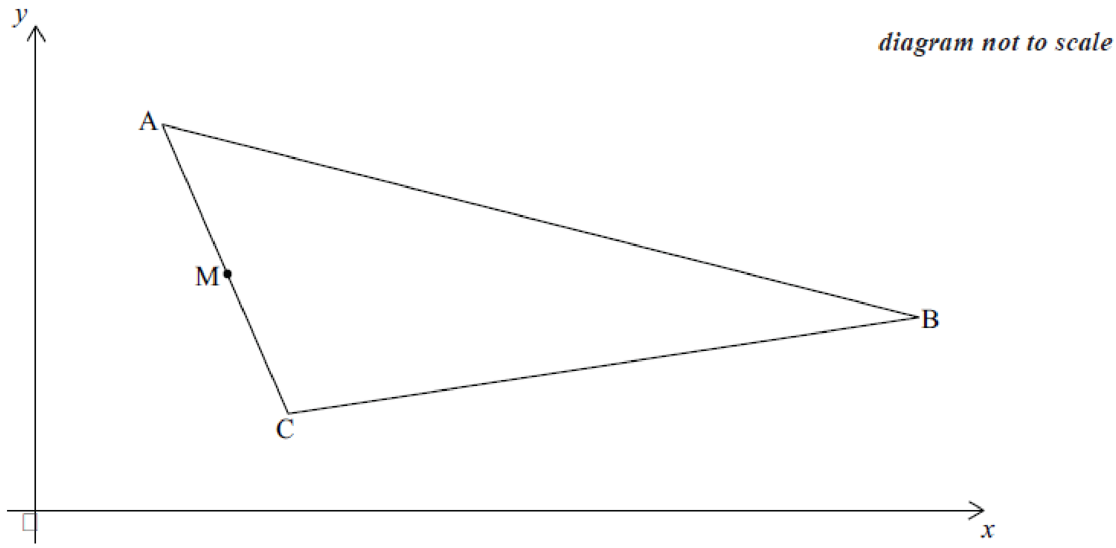
Topic 5 Part 2 *[251 marks]*

The diagram shows points A(2, 8), B(14, 4) and C(4, 2). M is the midpoint of AC.

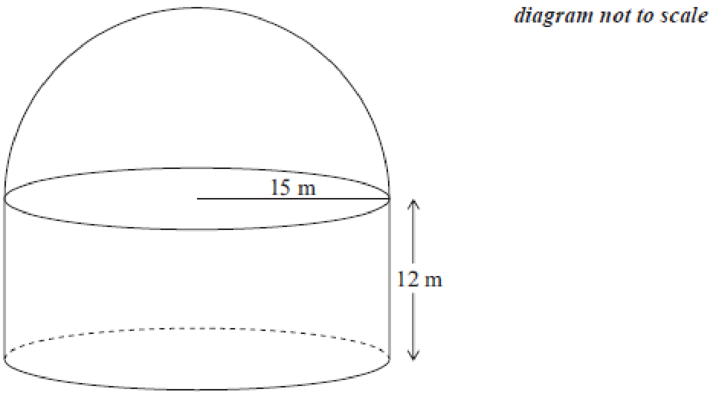


1a. Write down the coordinates of M. *[2 marks]*

1b. Calculate the gradient of the line AB. *[2 marks]*

1c. Find the equation of the line parallel to AB that passes through M. *[2 marks]*

An observatory is built in the shape of a cylinder with a hemispherical roof on the top as shown in the diagram. The height of the cylinder is 12 m and its radius is 15 m.



2a. Calculate the volume of the observatory. *[4 marks]*

2b. The hemispherical roof is to be painted. *[2 marks]*

Calculate the area that is to be painted.

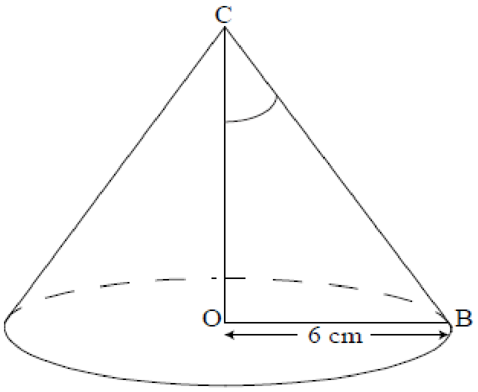
A solid metal **cylinder** has a base radius of 4 cm and a height of 8 cm.

3a. Find the area of the base of the cylinder. *[2 marks]*

3b. Show that the volume of the metal used in the cylinder is 402 cm^3, given correct to three significant figures. *[2 marks]*

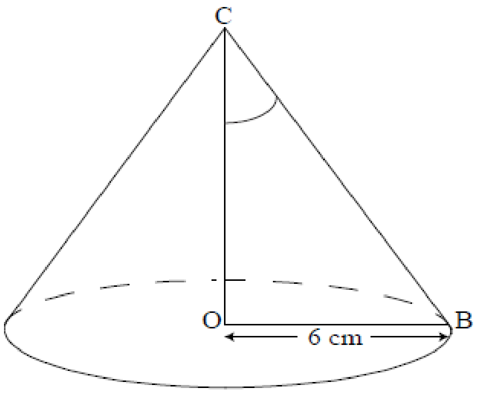
3c. Find the total surface area of the cylinder. *[3 marks]*

3d. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. *[3 marks]*



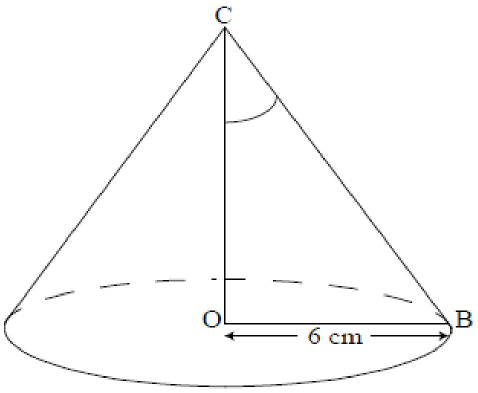
Find the height, OC, of the cone.

3e. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. *[2 marks]*



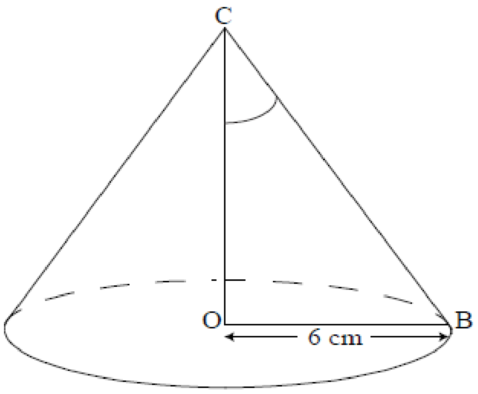
Find the size of angle BCO.

3f. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. *[2 marks]*



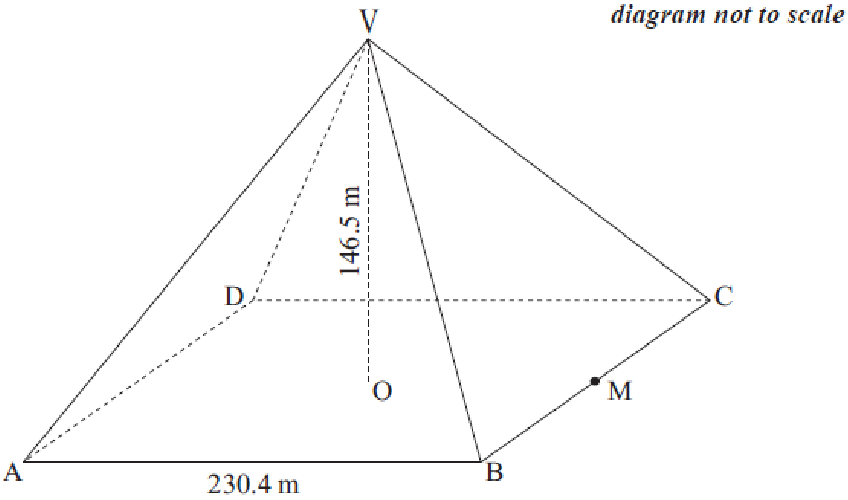
Find the slant height, CB.

3g. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. *[4 marks]*



Find the total surface area of the cone.

The Great Pyramid of Cheops in Egypt is a square based pyramid. The base of the pyramid is a square of side length 230.4 m and the vertical height is 146.5 m. The Great Pyramid is represented in the diagram below as ABCDV . The vertex V is directly above the centre O of the base. M is the midpoint of BC.



4a. (i) Write down the length of OM . *[3 marks]*

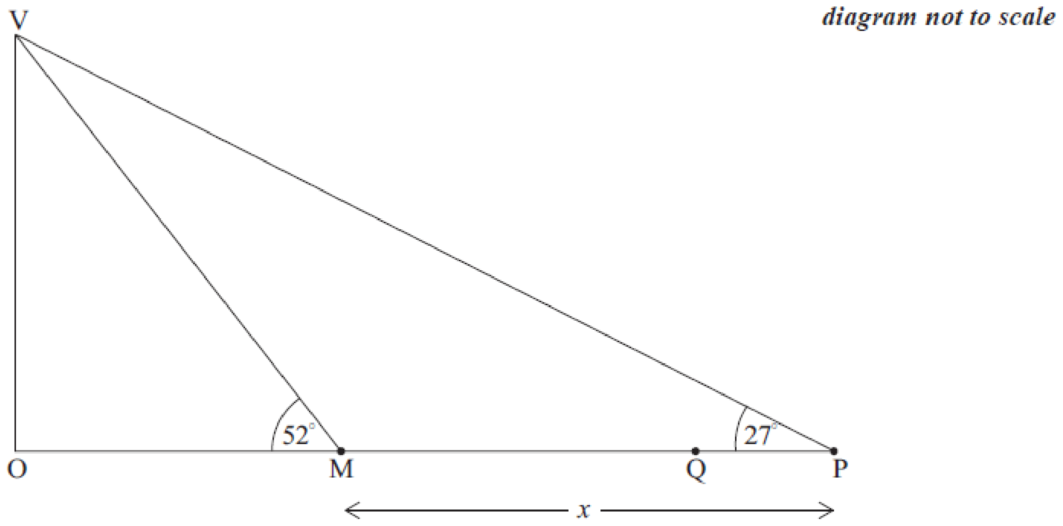
(ii) Find the length of VM .

4b. Find the area of triangle VBC . *[2 marks]*

4c. Calculate the volume of the pyramid. *[2 marks]*

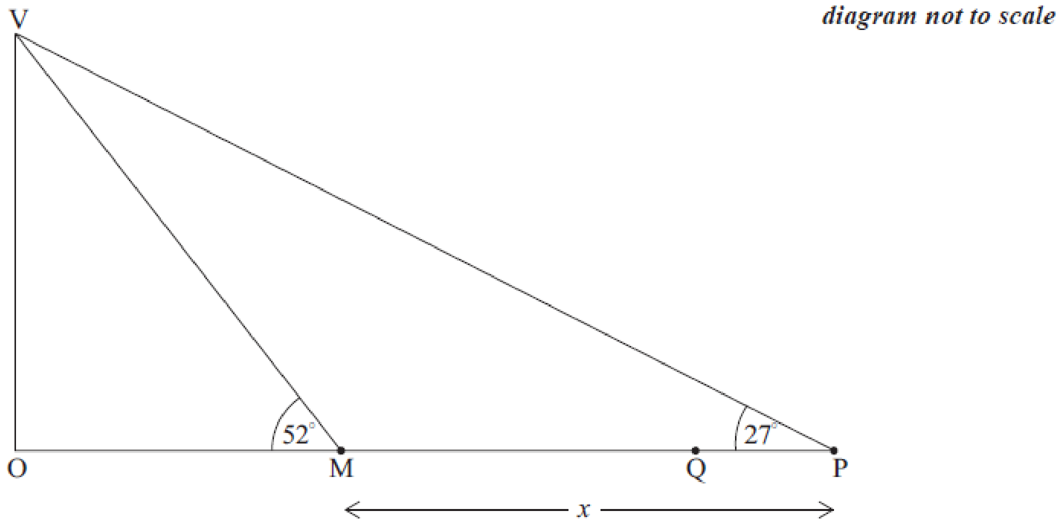
4d. Show that the angle between the line VM and the base of the pyramid is 52° correct to 2 significant figures. *[2 marks]*

4e. Ahmed is at point P , a distance x metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . *[1 mark]*



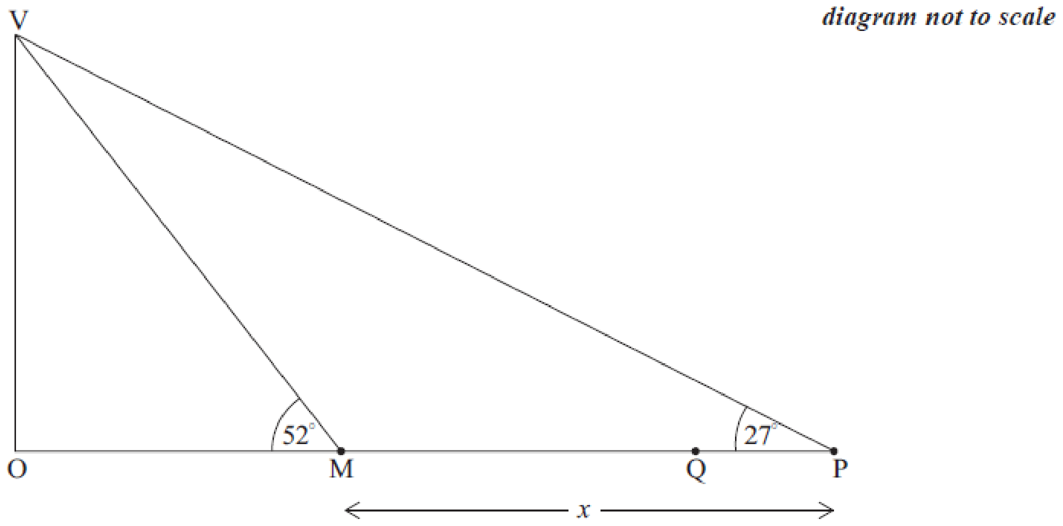
Write down the size of angle VMP .

4f. Ahmed is at point P , a distance x metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . *[4 marks]*



Using your value of VM from part (a)(ii), find the value of x.

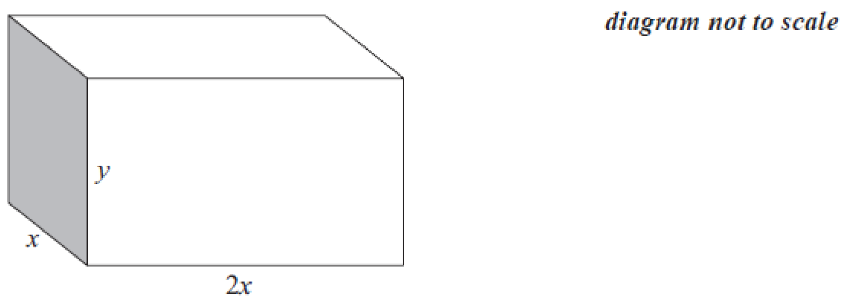
4g. Ahmed is at point P , a distance x metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . *[4 marks]*



Ahmed walks 50 m from P to Q.

Find the length of QV, the distance from Ahmed to the vertex of the pyramid.

A shipping container is to be made with six rectangular faces, as shown in the diagram.



The dimensions of the container are

length $2x$
width x
height y.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

5a. Show that $y = 12 - 3x$. *[3 marks]*

5b. Show that the volume V m^3 of the container is given by *[2 marks]*

$$V = 24x^2 - 6x^3$$

5c. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. *[2 marks]*

5d. Find the value of x for which V is a maximum. *[3 marks]*

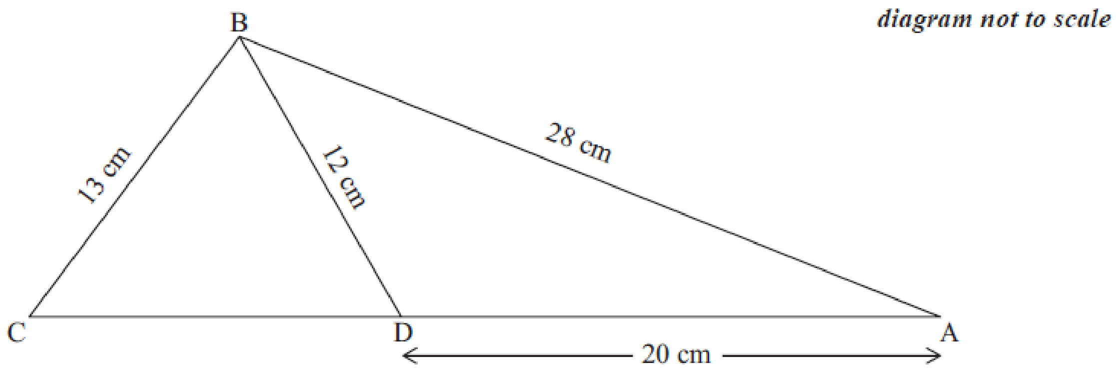
5e. Find the maximum volume of the container. *[2 marks]*

5f. Find the length and height of the container for which the volume is a maximum. *[3 marks]*

5g. The shipping container is to be painted. One litre of paint covers an area of 15 m^2 . Paint comes in tins containing four litres. *[4 marks]*

Calculate the number of tins required to paint the shipping container.

The diagram shows triangle ABC in which
AB = 28 cm,
BC = 13 cm,
BD = 12 cm and
AD = 20 cm.



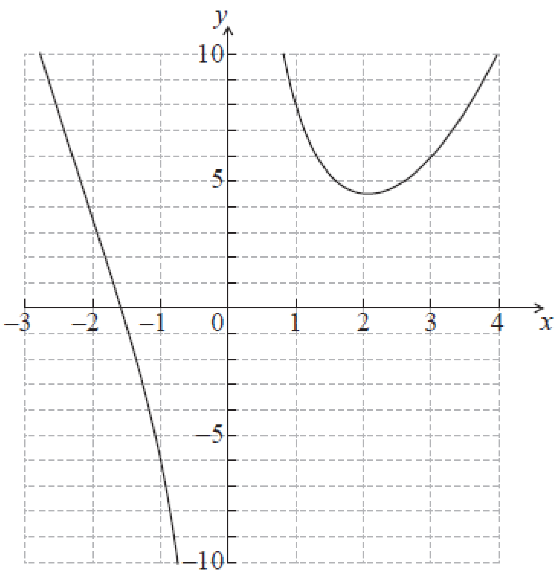
6a. Calculate the size of angle ADB. *[3 marks]*

6b. Find the area of triangle ADB. *[3 marks]*

6c. Calculate the size of angle BCD. *[4 marks]*

6d. Show that the triangle ABC is not right angled. *[4 marks]*

The diagram shows part of the graph of
$f(x) = x^2 - 2x + \frac{9}{x}$, where
$x \neq 0$.



7a. Write down *[5 marks]*

(i) the equation of the vertical asymptote to the graph of $y = f(x)$;

(ii) the solution to the equation $f(x) = 0$;

(iii) the coordinates of the local minimum point.

7b. Find $f'(x)$. *[4 marks]*

7c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3 - 2x^2 - 9}{x^2}$. *[2 marks]*

7d. Find the gradient of the tangent to $y = f(x)$ at the point $\mathrm{A}(1, 8)$. *[2 marks]*

7e. The line, L, passes through the point A and is perpendicular to the tangent at A. *[1 mark]*

Write down the gradient of L .

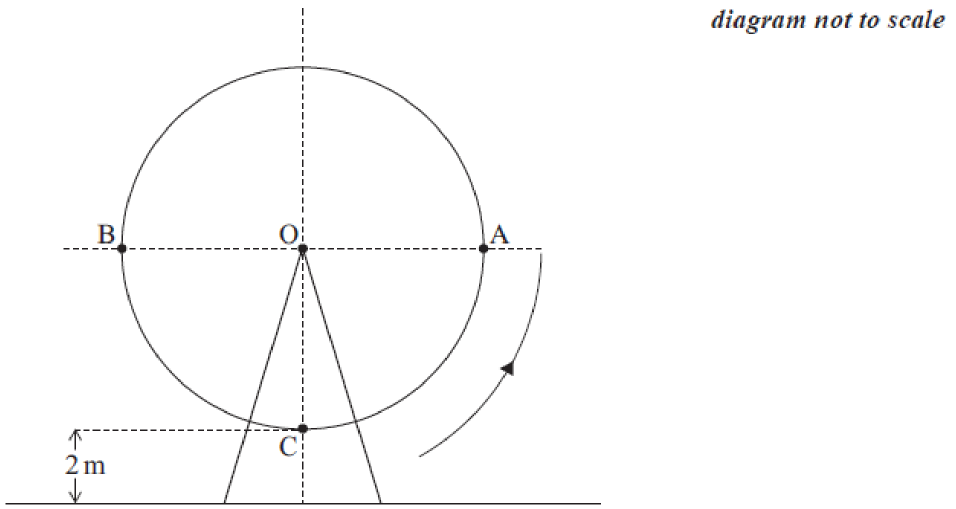
7f. The line, L , passes through the point A and is perpendicular to the tangent at A. *[3 marks]*

Find the equation of L . Give your answer in the form $y = mx + c$.

7g. The line, L , passes through the point A and is perpendicular to the tangent at A. *[2 marks]*

L also intersects the graph of $y = f(x)$ at points B and C . Write down the ***x*-coordinate** of B and of C .

The diagram shows a Ferris wheel that moves with constant speed and completes a rotation every 40 seconds. The wheel has a radius of 12 m and its lowest point is 2 m above the ground.



8a. Initially, a seat C is vertically below the centre of the wheel, O. It then rotates in an anticlockwise (counterclockwise) direction. *[2 marks]*

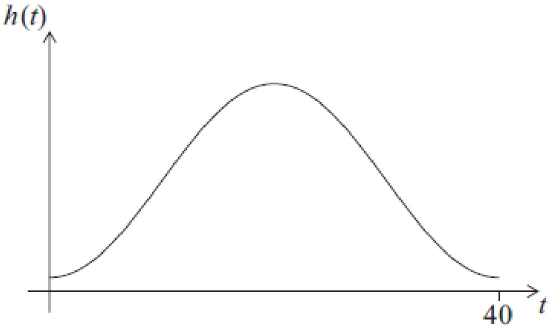
Write down

(i) the height of O above the ground;

(ii) the maximum height above the ground reached by C .

8b. In a revolution, C reaches points A and B , which are at the same height above the ground as the centre of the wheel. Write down the number of seconds taken for C to first reach A and then B . *[2 marks]*

8c. The sketch below shows the graph of the function, $h(t)$, for the height above ground of C, where h is measured in metres and t is the time in seconds, $0 \leqslant t \leqslant 40$. *[4 marks]*



Copy the sketch and show the results of part (a) and part (b) on your diagram. Label the points clearly with their coordinates.

A satellite travels around the Earth in a circular orbit 500 kilometres above the Earth's surface. The radius of the Earth is taken as 6400 kilometres.

9a. Write down the radius of the satellite's orbit. *[1 mark]*

9b. Calculate the distance travelled by the satellite in one orbit of the Earth. Give your answer correct to the nearest km. *[3 marks]*

9c. Write down your answer to (b) in the form $a \times 10^k$, where $1 \leqslant a < 10, k \in \mathbb{Z}$. *[2 marks]*

A room is in the shape of a cuboid. Its floor measures 7.2 m by 9.6 m and its height is 3.5 m.

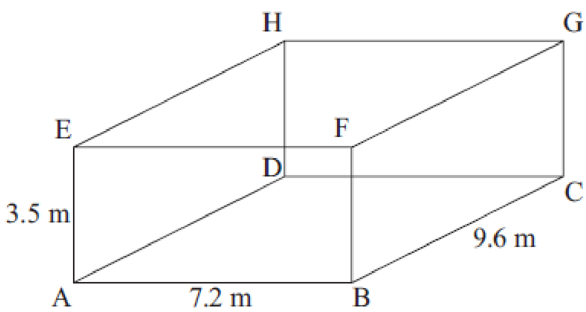


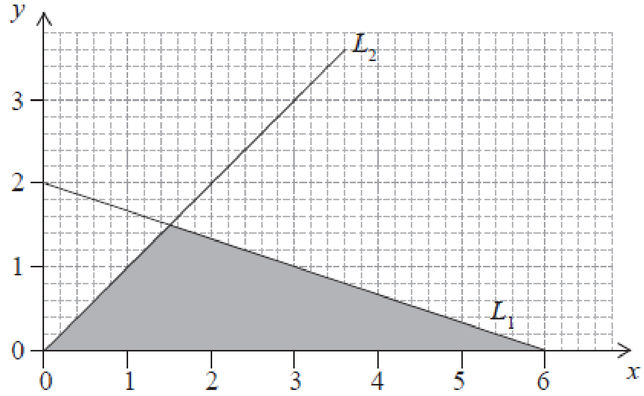
diagram not to scale

10a. Calculate the length of AC. *[2 marks]*

10b. Calculate the length of AG. *[2 marks]*

10c. Calculate the angle that AG makes with the floor. *[2 marks]*

The diagram shows the straight lines L_1 and L_2. The equation of L_2 is $y = x$.



11a. Find [3 marks]

(i) the gradient of L_1;

(ii) the equation of L_1.

11b. Find the area of the shaded triangle. [2 marks]

In a television show there is a transparent box completely filled with identical cubes. Participants have to estimate the number of cubes in the box. The box is 50 cm wide, 100 cm long and 40 cm tall.

12a. Find the volume of the box. [2 marks]

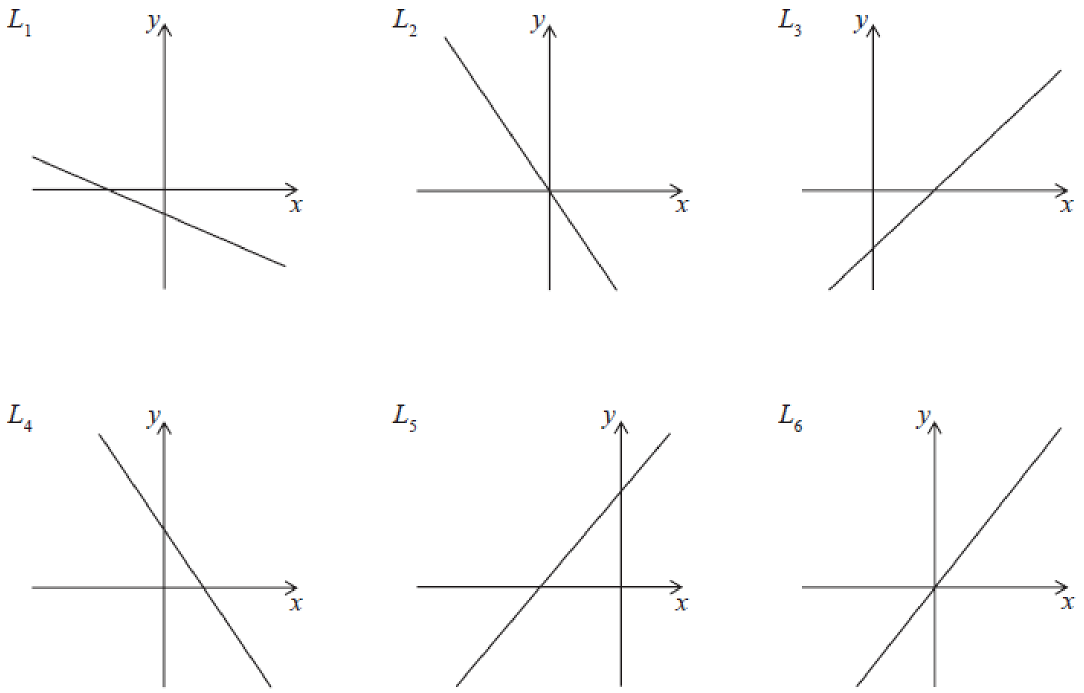
12b. Joaquin estimates the volume of one cube to be 500 cm^3. He uses this value to estimate the number of cubes in the box. [2 marks]

Find Joaquin's estimated number of cubes in the box.

12c. The actual number of cubes in the box is 350. [2 marks]

Find the percentage error in Joaquin's estimated number of cubes in the box.

13. The following diagrams show six lines with equations of the form $y = mx + c$. *[6 marks]*

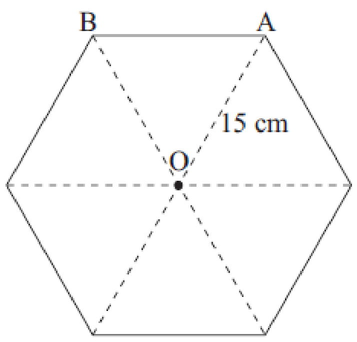


In the table below there are four possible conditions for the pair of values m and c. Match each of the given conditions with one of the lines drawn above.

Condition	Line
$m > 0$ and $c > 0$	
$m < 0$ and $c > 0$	
$m < 0$ and $c < 0$	
$m > 0$ and $c < 0$	

The base of a prism is a **regular hexagon**. The centre of the hexagon is O and the length of OA is 15 cm.

diagram not to scale



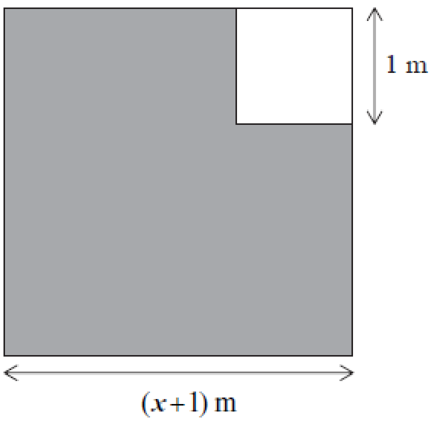
14a. Write down the size of angle AOB. *[1 mark]*

14b. Find the area of the triangle AOB. *[3 marks]*

14c. The height of the prism is 20 cm. *[2 marks]*

Find the volume of the prism.

The length of a square garden is $(x + 1)$ m. In one of the corners a square of 1 m length is used only for grass. The rest of the garden is only for planting roses and is shaded in the diagram below.



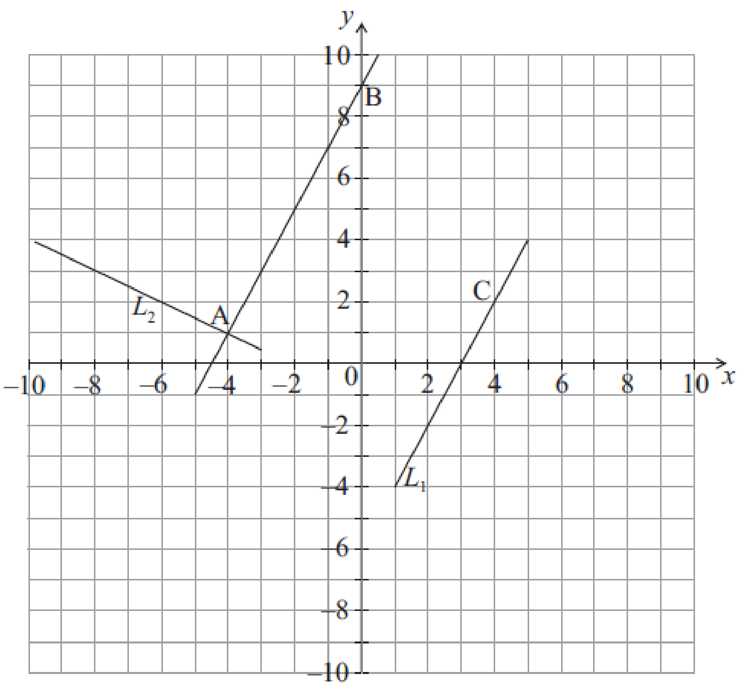
The area of the shaded region is A .

15a. Write down an expression for A in terms of x. *[1 mark]*

15b. Find the value of x given that $A = 109.25\text{ m}^2$. *[3 marks]*

15c. The owner of the garden puts a fence around the shaded region. Find the length of this fence. *[2 marks]*

The points A $(-4, 1)$, B $(0, 9)$ and C $(4, 2)$ are plotted on the diagram below. The diagram also shows the lines AB, L_1 and L_2.



16a. Find the gradient of AB. *[2 marks]*

16b. L_1 passes through C and is parallel to AB. *[1 mark]*

Write down the y-intercept of L_1.

16c. L_2 passes through A and is perpendicular to AB. *[3 marks]*

Write down the equation of L_2. Give your answer in the form $\boldsymbol{ax + by + d = 0}$ where a, b and d $\in \mathbb{Z}$.

16d. Write down the coordinates of the point D, the intersection of L_1 and L_2. *[1 mark]*

16e. There is a point R on L_1 such that ABRD is a rectangle. *[2 marks]*

Write down the coordinates of R.

16f. The distance between A and D is $\sqrt{45}$. *[4 marks]*

(i) Find the distance between D and R .

(ii) Find the area of the triangle BDR .

The equation of the line R_1 is $2x + y - 8 = 0$. The line R_2 is perpendicular to R_1 .

17a. Calculate the gradient of R_2 . *[2 marks]*

17b. The point of intersection of R_1 and R_2 is $(4, k)$. *[4 marks]*

Find

(i) the value of k ;

(ii) the equation of R_2 .

Consider the function $f(x) = x^3 + \frac{48}{x}, x \neq 0.$

18a. Calculate $f(2)$. *[2 marks]*

18b. Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-200 \leqslant y \leqslant 200$. *[4 marks]*

18c. Find $f'(x)$. *[3 marks]*

18d. Find $f'(2)$. *[2 marks]*

18e. Write down the coordinates of the local maximum point on the graph of f . *[2 marks]*

18f. Find the range of f . *[3 marks]*

18g. Find the gradient of the tangent to the graph of f at $x = 1$. *[2 marks]*

18h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. *[2 marks]*

Find the x-coordinate of this point.

Pauline owns a piece of land ABCD in the shape of a quadrilateral. The length of BC is 190 m , the length of CD is 120 m , the length of AD is 70 m , the size of angle BCD is 75° and the size of angle BAD is 115° .

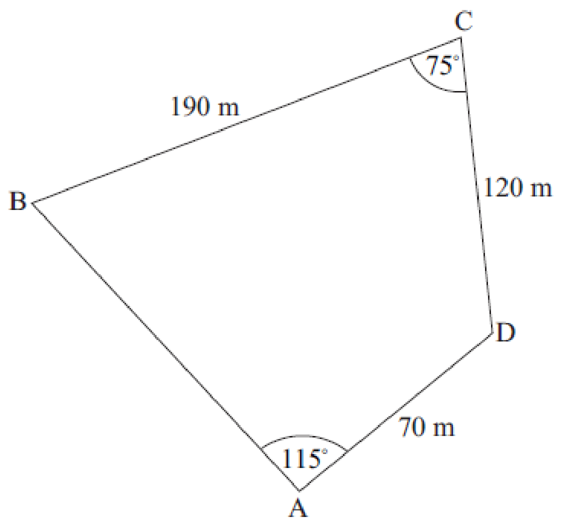


diagram not to scale

Pauline decides to sell the triangular portion of land ABD . She first builds a straight fence from B to D .

19a. Calculate the length of the fence. *[3 marks]*

19b. The fence costs 17 USD per metre to build. *[2 marks]*

Calculate the cost of building the fence. Give your answer correct to the nearest USD.

19c. Show that the size of angle ABD is 18.8° , correct to three significant figures. *[3 marks]*

19d. Calculate the area of triangle ABD . *[4 marks]*

19e. She sells the land for 120 USD per square metre. *[2 marks]*

Calculate the value of the land that Pauline sells. Give your answer correct to the nearest USD.

19f. Pauline invests 300000 USD from the sale of the land in a bank that pays compound interest compounded annually. *[4 marks]*

Find the interest rate that the bank pays so that the investment will double in value in 15 years.

The planet Earth takes one year to revolve around the Sun. Assume that a year is 365 days and the path of the Earth around the Sun is the circumference of a circle of radius
150000000 km.

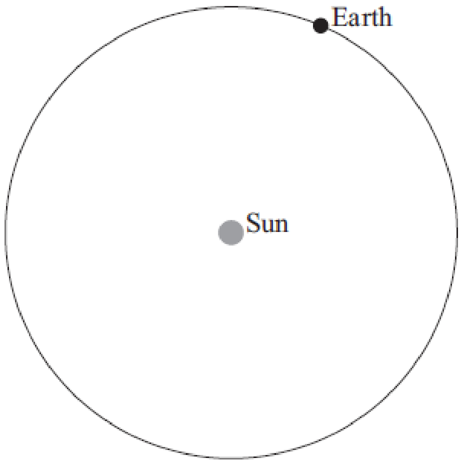


diagram not to scale

20a. Calculate the distance travelled by the Earth in **one day**. *[4 marks]*

20b. Give your answer to part (a) in the form $a \times 10^k$ where $1 \leqslant a \leqslant 10$ and $k \in \mathbb{Z}$. *[2 marks]*

The straight line L passes through the points $\mathrm{A}(-1, 4)$ and $\mathrm{B}(5, 8)$.

21a. Calculate the gradient of L . *[2 marks]*

21b. Find the equation of L . *[2 marks]*

21c. The line L also passes through the point $\mathrm{P}(8, y)$. Find the value of y . *[2 marks]*

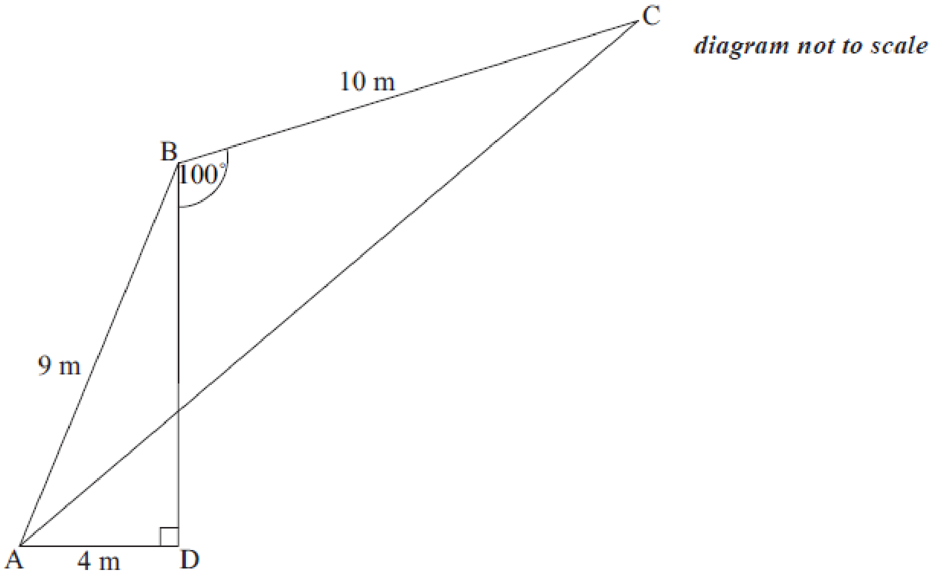
75 metal spherical cannon balls, each of diameter 10 cm, were excavated from a Napoleonic War battlefield.

22a. Calculate the total volume of all 75 metal cannon balls excavated. *[3 marks]*

22b. The cannon balls are to be melted down to form a sculpture in the shape of a cone. The base radius of the cone is 20 cm. *[3 marks]*

Calculate the height of the cone, assuming that no metal is wasted.

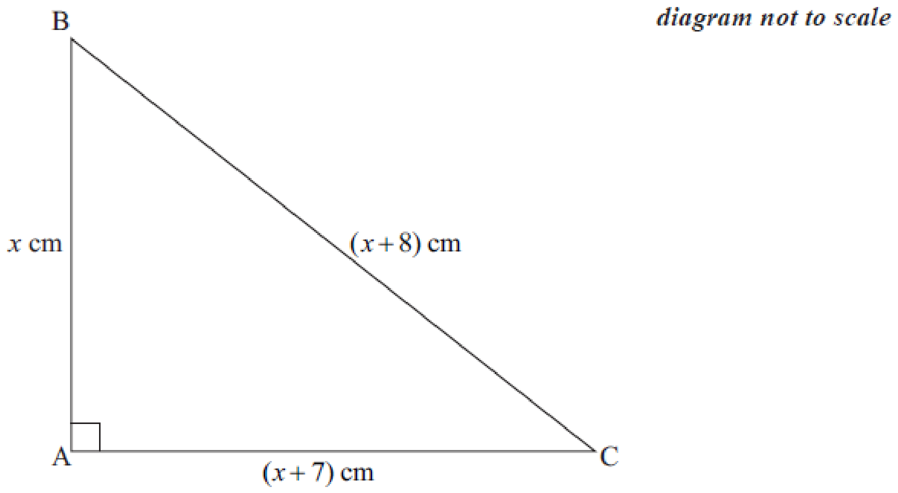
In the diagram,
$\text{AD} = 4$ m,
$\text{AB} = 9$ m,
$\text{BC} = 10$ m,
$\text{B}\hat{\text{D}}\text{A} = 90^\circ$ and
$\text{D}\hat{\text{B}}\text{C} = 100^\circ$.



23a. Calculate the size of $\text{A}\hat{\text{B}}\text{C}$. *[3 marks]*

23b. Calculate the length of AC. *[3 marks]*

In the diagram,
$\text{B}\hat{\text{A}}\text{C} = 90^\circ$. The length of the three sides are
x cm,
$(x + 7)$ cm and
$(x + 8)$ cm.

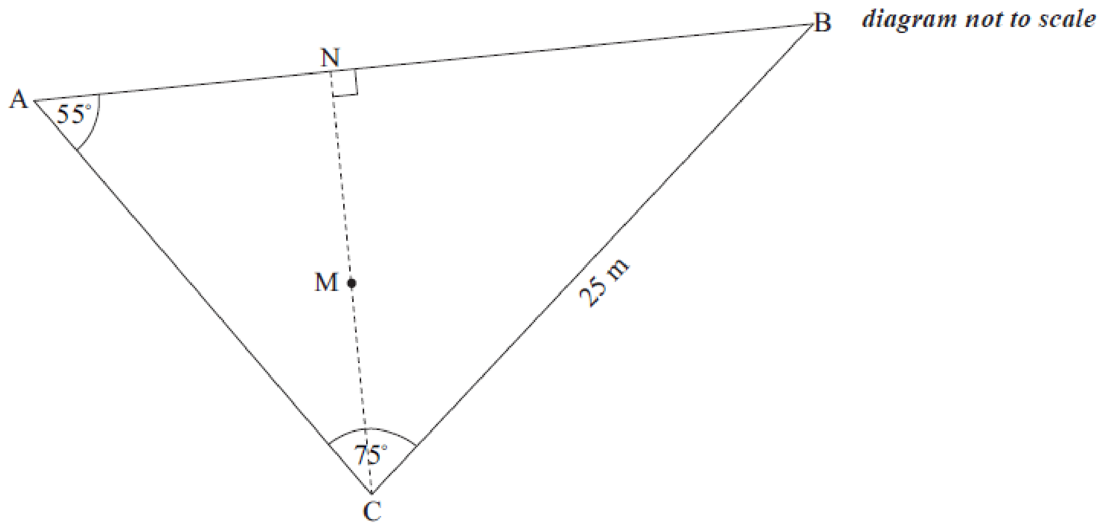


24a. Write down and **simplify** a quadratic equation in x which links the three sides of the triangle. *[3 marks]*

24b. Solve the quadratic equation found in part (a). *[2 marks]*

24c. Write down the value of the perimeter of the triangle. *[1 mark]*

The diagram represents a small, triangular field, ABC , with
$BC = 25$ m ,
angle $BAC = 55°$ and
angle $ACB = 75°$.



25a. Write down the size of angle ABC. *[1 mark]*

25b. Calculate the length of AC. *[3 marks]*

25c. Calculate the area of the field ABC. *[3 marks]*

25d. N is the point on AB such that CN is perpendicular to AB. M is the midpoint of CN. *[3 marks]*

Calculate the length of NM.

25e. A goat is attached to one end of a rope of length 7 m. The other end of the rope is attached to the point M. *[5 marks]*

Decide whether the goat can reach point P, the midpoint of CB. Justify your answer.