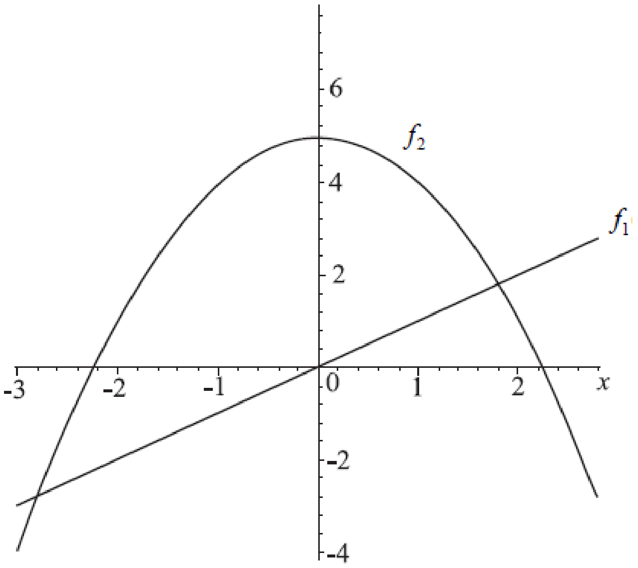
Topic 7 Part 4 *[73 marks]*

The figure below shows the graphs of functions
$f_1(x) = x$ and
$f_2(x) = 5 - x^2$.



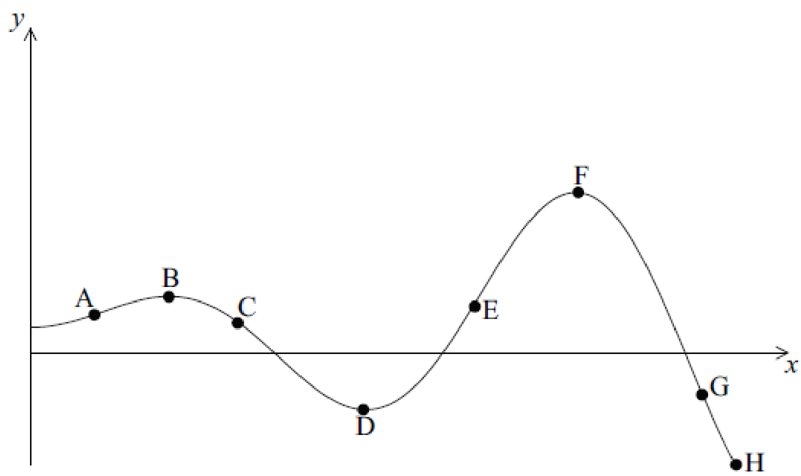
1a. (i) Differentiate $f_1(x)$ with respect to x. *[3 marks]*

(ii) Differentiate $f_2(x)$ with respect to x.

1b. Calculate the value of x for which the gradient of the two graphs is the same. *[2 marks]*

1c. Draw the tangent to the **curved** graph for this value of x on the figure, showing clearly the property in part (b). *[1 mark]*

Consider the graph of the function
$y = f(x)$ defined below.



Write down **all** the labelled points on the curve

2a. that are local maximum points; *[1 mark]*

2b. where the function attains its least value; *[1 mark]*

2c. where the function attains its greatest value; *[1 mark]*

2d. where the gradient of the tangent to the curve is positive; *[1 mark]*

2e. where $f(x) > 0$ and $f'(x) < 0$. *[2 marks]*

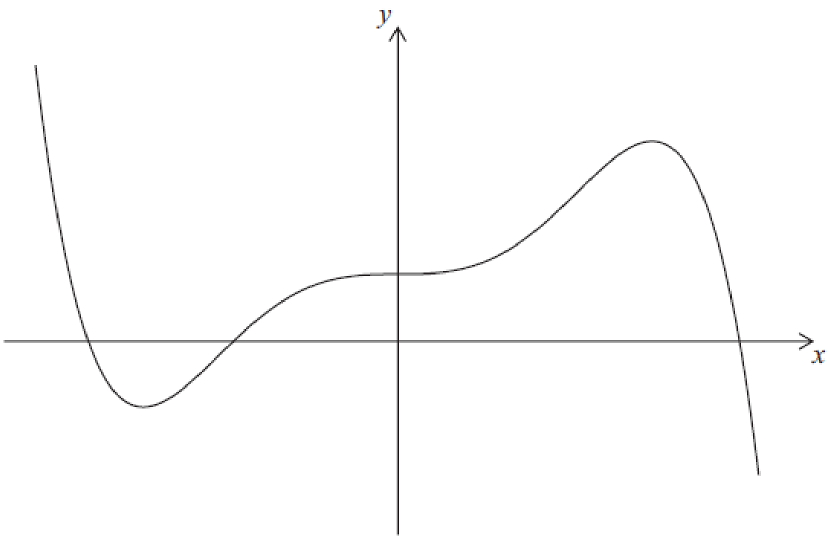
Consider the curve $y = x^2$.

3a. Write down $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[1 mark]*

3b. The point $\mathrm{P}(3, 9)$ lies on the curve $y = x^2$. Find the gradient of the tangent to the curve at P . *[2 marks]*

3c. The point $\mathrm{P}(3, 9)$ lies on the curve $y = x^2$. Find the equation of the normal to the curve at P . Give your answer in the form $y = mx + c$. *[3 marks]*

A sketch of the function $f(x) = 5x^3 - 3x^5 + 1$ is shown for $-1.5 \leqslant x \leqslant 1.5$ and $-6 \leqslant y \leqslant 6$.



4a. Write down $f'(x)$. *[2 marks]*

4b. Find the equation of the tangent to the graph of $y = f(x)$ at $(1, 3)$. *[2 marks]*

4c. Write down the coordinates of the second point where this tangent intersects the graph of $y = f(x)$. *[2 marks]*

A small manufacturing company makes and sells x machines each month. The monthly cost C , in dollars, of making x machines is given by

$$C(x) = 2600 + 0.4x^2.$$

The monthly income I , in dollars, obtained by selling x machines is given by

$$I(x) = 150x - 0.6x^2.$$

$P(x)$ is the monthly profit obtained by selling x machines.

5a. Find $P(x)$. *[2 marks]*

5b. Find the number of machines that should be made and sold each month to maximize $P(x)$. *[2 marks]*

5c. Use your answer to part (b) to find the selling price of **each machine** in order to maximize $P(x)$. *[2 marks]*

Given $f(x) = x^2 - 3x^{-1}, x \in \mathbb{R}, -5 \leqslant x \leqslant 5, x \neq 0,$

6a. Write down the equation of the vertical asymptote. *[1 mark]*

6b. Find $f'(x)$. *[2 marks]*

6c. Using your graphic display calculator or otherwise, write down the coordinates of any point where the graph of $y = f(x)$ has zero gradient. *[2 marks]*

6d. Write down all intervals in the given domain for which $f(x)$ is increasing. *[3 marks]*

A football is kicked from a point A (a, 0), 0 < a < 10 on the ground towards a goal to the right of A.

The ball follows a path that can be modelled by **part** of the graph

$$y = -0.021x^2 + 1.245x - 6.01, x \in \mathbb{R}, y \geqslant 0.$$

x is the horizontal distance of the ball from the origin

y is the height above the ground

Both *x* and *y* are measured in metres.

6e. Using your graphic display calculator or otherwise, find the value of *a*. *[1 mark]*

6f. Find $\frac{dy}{dx}$. *[2 marks]*

6g. (i) Use your answer to part (b) to calculate the horizontal distance the ball has travelled from A when its height is a maximum. *[4 marks]*

(ii) Find the maximum vertical height reached by the football.

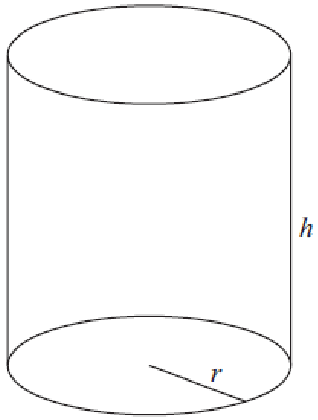
6h. Draw a graph showing the path of the football from the point where it is kicked to the point where it hits the ground again. Use 1 cm to represent 5 m on the horizontal axis and 1 cm to represent 2 m on the vertical scale. *[4 marks]*

6i. The goal posts are 35 m from **the point where the ball is kicked**. *[2 marks]*

At what height does the ball pass over the goal posts?

Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3.

diagram not to scale



Nadia decides to make the radius, r, of the bin 5 cm.

7a. Calculate *[7 marks]*

(i) the area of the base of the wastepaper bin;

(ii) the height, h, of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin.

7b. State whether Nadia's design is practical. Give a reason. *[2 marks]*

Merryn also designs a cylindrical wastepaper bin with a volume of
$8000\ \text{cm}^3$. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

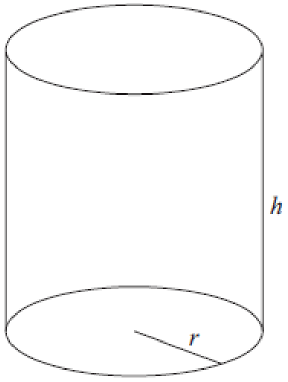


diagram not to scale

Let the radius of the base of Merryn's wastepaper bin be r, and let its height be h.

7c. Write down an equation in h and r, using the given volume of the bin. *[1 mark]*

7d. Show that the total external surface area, A, of the bin is $A = \pi r^2 + \frac{16000}{r}$. *[2 marks]*

7e. Write down $\frac{\mathrm{d}A}{\mathrm{d}r}$. *[3 marks]*

7f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin. *[5 marks]*

(ii) Calculate the value of h corresponding to this value of r.

7g. Determine whether Merryn's design is an improvement upon Nadia's. Give a reason. *[2 marks]*