Topic 7 Part 3 *[223 marks]*

1a. *[1 mark]*

Markscheme

30 ***(A1)***

[1 mark]

Examiners report

The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

1b. *[3 marks]*

Markscheme

$f'(x) = 3x^2 - 6x - 24$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each term. Award at most ***(A1)(A1)*** if extra terms present.

[3 marks]

Examiners report

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1c. *[2 marks]*

Markscheme

$f'(1) = -27$ ***(M1)(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for substituting $x = 1$ into their derivative.

[2 marks]

Examiners report

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1d. *[5 marks]*

Markscheme

(i) $f'(x) = 0$

$3x^2 - 6x - 24 = 0$ ***(M1)***

$x = 4$; $x = -2$ ***(A1)*****(ft)*****(A1)*****(ft)**

Notes: Award ***(M1)*** for either $f'(x) = 0$ or $3x^2 - 6x - 24 = 0$ seen. Follow through from their derivative. Do not award the two answer marks if derivative not used.

(ii) M(–2, 58) accept $x = -2$, $y = 58$ ***(A1)*****(ft)**

N(4, – 50) accept $x = 4$, $y = -50$ ***(A1)*****(ft)**

Note: Follow through from their answer to part (d) (i).

[5 marks]

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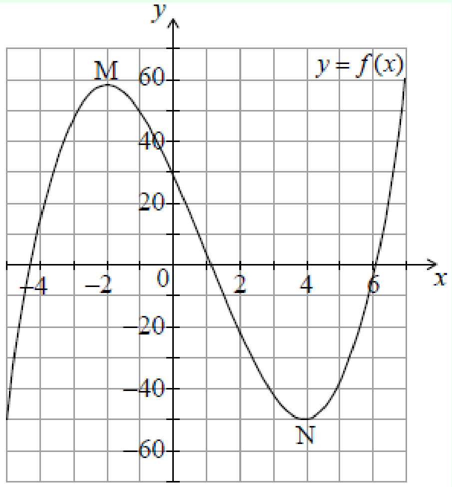
1e. *[4 marks]*

Markscheme



(A1) for window

(A1) for a smooth curve with the correct shape

(A1) for axes intercepts in approximately the correct positions

(A1) for M and N marked on diagram and in approximately correct position ***(A4)***

Note: If window is not indicated award at most ***(A0)(A1)(A0)(A1)*****(ft)**.

[4 marks]

Examiners report

The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

1f. *[6 marks]*

Markscheme

(i) $3x^2 - 6x - 24 = 21$ ***(M1)***

$3x^2 - 6x - 45 = 0$ ***(M1)***

$x = 5; x = -3$ ***(A1)*****(ft)*****(A1)*****(ft)*****(G3)***

Note: Follow through from their derivative.

OR

Award ***(A1)*** for L_1 drawn tangent to the graph of f on their sketch in approximately the correct position ($x = -3$), ***(A1)*** for a second tangent parallel to their L_1, ***(A1)*** for $x = -3$, ***(A1)*** for $x = 5$. ***(A1)*****(ft)*****(A1)*****(ft)*****(A1)(A1)***

Note: If only $x = -3$ is shown without working award ***(G2)*****.** If both answers are shown irrespective of workingaward ***(G3)***.

(ii) $f(5) = -40$ ***(M1)(A1)*****(ft)*****(G2)***

Notes: Award ***(M1)*** for attempting to find the image of their $x = 5$. Award ***(A1)*** only for (5, –40). Follow through from their x-coordinate of B only if it has **been clearly identified** in (f) (i).

[6 marks]

Examiners report

The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

2a. *[3 marks]*

Markscheme

$f'(x) = x^2 + 4x - 12$ ***(A1)(A1)(A1)*** ***(C3)***

Notes: Award ***(A1)*** for each term. Award at most ***(A1)(A1)(A0)*** if other terms are seen.

[3 marks]

Examiners report

This question was quite a good differentiator with many able to score at least one mark in part (a). Part (b) proved however to be quite a challenge as many candidates did not seem to understand what was required and were unable to use their answer to part (a) to help them to meet the demands of this question part. The top quartile scored well with virtually everyone scoring at least three marks. The picture was somewhat reversed with the lower quartile with the majority of candidates scoring 2 or fewer marks.

2b. *[3 marks]*

Markscheme

$-6 \leqslant x \leqslant 2$ **OR**

$-6 < x < 2$ ***(A1)*(ft)*(A1)*(ft)*(A1)*** ***(C3)***

Notes: Award ***(A1)*(ft)** for
-6, ***(A1)*(ft)** for
2, ***(A1)*** for consistent use of strict (
$<$) or weak (
$\leqslant$) inequalities. Final ***(A1)*** for correct interval notation (accept alternative forms). This can only be awarded when the left hand side of the inequality is less than the right hand side of the inequality. Follow through from their solutions to their
$f'(x) = 0$ only if working seen.

[3 marks]

Examiners report

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3a. *[2 marks]*

Markscheme

$x = 0$ ***(A1)(A1)***

Notes: Award ***(A1)*** for x=constant, ***(A1)*** for 0. Award ***(A0)(A0)*** if answer is not an equation.

[2 marks]

Examiners report

This question was moderately well answered. The concept of vertical asymptote in part (a) seemed to be problematic for a great number of candidates. In many cases students showed partial understanding of the vertical asymptote but found it difficult to write a correct equation.

3b. *[3 marks]*

Markscheme

$b - \frac{2}{x^3}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for b, ***(A1)*** for -2, ***(A1)*** for
$\frac{1}{x^3}$ (or x^{-3}). Award at most ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

Examiners report

This question was moderately well answered. The concept of vertical asymptote in part (a) seemed to be problematic for a great number of candidates. In many cases students showed partial understanding of the vertical asymptote but found it difficult to write a correct equation. Finding the derivative in part (b) proved problematic as well. It seems that the presence of the parameter b in the function may have contributed to this.

3c. *[2 marks]*

Markscheme

$3 = b - \frac{2}{(1)^3}$ ***(M1)(M1)***

Note: Award ***(M1)*** for substituting 1 into their gradient function, ***(M1)*** for equating their gradient function to 3.

$b = 5$ ***(AG)***

Note: Award at most ***(M1)(A0)*** if final line is not seen or b does not equal 5.

[2 marks]

Examiners report

This question was moderately well answered. In part (c) a great number of students substituted $b = 5$ in the equation of the function instead of substituting it in the equation of their derivative.

3d. *[3 marks]*

Markscheme

$g(1) = 3$ **or** $(1, 3)$ *(seen or implied from the line below)* ***(A1)***

$3 = 3 \times 1 + c$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their point (1, 3) and gradient 3 into equation $y = mx + c$. Follow through from their point of tangency.

$y = 3x$ ***(A1)*(ft)*(G2)***

OR

$y - 3 = 3(x - 1)$ ***(M1)(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for substitution of gradient 3 and their point (1, 3) into $y - y_1 = m(x - x_1)$, ***(A1)*(ft)** for correct substitutions. Follow through from their point of tangency. Award at most ***(A1)(M1)(A0)*(ft)** if further incorrect working seen.

[3 marks]

Examiners report

This question was moderately well answered. Very few students used the GDC to find the equation of the tangent at $x = 1$ in part (d).

3e. *[2 marks]*

Markscheme

$(-0.439, 0)$ $((-0.438785..., 0))$ ***(G1)(G1)***

Notes: If no parentheses award at most ***(G1)(G0)***. Accept $x = 0.439$, $y = 0$.

[2 marks]

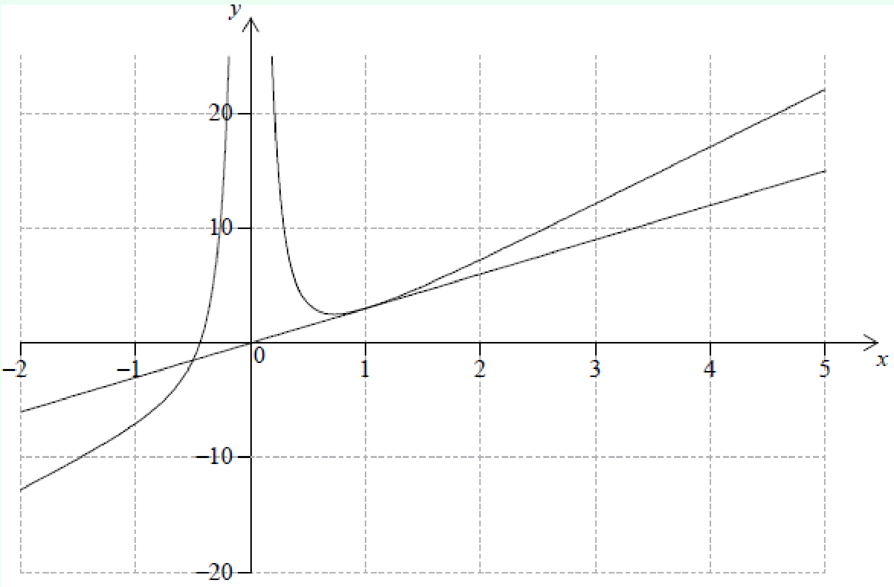
Examiners report

This question was moderately well answered. Good use of the GDC was seen in part (e), although some students wrote the x-coordinates of the point of intersection and neglected to write the y-coordinate.

3f. [6 marks]

Markscheme

(i)



Award *(A1)* for labels and some indication of scale in the stated window.

Award *(A1)* for correct general shape (curve must be smooth and must not cross the y-axis)

Award *(A1)***(ft)** for x-intercept consistent with their part (e).

Award *(A1)* for local minimum in the first quadrant. *(A1)(A1)(A1)***(ft)***(A1)*

(ii) Tangent to curve drawn at approximately $x = 1$ *(A1)(A1)*

Note: Award *(A1)* for a line tangent to curve approximately at $x = 1$. Must be a straight line for the mark to be awarded. Award *(A1)***(ft)** for line passing through the origin. Follow through from their answer to part (d).

[6 marks]

Examiners report

This question was moderately well answered. The sketch in part (f) was, for the most part, not well done. Often the axes labels were missing. Very few tangents to the curve at the correct point were seen. Often the intended tangent lines intersected the curve, which shows that candidates either did not know what a tangent is or did not make sense of the sketch.

3g. [2 marks]

Markscheme

(0.737, 2.53) ((0.736806..., 2.52604...)) *(G1)(G1)*

Notes: Do not penalize for lack of parentheses if already penalized in (e). Accept $x = 0.737$, $y = 2.53$.

[2 marks]

Examiners report

This question was moderately well answered. Good use of the GDC was shown in part (g) for finding the coordinates of the minimum point.

3h. *[2 marks]*

Markscheme

$0.737 < x < 5$ **OR** $(0.737;5)$ ***(A1)(A1)*(ft)**

Notes: Award ***(A1)*** for correct strict or weak inequalities with x seen if the interval is given as inequalities, ***(A1)*(ft)** for 0.737 and 5 or their value from part (g).

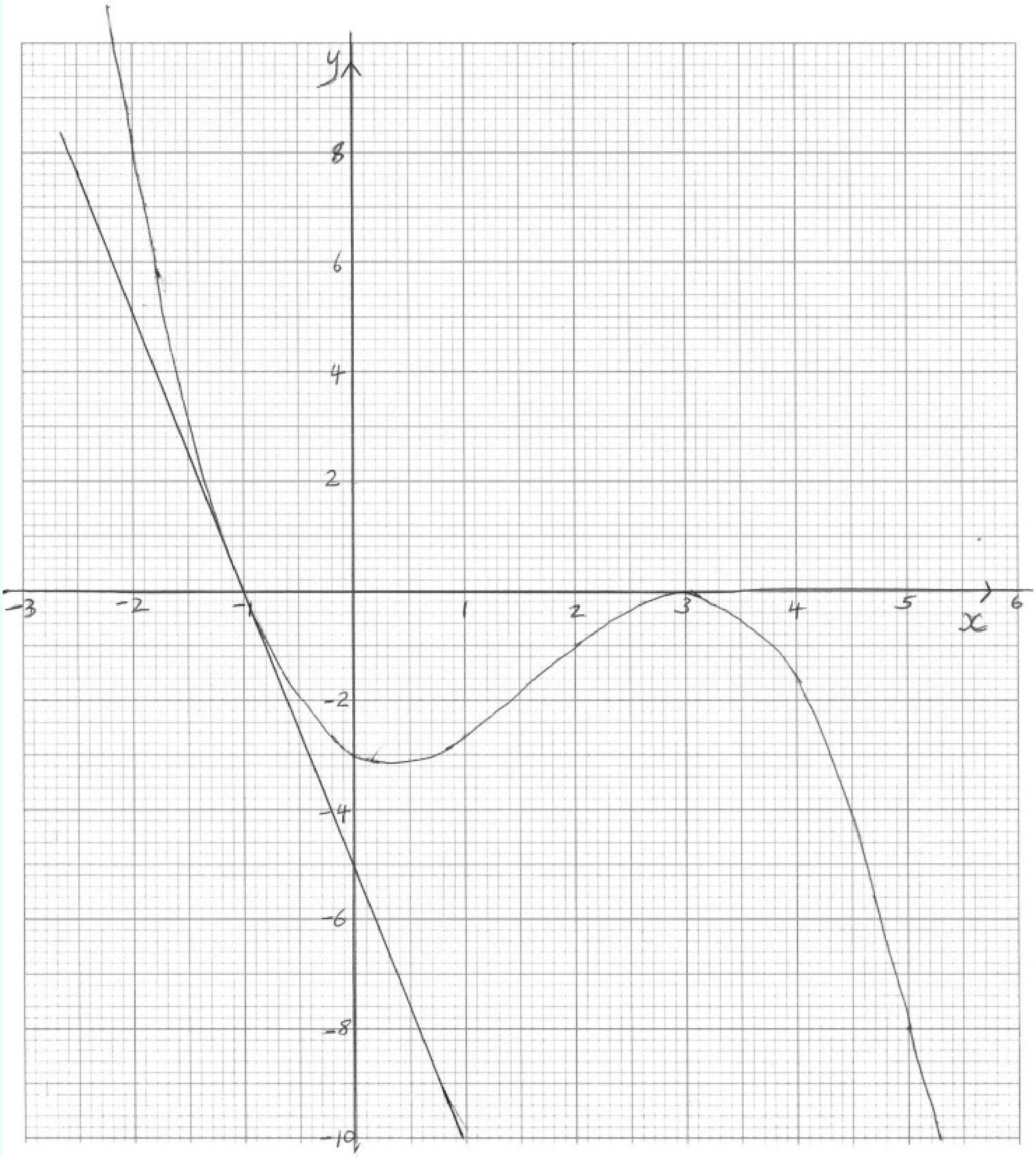
[2 marks]

Examiners report

This question was moderately well answered. Few acceptable answers were given in part (h).

4a. [4 marks]

Markscheme



(A1) for indication of window and labels. *(A1)* for smooth curve that does not enter the first quadrant, the curve must consist of one line only.

(A1) for x and y intercepts in approximately correct positions (allow ±0.5).

(A1) for local maximum and minimum in approximately correct position. (minimum should be $0 \le x \le 1$ and $-2 \le y \le -4$), the y-coordinate of the maximum should be 0 ± 0.5. *(A4)*

[4 marks]

Examiners report

This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

Drawing/sketching graphs is perhaps the area of the course that results in the poorest responses. It is also the area of the course that results in the best. It is therefore the area of the course that good teaching can influence the most.

Candidates should:

- Use the correct scale and window. Label the axes.
- Enter the formula into the GDC and use the table function to determine the points to be plotted.
- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

4b. *[2 marks]*

Markscheme

$-\frac{1}{3}(-1)^3 + \frac{5}{3}(-1)^2 - (-1) - 3$ ***(M1)***

Note: Award ***(M1)*** for substitution of –1 into $f(x)$

$= 0$ ***(A1)(G2)***

[2 marks]

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In part (b) the answer could have been checked using the table on the GDC.

4c. *[1 mark]*

Markscheme

$(0, -3)$ ***(A1)***

OR

$x = 0, y = -3$ ***(A1)***

Note: Award ***(A0)*** if brackets are omitted.

[1 mark]

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In part (c) **coordinates** were required.

4d. *[3 marks]*

Markscheme

$f'(x) = -x^2 + \frac{10}{3}x - 1$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term. Award ***(A1)(A1)(A0)*** at most if there are extra terms.

[3 marks]

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The responses to part (d) were generally correct.

4e. *[1 mark]*

Markscheme

$f'(-1) = -(-1)^2 + \frac{10}{3}(-1) - 1$ ***(M1)***

$= -\frac{16}{3}$ ***(AG)***

Note: Award ***(M1)*** for substitution of $x = -1$ into correct derivative only. The final answer must be seen.

[1 mark]

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The “show that” nature of part (e) meant that the final answer had to be stated.

4f. *[2 marks]*

Markscheme

$f'(-1)$ gives the gradient of the tangent to the curve at the point with $x = -1$. ***(A1)(A1)***

Note: Award ***(A1)*** for “gradient (of curve)”, ***(A1)*** for “at the point with $x = -1$”. Accept “the instantaneous rate of change of y” or “the (first) derivative”.

[2 marks]

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- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

The interpretive nature of part (f) was not understood by the majority.

4g.

[2 marks]

Markscheme

$y = -\frac{16}{3}x + c$ ***(M1)***

Note: Award ***(M1)*** for $-\frac{16}{3}$ substituted in equation.

$0 = -\frac{16}{3} \times (-1) + c$

$c = -\frac{16}{3}$

$y = -\frac{16}{3}x - \frac{16}{3}$ ***(A1)(G2)***

Note: Accept $y = -5.33x - 5.33$.

OR

$(y - 0) = \frac{-16}{3}(x + 1)$ ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for $-\frac{16}{3}$ substituted in equation, ***(A1)*** for correct equation. Follow through from their answer to part (b). Accept $y = -5.33\ (x + 1)$. Accept equivalent equations.

[2 marks]

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4h.

[2 marks]

Markscheme

(A1)**(ft)** for a tangent to their curve drawn.

(A1)**(ft)** for their tangent drawn at the point $x = -1$. ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Follow through from their graph. The tangent must be a straight line otherwise award at most ***(A0)(A1)***.

[2 marks]

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- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

4i. *[2 marks]*

Markscheme

(i)
$a = \frac{1}{3}$ ***(G1)***

(ii)
$b = 3$ ***(G1)***

Note: If a and b are reversed award ***(A0)(A1)***.

[2 marks]

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Parts (i) and (j) had many candidates floundering; there were few good responses to these parts.

4j. *[1 mark]*

Markscheme

$f(x)$ is increasing ***(A1)***

[1 mark]

Examiners report

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Parts (i) and (j) had many candidates floundering; there were few good responses to these parts.

5a. *[2 marks]*

Markscheme

$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 5$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)*** for each correct term. Award ***(A1)(A0)*** if any other terms are given.

[2 marks]

Examiners report

The derivative of the function was correctly found by most candidates. Rearranging the equation of the line to find the gradient was also successfully performed. Most candidates could not find the *x*-coordinate of the point on the curve whose tangent was parallel to a given line. To most candidates, part (b) appeared to be disconnected to part (a).

5b. *[4 marks]*

Markscheme

$y = -3x - \frac{1}{2}$ ***(M1)***

Note: Award ***(M1)*** for rearrangement of equation

gradient of line is –3 ***(A1)***

$4x - 5 = -3$ ***(M1)***

Notes: Award ***(M1)*** for equating their gradient to their derivative from part (a). If $4x - 5 = -3$ is seen with no working award ***(M1)(A1)(M1)***.

$x = \frac{1}{2}$ ***(A1)*****(ft)** ***(C4)***

Note: Follow through from their part (a). If answer is given as (0.5, 2) with no working award the final ***(A1)*** only.

[4 marks]

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The derivative of the function was correctly found by most candidates. Rearranging the equation of the line to find the gradient was also successfully performed. Most candidates could not find the *x*-coordinate of the point on the curve whose tangent was parallel to a given line. To most candidates, part (b) appeared to be disconnected to part (a).

6a.

[3 marks]

Markscheme

$4(2x) + 4y + 4x = 48$ ***(M1)***

Note: Award ***(M1)*** for setting up the equation.

$12x + 4y = 48$ ***(M1)***

Note: Award ***(M1)*** for simplifying (can be implied).

$y = \frac{48-12x}{4}$ **OR**

$3x + y = 12$ ***(A1)***

$y = 12 - 3x$ ***(AG)***

Note: The last line must be seen for the ***(A1)*** to be awarded.

[3 marks]

Examiners report

Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(a) This was very poorly done. Most candidates had no idea what they were supposed to do here. Many tried to find values for *x*.

6b.

[2 marks]

Markscheme

$V = 2x \times x \times (12 - 3x)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into volume equation, ***(A1)*** for correct substitution.

$= 24x^2 - 6x^3$ ***(AG)***

Note: The last line must be seen for the ***(A1)*** to be awarded.

[2 marks]

Examiners report

Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(a) This was very poorly done. Most candidates had no idea what they were supposed to do here. Many tried to find values for *x*.

(b) Similar comment as for part (a) although more candidates made an attempt at finding the Volume.

6c.

[2 marks]

Markscheme

$\frac{\mathrm{d}V}{\mathrm{d}x} = 48x - 18x^2$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

[2 marks]

Examiners report

Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(c) This part was very well done.

6d. *[3 marks]*

Markscheme

$48x - 18x^2 = 0$ ***(M1)(M1)***

Note: Award ***(M1)*** for using their derivative, ***(M1)*** for equating their answer to part (c) to 0.

OR

(M1) for sketch of
$V = 24x^2 - 6x^3$, ***(M1)*** for the maximum point indicated ***(M1)(M1)***

OR

(M1) for sketch of
$\frac{\mathrm{d}V}{\mathrm{d}x} = 48x - 18x^2$, ***(M1)*** for the positive root indicated ***(M1)(M1)***

$2.67\left(\frac{24}{9}, \frac{8}{3}, 2.66666...\right)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their part (c).

[3 marks]

Examiners report

Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(d) Not many correct answers seen. Many candidates graphed the wrong equation and found 1.333 as their answer.

6e. *[2 marks]*

Markscheme

$V = 24 \times \left(\frac{8}{3}\right)^2 - 6 \times \left(\frac{8}{3}\right)^3$ ***(M1)***

Note: Award ***(M1)*** for substitution of their value from part (d) into volume equation.

$56.9(\mathrm{m}^3)\left(\frac{512}{9}, 56.8888...\right)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answer to part (d).

[2 marks]

Examiners report

Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(e) Some managed to gain follow through marks for this part.

6f. *[3 marks]*

Markscheme

length $= \frac{16}{3}$ ***(A1)*(ft)*(G1)***

Note: Follow through from their answer to part (d). Accept 5.34 from use of 2.67

height $= 12 - 3 \times \left(\frac{8}{3}\right) = 4$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for substitution of their answer to part (d), ***(A1)*(ft)** for answer. Accept 3.99 from use of 2.67.

[3 marks]

Examiners report

Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(f) Again here follow through marks were gained by those who attempted it.

6g. *[4 marks]*

Markscheme

SA $= 2 \times \frac{16}{3} \times 4 + 2 \times \frac{8}{3} \times 4 + 2 \times \frac{16}{3} \times \frac{8}{3}$ ***(M1)***

OR

SA $= 4\left(\frac{8}{3}\right)^2 + 6 \times \frac{8}{3} \times 4$ ***(M1)***

Note: Award ***(M1)*** for substitution of their values from parts (d) and (f) into formula for surface area.

92.4 (m^2) (92.4444...(m^2)) ***(A1)***

Note: Accept 92.5 (92.4622...) from use of 3 sf answers.

Number of tins $= \frac{92.4444...}{15 \times 4} (= 1.54)$ ***(M1)***

[4 marks]

Note: Award ***(M1)*** for division of their surface area by 60.

2 tins required ***(A1)*(ft)**

Note: Follow through from their answers to parts (d) and (f).

Examiners report

Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(g) Very few correct answers for the surface area were seen. Most candidates thought that there were 4 equal faces 2 xy and 2 faces xy. Some managed to get follow through marks for the last part if they divided by 60.

7a. *[5 marks]*

Markscheme

(i)
$x = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for
$x =$ a constant, ***(A1)*** for the constant in their equation being
0.

(ii)
-1.58 (
$-1.58454\ldots$) ***(G1)***

Note: Accept
-1.6, do not accept
-2 or
-1.59.

(iii)
$(2.06, 4.49)$
$(2.06020\ldots, 4.49253\ldots)$ ***(G1)(G1)***

Note: Award at most ***(G1)(G0)*** if brackets not used. Award ***(G0)(G1)*(ft)** if coordinates are reversed.

Note: Accept
$x = 2.06$,
$y = 4.49$.

Note: Accept
2.1, do not accept
2.0 or
2. Accept
4.5, do not accept
5 or
4.50.

[5 marks]

Examiners report

As usual, the content in this question caused difficulty for many candidates. However, for those with a sound grasp of the topic, there were many very successful attempts. The curve was given so that a comparison could be made to a GDC version and the correct form of the derivative was also given to permit weaker candidates to progress to the latter stages. Unfortunately, some decided to proceed with their own incorrect versions, in which case **very limited follow through accrued**. It should be emphasized to candidates that when an answer is given in this way it should be used in subsequent parts of the question.

As in previous years, much of the question could have been answered successfully by using the GDC. However, it was also clear that a large number of candidates did not attempt either to verify their work with their GDC or to use it in place of an algebraic approach.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

7b. *[4 marks]*

Markscheme

$f'(x) = 2x - 2 - \frac{9}{x^2}$ ***(A1)(A1)(A1)(A1)***

Notes: Award ***(A1)*** for
$2x$, ***(A1)*** for
-2, ***(A1)*** for
-9, ***(A1)*** for
x^{-2} . Award a maximum of ***(A1)(A1)(A1)(A0)*** if there are extra terms present.

[4 marks]

Examiners report

As usual, the content in this question caused difficulty for many candidates. However, for those with a sound grasp of the topic, there were many very successful attempts. The curve was given so that a comparison could be made to a GDC version and the correct form of the derivative was also given to permit weaker candidates to progress to the latter stages. Unfortunately, some decided to proceed with their own incorrect versions, in which case **very limited follow through accrued**. It should be emphasized to candidates that when an answer is given in this way it should be used in subsequent parts of the question.

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7c.

[2 marks]

Markscheme

$f'(x) = \frac{x^2(2x-2)}{x^2} - \frac{9}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for taking the correct common denominator.

$= \frac{(2x^3-2x^2)}{x^2} - \frac{9}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for multiplying brackets or equivalent.

$= \frac{2x^3-2x^2-9}{x^2}$ ***(AG)***

Note: The final ***(M1)*** is not awarded if the given answer is not seen.

[2 marks]

Examiners report

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7d.

[2 marks]

Markscheme

$f'(1) = \frac{2(1)^3-2(1)-9}{(1)^2}$ ***(M1)***

$= -9$ ***(A1)(G2)***

Note: Award ***(M1)*** for substitution into **given** (or their correct)
$f'(x)$. There is no follow through for use of their incorrect derivative.

[2 marks]

Examiners report

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7e. *[1 mark]*

Markscheme

$\frac{1}{9}$ ***(A1)*(ft)**

Note: Follow through from part (d).

[1 mark]

Examiners report

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7f. *[3 marks]*

Markscheme

$y - 8 = \frac{1}{9}(x - 1)$ ***(M1)(M1)***

Notes: Award ***(M1)*** for substitution of their gradient from (e), ***(M1)*** for substitution of given point. Accept all forms of straight line.

$y = \frac{1}{9}x + \frac{71}{9}$ (
$y = 0.111111\ldots x + 7.88888\ldots$) ***(A1)*(ft)*(G3)***

Note: Award the final ***(A1)*(ft)** for a correctly rearranged formula of **their** straight line in (f). Accept
$0.11x$, do not accept
$0.1x$. Accept
7.9, do not accept
7.88, do not accept
7.8.

[3 marks]

Examiners report

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7g. *[2 marks]*

Markscheme

-2.50,
3.61 (
$-2.49545\ldots$,
$3.60656\ldots$)   ***(A1)*(ft)*(A1)*(ft)**

Notes: Follow through from their line
L from part (f) even if no working shown. Award at most ***(A0)(A1)*(ft)** if their correct coordinate pairs given.

Note: Accept
-2.5, do not accept
-2.49. Accept
3.6, do not accept
3.60.

[2 marks]

Examiners report

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8a. *[2 marks]*

Markscheme

$f(2) = 2^3 + \frac{48}{2}$ ***(M1)***

$= 32$ ***(A1)(G2)***

[2 marks]

Examiners report

As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

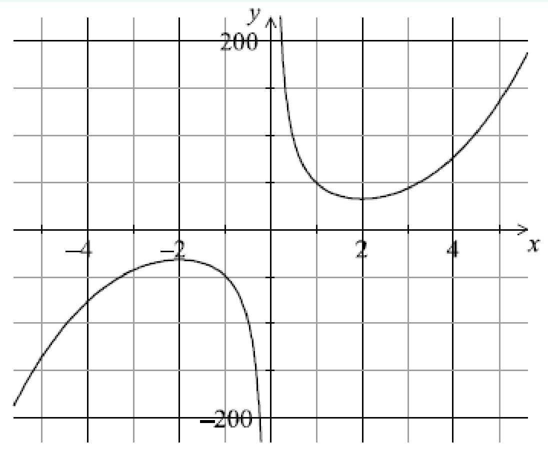
The majority of sketches were drawn sloppily and with little attention to detail. Teachers must impress on their students that a mathematical sketch is designed to illustrate the main points of a curve – the smooth nature by which it changes, any symmetries (reflectional or rotational), positions of turning points, intercepts with axes and the behaviour of a curve as it approaches an asymptote. There must also be some indication of the dimensions used for the “window”.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

It was also evident that some centres do not teach the differential calculus.

8b. *[4 marks]*

Markscheme



(A1) for labels and some indication of scale in an appropriate window
(A1) for correct shape of the two unconnected and smooth branches
(A1) for maximum and minimum in approximately correct positions
(A1) for asymptotic behaviour at
y-axis ***(A4)***

Notes: Please be rigorous.
The axes need not be drawn with a ruler.
The branches must be smooth: a single continuous line that does not deviate from its proper direction.
The position of the maximum and minimum points must be symmetrical about the origin.
The
y-axis must be an asymptote for both branches. Neither branch should touch the axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

Examiners report

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8c.

[3 marks]

Markscheme

$f'(x) = 3x^2 - \frac{48}{x^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for
$3x^2$, ***(A1)*** for
-48 , ***(A1)*** for
x^{-2} . Award a maximum of ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

Examiners report

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8d.

[2 marks]

Markscheme

$f'(2) = 3(2)^2 - \frac{48}{(2)^2}$ ***(M1)***

Note: Award ***(M1)*** for substitution of
$x = 2$ into their derivative.

$= 0$ ***(A1)*****(ft)*****(G1)***

[2 marks]

Examiners report

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8e. *[2 marks]*

Markscheme

$(-2,\ -32)$ or
$x = -2,$
$y = -32$ ***(G1)(G1)***

Notes: Award ***(G0)(G0)*** for
$x = -32,$
$y = -2$. Award at most ***(G0)(G1)*** if parentheses are omitted.

[2 marks]

Examiners report

As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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8f. *[3 marks]*

Markscheme

$\{y \geqslant 32\} \cup \{y \leqslant -32\}$ ***(A1)(A1)*(ft)*(A1)*(ft)**

Notes: Award ***(A1)*(ft)**
$y \geqslant 32$ or
$y > 32$ seen, ***(A1)*(ft)** for
$y \leqslant -32$ or
$y < -32$, ***(A1)*** for weak (non-strict) inequalities used in both of the above.
Accept use of
f in place of
y. Accept alternative interval notation.
Follow through from their (a) and (e).
If domain is given award ***(A0)(A0)(A0)***.
Award ***(A0)(A1)*(ft)*(A1)*(ft)** for
$[-200,\ -32]$,
$[32,\ 200]$.
Award ***(A0)(A1)*(ft)*(A1)*(ft)** for
$]-200,\ -32]$,
$[32,\ 200[$.

[3 marks]

Examiners report

As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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8g. *[2 marks]*

Markscheme

$f'(1) = -45$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for
$f'(1)$ seen or substitution of
$x = 1$ into their derivative. Follow through from their derivative if working is seen.

[2 marks]

Examiners report

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8h. *[2 marks]*

Markscheme

$x = -1$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for equating their derivative to their
-45 or for seeing parallel lines on their graph in the approximately correct position.

[2 marks]

Examiners report

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9a. *[1 mark]*

$\frac{1}{2}x \ \left(\frac{2}{4}x\right)$

$2 \times \frac{1}{4}x$

Examiners report

Parts (a) and (b) were reasonably well attempted indicating that candidates are well drilled in the process of differentiation. Correct answers however in part (c) proved elusive to many as frequent attempts to equate the two given functions rather than the gradients of the given functions resulted in a popular, but incorrect, answer of $x = -1.46$. Part (d) was poorly attempted with many candidates simply either not attempting to draw a tangent or drawing it in the wrong place.

9b.

[1 mark]

Markscheme

1 ***(A1)*** ***(C1)***

[1 mark]

Examiners report

Parts (a) and (b) were reasonably well attempted indicating that candidates are well drilled in the process of differentiation. Correct answers however in part (c) proved elusive to many as frequent attempts to equate the two given functions rather than the gradients of the given functions resulted in a popular, but incorrect, answer of $x = -1.46$. Part (d) was poorly attempted with many candidates simply either not attempting to draw a tangent or drawing it in the wrong place.

9c.

[2 marks]

Markscheme

$\frac{1}{2}x = 1$ ***(M1)***

$x = 2$ ***(A1)*(ft)** ***(C2)***

Notes: Award ***(M1)(A0)*** for coordinate pair $(2,\ -1)$ seen with or without working. Follow through from their answers to parts (a) and (b).

[2 marks]

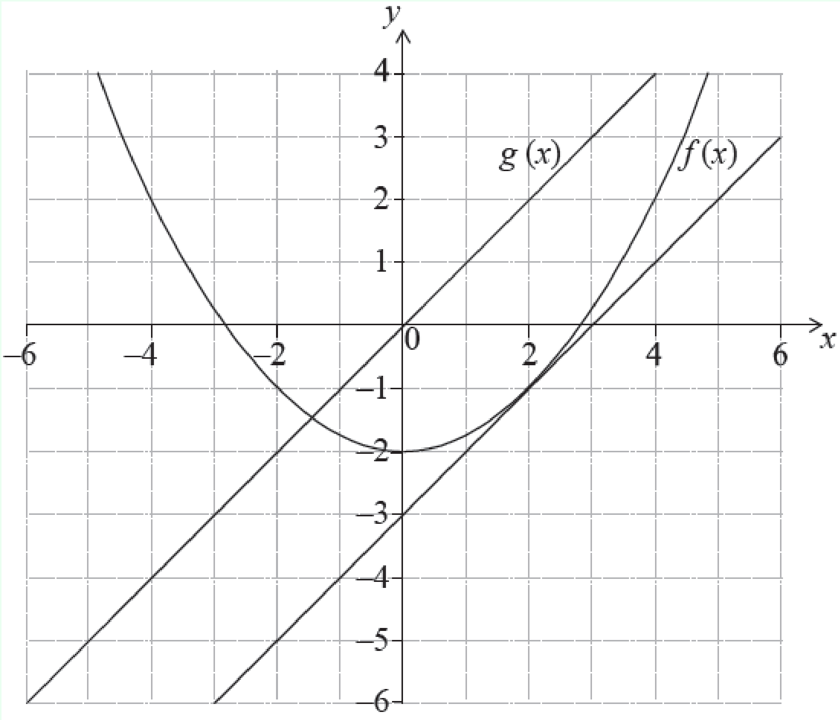
Examiners report

Parts (a) and (b) were reasonably well attempted indicating that candidates are well drilled in the process of differentiation. Correct answers however in part (c) proved elusive to many as frequent attempts to equate the two given functions rather than the gradients of the given functions resulted in a popular, but incorrect, answer of $x = -1.46$. Part (d) was poorly attempted with many candidates simply either not attempting to draw a tangent or drawing it in the wrong place.

9d.

[2 marks]

Markscheme



tangent drawn to the parabola at the x-coordinate found in part (c) ***(A1)*(ft)**

candidate's attempted tangent drawn parallel to the graph of $g(x)$ ***(A1)*(ft)** ***(C2)***

[2 marks]

Examiners report

Parts (a) and (b) were reasonably well attempted indicating that candidates are well drilled in the process of differentiation. Correct answers however in part (c) proved elusive to many as frequent attempts to equate the two given functions rather than the gradients of the given functions resulted in a popular, but incorrect, answer of $x = -1.46$. Part (d) was poorly attempted with many candidates simply either not attempting to draw a tangent or drawing it in the wrong place.

10a.

[2 marks]

Markscheme

$x = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for $x =$ constant, ***(A1)*** for 0.

[2 marks]

Examiners report

Part a) was either answered well or poorly.

10b. *[3 marks]*

Markscheme

$f'(x) = 1.5 - \frac{6}{x^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for
1.5, ***(A1)*** for
-6, ***(A1)*** for
x^{-2} . Award ***(A1)(A1)(A0)*** at most if any other term present.

[3 marks]

Examiners report

Most candidates found the first term of the derivative in part b) correctly, but the rest of the terms were incorrect.

10c. *[2 marks]*

Markscheme

$1.5 - \frac{6}{(-1)}$ ***(M1)***

$= -4.5$ ***(A1)*(ft)*(G2)***

Note: Follow through from their derivative function.

[2 marks]

Examiners report

The gradient in c) was for the most part correctly calculated, although some candidates substituted incorrectly in
$f(x)$ instead of in
$f'(x)$.

10d. *[2 marks]*

Markscheme

Decreasing, the derivative (gradient or slope) is negative (at
$x = -1$) ***(A1)(R1)*(ft)**

Notes: Do not award ***(A1)(R0)***. Follow through from their answer to part (c).

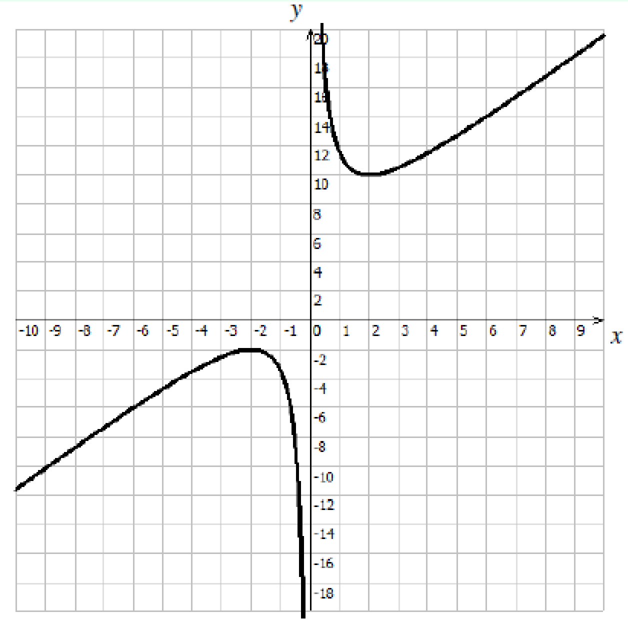
[2 marks]

Examiners report

Part d) had mixed responses.

10e. *[4 marks]*

Markscheme



(A4)

Notes: Award *(A1)* for labels and some indication of scales and an appropriate window.

Award *(A1)* for correct shape of the two unconnected, and smooth branches.

Award *(A1)* for the maximum and minimum points in the approximately correct positions.

Award *(A1)* for correct asymptotic behaviour at $x = 0$.

Notes: Please be rigorous.

The axes need not be drawn with a ruler.

The branches must be smooth and single continuous lines that do not deviate from their proper direction.

The max and min points must be symmetrical about point $(0, 4)$.

The y-axis must be an asymptote for **both** branches.

[4 marks]

Examiners report

Lack of labels of the axes, appropriate scale, window, incorrect maximum and minimum and incorrect asymptotic behaviour were the main problems with the sketches in e).

10f.

[4 marks]

Markscheme

(i)
$(-2,\ -2)$ or
$x = -2$,
$y = -2$ ***(G1)(G1)***

(ii)
$(2, 10)$ or
$x = 2$,
$y = 10$ ***(G1)(G1)***

[4 marks]

Examiners report

Part f) was also either answered correctly or entirely incorrectly. Some candidates used the trace function on the GDC instead of the min and max functions, and thus acquired coordinates with unacceptable accuracy. Some were unclear that a point of local maximum may be positioned on the coordinate system "below" the point of local minimum, and exchanged the pairs of coordinates of those points in f(i) and f(ii).

10g.

[3 marks]

Markscheme

$\{-2 \geqslant y\}$ or
$\{y \geqslant 10\}$ ***(A1)(A1)*(ft)*(A1)***

Notes: ***(A1)*(ft)** for
$y > 10$ or
$y \geqslant 10$. ***(A1)*(ft)** for
$y < -2$ or
$y \leqslant -2$. ***(A1)*** for weak (non-strict) inequalities used in **both** of the above. Follow through from their (e) and (f).

[3 marks]

Examiners report

Very few candidates were able to identify the range of the function in (g) irrespective of whether or not they had the sketches drawn correctly.

11a.

[3 marks]

Markscheme

$f'(x) = 3 - \frac{24}{x^3}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for 3, ***(A1)*** for –24, ***(A1)*** for x^3 (or x^{-3}). If extra terms present award at most ***(A1)(A1)(A0)***.

[3 marks]

Examiners report

Many students did not know the term "differentiate" and did not answer part (a).

11b. *[2 marks]*

Markscheme

$f'(1) = -21$ ***(M1)(A1)*(ft)*(G2)***

Note: (ft) from their derivative only if working seen.

[2 marks]

Examiners report

However, the derivative was seen in (b) when finding the gradient at $x = 1$. The negative index of the formula did cause problems for many when finding the derivative. The meaning of the derivative was not clear for a number of students.

11c. *[2 marks]*

Markscheme

Derivative (gradient, slope) is negative. Decreasing. ***(R1)(A1)*(ft)**

Note: Do not award ***(R0)(A1)***.

[2 marks]

Examiners report

[N/A]

11d. *[3 marks]*

Markscheme

$3 - \frac{24}{x^3} = 0$ ***(M1)***

$x^3 = 8$ ***(A1)***

$x = 2$ ***(A1)*(ft)*(G2)***

[3 marks]

Examiners report

Part (d) was handled well by some but many substituted $x = 0$ into $f'(x)$.

11e. *[2 marks]*

Markscheme

(2, 9) (Accept $x = 2$, $y = 9$) ***(A1)(A1)(G2)***

Notes: (ft) from their answer in (d).

Award ***(A1)(A0)*** if brackets not included and not previously penalized.

[2 marks]

Examiners report

It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

11f. *[1 mark]*

Markscheme

0 ***(A1)***

[1 mark]

Examiners report

It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

11g. *[2 marks]*

Markscheme

$y = 9$ ***(A1)(A1)*****(ft)*****(G2)***

Notes: Award ***(A1)*** for y = constant, ***(A1)*** for 9.

Award ***(A1)*****(ft)** for their value of y in (e)(i).

[2 marks]

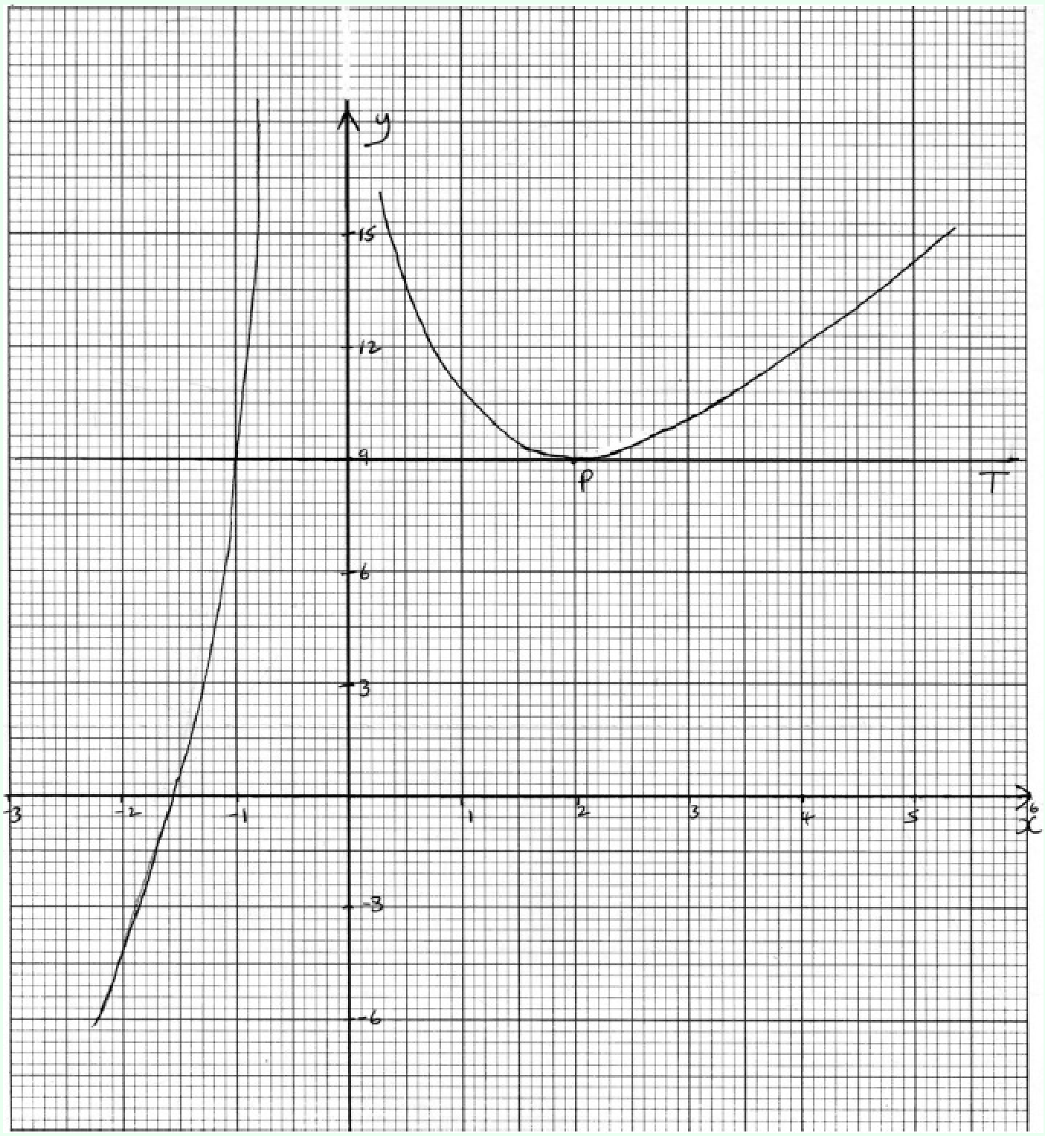
Examiners report

It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

11h.

[4 marks]

Markscheme



(A4)

Notes: Award *(A1)* for labels and some indication of scale in the stated window.

Award *(A1)* for correct general shape (curve must be smooth and must not cross the y-axis).

Award *(A1)* for x-intercept seen in roughly the correct position.

Award *(A1)* for minimum (P).

[4 marks]

Examiners report

There were good answers to the sketch though setting out axes and a scale seemed not to have had enough practise.

11i. *[2 marks]*

Markscheme

Tangent drawn at P (line must be a tangent and horizontal). ***(A1)***

Tangent labeled T. ***(A1)***

Note: (ft) from their tangent equation only if tangent is drawn and answer is consistent with graph.

[2 marks]

Examiners report

Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.

11j. *[1 mark]*

Markscheme

$x = -1$ ***(G1)*****(ft)**

[1 mark]

Examiners report

Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.

12a. *[4 marks]*

Markscheme

$f'(x) = \frac{-10}{x^3} + 3$ ***(A1)(A1)(A1)(A1)***

Note: Award ***(A1)*** for -10, ***(A1)*** for x^3 (or x^{-3}), ***(A1)*** for 3, ***(A1)*** for no other constant term.

[4 marks]

Examiners report

A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Most candidates could score 4 marks.

12b. *[2 marks]*

Markscheme

$4 = 5 + 3 + c$ ***(M1)***

Note: Award ***(M1)*** for substitution in $f(x)$.

$c = -4$ ***(A1)(G2)***

[2 marks]

Examiners report

A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

A good number of candidates correctly substituted into the original function.

12c. *[4 marks]*

Markscheme

$f'(x) = 0$ ***(M1)***

$0 = \frac{-10}{x^3} + 3$ ***(A1)*(ft)**

$(1.49, 2.72)$ (accept $x = 1.49$ $y = 2.72$) ***(A1)*(ft)*(A1)*(ft)*(G3)***

Notes: If answer is given as $(1.5, 2.7)$ award ***(A0)(AP)(A1)***.

Award at most ***(M1)(A1)(A1)(A0)*** if parentheses not included. **(ft)** from their (a).

If no working shown award ***(G2)(G0)*** if parentheses are not included.

OR

Award ***(M2)*** for sketch, ***(A1)*(ft)*(A1)*(ft)** for correct coordinates. **(ft)** from their (b). ***(M2)(A1)*(ft)*(A1)*(ft)**

Note: Award at most ***(M2)(A1)*(ft)*(A0)*** if parentheses not included.

[4 marks]

Examiners report

A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Very few managed to answer this part algebraically. Those candidates who were aware that they could read the values from their GDC gained some easy marks.

12d. *[3 marks]*

Markscheme

$0 < x < 1.49$ **OR** $0 < x \le 1.49$ ***(A1)(A1)*(ft)*(A1)***

Notes: Award ***(A1)*** for 0, ***(A1)*(ft)** for 1.49 and ***(A1)*** for correct inequality signs.

(ft) from their x value in (c) (i).

[3 marks]

Examiners report

A large number of candidates were not able to answer this question well because of the firstterm of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Proved to be difficult for many candidates as increasing/decreasing intervals were not well understood by many.

12e. *[2 marks]*

Markscheme

For P(1, 4)
$f'(1) = -10 + 3$ ***(M1)(A1)***

$= -7$ ***(AG)***

Note: Award ***(M1)*** for substituting
$x = 1$ into their
$f'(x)$. ***(A1)*** for
$-10 + 3$.

-7 must be seen for ***(A1)*** to be awarded.

[2 marks]

Examiners report

A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

As this part was linked to part (a), candidates who could not find the correct derivative could not show that the gradient of *T* was −7. Many candidates did not realize that they had to substitute into the first derivative. For those who did, finding the equation of *T* was a simple task.

12f. *[2 marks]*

Markscheme

$4 = -7 \times 1 + c$
$11 = c$ ***(A1)***

$y = -7x + 11$ ***(A1)***

[2 marks]

Examiners report

A large number of candidates were not able to answer this question well because of the firstterm of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

As this part was linked to part (a), candidates who could not find the correct derivative could not show that the gradient of *T* was −7. Many candidates did not realize that they had to substitute into the first derivative. For those who did, finding the equation of *T* was a simple task.

12g.

[2 marks]

Markscheme

Point of intersection is R(–0.5, 14.5)   ***(A1)*****(ft)*****(A1)*****(ft)*****(G2)*****(ft)**

Notes: Award ***(A1)*** for the *x* coordinate, ***(A1)*** for the *y* coordinate.

Allow **(ft)** from candidate's (d)(ii) equation and their (b) even with no working seen.

Award ***(A1)*****(ft)*****(A0)*** if brackets not included and not previously penalised.

[2 marks]

Examiners report

A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

For those who had part (d) (ii) correct, this allowed them to score easy marks. For the others, it proved a difficult task because their equation in (d) would not be a tangent.

13a.

[3 marks]

Markscheme

(i)
$y = 0$ ***(A1)***

(ii)
$(0,\ -2)$ ***(A1)(A1)***

Note: Award ***(A1)(A0)*** if brackets missing.

OR

$x = 0,\ y = -2$ ***(A1)(A1)***

Note: If coordinates reversed award ***(A0)(A1)*****(ft)**. Two coordinates must be given.

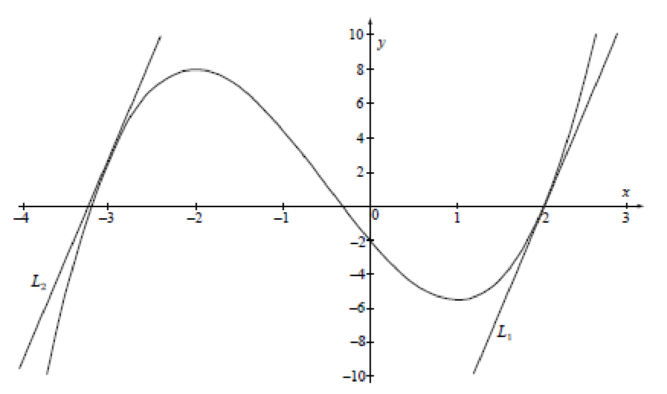
[3 marks]

Examiners report

Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x . In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.

13b. *[4 marks]*

Markscheme



(A4)

Notes: *(A1)* for appropriate window. Some indication of scale on the x-axis must be present (for example ticks). Labels not required. *(A1)* for smooth curve and shape, *(A1)* for maximum and minimum in approximately correct position, *(A1)* for x and y intercepts found in (a) in approximately correct position.

[4 marks]

Examiners report

Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x . In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.

13c. *[3 marks]*

Markscheme

$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 3x - 6$ *(A1)(A1)(A1)*

Note: *(A1)* for each correct term. Award *(A1)(A1)(A0)* at most if any other term is present.

[3 marks]

Examiners report

Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x . In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.

13d. *[8 marks]*

Markscheme

(i)

$3 \times 4 + 3 \times 2 - 6 = 12$ ***(M1)(A1)(AG)***

Note: ***(M1)*** for using the derivative and substituting $x = 2$. ***(A1)*** for correct (and clear) substitution. The 12 must be seen.

(ii) Gradient of L_2 is 12 (can be implied) ***(A1)***

$3x^2 + 3x - 6 = 12$ ***(M1)***

$x = -3$ ***(A1)(G2)***

Note: ***(M1)*** for equating the derivative to 12 or showing a sketch of the derivative together with a line at $y = 12$ or a table of values showing the 12 in the derivative column.

(iii) ***(A1)*** for L_1 correctly drawn at approx the correct point ***(A1)***

(A1) for L_2 correctly drawn at approx the correct point ***(A1)***

(A1) for 2 parallel lines ***(A1)***

Note: If lines are not labelled award at most ***(A1)(A1)(A0)***. Do not accept 2 horizontal or 2 vertical parallel lines.

[8 marks]

Examiners report

Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x . In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.

13e. *[5 marks]*

Markscheme

(i)
$b = 1$ ***(G2)***

(ii) The curve is decreasing. ***(A1)***

Note: Accept any valid description.

(iii)
$y = 8$ ***(A1)(A1)(G2)***

Note: ***(A1)*** for "
$y =$ a constant", ***(A1)*** for
8.

[5 marks]

Examiners report

Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x. In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.