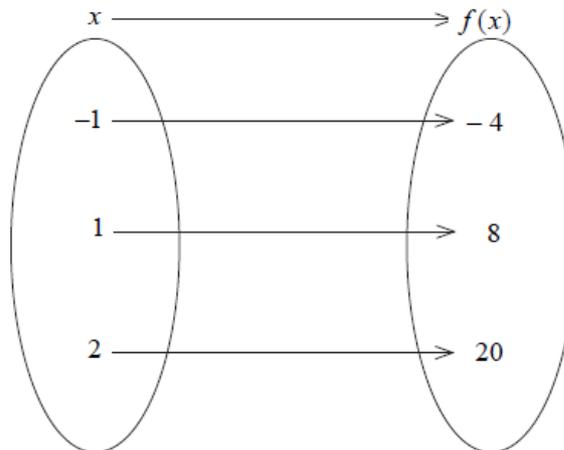


## Topic 6 Part 5 [189 marks]

A quadratic function,  
 $f(x) = ax^2 + bx$ , is represented by the mapping diagram below.



- 1a. Use the mapping diagram to write down **two** equations in terms of  $a$  and  $b$ . [2 marks]
- 1b. Find the value of  $a$ . [1 mark]
- 1c. Find the value of  $b$ . [1 mark]
- 1d. Calculate the  $x$ -coordinate of the vertex of the graph of  $f(x)$ . [2 marks]

The function  
 $f(x) = 5 - 3(2^{-x})$  is defined for  
 $x \geq 0$ .

- 2a. On the axes below sketch the graph of  $f(x)$  and show the behaviour of the curve as  $x$  increases. [3 marks]
- 2b. Write down the coordinates of any intercepts with the axes. [1 mark]
- 
- 2c. Draw the line  $y = 5$  on your sketch. [1 mark]
- 2d. Write down the number of solutions to the equation  $f(x) = 5$ . [1 mark]

A rumour spreads through a group of teenagers according to the exponential model

$$N = 2 \times (1.81)^{0.7t}$$

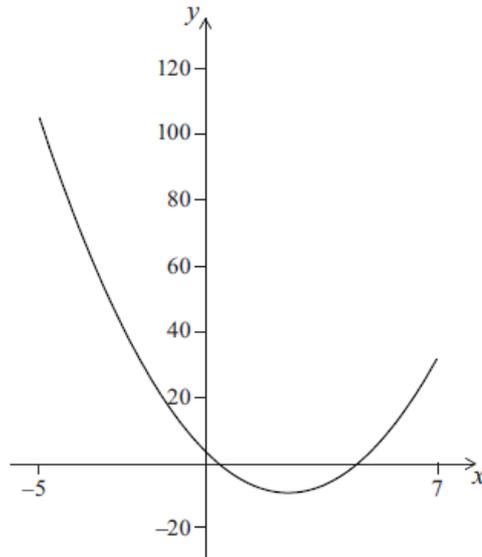
where  $N$  is the number of teenagers who have heard the rumour  $t$  hours after it is first started.

- 3a. Find the number of teenagers who started the rumour. [2 marks]

3b. Write down the number of teenagers who have heard the rumour five hours after it is first started. [1 mark]

3c. Determine the length of time it would take for 150 teenagers to have heard the rumour. Give your answer correct to the nearest minute. [3 marks]

The graph of  $y = 2x^2 - rx + q$  is shown for  $-5 \leq x \leq 7$ .



The graph cuts the y axis at (0, 4).

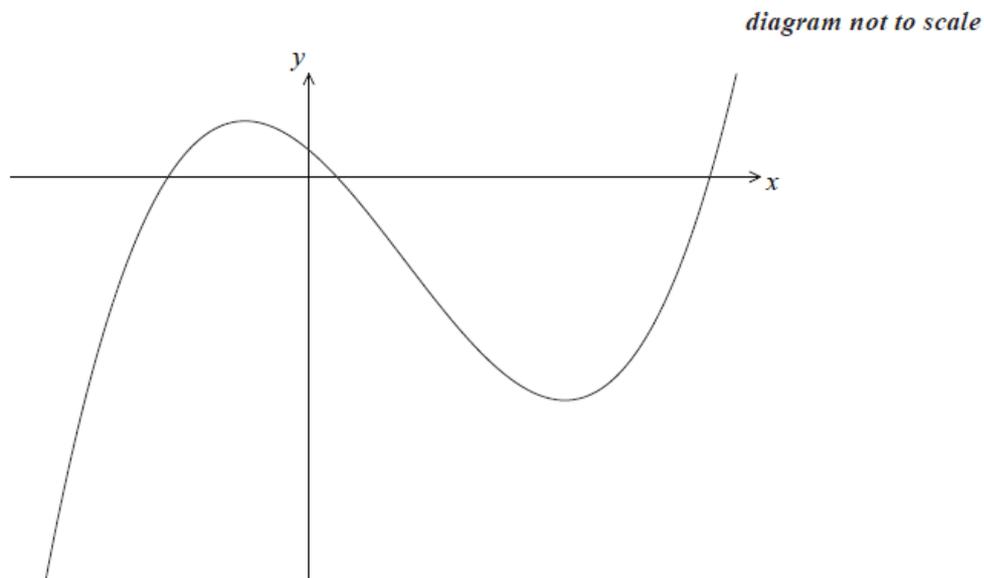
4a. Write down the value of  $q$ . [1 mark]

4b. The axis of symmetry is  $x = 2.5$ . [2 marks]  
Find the value of  $r$ .

4c. The axis of symmetry is  $x = 2.5$ . [1 mark]  
Write down the minimum value of  $y$ .

4d. The axis of symmetry is  $x = 2.5$ . [2 marks]  
Write down the range of  $y$ .

The diagram shows a sketch of the function  $f(x) = 4x^3 - 9x^2 - 12x + 3$ .



- 5a. Write down the values of  $x$  where the graph of  $f(x)$  intersects the  $x$ -axis. [3 marks]
- 5b. Write down  $f'(x)$ . [3 marks]
- 5c. Find the value of the local maximum of  $y = f(x)$ . [4 marks]
- 5d. Let P be the point where the graph of  $f(x)$  intersects the  $y$  axis. [1 mark]  
Write down the coordinates of P.
- 5e. Let P be the point where the graph of  $f(x)$  intersects the  $y$  axis. [2 marks]  
Find the gradient of the curve at P.
- 5f. The line,  $L$ , is the tangent to the graph of  $f(x)$  at P. [2 marks]  
Find the equation of  $L$  in the form  $y = mx + c$ .
- 5g. There is a second point, Q, on the curve at which the tangent to  $f(x)$  is parallel to  $L$ . [1 mark]  
Write down the gradient of the tangent at Q.
- 5h. There is a second point, Q, on the curve at which the tangent to  $f(x)$  is parallel to  $L$ . [3 marks]  
Calculate the  $x$ -coordinate of Q.

Daniel wants to invest  
\$25 000 for a total of three years. There are two investment options.

**Option One** pays compound interest at a nominal annual rate of interest of 5 %, compounded **annually**.

**Option Two** pays compound interest at a nominal annual rate of interest of 4.8 %, compounded **monthly**.

- 6a. Calculate the value of his investment at the end of the third year for each investment option, **correct to two decimal places**. [8 marks]
- 6b. Determine Daniel's best investment option. [1 mark]

An arithmetic sequence is defined as

$$u_n = 135 + 7n, \quad n = 1, 2, 3, \dots$$

6c. Calculate  $u_1$ , the first term in the sequence. [2 marks]

6d. Show that the common difference is 7. [2 marks]

6e.  $S_n$  is the sum of the first  $n$  terms of the sequence. [3 marks]

Find an expression for  $S_n$ . Give your answer in the form  $S_n = An^2 + Bn$ , where  $A$  and  $B$  are constants.

6f. The first term,  $v_1$ , of a geometric sequence is 20 and its fourth term  $v_4$  is 67.5. [2 marks]

Show that the common ratio,  $r$ , of the geometric sequence is 1.5.

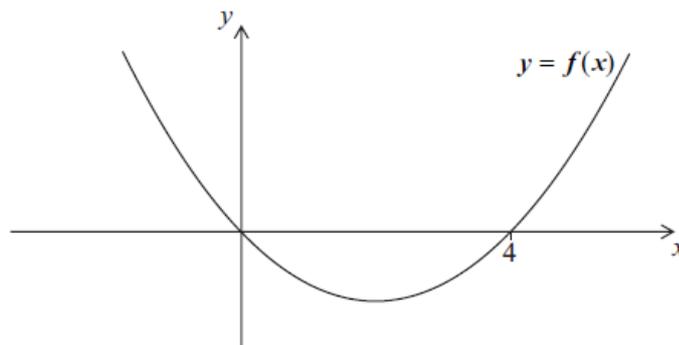
6g.  $T_n$  is the sum of the first  $n$  terms of the geometric sequence. [2 marks]

Calculate  $T_7$ , the sum of the first seven terms of the geometric sequence.

6h.  $T_n$  is the sum of the first  $n$  terms of the geometric sequence. [2 marks]

Use your graphic display calculator to find the smallest value of  $n$  for which  $T_n > S_n$ .

The following is the graph of the quadratic function  $y = f(x)$ .



7a. Write down the solutions to the equation  $f(x) = 0$ . [2 marks]

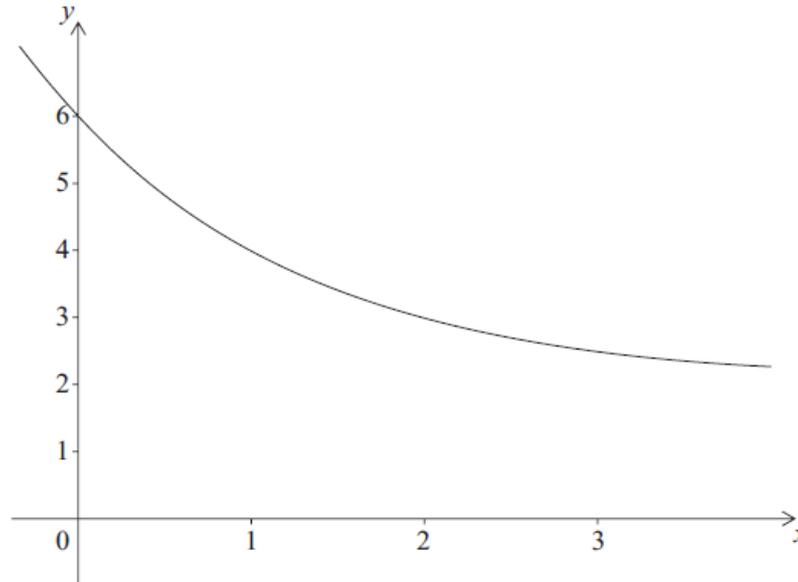
7b. Write down the equation of the axis of symmetry of the graph of  $f(x)$ . [2 marks]

7c. The equation  $f(x) = 12$  has two solutions. One of these solutions is  $x = 6$ . Use the symmetry of the graph to find the other solution. [1 mark]

7d. The minimum value for  $y$  is  $-4$ . Write down the range of  $f(x)$ . [1 mark]

Consider the function

$f(x) = p(0.5)^x + q$  where  $p$  and  $q$  are constants. The graph of  $f(x)$  passes through the points  $(0, 6)$  and  $(1, 4)$  and is shown below.



- 8a. Write down two equations relating  $p$  and  $q$ . [2 marks]
- 8b. Find the value of  $p$  and of  $q$ . [2 marks]
- 8c. Write down the equation of the horizontal asymptote to the graph of  $f(x)$ . [2 marks]

Consider the function  $f(x) = x^3 - 3x^2 - 24x + 30$ .

- 9a. Write down  $f(0)$ . [1 mark]
- 9b. Find  $f'(x)$ . [3 marks]
- 9c. Find the gradient of the graph of  $f(x)$  at the point where  $x = 1$ . [2 marks]
- 9d. (i) Use  $f'(x)$  to find the  $x$ -coordinate of M and of N. [5 marks]  
(ii) Hence or otherwise write down the coordinates of M and of N.
- 9e. Sketch the graph of  $f(x)$  for  $-5 \leq x \leq 7$  and  $-60 \leq y \leq 60$ . Mark clearly M and N on your graph. [4 marks]
- 9f. Lines  $L_1$  and  $L_2$  are parallel, and they are tangents to the graph of  $f(x)$  at points A and B respectively.  $L_1$  has equation  $y = 21x + 111$ . [6 marks]
- (i) Find the  $x$ -coordinate of A and of B.
- (ii) Find the  $y$ -coordinate of B.

Give all answers in this question to the nearest whole currency unit.

Ying and Ruby each have 5000 USD to invest.

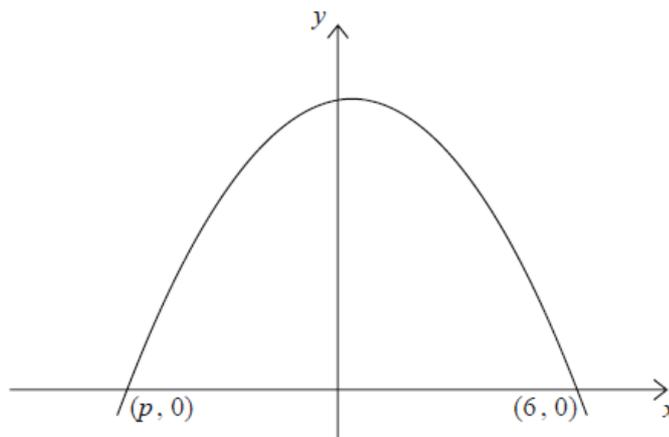
Ying invests his 5000 USD in a bank account that pays a nominal annual interest rate of 4.2 % **compounded yearly**. Ruby invests her 5000 USD in an account that offers a fixed interest of 230 USD each year.

- 10a. Find the amount of money that Ruby will have in the bank after 3 years. [2 marks]
- 10b. Show that Ying will have 7545 USD in the bank at the end of 10 years. [3 marks]
- 10c. Find the number of complete years it will take for Ying's investment to first exceed 6500 USD. [3 marks]
- 10d. Find the number of complete years it will take for Ying's investment to exceed Ruby's investment. [3 marks]
- 10e. Ruby moves from the USA to Italy. She transfers 6610 USD into an Italian bank which has an exchange rate of 1 USD = 0.735 Euros. The bank charges 1.8 % commission. [4 marks]
- Calculate the amount of money Ruby will invest in the Italian bank after commission.
- 10f. Ruby returns to the USA for a short holiday. She converts 800 Euros at a bank in Chicago and receives 1006.20 USD. The bank advertises an exchange rate of 1 Euro = 1.29 USD. [5 marks]
- Calculate the percentage commission Ruby is charged by the bank.

Given the function  
 $f(x) = 2 \times 3^x$  for  $-2 \leq x \leq 5$ ,

- 11a. find the range of  $f$ . [4 marks]
- 11b. find the value of  $x$  given that  $f(x) = 162$ . [2 marks]

The diagram below shows the graph of a quadratic function. The graph passes through the points  $(6, 0)$  and  $(p, 0)$ . The maximum point has coordinates  $(0.5, 30.25)$ .



- 12a. Calculate the value of  $p$ . [2 marks]

- 12b. Given that the quadratic function has an equation  $y = -x^2 + bx + c$  where  $b, c \in \mathbb{Z}$ , find  $b$  and  $c$ . [4 marks]

- 13a. Sketch the graph of  $y = 2^x$  for  $-2 \leq x \leq 3$ . Indicate clearly where the curve intersects the  $y$ -axis. [3 marks]

- 13b. Write down the equation of the asymptote of the graph of  $y = 2^x$ . [2 marks]

- 13c. On the same axes sketch the graph of  $y = 3 + 2x - x^2$ . Indicate clearly where this curve intersects the  $x$  and  $y$  axes. [3 marks]

- 13d. Using your graphic display calculator, solve the equation  $3 + 2x - x^2 = 2^x$ . [2 marks]

- 13e. Write down the maximum value of the function  $f(x) = 3 + 2x - x^2$ . [1 mark]

- 13f. Use Differential Calculus to verify that your answer to (e) is correct. [5 marks]

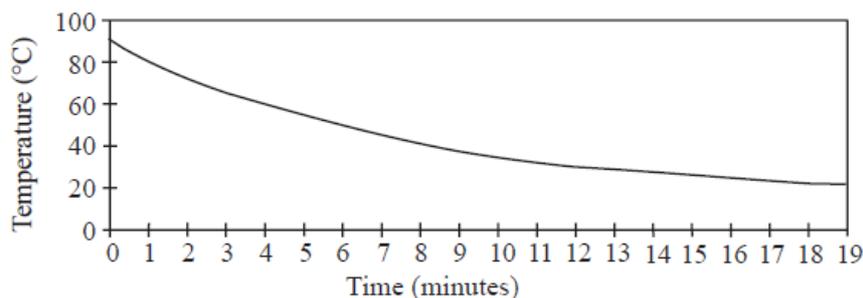
- 13g. The curve  $y = px^2 + qx - 4$  passes through the point  $(2, -10)$ . Use the above information to write down an equation in  $p$  and  $q$ . [2 marks]

- 13h. The gradient of the curve  $y = px^2 + qx - 4$  at the point  $(2, -10)$  is 1. Find  $\frac{dy}{dx}$ . [2 marks]

- 13i. The gradient of the curve  $y = px^2 + qx - 4$  at the point  $(2, -10)$  is 1. Hence, find a second equation in  $p$  and  $q$ . [1 mark]

- 13j. The gradient of the curve  $y = px^2 + qx - 4$  at the point  $(2, -10)$  is 1. Solve the equations to find the value of  $p$  and of  $q$ . [3 marks]

The following graph shows the temperature in degrees Celsius of Robert's cup of coffee,  $t$  minutes after pouring it out. The equation of the cooling graph is  $f(t) = 16 + 74 \times 2.8^{-0.2t}$  where  $f(t)$  is the temperature and  $t$  is the time in minutes after pouring the coffee out.



- 14a. Find the initial temperature of the coffee. [1 mark]

14b. Write down the equation of the horizontal asymptote. [1 mark]

14c. Find the room temperature. [1 mark]

14d. Find the temperature of the coffee after 10 minutes. [1 mark]

14e. Find the temperature of Robert's coffee after being heated in the microwave for 30 **seconds** after it has reached the temperature in part (d). [3 marks]

14f. Calculate the length of time it would take a similar cup of coffee, initially at  $20^{\circ}\text{C}$ , to be heated in the microwave to reach  $100^{\circ}\text{C}$ . [4 marks]

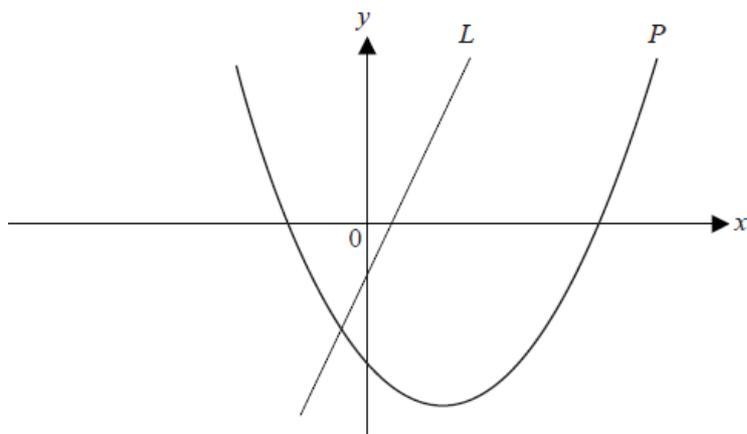
Robert, who lives in the UK, travels to Belgium. The exchange rate is 1.37 euros to one British Pound (GBP) with a commission of 3 GBP, which is subtracted before the exchange takes place. Robert gives the bank 120 GBP.

14g. Calculate **correct to 2 decimal places** the amount of euros he receives. [3 marks]

14h. He buys 1 kilogram of Belgian chocolates at 1.35 euros per 100 g. [3 marks]

Calculate the cost of his chocolates in GBP **correct to 2 decimal places**.

The diagram below shows the graph of a line  $L$  passing through  $(1, 1)$  and  $(2, 3)$  and the graph  $P$  of the function  $f(x) = x^2 - 3x - 4$



15a. Find the gradient of the line  $L$ . [2 marks]

15b. Differentiate  $f(x)$ . [2 marks]

15c. Find the coordinates of the point where the tangent to  $P$  is parallel to the line  $L$ . [3 marks]

15d. Find the coordinates of the point where the tangent to  $P$  is perpendicular to the line  $L$ . [4 marks]

15e. Find [3 marks]

(i) the gradient of the tangent to  $P$  at the point with coordinates  $(2, -6)$ .

(ii) the equation of the tangent to  $P$  at this point.

15f. State the equation of the axis of symmetry of  $P$ . [1 mark]

15g. Find the coordinates of the vertex of  $P$  and state the gradient of the curve at this point. [3 marks]

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