourists in the group and every tourist saw at least one of the three types of animal. *[2 marks]*
If a tourist chosen at random from the group saw Leopard, find the probability that he also saw Cheetah.

$U = \{x | x \text{ is an integer, } 2 < x < 10\}$

A and B are subsets of U such that A = {multiples of 3}, B = {factors of 24}.

20a. List the elements of *[2 marks]*
(i) U ;
(ii) B .

20b. Write down the elements of U on the Venn diagram. *[3 marks]*



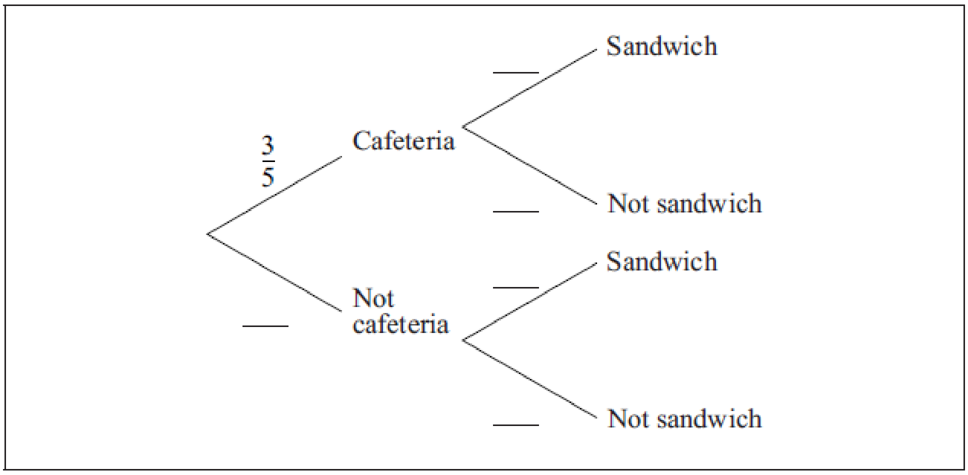
20c. Write down *[1 mark]*
$n(A \cap B)$.

The probability that Tanay eats lunch in the school cafeteria is
$\frac{3}{5}$.

If he eats lunch in the school cafeteria, the probability that he has a sandwich is
$\frac{3}{10}$.

If he does not eat lunch in the school cafeteria the probability that he has a sandwich is
$\frac{9}{10}$.

21a. Complete the tree diagram below. *[3 marks]*



21b. Find the probability that Tanay has a sandwich for his lunch. *[3 marks]*

22a. Complete the truth table. *[2 marks]*

p	q	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

22b. Consider the propositions p and q: *[2 marks]*

p: x is a number less than 10.

q: x2 is a number greater than 100.

Write in words the compound proposition
$\neg p \vee q$.

22c. Using part (a), determine whether
$\neg p \vee q$ is true or false, for the case where
x is a number less than 10 and
x^2 is a number greater than 100. *[1 mark]*

22d. Write down a value of
x for which
$\neg p \vee q$ is false. *[1 mark]*

Consider the following propositions.

p : *Students stay up late.*

q : *Students fall asleep in class.*

23a. Write the following compound proposition in symbolic form. *[2 marks]*

If students do not stay up late then they will not fall asleep in class.

23b. Complete the following truth table. *[3 marks]*

p	q	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$
T	T			
T	F			
F	T			
F	F			

23c. Write down a reason why the statement $\neg(p \vee \neg q)$ is not a contradiction. *[1 mark]*

Consider the following logic propositions:

p : *Yuiko is studying French.*

q : *Yuiko is studying Chinese.*

24a. Write down the following compound propositions in symbolic form. *[3 marks]*

(i) Yuiko is studying French but not Chinese.

(ii) Yuiko is studying French or Chinese, but not both.

24b. Write down in words the **inverse** of the following compound proposition. *[3 marks]*

If Yuiko is studying Chinese, then she is not studying French.

Alan's laundry basket contains two green, three red and seven black socks. He selects one sock from the laundry basket at random.

25a. Write down the probability that the sock is red. *[1 mark]*

25b. Alan returns the sock to the laundry basket and selects two socks at random. *[2 marks]*

Find the probability that the first sock he selects is green and the second sock is black.

25c. Alan returns the socks to the laundry basket and again selects two socks at random. *[3 marks]*

Find the probability that he selects two socks of the same colour.

Consider the propositions

p: *I have a bowl of soup.*

q: *I have an ice cream.*

26a. Write down, in words, the compound proposition $\neg p \Rightarrow q$. *[2 marks]*

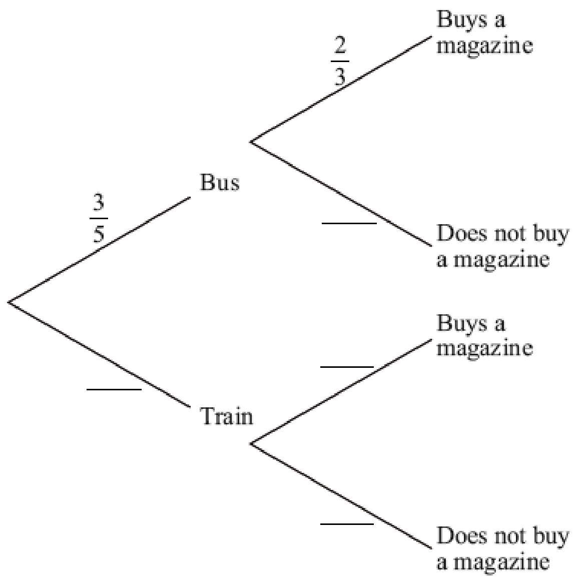
26b. Complete the truth table. *[2 marks]*

p	q	$\neg p$	$\neg p \Rightarrow q$
T	T		
T	F		
F	T		
F	F		

26c. Write down, in symbolic form, the converse of $\neg p \Rightarrow q$. *[2 marks]*

Ramzi travels to work each day, either by bus or by train. The probability that he travels by bus is $\frac{3}{5}$. If he travels by bus, the probability that he buys a magazine is $\frac{2}{3}$. If he travels by train, the probability that he buys a magazine is $\frac{3}{4}$.

27a. Complete the tree diagram. *[3 marks]*



27b. Find the probability that Ramzi buys a magazine when he travels to work. *[3 marks]*

A group of 120 women in the USA were asked whether they had visited the continents of Europe (E) or South America (S) or Asia (A).

7 had visited all three continents

28 had visited Europe only

22 had visited South America only

16 had visited Asia only

15 had visited Europe and South America but had not visited Asia

x had visited South America and Asia but had not visited Europe

$2x$ had visited Europe and Asia but had not visited South America

20 had not visited any of these continents

28a. Draw a Venn diagram, using sets labelled E, S and A, to show this information. *[5 marks]*

28b. Calculate the value of x. *[2 marks]*

28c. Explain, in words, the meaning of $(E \cup S) \cap A'$. *[2 marks]*

28d. Write down $n\left((E \cup S \cup A)'\right)$. *[1 mark]*

28e. Find the probability that a woman selected at random from the group had visited Europe. *[2 marks]*

28f. Find the probability that a woman selected at random from the group had visited Europe, given that she had visited Asia. *[2 marks]*

28g. Two women from the group are selected at random.
Find the probability that both women selected had visited South America. *[3 marks]*