

## Topic 3 Part 4 [163 marks]

Consider the statement  $p$ :

“If a quadrilateral is a square then the four sides of the quadrilateral are equal”.

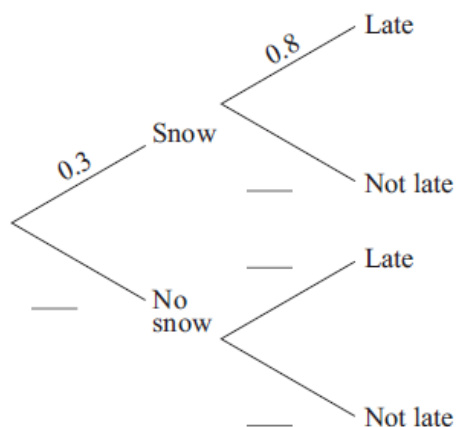
- 1a. Write down the inverse of statement  $p$  in words. [2 marks]
- 1b. Write down the converse of statement  $p$  in words. [2 marks]
- 1c. Determine whether the converse of statement  $p$  is always true. Give an example to justify your answer. [2 marks]

The probability that it will snow tomorrow is 0.3.

If it snows tomorrow the probability that Chuck will be late for school is 0.8.

If it does not snow tomorrow the probability that Chuck will be late for school is 0.1.

- 2a. Complete the tree diagram below. [3 marks]



- 2b. Find the probability that it does not snow tomorrow and Chuck is late for school. [1 mark]
- 2c. Find the probability that Chuck is late for school. [2 marks]

A class consists of students studying Spanish or French or both. Fifteen students study Spanish and twelve study French.

The probability that a student studies French given that she studies Spanish is

$$\frac{7}{15}.$$

- 3a. Draw a Venn diagram in the space below to illustrate this information. [3 marks]
- 3b. Find the probability that a student studies Spanish given that she studies one language only. [3 marks]

The Venn diagram below represents the students studying Mathematics ( $A$ ), Further Mathematics ( $B$ ) and Physics ( $C$ ) in a school.

50 students study Mathematics

38 study Physics

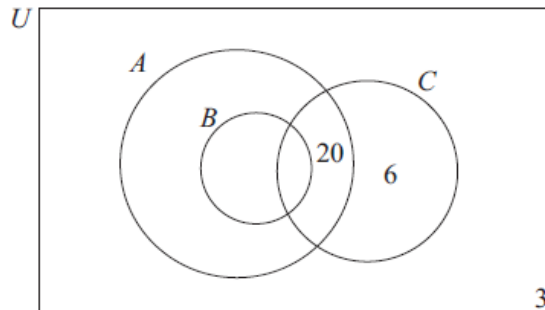
20 study Mathematics and Physics but not Further Mathematics

10 study Further Mathematics but not Physics

12 study Further Mathematics and Physics

6 study Physics but not Mathematics

3 study none of these three subjects.



4a. Copy and complete the Venn diagram **on your answer paper**. [3 marks]

4b. Write down the number of students who study Mathematics but not Further Mathematics. [1 mark]

4c. Write down the total number of students in the school. [1 mark]

4d. Write down  $n(B \cup C)$ . [2 marks]

Three propositions are given as

$p$  : It is snowing     $q$  : The roads are open     $r$  : We will go skiing

4e. Write the following compound statement in symbolic form. [2 marks]

“It is snowing and the roads are not open.”

4f. Write the following compound statement in words. [3 marks]

$$(\neg p \wedge q) \Rightarrow r$$

- 4g. An incomplete truth table for the compound proposition  $(\neg p \wedge q) \Rightarrow r$  is given below.

[3 marks]

Copy and complete the truth table **on your answer paper**.

$p$	$q$	$r$	$\neg p$	$\neg p \wedge q$	$(\neg p \wedge q) \Rightarrow r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Consider the two propositions  $p$  and  $q$ .

$p$ : The sun is shining     $q$ : I will go swimming

- 5a. Write in words the compound proposition

[2 marks]

$p \Rightarrow q$ ;

- 5b. Write in words the compound proposition

[2 marks]

$\neg p \vee q$ .

- 5c. The truth table for these compound propositions is given below.

[1 mark]

$p$	$q$	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		T

Complete the column for

$\neg p$ .

- 5d. The truth table for these compound propositions is given below.

[1 mark]

$p$	$q$	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		T

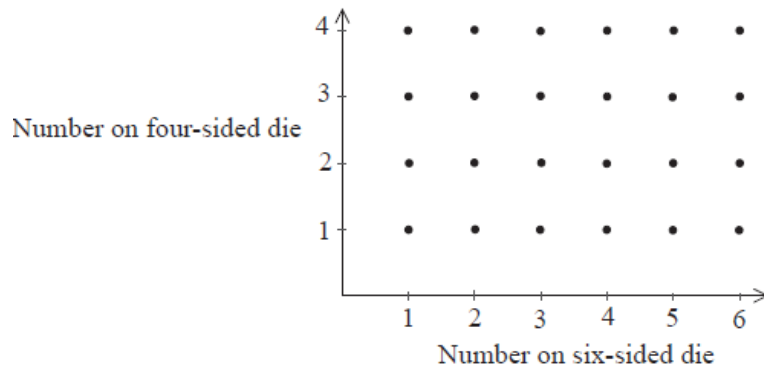
State the relationship between the compound propositions

$p \Rightarrow q$  and

$\neg p \vee q$ .

A fair six-sided die has the numbers 1, 2, 3, 4, 5, 6 written on its faces. A fair four-sided die has the numbers 1, 2, 3, and 4 written on its faces. The two dice are rolled.

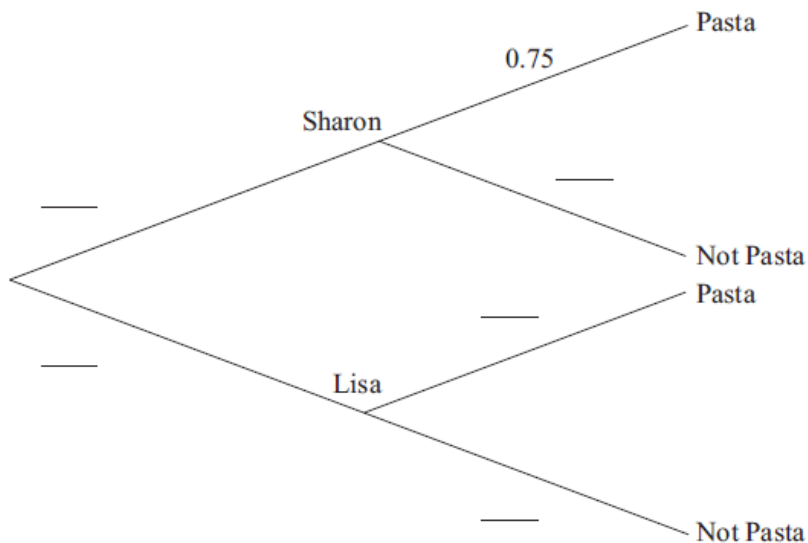
The following diagram shows the possible outcomes.



- 6a. Find the probability that the two dice show the same number. [2 marks]
- 6b. Find the probability that the difference between the two numbers shown on the dice is 1. [2 marks]
- 6c. Find the probability that the number shown on the four-sided die is greater than the number shown on the six-sided die, given that the difference between the two numbers is 1. [2 marks]

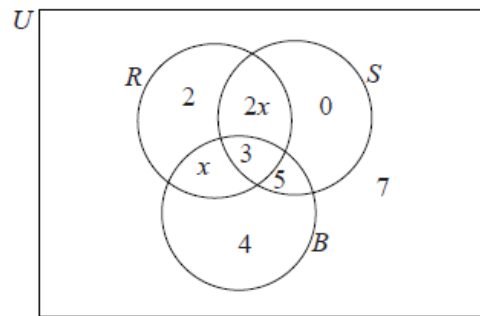
Sharon and Lisa share a flat. Sharon cooks dinner three nights out of ten. If Sharon does not cook dinner, then Lisa does. If Sharon cooks dinner the probability that they have pasta is 0.75. If Lisa cooks dinner the probability that they have pasta is 0.12.

- 7a. **Copy and complete** the tree diagram to represent this information. [3 marks]



- 7b. Find the probability that Lisa cooks dinner and they do not have pasta. [2 marks]
- 7c. Find the probability that they do not have pasta. [3 marks]
- 7d. Given that they do not have pasta, find the probability that Lisa cooked dinner. [3 marks]

A survey was carried out in a year 12 class. The pupils were asked which pop groups they like out of the *Rockers* ( $R$ ), the *Salseros* ( $S$ ), and the *Bluers* ( $B$ ). The results are shown in the following diagram.



7e. Write down  $n(R \cap S \cap B)$ . [1 mark]

7f. Find  $n(R')$ . [2 marks]

7g. Describe which groups the pupils in the set  $S \cap B$  like. [2 marks]

7h. Use set notation to describe the group of pupils who like the *Rockers* and the *Bluers* but do not like the *Salseros*. [2 marks]

7i. There are 33 pupils in the class.  
Find  $x$ . [2 marks]

7j. There are 33 pupils in the class.  
Find the number of pupils who like the *Rockers*. [1 mark]

For events  $A$  and  $B$ , the probabilities are

$$P(A) = \frac{4}{13} \text{ and}$$

$$P(B) = \frac{5}{13}.$$

8a. If events  $A$  and  $B$  are mutually exclusive, write down the value of  $P(A \cap B)$ . [1 mark]

8b. If events  $A$  and  $B$  are independent, find the value of  $P(A \cap B)$ . [2 marks]

8c. If  $P(A \cup B) = \frac{7}{13}$ , find the value of  $P(A \cap B)$ . [3 marks]

A **weighted** die has 2 red faces, 3 green faces and 1 black face. When the die is thrown, the black face is three times as likely to appear on top as one of the other five faces. The other five faces have equal probability of appearing on top.

The following table gives the probabilities.

Red 1	Red 2	Green 1	Green 2	Green 3	Black
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{m}{8}$	$\frac{1}{8}$	$\frac{n}{8}$

9a. Find the value of [2 marks]

(i)  $m$ ;

(ii)  $n$ .

9b. The die is thrown once. [2 marks]

Given that the face on top is not red, find the probability that it is black.

9c. The die is now thrown twice. [2 marks]

Calculate the probability that black appears on top both times.

A survey of 100 families was carried out, asking about the pets they own. The results are given below.

56 owned dogs ( $S$ )

38 owned cats ( $Q$ )

22 owned birds ( $R$ )

16 owned dogs and cats, but not birds

8 owned birds and cats, but not dogs

3 owned dogs and birds, but not cats

4 owned all three types of pets

10a. Draw a Venn diagram to represent this information. [5 marks]

10b. Find the number of families who own no pets. [2 marks]

10c. Find the percentage of families that own exactly one pet. [3 marks]

10d. A family is chosen at random. Find the probability that they own a cat, given that they own a bird. [2 marks]

A geometric sequence has second term 12 and fifth term 324.

11a. Calculate the value of the common ratio. [4 marks]

11b. Calculate the 10<sup>th</sup> term of this sequence. [3 marks]

11c. The  $k^{\text{th}}$  term is the first term which is greater than 2000. Find the value of  $k$ . [3 marks]

Consider the following propositions

$p$ : The number is a multiple of five.

$q$ : The number is even.

$r$ : The number ends in zero.

11d. Write in words [3 marks]  
 $(q \wedge \neg r) \Rightarrow \neg p$ .

11e. Consider the statement “If the number is a multiple of five, and is not even then it will not end in zero”. [4 marks]  
Write this statement in symbolic form.

11f. Consider the statement “If the number is a multiple of five, and is not even then it will not end in zero”. [2 marks]  
Write the contrapositive of this statement in symbolic form.

The grades obtained by a group of  
20 IB students are listed below:

6   2   5   3   5   5   6   2   6   1  
7   6   2   4   2   4   3   4   5   6

12a. Complete the following table for the grades obtained by the students. [2 marks]

Grade	Frequency
1	
2	
3	2
4	
5	4
6	
7	1

12b. Write down the modal grade obtained by the students. [1 mark]

12c. Calculate the median grade obtained by the students. [2 marks]

12d. One student is chosen at random from the group. [1 mark]  
Find the probability that this student obtained either grade  
4 or grade  
5.

Let  
 $p$  and  
 $q$  represent the propositions

$p$ : food may be taken into the cinema

$q$ : drinks may be taken into the cinema

- 13a. Complete the truth table below for the symbolic statement  $\neg(p \vee q)$ .

[2 marks]

$p$	$q$	$p \vee q$	$\neg(p \vee q)$
T	T		
T	F		
F	T		
F	F		

- 13b. Write down in words the meaning of the symbolic statement  $\neg(p \vee q)$ .

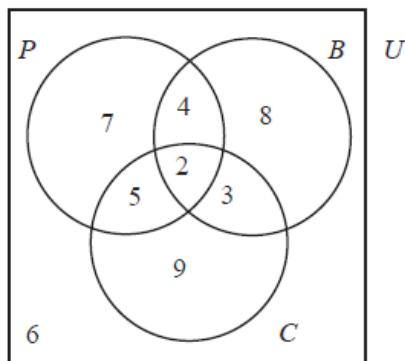
[2 marks]

- 13c. Write in symbolic form the compound statement:

[2 marks]

“no food and no drinks may be taken into the cinema”.

The Venn diagram shows the numbers of pupils in a school according to whether they study the sciences Physics ( $P$ ), Chemistry ( $C$ ), Biology ( $B$ ).



- 14a. Write down the number of pupils that study Chemistry only.

[1 mark]

- 14b. Write down the number of pupils that study **exactly** two sciences.

[1 mark]

- 14c. Write down the number of pupils that do not study Physics.

[2 marks]

- 14d. Find  $n[(P \cup B) \cap C]$ .

[2 marks]



Jorge conducted a survey of 200 drivers. He asked two questions:

How long have you had your driving licence?  
Do you wear a seat belt when driving?

The replies are summarized in the table below.

	<b>Wear a seat belt</b>	<b>Do not wear a seat belt</b>
<b>Licence less than 2 years</b>	38	42
<b>Licence between 2 and 15 years</b>	30	45
<b>Licence more than 15 years</b>	30	15

15a. Jorge applies a  $\chi^2$  test at the 5% level to investigate whether wearing a seat belt is associated with the time a driver has had their licence. [8 marks]

- (i) Write down the null hypothesis,  $H_0$ .
- (ii) Write down the number of degrees of freedom.
- (iii) Show that the expected number of drivers that wear a seat belt and have had their driving licence for more than 15 years is 22, correct to the nearest whole number.
- (iv) Write down the  $\chi^2$  test statistic for this data.
- (v) Does Jorge accept  $H_0$ ? Give a reason for your answer.

15b. Consider the 200 drivers surveyed. One driver is chosen at random. Calculate the probability that [4 marks]

- (i) this driver wears a seat belt;
- (ii) the driver does not wear a seat belt, **given that** the driver has held a licence for more than 15 years.

15c. Two drivers are chosen at random. Calculate the probability that [6 marks]

- (i) both wear a seat belt.
- (ii) at least one wears a seat belt.

Consider the following logic propositions:

$p$  : Sean is at school

$q$  : Sean is playing a game on his computer.

16a. Write in words,  $p \vee q$ . [2 marks]

16b. Write in words, the converse of  $p \Rightarrow \neg q$ . [2 marks]

- 16c. Complete the following truth table for  $p \Rightarrow \neg q$ .

[2 marks]

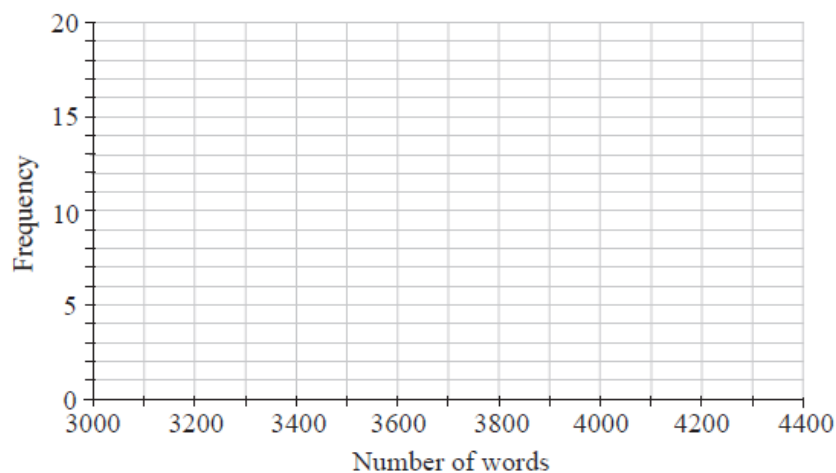
$p$	$q$	$\neg q$	$p \Rightarrow \neg q$
T	T		
T	F		
F	T		
F	F		

The table below shows the number of words in the extended essays of an IB class.

Number of words	$3200 \leq w < 3400$	$3400 \leq w < 3600$	$3600 \leq w < 3800$	$3800 \leq w < 4000$	$4000 \leq w < 4200$
Frequency	2	5	8	17	3

- 17a. Draw a histogram on the grid below for the data in this table.

[3 marks]



- 17b. Write down the modal group.

[1 mark]

- 17c. The maximum word count is 4000 words.

[2 marks]

Write down the probability that a student chosen at random is on or over the word count.

- 18a. (i) Complete the truth table below.

[4 marks]

$p$	$q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T			F	F	
T	F			F	T	
F	T			T	F	
F	F			T	T	

- (ii) State whether the compound propositions

$\neg(p \wedge q)$  and

$\neg p \vee \neg q$  are equivalent.

18b. Consider the following propositions.

[2 marks]

$p$  : Amy eats sweets

$q$  : Amy goes swimming.

Write, in symbolic form, the following proposition.

*Amy either eats sweets or goes swimming, but not both.*