

## Topic 3 Part 5 [102 marks]

You may choose from three courses on a lunchtime menu at a restaurant.

$s$ : you choose a salad,

$m$ : you choose a meat dish (main course),

$d$ : you choose a dessert.

You choose a **two** course meal which **must** include a main course and either a salad or a dessert, but not both.

1a. Write the sentence above using logic symbols. [2 marks]

1b. Write in words  $s \Rightarrow \neg d$ . [2 marks]

1c. Complete the following truth table. [2 marks]

$s$	$d$	$\neg s$	$\neg s \Rightarrow d$
T	T		
T	F		
F	T		
F	F		

A school offers three activities, basketball ( $B$ ), choir ( $C$ ) and drama ( $D$ ). Every student must participate in at least one activity.

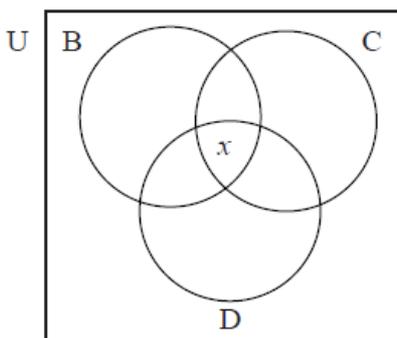
16 students play basketball only.

18 students play basketball and sing in the choir but do not do drama.

34 students play basketball and do drama but do not sing in the choir.

27 students are in the choir and do drama but do not play basketball.

2a. Enter the above information on the Venn diagram below. [2 marks]

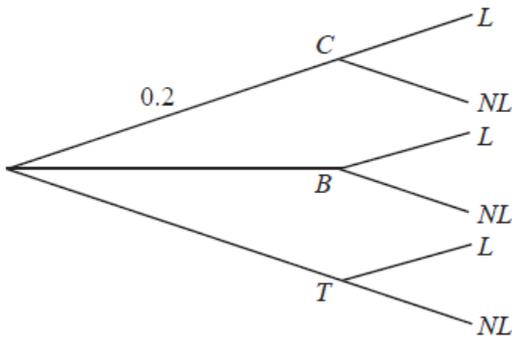


2b. 99 of the students play basketball, 88 sing in the choir and 110 do drama. [1 mark]  
Calculate the number of students  $x$  participating in all three activities.

2c. 99 of the students play basketball, 88 sing in the choir and 110 do drama. [3 marks]  
Calculate the total number of students in the school.

When Geraldine travels to work she can travel either by car ( $C$ ), bus ( $B$ ) or train ( $T$ ). She travels by car on one day in five. She uses the bus 50 % of the time. The probabilities of her being late ( $L$ ) when travelling by car, bus or train are 0.05, 0.12 and 0.08 respectively.

- 3a. Copy the tree diagram below and fill in all the probabilities, where  $NL$  represents not late, to represent this information. [5 marks]



- 3b. Find the probability that Geraldine travels by bus and is late. [1 mark]

- 3c. Find the probability that Geraldine is late. [3 marks]

- 3d. Find the probability that Geraldine travelled by train, given that she is late. [3 marks]

*It is not necessary to use graph paper for this question.*

- 3e. Sketch the curve of the function  $f(x) = x^3 - 2x^2 + x - 3$  for values of  $x$  from  $-2$  to  $4$ , giving the intercepts with both axes. [3 marks]

- 3f. On the same diagram, sketch the line  $y = 7 - 2x$  and find the coordinates of the point of intersection of the line with the curve. [3 marks]

- 3g. Find the value of the gradient of the curve where  $x = 1.7$ . [2 marks]

The truth table below shows the truth-values for the proposition

$$p \vee q \Rightarrow \neg p \vee \neg q$$

$p$	$q$	$\neg p$	$\neg q$	$p \vee q$	$\neg p \vee \neg q$	$p \vee q \Rightarrow \neg p \vee \neg q$
T	T	F	F		F	
T	F	F		T	T	T
F	T	T	F	T	T	T
F	F	T	T	F		T

- 4a. Explain the distinction between the compound propositions,  $p \vee q$  and  $p \vee q$ . [1 mark]

- 4b. Fill in the four missing truth-values on the table. [4 marks]

- 4c. State whether the proposition  $p \vee q \Rightarrow \neg p \vee \neg q$  is a tautology, a contradiction or neither. [1 mark]

The universal set  $U$  is the set of integers from 1 to 20 inclusive.

$A$  and  $B$  are subsets of  $U$  where:

$A$  is the set of even numbers between 7 and 17.

$B$  is the set of multiples of 3.

5a. List the elements of the following sets: [1 mark]  
 $A$ ,

5b. List the elements of the following sets: [1 mark]  
 $B$ ,

5c. List the elements of the following sets: [2 marks]  
 $A \cup B$ ,

5d. List the elements of the following sets: [2 marks]  
 $A \cap B'$ .

A group of 50 students completed a questionnaire for a Mathematical Studies project. The following data was collected.

18 students own a digital camera (D)

15 students own an iPod (I)

26 students own a cell phone (C)

1 student owns all three items

5 students own a digital camera and an iPod but not a cell phone

2 students own a cell phone and an iPod but not a digital camera

3 students own a cell phone and a digital camera but not an iPod

6a. Represent this information on a Venn diagram. [4 marks]

6b. Calculate the number of students who own none of the items mentioned above. [2 marks]

6c. If a student is chosen at random, write down the probability that the student owns a digital camera **only**. [1 mark]

6d. If two students are chosen at random, calculate the probability that they both own a cell phone **only**. [3 marks]

6e. If a student owns an iPod, write down the probability that the student also owns a digital camera. [2 marks]

Claire and Kate both wish to go to the cinema but one of them has to stay at home to baby-sit.

The probability that Kate goes to the cinema is

0.2. If Kate does not go Claire goes.

If Kate goes to the cinema the probability that she is late home is

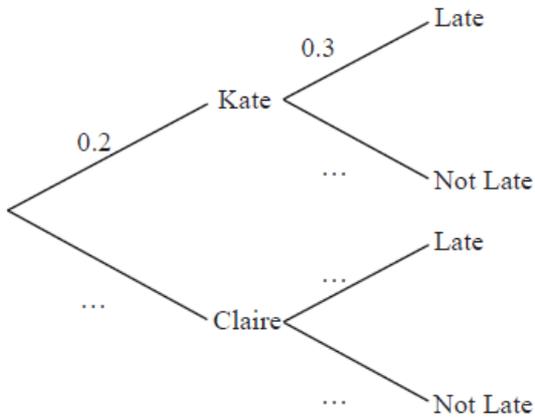
0.3.

If Claire goes to the cinema the probability that she is late home is

0.6.

6f. Copy and complete the probability tree diagram below.

[3 marks]



6g. Calculate the probability that

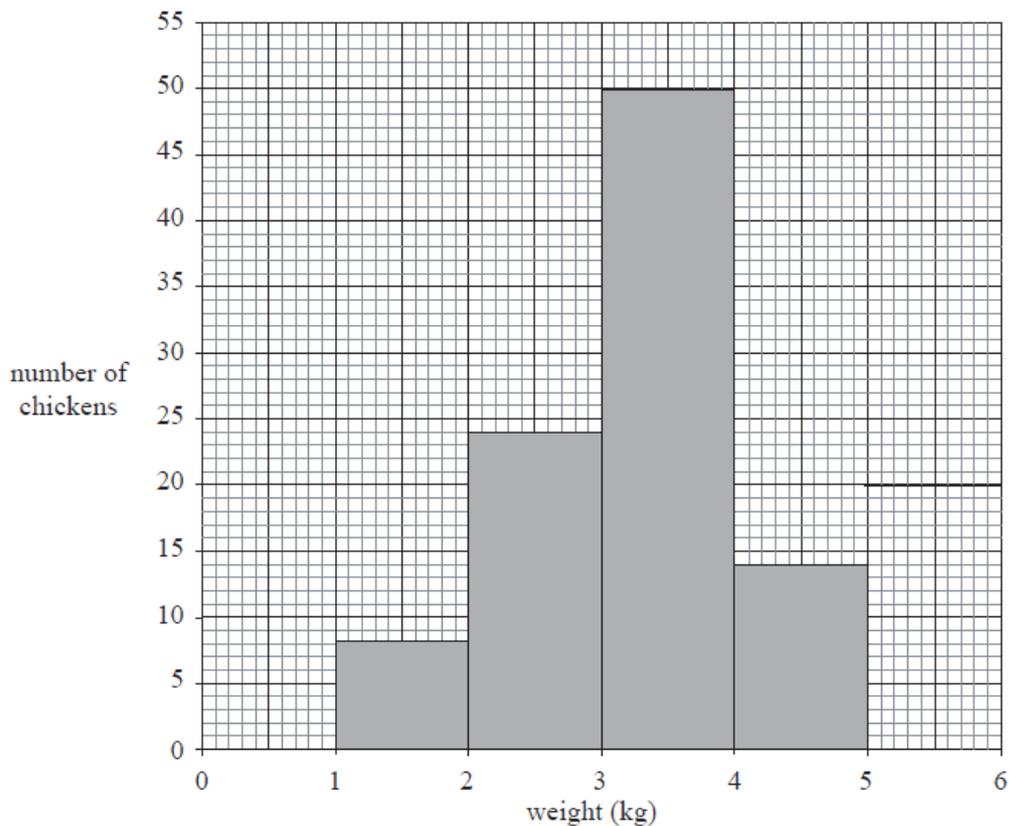
[5 marks]

- (i) Kate goes to the cinema and is not late;
- (ii) the person who goes to the cinema arrives home late.

The following histogram shows the weights of a number of frozen chickens in a supermarket. The weights are grouped such that

$1 \leq \text{weight} < 2$ ,

$2 \leq \text{weight} < 3$  and so on.



7a. Find the total number of chickens.

[1 mark]

7b. Write down the modal group.

[1 mark]

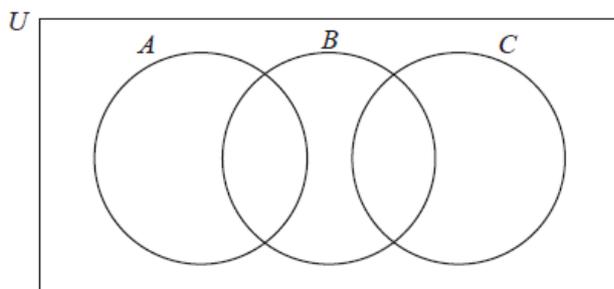
7c. Gabriel chooses a chicken at random.

[2 marks]

Find the probability that this chicken weighs less than 4 kg.

8a. Shade  $(A \cup B) \cap C'$  on the diagram below.

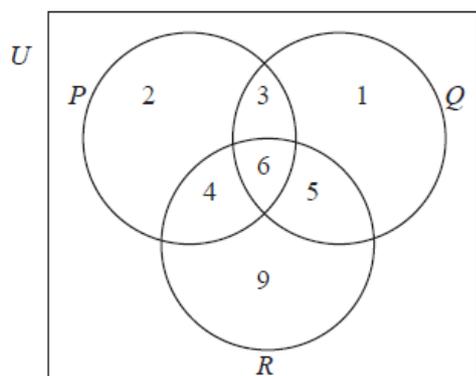
[2 marks]



8b. In the Venn diagram below, the number of elements in each region is given.

[2 marks]

Find  $n((P \cap Q) \cup R)$ .



8c.  $U$  is the set of positive integers,  $\mathbb{Z}^+$ .

[2 marks]

$E$  is the set of even numbers.

$M$  is the set of multiples of 3.

(i) List the first six elements of the set  $M$ .

(ii) List the first six elements of the set  $E' \cap M$ .

When Andy plays tennis,  
65% of his first serves go into the correct area of the court.

If the first serve goes into the correct area, his chance of winning the point is  
90%.

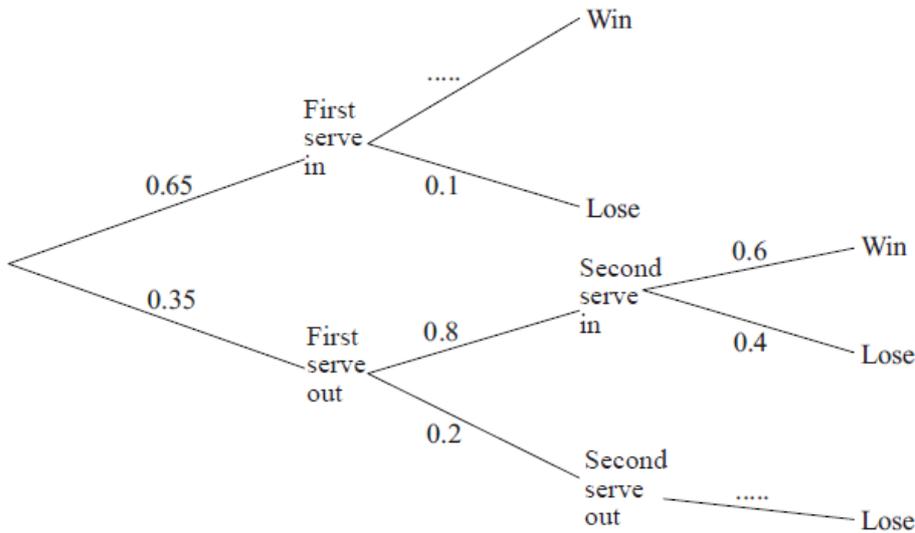
If his first serve does not go into the correct area, Andy is allowed a second serve and, of these,  
80% go into the correct area.

If the second serve goes into the correct area, his chance of winning the point is  
60%.

If neither serve goes into the correct area, Andy loses the point.

9a. Complete the tree diagram below.

[2 marks]



9b. Find the probability that Andy loses the point.

[4 marks]

A survey of  
400 people is carried out by a market research organization in two different cities, Buenos Aires and Montevideo. The people are  
asked which brand of cereal they prefer out of Chocos, Zucos or Fruti. The table below summarizes their responses.

	Chocos	Zucos	Fruti	Total
Buenos Aires	43	85	62	190
Montevideo	57	35	118	210
Total	100	120	180	400

10a. One person is chosen at random from those surveyed. Find the probability that this person

[4 marks]

- (i) does not prefer Zucos;
- (ii) prefers Chocos, given that they live in Montevideo.

10b. Two people are chosen at random from those surveyed. Find the probability that they both prefer Fruti.

[3 marks]

10c. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-squared test at the  
5% level of significance is performed.

[1 mark]

State the null hypothesis.

- 10d. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-squared test at the 5% level of significance is performed. [1 mark]

State the number of degrees of freedom.

- 10e. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-squared test at the 5% level of significance is performed. [2 marks]

Show that the expected frequency for the number of people who live in Montevideo and prefer Zucos is 63.

- 10f. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-squared test at the 5% level of significance is performed. [2 marks]

Write down the chi-squared statistic for this data.

- 10g. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-squared test at the 5% level of significance is performed. [2 marks]

State whether the market research organization would accept the null hypothesis. Clearly justify your answer.

The following table shows the cost in AUD of seven paperback books chosen at random, together with the number of pages in each book.

Book	1	2	3	4	5	6	7
Number of pages ( $x$ )	50	120	200	330	400	450	630
Cost ( $y$ AUD)	6.00	5.40	7.20	4.60	7.60	5.80	5.20

- 10h. Plot these pairs of values on a scatter diagram. Use a scale of 1 cm to represent 50 pages on the horizontal axis and 1 cm to represent 1 AUD on the vertical axis. [3 marks]

- 10i. Write down the linear correlation coefficient,  $r$ , for the data. [2 marks]

- 10j. Stephen wishes to buy a paperback book which has 350 pages in it. He plans to draw a line of best fit to determine the price. State whether or not this is an appropriate method in this case and justify your answer. [2 marks]