Topic 7 Part 1 *[230 marks]*

The table given below describes the behaviour of $f'(x)$, the derivative function of $f(x)$, in the domain $-4 < x < 2$.

x	$f'(x)$
$-4 < x < -2$	< 0
-2	0
$-2 < x < 1$	> 0
1	0
$1 < x < 2$	> 0

1a. State whether $f(0)$ is greater than, less than or equal to $f(-2)$. Give a reason for your answer. *[2 marks]*

1b. The point P$(-2, 3)$ lies on the graph of $f(x)$. *[2 marks]*

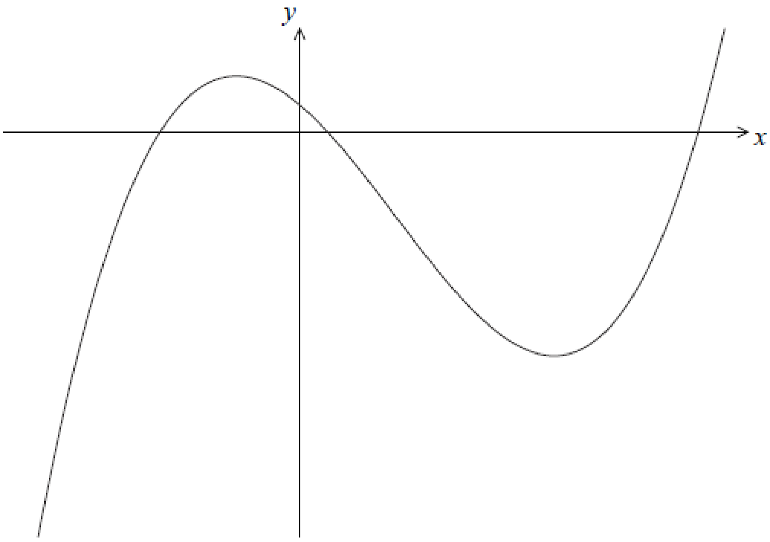
Write down the equation of the tangent to the graph of $f(x)$ at the point P.

1c. The point P$(-2, 3)$ lies on the graph of $f(x)$. *[2 marks]*

From the information given about $f'(x)$, state whether the point $(-2, 3)$ is a maximum, a minimum or neither. Give a reason for your answer.

The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

diagram not to scale



2a. Write down the values of x where the graph of $f(x)$ intersects the x-axis. *[3 marks]*

2b. Write down $f'(x)$. *[3 marks]*

2c. Find the value of the local maximum of $y = f(x)$. *[4 marks]*

2d. Let P be the point where the graph of $f(x)$ intersects the y axis. *[1 mark]*

Write down the coordinates of P.

2e. Let P be the point where the graph of $f(x)$ intersects the y axis. *[2 marks]*

Find the gradient of the curve at P.

2f. The line, L, is the tangent to the graph of $f(x)$ at P. *[2 marks]*

Find the equation of L in the form $y = mx + c$.

2g. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. *[1 mark]*

Write down the gradient of the tangent at Q.

2h. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. *[3 marks]*

Calculate the x-coordinate of Q.

Consider the function $f(x) = x^3 - 3x - 24x + 30$.

3a. Write down $f(0)$. *[1 mark]*

3b. Find $f'(x)$. *[3 marks]*

3c. Find the gradient of the graph of $f(x)$ at the point where $x = 1$. *[2 marks]*

3d. (i) Use $f'(x)$ to find the x-coordinate of M and of N. *[5 marks]*

(ii) Hence or otherwise write down the coordinates of M and of N.

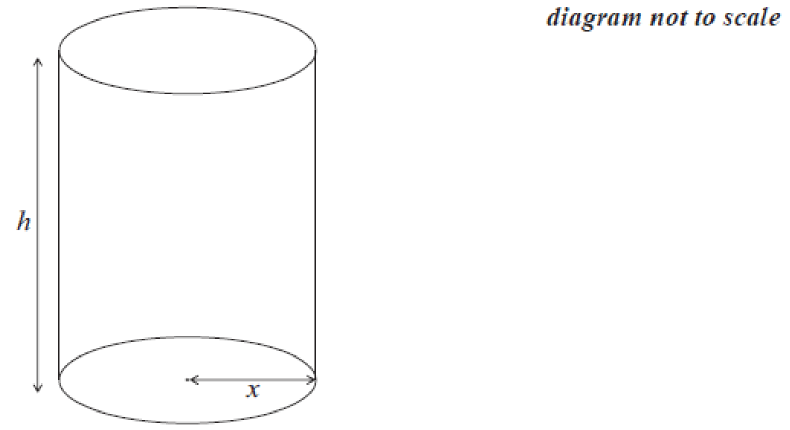
3e. Sketch the graph of $f(x)$ for $-5 \leqslant x \leqslant 7$ and $-60 \leqslant y \leqslant 60$. Mark clearly M and N on your graph. *[4 marks]*

3f. Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$. *[6 marks]*

(i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B.

A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm and x, the radius of the base of the can in cm.



The volume of the dog food cans is 600 cm^3.

4a. Show that $h = \frac{600}{\pi x^2}$. [2 marks]

4b. Find an expression for the curved surface area of the can, in terms of x. Simplify your answer. [2 marks]

4c. Hence write down an expression for A, the total surface area of the can, in terms of x. [2 marks]

4d. Differentiate A in terms of x. [3 marks]

4e. Find the value of x that makes A a minimum. [3 marks]

4f. Calculate the minimum total surface area of the dog food can. [2 marks]

Let $f(x) = 2x^2 + x - 6$

5a. Find $f'(x)$. [3 marks]

5b. Find the value of $f'(-3)$. [1 mark]

5c. Find the value of x for which $f'(x) = 0$. [2 marks]

6a. Sketch the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$. Indicate clearly where the curve intersects the y-axis. [3 marks]

6b. Write down the equation of the asymptote of the graph of $y = 2^x$. [2 marks]

6c. On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. [3 marks]

6d. Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. [2 marks]

6e. Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. *[1 mark]*

6f. Use Differential Calculus to verify that your answer to (e) is correct. *[5 marks]*

6g. The curve $y = px^2 + qx - 4$ passes through the point (2, –10). *[2 marks]*

Use the above information to write down an equation in p and q.

6h. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. *[2 marks]*

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

6i. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. *[1 mark]*

Hence, find a second equation in p and q.

6j. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. *[3 marks]*

Solve the equations to find the value of p and of q.

The straight line, L, has equation $2y - 27x - 9 = 0$.

7a. Find the gradient of L. *[2 marks]*

7b. Sarah wishes to draw the tangent to $f(x) = x^4$ parallel to L. *[1 mark]*

Write down $f'(x)$.

7c. Find the x coordinate of the point at which the tangent must be drawn. *[2 marks]*

7d. Write down the value of $f(x)$ at this point. *[1 mark]*

Consider the function $f(x) = 3x + \frac{12}{x^2}$, $x \neq 0$.

8a. Differentiate $f(x)$ with respect to x. *[3 marks]*

8b. Calculate $f'(x)$ when $x = 1$. *[2 marks]*

8c. Use your answer to part (b) to decide whether the function, f, is increasing or decreasing at $x = 1$. Justify your answer. *[2 marks]*

8d. Solve the equation $f'(x) = 0$. *[3 marks]*

8e. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. *[2 marks]*

Write down the coordinates of P.

8f. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. *[1 mark]*

Write down the gradient of T.

8g. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. *[2 marks]*

Write down the equation of T.

8h. Sketch the graph of the function f, for $-3 \le x \le 6$ and $-7 \le y \le 15$. Indicate clearly the point P and any intercepts of the curve with the axes. *[4 marks]*

8i. On your graph draw and label the tangent T. *[2 marks]*

8j. T intersects the graph of f at a second point. Write down the x-coordinate of this point of intersection. *[1 mark]*

Consider $f : x \mapsto x^2 - 4$.

9a. Find $f'(x)$. *[1 mark]*

9b. Let L be the line with equation $y = 3x + 2$. *[1 mark]*

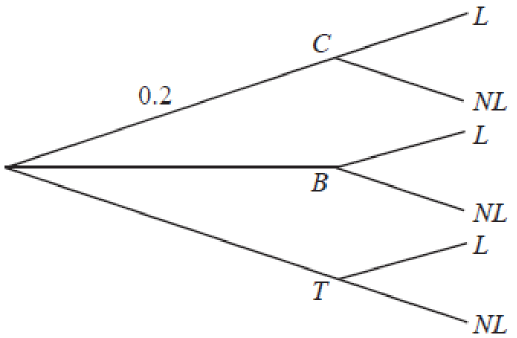
Write down the gradient of a line parallel to L.

9c. Let L be the line with equation $y = 3x + 2$. *[4 marks]*

Let P be a point on the curve of f. At P, the tangent to the curve is parallel to L. Find the coordinates of P.

When Geraldine travels to work she can travel either by car (C), bus (B) or train (T). She travels by car on one day in five. She uses the bus 50 % of the time. The probabilities of her being late (L) when travelling by car, bus or train are 0.05, 0.12 and 0.08 respectively.

10a. Copy the tree diagram below and fill in all the probabilities, where NL represents not late, to represent this information. *[5 marks]*



10b. Find the probability that Geraldine travels by bus and is late. *[1 mark]*

10c. Find the probability that Geraldine is late. *[3 marks]*

10d. Find the probability that Geraldine travelled by train, given that she is late. *[3 marks]*

*It is **not** necessary to use graph paper for this question.*

10e. Sketch the curve of the function $f(x) = x^3 - 2x^2 + x - 3$ for values of x from −2 to 4, giving the intercepts with both axes. *[3 marks]*

10f. On the same diagram, sketch the line $y = 7 - 2x$ and find the coordinates of the point of intersection of the line with the curve. *[3 marks]*

10g. Find the value of the gradient of the curve where $x = 1.7$. *[2 marks]*

A function is represented by the equation

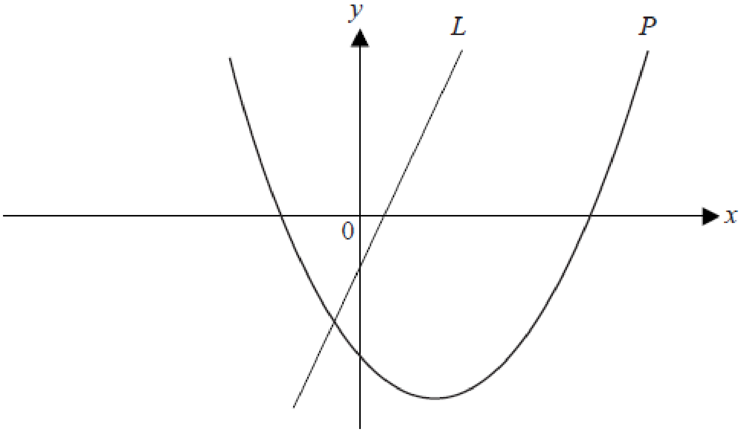
$$f(x) = ax^2 + \frac{4}{x} - 3$$

11a. Find $f'(x)$. *[3 marks]*

11b. The function $f(x)$ has a local maximum at the point where $x = -1$. *[3 marks]*

Find the value of a.

The diagram below shows the graph of a line L passing through (1, 1) and (2 , 3) and the graph P of the function $f(x) = x^2 - 3x - 4$



12a. Find the gradient of the line L. *[2 marks]*

12b. Differentiate $f(x)$. *[2 marks]*

12c. Find the coordinates of the point where the tangent to P is parallel to the line L. *[3 marks]*

12d. Find the coordinates of the point where the tangent to P is perpendicular to the line L. *[4 marks]*

12e. Find *[3 marks]*

(i) the gradient of the tangent to P at the point with coordinates $(2, -6)$.

(ii) the equation of the tangent to P at this point.

12f. State the equation of the axis of symmetry of P. *[1 mark]*

12g. Find the coordinates of the vertex of P and state the gradient of the curve at this point. *[3 marks]*

Consider the function $f(x) = 2x^3 - 5x^2 + 3x + 1$.

13a. Find $f'(x)$. *[3 marks]*

13b. Write down the value of $f'(2)$. *[1 mark]*

13c. Find the equation of the tangent to the curve of $y = f(x)$ at the point $(2, 3)$. *[2 marks]*

14a. Factorise $3x^2 + 13x - 10$. *[2 marks]*

14b. Solve the equation $3x^2 + 13x - 10 = 0$. *[2 marks]*

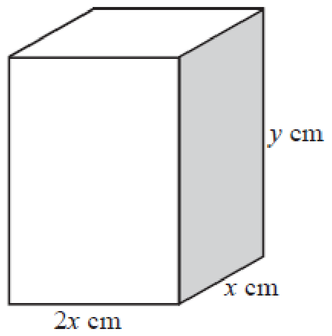
14c. Consider a function $f(x) = 3x^2 + 13x - 10$. *[2 marks]*

Find the equation of the axis of symmetry on the graph of this function.

14d. Consider a function $f(x) = 3x^2 + 13x - 10$. *[2 marks]*

Calculate the minimum value of this function.

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



14e. Show that $4x^2 + 6xy = 300$. *[2 marks]*

14f. Find an expression for y in terms of x. *[2 marks]*

14g. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. *[2 marks]*

14h. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. *[2 marks]*

14i. (i) Hence find the value of x and of y required to make the volume of the box a maximum. *[5 marks]*

(ii) Calculate the maximum volume.

Consider the function $f(x) = \frac{1}{2}x^3 - 2x^2 + 3$.

15a. Find $f'(x)$. *[2 marks]*

15b. Find $f''(x)$. *[2 marks]*

15c. Find the equation of the tangent to the curve of f at the point $(1, 1.5)$. *[2 marks]*

The function
$f(x)$ is such that
$f'(x) < 0$ for
$1 < x < 4$. At the point
$\text{P}(4, 2)$ on the graph of
$f(x)$ the gradient is zero.

16a. Write down the equation of the tangent to the graph of $f(x)$ at P. *[2 marks]*

16b. State whether $f(4)$ is greater than, equal to or less than $f(2)$. *[2 marks]*

16c. Given that $f(x)$ is increasing for $4 \leqslant x < 7$, what can you say about the point P? *[2 marks]*

Consider the function
$f : x \mapsto \frac{kx}{2^x}$.

17a. Given that $f(1) = 2$, show that $k = 4$. *[2 marks]*

17b. Write down the values of q and r for the following table. *[2 marks]*

x	–1	0	1	2	4	8
$f(x)$	–8	0	2	q	1	r

17c. As x increases from -1, the graph of $y = f(x)$ reaches a maximum value and then decreases, behaving asymptotically. *[4 marks]*

Draw the graph of $y = f(x)$ for $-1 \leqslant x \leqslant 8$. Use a scale of 1 cm to represent 1 unit on both axes. The position of the maximum, \({\text{M}}\), the y-intercept and the asymptotic behaviour should be clearly shown.

17d. Using your graphic display calculator, find the coordinates of M, the maximum point on the graph of $y = f(x)$. *[2 marks]*

17e. Write down the equation of the horizontal asymptote to the graph of $y = f(x)$. *[2 marks]*

17f. (i) Draw and label the line $y = 1$ on your graph. *[4 marks]*

(ii) The equation $f(x) = 1$ has two solutions. One of the solutions is $x = 4$. Use your **graph** to find the other solution.

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x}$$

17g. Find $C'(x)$. *[3 marks]*

17h. Show that the cost per person is a minimum when 10 people are invited to the party. *[2 marks]*

17i. Calculate the minimum cost per person. *[2 marks]*