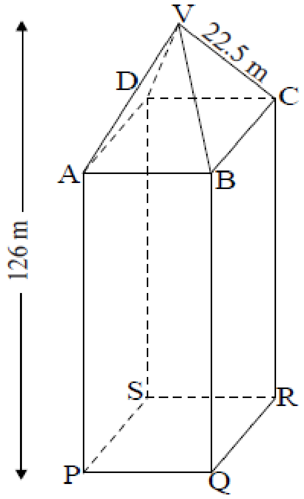
Topic 5 Part 1 *[240 marks]*

The diagram shows an office tower of total height 126 metres. It consists of a square based pyramid VABCD on top of a cuboid ABCDPQRS.

V is directly above the centre of the base of the office tower.

The length of the sloping edge VC is 22.5 metres and the angle that VC makes with the base ABCD (angle VCA) is 53.1°.

diagram not to scale



1a. Write down the length of VA in metres. *[1 mark]*

1b. Sketch the triangle VCA showing clearly the length of VC and the size of angle VCA. *[1 mark]*

1c. Show that the height of the pyramid is 18.0 metres correct to 3 significant figures. *[2 marks]*

1d. Calculate the length of AC in metres. *[3 marks]*

1e. Show that the length of BC is 19.1 metres correct to 3 significant figures. *[2 marks]*

1f. Calculate the volume of the tower. *[4 marks]*

1g. To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90 % of the volume of the tower is occupied by air and they know that 1 m^3 of air weighs 1.2 kg. *[3 marks]*

Calculate the weight of air in the tower.

A gardener has to pave a rectangular area 15.4 metres long and 5.5 metres wide using rectangular bricks. The bricks are 22 cm long and 11 cm wide.

2a. Calculate the total area to be paved. Give your answer in cm^2. *[3 marks]*

2b. Write down the area of each brick. *[1 mark]*

2c. Find how many bricks are required to pave the total area. *[2 marks]*

The gardener decides to have a triangular lawn ABC, instead of paving, in the middle of the rectangular area, as shown in the diagram below.

The distance AB is 4 metres, AC is 6 metres and angle BAC is 40°.

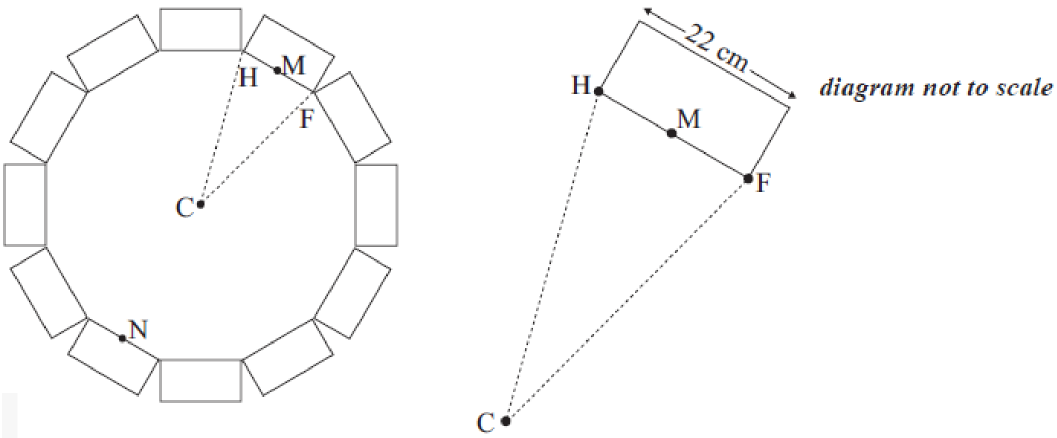
2d. Find the length of BC. *[3 marks]*

2e. Hence write down the perimeter of the triangular lawn. *[1 mark]*

2f. Calculate the area of the lawn. *[2 marks]*

2g. Find the percentage of the rectangular area which is to be lawn. *[3 marks]*

In another garden, twelve of the same rectangular bricks are to be used to make an edge around a small garden bed as shown in the diagrams below. FH is the length of a brick and C is the centre of the garden bed. M and N are the midpoints of the long edges of the bricks on opposite sides of the garden bed.



2h. Find the angle FCH. *[2 marks]*

2i. Calculate the distance MN from one side of the garden bed to the other, passing through C. *[3 marks]*

The garden bed has an area of 5419 cm^2. It is covered with soil to a depth of 2.5 cm.

2j. Find the volume of soil used. *[2 marks]*

It is estimated that 1 kilogram of soil occupies 514 cm^3.

2k. Find the number of kilograms of soil required for this garden bed. *[2 marks]*

A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm and x, the radius of the base of the can in cm.

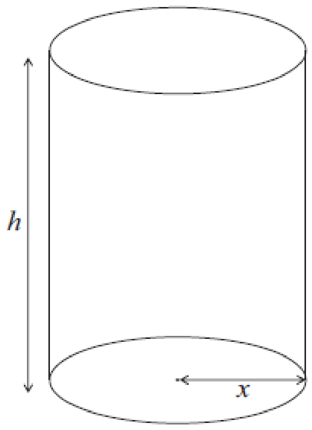


diagram not to scale

The volume of the dog food cans is 600 cm^3.

3a. Show that $h = \frac{600}{\pi x^2}$. [2 marks]

3b. Find an expression for the curved surface area of the can, in terms of x. Simplify your answer. [2 marks]

3c. Hence write down an expression for A, the total surface area of the can, in terms of x. [2 marks]

3d. Differentiate A in terms of x. [3 marks]

3e. Find the value of x that makes A a minimum. [3 marks]

3f. Calculate the minimum total surface area of the dog food can. [2 marks]

Consider the statement p:

“If a quadrilateral is a square then the four sides of the quadrilateral are equal”.

4a. Write down the inverse of statement p in words. [2 marks]

4b. Write down the converse of statement p in words. [2 marks]

4c. Determine whether the converse of statement p is always true. Give an example to justify your answer. [2 marks]

A line joins the points A(2, 1) and B(4, 5).

5a. Find the gradient of the line AB. [2 marks]

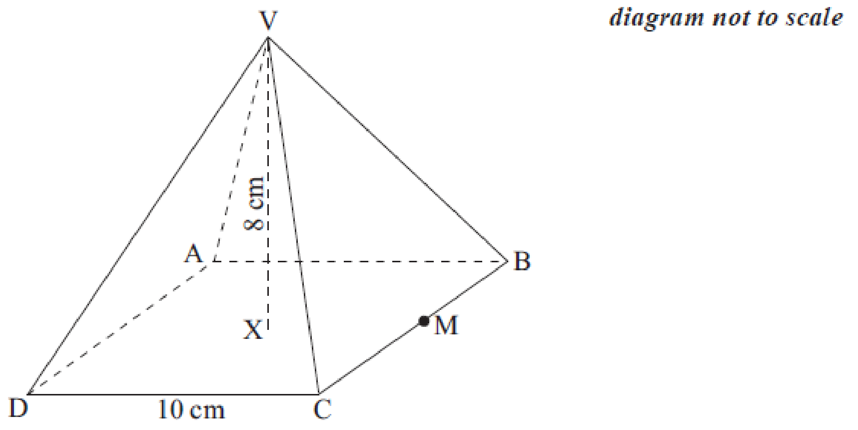
5b. Let M be the midpoint of the line segment AB. [1 mark]

Write down the coordinates of M.

5c. Let M be the midpoint of the line segment AB. [3 marks]

Find the equation of the line perpendicular to AB and passing through M.

The diagram below shows a square based right pyramid. ABCD is a square of side 10 cm. VX is the perpendicular height of 8 cm. M is the midpoint of BC.



6a. Write down the length of XM. *[1 mark]*

In a mountain region there appears to be a relationship between the number of trees growing in the region and the depth of snow in winter. A set of 10 areas was chosen, and in each area the number of trees was counted and the depth of snow measured. The results are given in the table below.

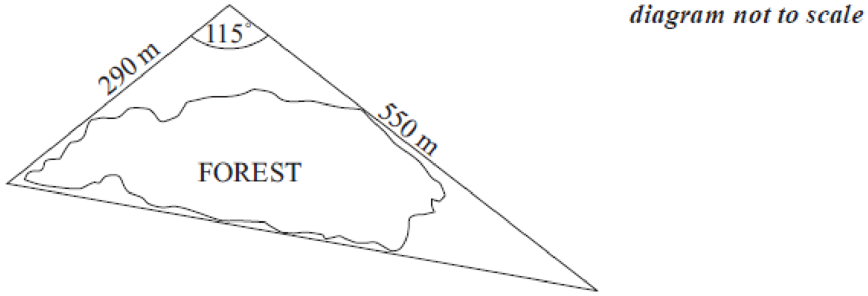
Number of trees (x)	Depth of snow in cm (y)
45	30
75	50
66	40
27	25
44	30
28	5
60	35
35	20
73	45
47	25

6b. Use your graphic display calculator to find the standard deviation of the number of trees. *[1 mark]*

6c. Calculate the length of VM. *[2 marks]*

6d. Calculate the angle between VM and ABCD. *[2 marks]*

A path goes around a forest so that it forms the three sides of a triangle. The lengths of two sides are 550 m and 290 m. These two sides meet at an angle of 115°. A diagram is shown below.



6e. Calculate the length of the third side of the triangle. Give your answer correct to the nearest 10 m. *[4 marks]*

6f. Calculate the area enclosed by the path that goes around the forest. *[3 marks]*

6g. Inside the forest a second path forms the three sides of another triangle named ABC. Angle BAC is 53°, AC is 180 m and BC is 230 m. *[4 marks]*

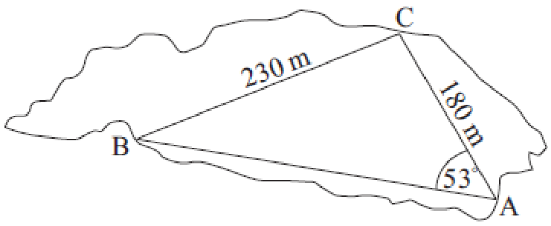
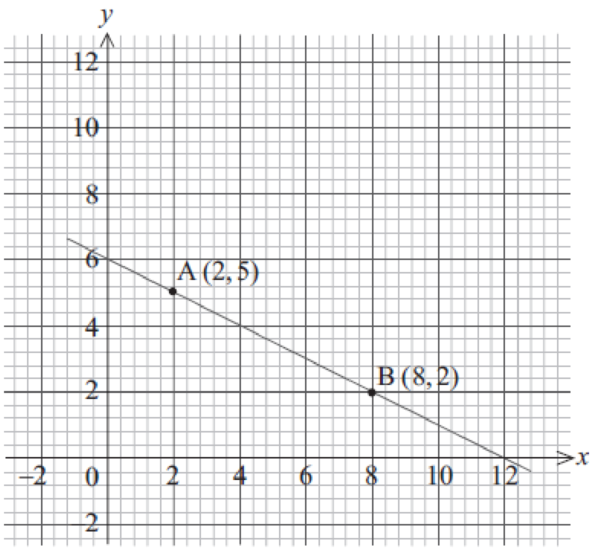


diagram not to scale

Calculate the size of angle ACB.

A and B are points on a straight line as shown on the graph below.



7a. Write down the *y*-intercept of the line AB. *[1 mark]*

7b. Calculate the gradient of the line AB. *[2 marks]*

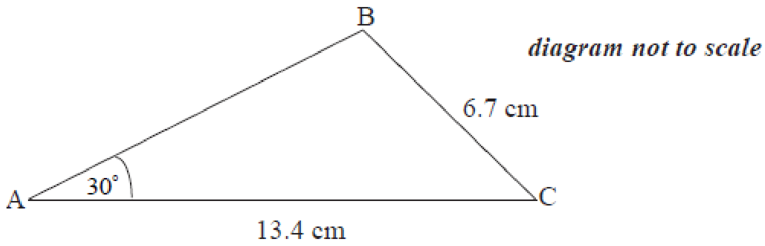
7c. The acute angle between the line AB and the *x*-axis is θ. *[1 mark]*

Show θ on the diagram.

7d. The acute angle between the line AB and the *x*-axis is θ. *[2 marks]*

Calculate the size of θ.

The diagram shows triangle ABC in which angle BAC
$= 30°$, BC
$= 6.7$ cm and AC
$= 13.4$ cm.



8a. Calculate the size of angle ACB. *[4 marks]*

8b. Nadia makes an accurate drawing of triangle ABC. She measures angle BAC and finds it to be 29°. *[2 marks]*

Calculate the percentage error in Nadia's measurement of angle BAC.

Tennis balls are sold in cylindrical tubes that contain four balls. The radius of each tennis ball is 3.15 cm and the radius of the tube is 3.2 cm. The length of the tube is 26 cm.

9a. Find the volume of one tennis ball. *[2 marks]*

9b. Calculate the volume of the empty space in the tube when four tennis balls have been placed in it. *[4 marks]*

The straight line, *L*, has equation
$2y - 27x - 9 = 0$.

10a. Find the gradient of *L*. *[2 marks]*

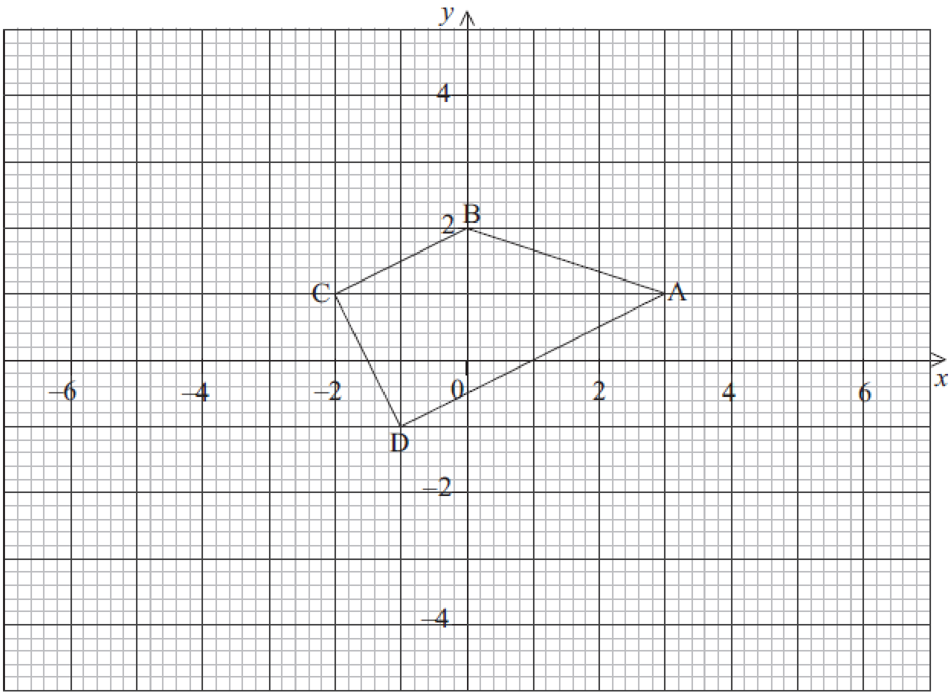
10b. Sarah wishes to draw the tangent to
$f(x) = x^4$ parallel to *L*. *[1 mark]*

Write down
$f'(x)$.

10c. Find the *x* coordinate of the point at which the tangent must be drawn. *[2 marks]*

10d. Write down the value of
$f(x)$ at this point. *[1 mark]*

The vertices of quadrilateral ABCD as shown in the diagram are A (3, 1), B (0, 2), C (–2, 1) and D (–1, –1).



11a. Calculate the gradient of line CD. *[2 marks]*

11b. Show that line AD is perpendicular to line CD. *[2 marks]*

11c. Find the equation of line CD. Give your answer in the form $ax + by = c$ where $a,\ b,\ c \in \mathbb{Z}$. *[3 marks]*

11d. Lines AB and CD intersect at point E. The equation of line AB is $x + 3y = 6$. *[2 marks]*

Find the coordinates of E.

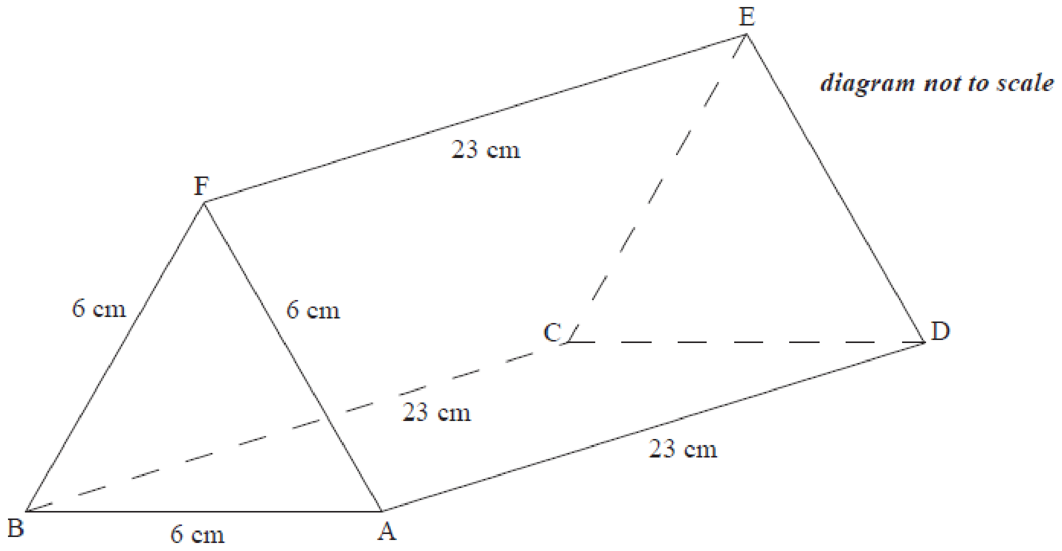
11e. Lines AB and CD intersect at point E. The equation of line AB is $x + 3y = 6$. *[2 marks]*

Find the distance between A and D.

11f. The distance between D and E is $\sqrt{20}$. *[2 marks]*

Find the area of triangle ADE.

A chocolate bar has the shape of a triangular right prism ABCDEF as shown in the diagram. The ends are equilateral triangles of side 6 cm and the length of the chocolate bar is 23 cm.



12a. Write down the size of angle BAF. *[1 mark]*

12b. Hence or otherwise find the area of the triangular end of the chocolate bar. *[3 marks]*

12c. Find the total surface area of the chocolate bar. *[3 marks]*

12d. It is known that 1 cm^3 of this chocolate weighs 1.5 g. Calculate the weight of the chocolate bar. *[3 marks]*

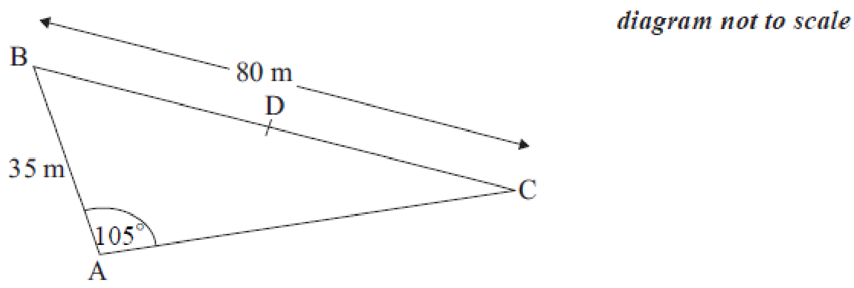
12e. A different chocolate bar made with the same mixture also has the shape of a triangular prism. The ends are triangles with sides of length 4 cm, 6 cm and 7 cm. *[3 marks]*

Show that the size of the angle between the sides of 6 cm and 4 cm is 86.4° correct to 3 significant figures.

12f. The weight of this chocolate bar is 500 g. Find its length. *[4 marks]*

A farmer has a triangular field, ABC, as shown in the diagram.

AB = 35 m, BC = 80 m and $B\hat{A}C = 105°$, and D is the midpoint of BC.



13a. Find the size of $B\hat{C}A$. *[3 marks]*

13b. Calculate the length of AD. *[5 marks]*

13c. The farmer wants to build a fence around ABD. *[2 marks]*

Calculate the total length of the fence.

13d. The farmer wants to build a fence around ABD. *[2 marks]*

The farmer pays 802.50 USD for the fence. Find the cost per metre.

13e. Calculate the area of the triangle ABD. *[3 marks]*

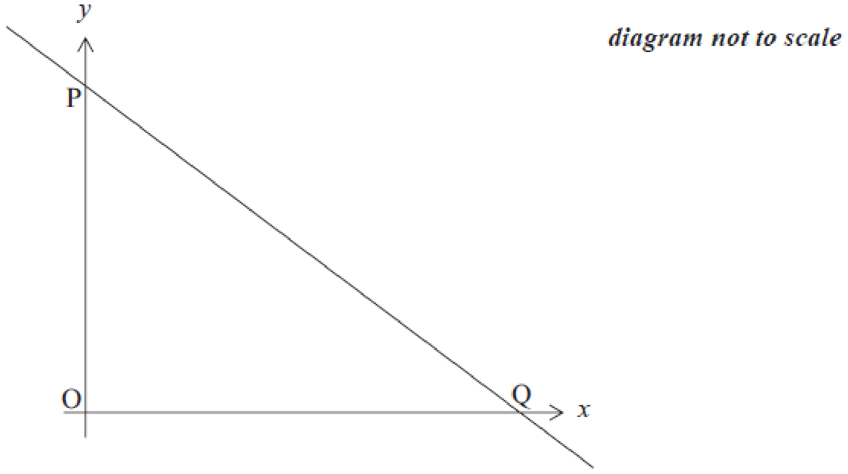
13f. A layer of earth 3 cm thick is removed from ABD. Find the volume removed in cubic metres. *[3 marks]*

A rectangle is 2680 cm long and 1970 cm wide.

14a. Find the perimeter of the rectangle, giving your answer in the form $a \times 10^k$, where $1 \leqslant a \leqslant 10$ and $k \in \mathbb{Z}$. *[3 marks]*

14b. Find the area of the rectangle, giving your answer correct to the nearest thousand square centimetres. *[3 marks]*

The diagram below shows the line PQ, whose equation is $x + 2y = 12$. The line intercepts the axes at P and Q respectively.



15a. Find the coordinates of P and of Q. *[3 marks]*

15b. A second line with equation $x - y = 3$ intersects the line PQ at the point A. Find the coordinates of A. *[3 marks]*

The coordinates of the vertices of a triangle ABC are A (4, 3), B (7, –3) and C (0.5, p).

16a. Calculate the gradient of the line AB. *[2 marks]*

16b. Given that the line AC is perpendicular to the line AB *[1 mark]*

write down the gradient of the line AC.

16c. Given that the line AC is perpendicular to the line AB *[3 marks]*

find the value of p.

Consider
$f : x \mapsto x^2 - 4$.

17a. Find $f'(x)$. *[1 mark]*

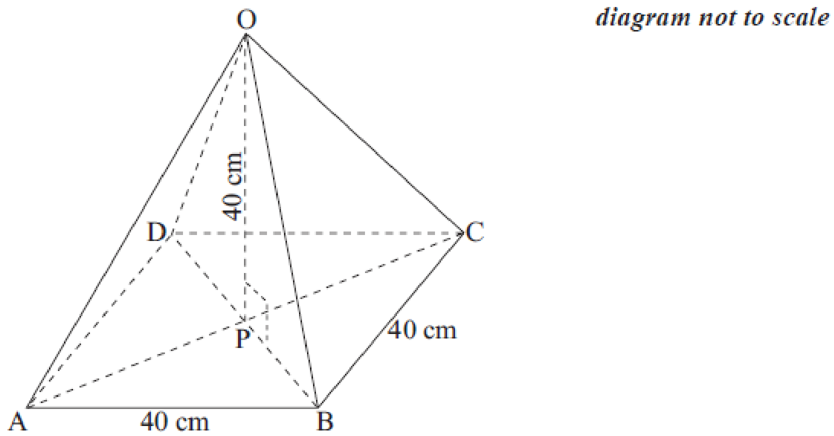
17b. Let L be the line with equation $y = 3x + 2$. *[1 mark]*

Write down the gradient of a line parallel to L.

17c. Let L be the line with equation $y = 3x + 2$. *[4 marks]*

Let P be a point on the curve of f. At P, the tangent to the curve is parallel to L. Find the coordinates of P.

The right pyramid shown in the diagram has a square base with sides of length 40 cm. The height of the pyramid is also 40 cm.



18a. Find the length of OB. *[4 marks]*

18b. Find the size of angle OBP. *[2 marks]*

A straight line, L_1, has equation $x + 4y + 34 = 0$.

19a. Find the gradient of L_1. *[2 marks]*

19b. The equation of line L_2 is $y = mx$. L_2 is perpendicular to L_1. *[2 marks]*

Find the value of m.

19c. The equation of line L_2 is $y = mx$. L_2 is perpendicular to L_1. *[2 marks]*

Find the coordinates of the point of intersection of the lines L_1 and L_2.

The diagram shows a rectangular based right pyramid VABCD in which
AD = 20 cm,
DC = 15 cm and the height of the pyramid,
VN = 30 cm.

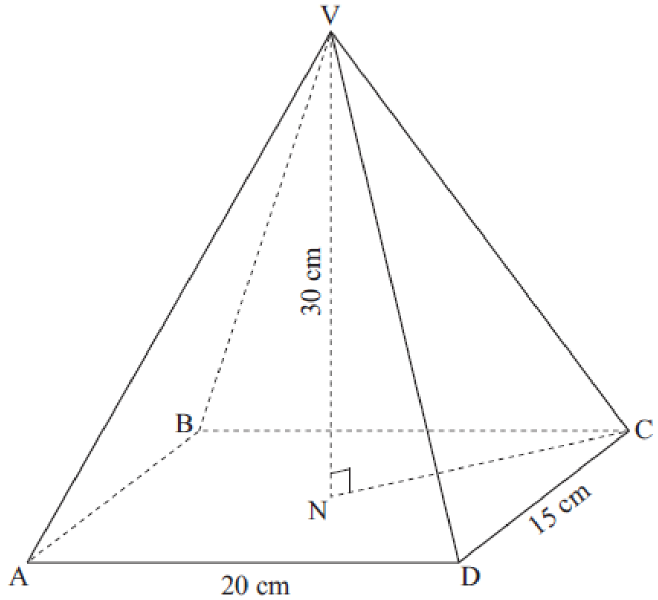


diagram not to scale

20a. Calculate [4 marks]
(i) the length of AC;
(ii) the length of VC.

20b. Calculate the angle between VC and the base ABCD. [2 marks]

A child's toy consists of a hemisphere with a right circular cone on top. The height of the cone is 12 cm and the radius of its base is
5 cm . The toy is painted red.

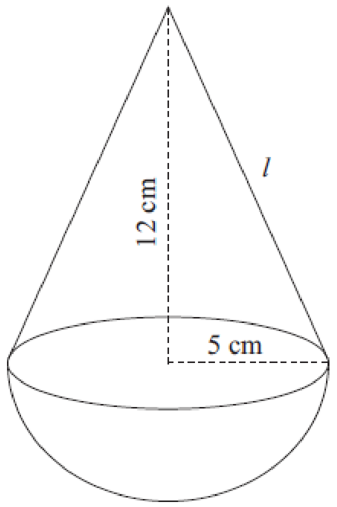


diagram not to scale

21a. Calculate the length, [2 marks]
l, of the slant height of the cone.

21b. Calculate the area that is painted red. [4 marks]

An office block, ABCPQR, is built in the shape of a triangular prism with its "footprint", ABC, on horizontal ground.
AB = 70 m,
BC = 50 m and
AC = 30 m. The vertical height of the office block is
120 m .

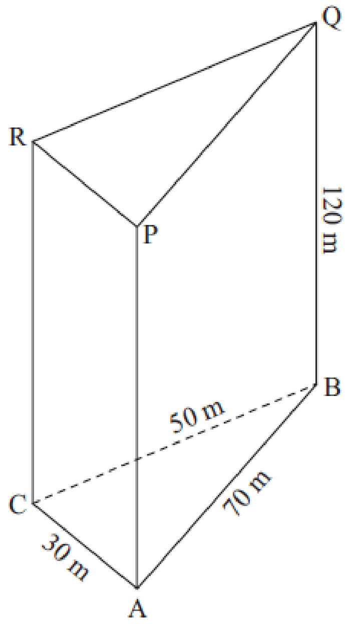


diagram not to scale

22a. Calculate the size of angle ACB. *[3 marks]*

22b. Calculate the area of the building's footprint, ABC. *[3 marks]*

22c. Calculate the volume of the office block. *[2 marks]*

22d. To stabilize the structure, a steel beam must be made that runs from point C to point Q. *[2 marks]*

Calculate the length of CQ.

22e. Calculate the angle CQ makes with BC. *[2 marks]*

Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3.

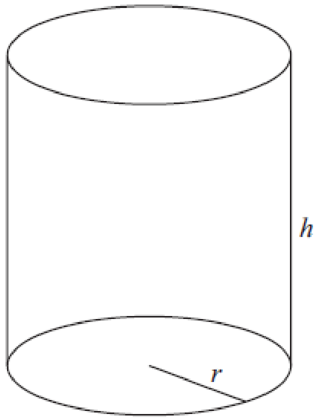


diagram not to scale

Nadia decides to make the radius, r, of the bin 5 cm.

23a. Calculate [7 marks]

(i) the area of the base of the wastepaper bin;

(ii) the height, h, of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin.

23b. State whether Nadia's design is practical. Give a reason. [2 marks]

Merryn also designs a cylindrical wastepaper bin with a volume of 8000 cm^3. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

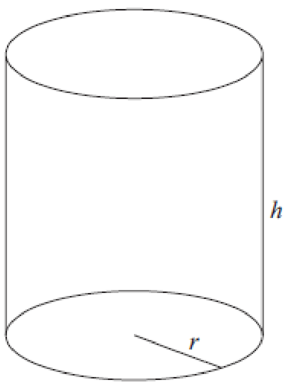


diagram not to scale

Let the radius of the base of Merryn's wastepaper bin be r, and let its height be h.

23c. Write down an equation in h and r, using the given volume of the bin. [1 mark]

23d. Show that the total external surface area, A, of the bin is $A = \pi r^2 + \frac{16000}{r}$. [2 marks]

23e. Write down $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3 marks]

23f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin. *[5 marks]*

(ii) Calculate the value of h corresponding to this value of r.

23g. Determine whether Merryn's design is an improvement upon Nadia's. Give a reason. *[2 marks]*

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.

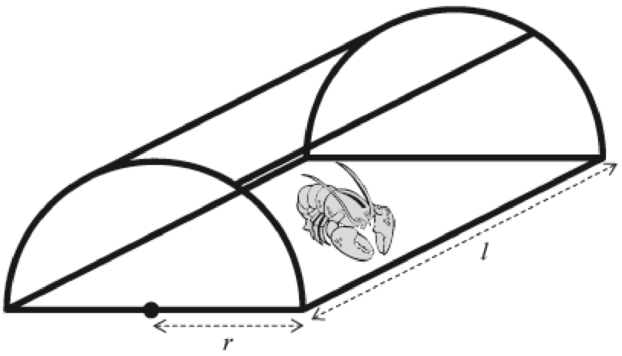


diagram not to scale

The semicircular ends each have radius r and the support rods each have length l.

Let T be the total length of steel used in the frame of the lobster trap.

24. Write down an expression for T in terms of r, l and π. *[3 marks]*