Topic 5 Part 5 *[219 marks]*

The area of a circle is equal to 8 cm^2.

1a. Find the radius of the circle. *[2 marks]*

1b. This circle is the base of a **solid** cylinder of height 25 cm. *[1 mark]*

Write down the volume of the **solid** cylinder.

1c. This circle is the base of a **solid** cylinder of height 25 cm. *[3 marks]*

Find the **total** surface area of the **solid** cylinder.

The straight line, L_1, has equation
$2y - 3x = 11$. The point A has coordinates (6, 0).

2a. Give a reason why L_1 **does not** pass through A. *[1 mark]*

2b. Find the gradient of L_1. *[2 marks]*

2c. L_2 is a line perpendicular to L_1. The equation of L_2 is $y = mx + c$. *[1 mark]*

Write down the value of m.

2d. L_2 **does** pass through A. *[2 marks]*

Find the value of c.

Francesca is a chef in a restaurant. She cooks eight chickens and records their masses and cooking times. The mass m of each chicken, in kg, and its cooking time t, in minutes, are shown in the following table.

Mass m (kg)	**Cooking time t (minutes)**
1.5	62
1.6	75
1.8	82
1.9	83
2.0	86
2.1	87
2.1	91
2.3	98

3a. Draw a scatter diagram to show the relationship between the mass of a chicken and its cooking time. Use 2 cm to represent 0.5 kg on the horizontal axis and 1 cm to represent 10 minutes on the vertical axis. *[4 marks]*

3b. Write down for this set of data [2 marks]

(i) the mean mass,
$\bar{m}$;

(ii) the mean cooking time,
$\bar{t}$.

3c. Label the point
$\mathrm{M}(\bar{m}, \bar{t})$ on the scatter diagram. [1 mark]

3d. Draw the line of best fit on the scatter diagram. [2 marks]

3e. Using your line of best fit, estimate the cooking time, in minutes, for a 1.7 kg chicken. [2 marks]

3f. Write down the Pearson's product–moment correlation coefficient, r . [2 marks]

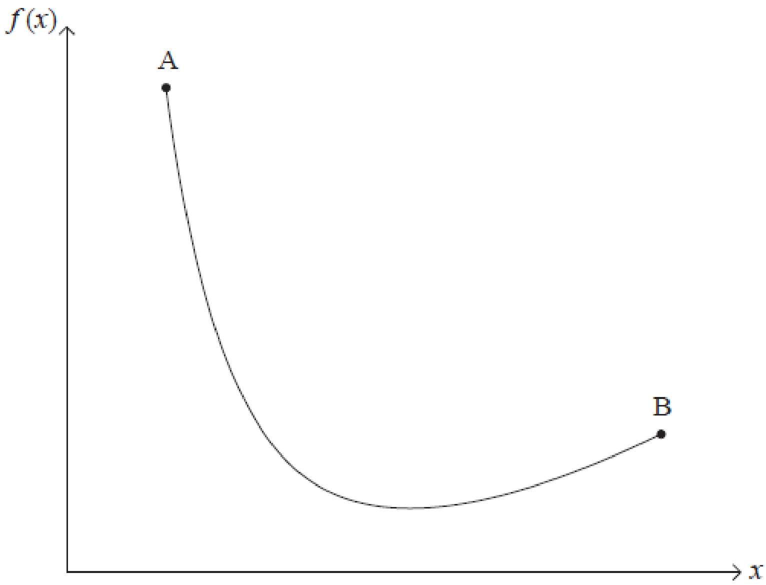
3g. Using your value for r , comment on the correlation. [2 marks]

3h. The cooking time of an additional 2.0 kg chicken is recorded. If the mass and cooking time of this chicken is included in the data, the correlation is weak. [2 marks]

(i) Explain how the cooking time of this additional chicken might differ from that of the other eight chickens.

(ii) Explain how a new line of best fit might differ from that drawn in part (d).

The graph of the function
$f(x) = \frac{14}{x} + x - 6$, for $1 \le x \le 7$ is given below.



4a. Calculate
$f(1)$. [2 marks]

4b. Find
$f'(x)$. [3 marks]

4c. **Use your answer to part (b)** to show that the *x*-coordinate of the local minimum point of the graph of
f is 3.7 correct to 2 significant figures. [3 marks]

4d. Find the range of f. *[3 marks]*

4e. Points A and B lie on the graph of f. The x-coordinates of A and B are 1 and 7 respectively. *[1 mark]*

Write down the y-coordinate of B.

4f. Points A and B lie on the graph of f . The x-coordinates of A and B are 1 and 7 respectively. *[2 marks]*

Find the gradient of the straight line passing through A and B.

4g. M is the midpoint of the line segment AB. *[2 marks]*

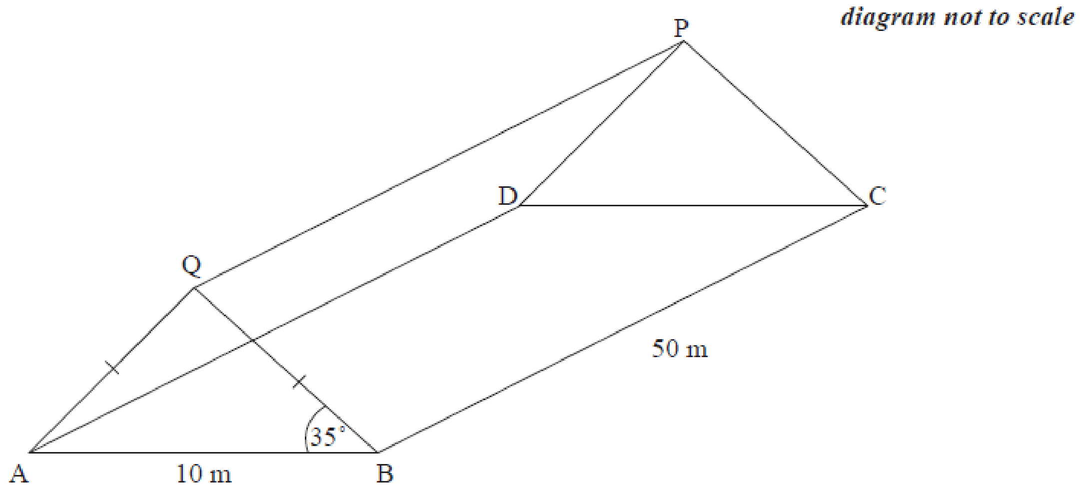
Write down the coordinates of M.

4h. L is the tangent to the graph of the function $y = f(x)$, at the point on the graph with the same x-coordinate as M. *[2 marks]*

Find the gradient of L.

4i. Find the equation of L. Give your answer in the form $y = mx + c$. *[3 marks]*

A greenhouse ABCDPQ is constructed on a rectangular concrete base ABCD and is made of glass. Its shape is a right prism, with cross section, ABQ, an isosceles triangle. The length of BC is 50 m, the length of AB is 10 m and the size of angle QBA is 35°.



5a. Write down the size of angle AQB. *[1 mark]*

5b. Calculate the length of AQ. *[3 marks]*

5c. Calculate the length of AC. *[2 marks]*

5d. Show that the length of CQ is 50.37 m, correct to 4 significant figures. *[2 marks]*

5e. Find the size of the angle AQC. *[3 marks]*

5f. Calculate the total area of the glass needed to construct *[5 marks]*

(i) the two rectangular faces of the greenhouse;

(ii) the two triangular faces of the greenhouse.

5g. The cost of one square metre of glass used to construct the greenhouse is 4.80 USD. *[3 marks]*

Calculate the cost of glass to make the greenhouse. Give your answer correct to the nearest 100 USD.

A cuboid has the following dimensions: length = 8.7 cm, width = 5.6 cm and height = 3.4 cm.

6a. Calculate the **exact** value of the volume of the cuboid, in cm^3. *[2 marks]*

6b. Write your answer to part (a) correct to *[2 marks]*

(i) one decimal place;

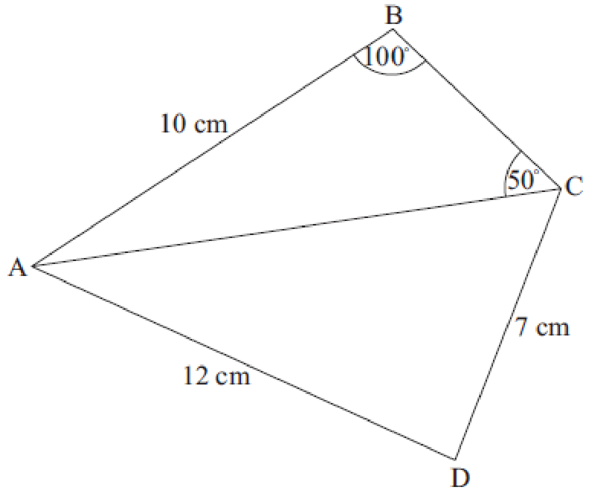
(ii) three significant figures.

6c. Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \leqslant a < 10, k \in \mathbb{Z}$. *[2 marks]*

The quadrilateral ABCD has AB = 10 cm, AD = 12 cm and CD = 7 cm.

The size of angle ABC is 100° and the size of angle ACB is 50°.

diagram not to scale



7a. Find the length of AC in centimetres. *[3 marks]*

7b. Find the size of angle ADC. *[3 marks]*

The equation of a line L_1 is $2x + 5y = -4$.

8a. Write down the gradient of the line L_1. *[1 mark]*

8b. A second line L_2 is perpendicular to L_1. *[1 mark]*

Write down the gradient of L_2.

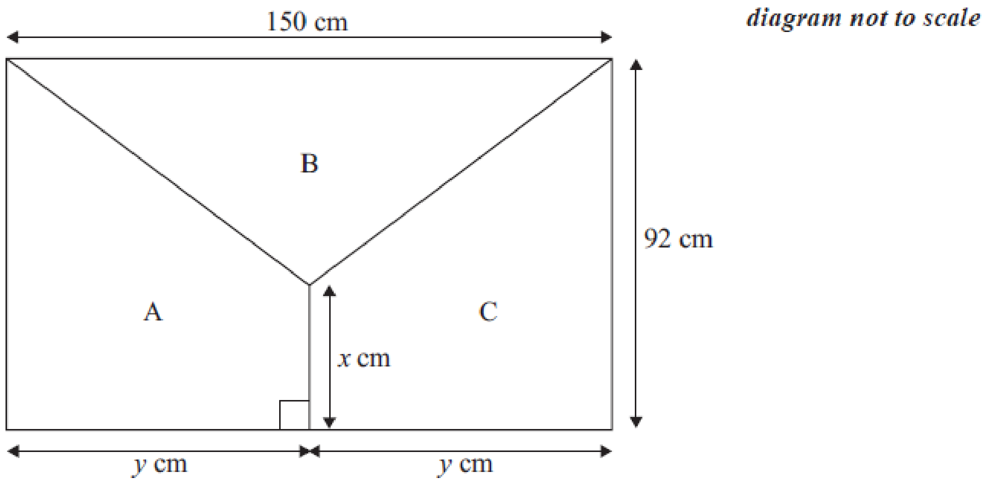
8c. The point (5, 3) is on L_2. [2 marks]

Determine the equation of L_2.

8d. Lines L_1 and L_2 intersect at point P. [2 marks]

Using your graphic display calculator or otherwise, find the coordinates of P.

The diagram below represents a rectangular flag with dimensions 150 cm by 92 cm. The flag is divided into three regions A, B and C.



9a. Write down the total area of the flag. [1 mark]

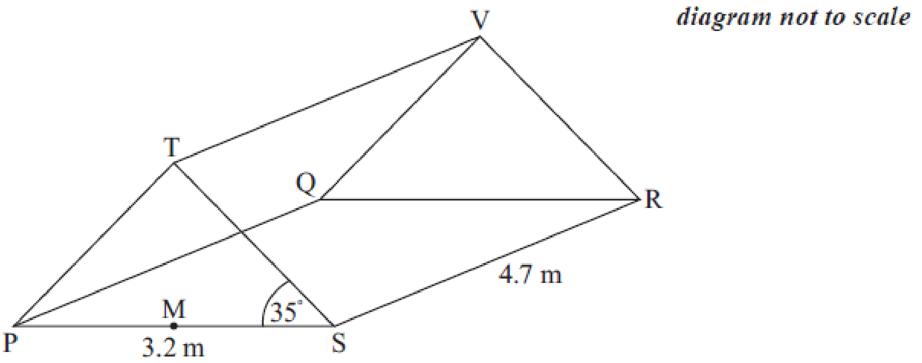
9b. Write down the value of y. [1 mark]

9c. The areas of regions A, B, and C are equal. [1 mark]

Write down the area of region A.

9d. Using your answers to **parts (b) and (c)**, find the value of x. [3 marks]

A tent is in the shape of a triangular right prism as shown in the diagram below.



The tent has a rectangular base PQRS .

PTS and QVR are isosceles triangles such that PT = TS and QV = VR .

PS is 3.2 m , SR is 4.7 m and the angle TSP is 35°.

10a. Show that the length of side ST is 1.95 m, correct to 3 significant figures. [3 marks]

10b. Calculate the area of the triangle PTS. [3 marks]

10c. Write down the area of the rectangle STVR. *[1 mark]*

10d. Calculate the **total** surface area of the tent, including the base. *[3 marks]*

10e. Calculate the volume of the tent. *[2 marks]*

10f. A pole is placed from V to M, the midpoint of PS. *[4 marks]*

Find in metres,

(i) the height of the tent, TM;

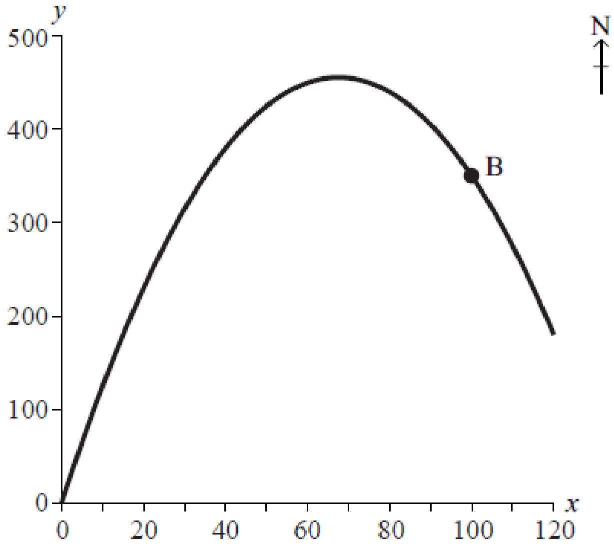
(ii) the length of the pole, VM.

10g. Calculate the angle between VM and the base of the tent. *[2 marks]*

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$y = \frac{-x^2}{10} + \frac{27}{2}x$, where
$x \geqslant 0,\ y \geqslant 0$

(x , y) are the coordinates of a point x metres east and y metres north of O , where O is the origin (0, 0) . B is a point on the bicycle track with coordinates (100, 350) .



11a. The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer. *[3 marks]*

11b. Find the derivative of *[2 marks]*
$y = \frac{-x^2}{10} + \frac{27}{2}x$.

11c. Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between O and B. Give a reason for your answer. *[4 marks]*

11d. (i) Write down the midpoint of the line segment OB. *[3 marks]*

(ii) Find the gradient of the line segment OB.

11e. Scott starts from a point C(0,150) . He hikes along a straight road towards the bicycle track, parallel to the line segment OB. *[3 marks]*

Find the equation of Scott's road. Express your answer in the form
$ax + by = c$, where
a, b and $c \in \mathbb{R}$.

11f. Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track. *[2 marks]*

The straight line, L_1, has equation $y = -2x + 5$.

12a. Write down the gradient of L_1 . *[1 mark]*

12b. Line L_2, is perpendicular to line L_1, and passes through the point $(4, 5)$. *[3 marks]*

(i) Write down the gradient of L_2 .

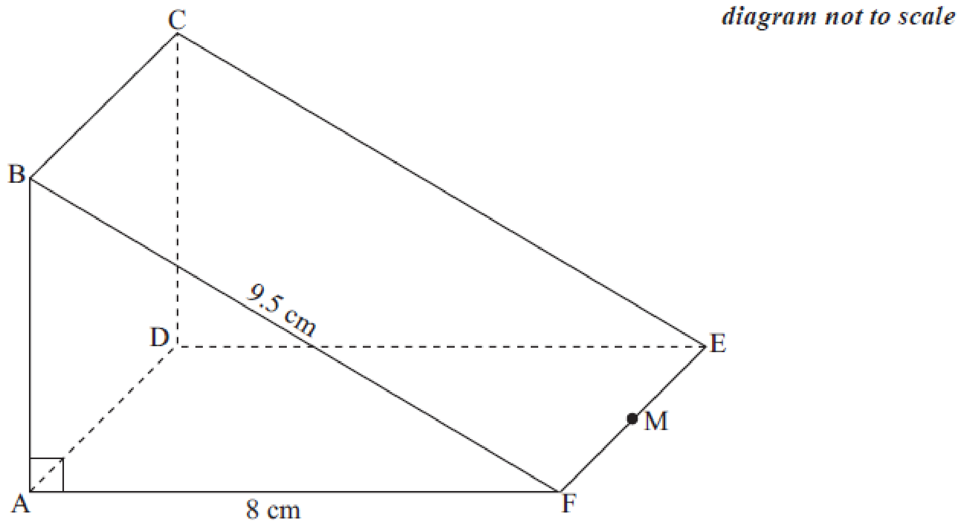
(ii) Find the equation of L_2 .

12c. Line L_2, is perpendicular to line L_1, and passes through the point $(4, 5)$. *[2 marks]*

Write down the coordinates of the point of intersection of L_1 and L_2 .

The diagram shows a right triangular prism, ABCDEF, in which the face ABCD is a square.

AF = 8 cm, BF = 9.5 cm, and angle BAF is 90°.



13a. Calculate the length of AB . *[2 marks]*

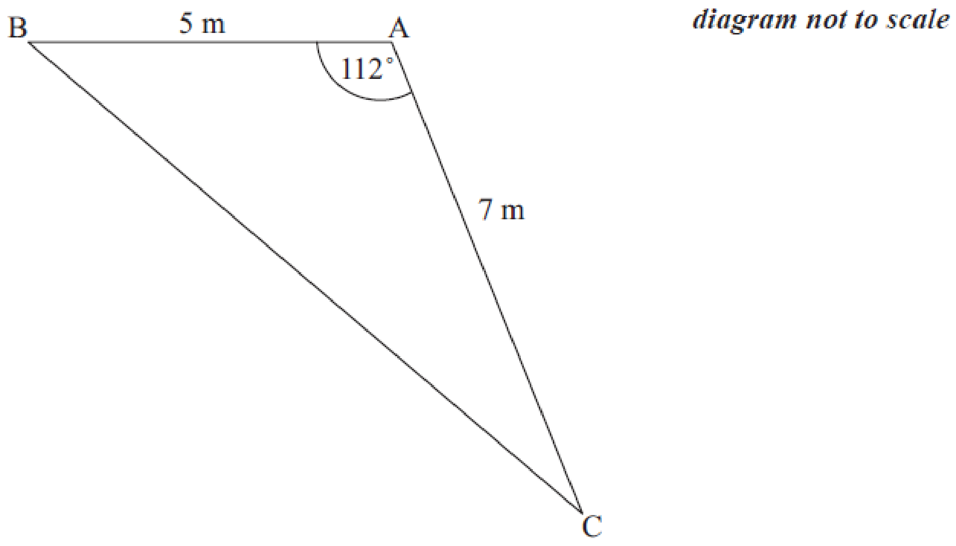
13b. M is the midpoint of EF . *[2 marks]*

Calculate the length of BM .

13c. M is the midpoint of EF . *[2 marks]*

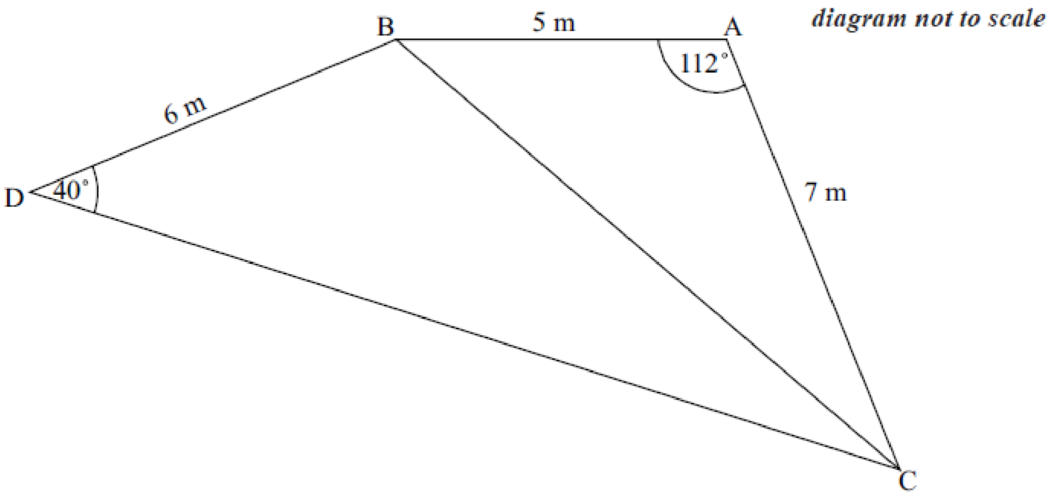
Find the size of the angle between BM and the face ADEF .

A contractor is building a house. He first marks out three points A , B and C on the ground such that AB = 5 m , AC = 7 m and angle BAC = 112°.



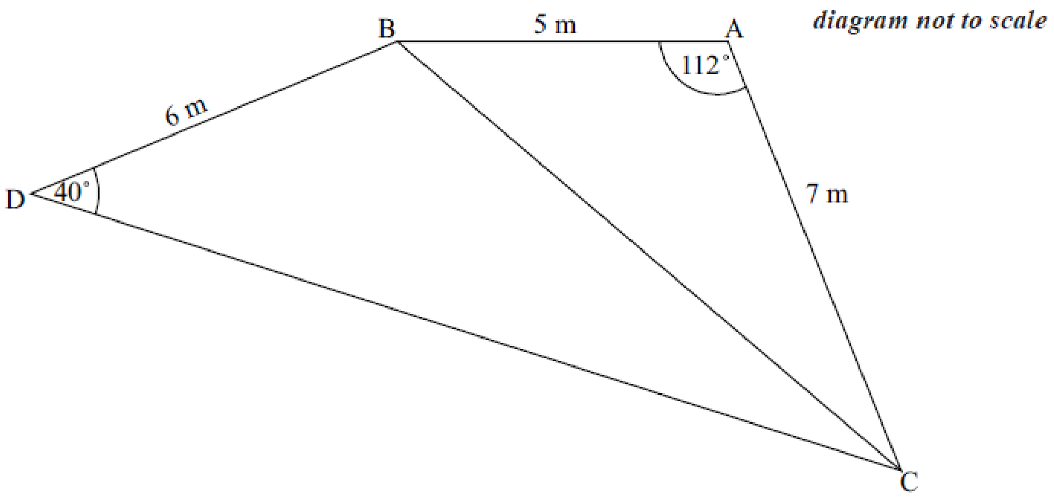
14a. Find the length of BC. *[3 marks]*

14b. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . *[4 marks]*



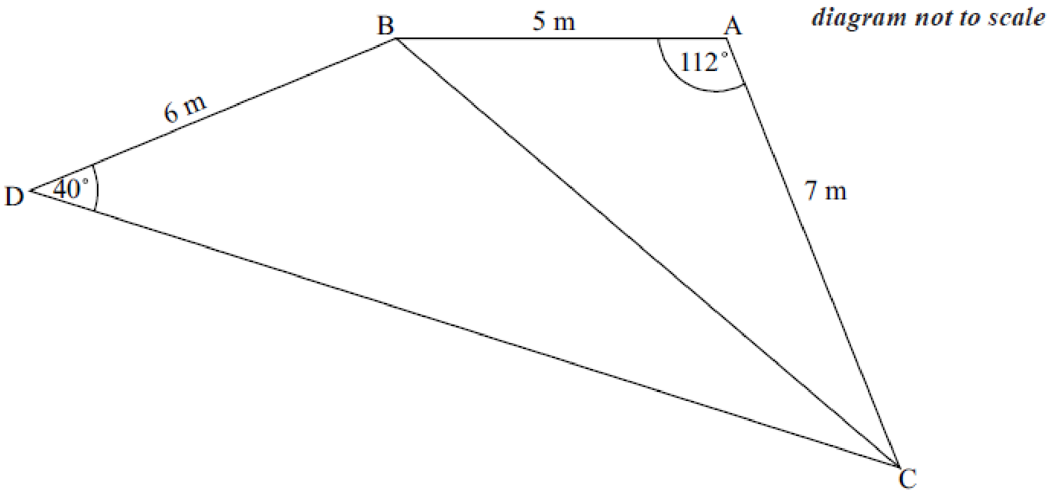
Find the size of angle DBC .

14c. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . *[4 marks]*



Find the area of the quadrilateral ABDC.

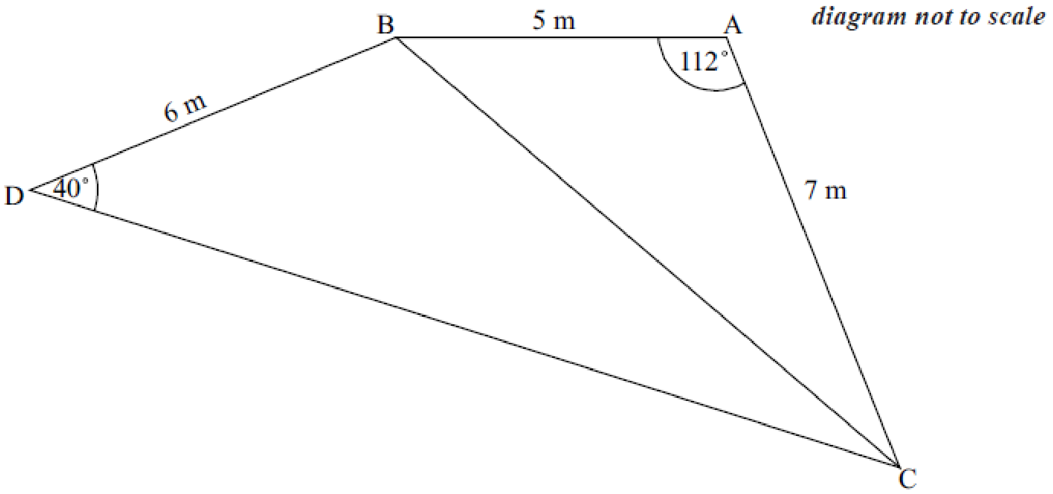
14d. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . *[3 marks]*



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

Find the volume of the soil removed. Give your answer in $\mathbf{m^3}$.

14e. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . *[5 marks]*



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

To transport the soil removed, the contractor uses cylindrical drums with a diameter of 30 cm and a height of 40 cm.

(i) Find the volume of a drum. Give your answer in $\mathbf{m^3}$.

(ii) Find the minimum number of drums required to transport the soil removed.

The coordinates of point A are $(-4, p)$ and the coordinates of point B are $(2, -3)$.

The mid-point of the line segment AB, has coordinates $(q, 1)$.

15a. Find the value of *[4 marks]*

(i) q ;

(ii) p .

15b. Calculate the distance AB. *[2 marks]*

Line L is given by the equation $3y + 2x = 9$ and point P has coordinates $(6, -5)$.

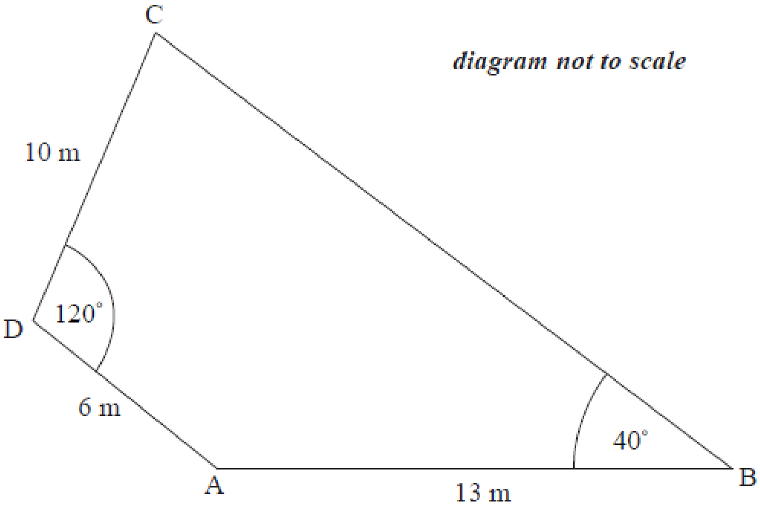
16a. Explain why point P is not on the line L. *[1 mark]*

16b. Find the gradient of line L. *[2 marks]*

16c. (i) Write down the gradient of a line perpendicular to line L. *[3 marks]*

(ii) Find the equation of the line perpendicular to L and passing through point P.

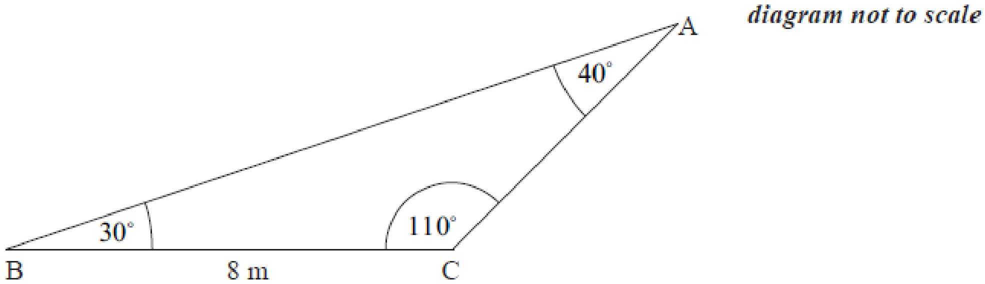
The diagram shows quadrilateral ABCD in which AB = 13 m , AD = 6 m and DC = 10 m. Angle ADC =120° and angle ABC = 40°.



17a. Calculate the length of AC. *[3 marks]*

17b. Calculate the size of angle ACB. *[3 marks]*

In triangle ABC, BC = 8 m, angle ACB = 110°, angle CAB = 40°, and angle ABC = 30°.



18a. Find the length of AC. *[3 marks]*

18b. Find the area of triangle ABC. *[3 marks]*

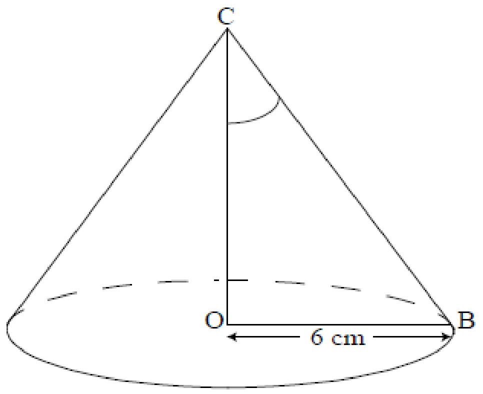
A solid metal **cylinder** has a base radius of 4 cm and a height of 8 cm.

19a. Find the area of the base of the cylinder. *[2 marks]*

19b. Show that the volume of the metal used in the cylinder is 402 cm^3, given correct to three significant figures. *[2 marks]*

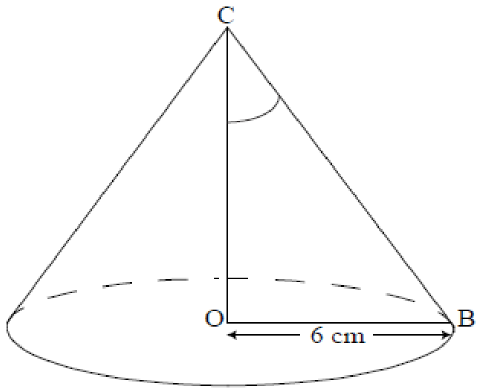
19c. Find the total surface area of the cylinder. *[3 marks]*

19d. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. *[3 marks]*



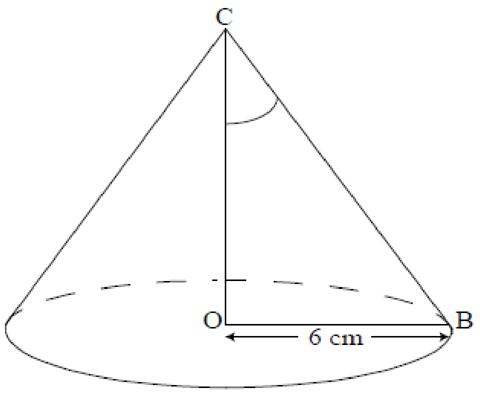
Find the height, OC, of the cone.

19e. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. *[2 marks]*



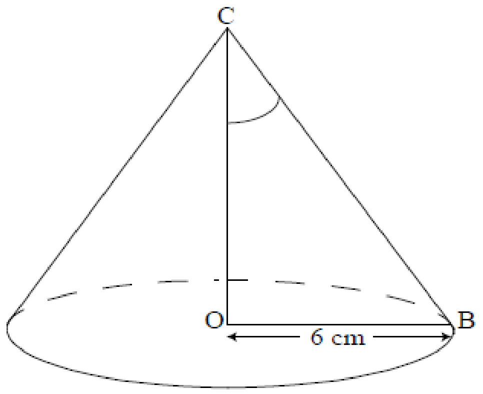
Find the size of angle BCO.

19f. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. *[2 marks]*



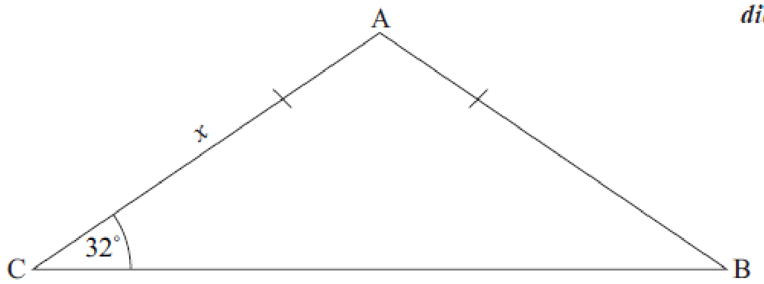
Find the slant height, CB.

19g. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. *[4 marks]*



Find the total surface area of the cone.

In the diagram, triangle ABC is isosceles. AB = AC and angle ACB is 32°. The length of side AC is *x* cm.



20a. Write down the size of angle CBA. *[1 mark]*

20b. Write down the size of angle CAB. *[1 mark]*

20c. The area of triangle ABC is 360 cm^2. Calculate the length of side AC. Express your answer in **millimetres**. *[4 marks]*

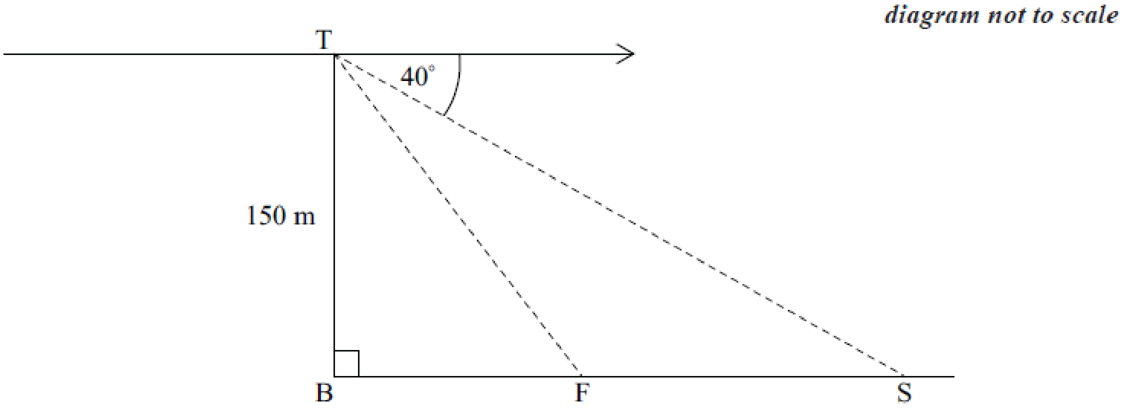
The equation of a curve is given as
$y = 2x^2 - 5x + 4$.

21a. Find *[2 marks]*
$\frac{\mathrm{d}y}{\mathrm{d}x}$.

21b. The equation of the line L is *[4 marks]*
$6x + 2y = -1$.

Find the x-coordinate of the point on the curve
$y = 2x^2 - 5x + 4$ where the tangent is parallel to L.

Tom stands at the top, T , of a vertical cliff
150 m high and sees a fishing boat, F , and a ship, S . B represents a point at the bottom of the cliff directly below T . The angle of depression of the ship is
40° and the angle of depression of the fishing boat is
55° .



22a. Calculate, SB, the distance between the ship and the bottom of the cliff. *[2 marks]*

22b. Calculate, SF, the distance between the ship and the fishing boat. Give your answer correct to the nearest metre. *[4 marks]*