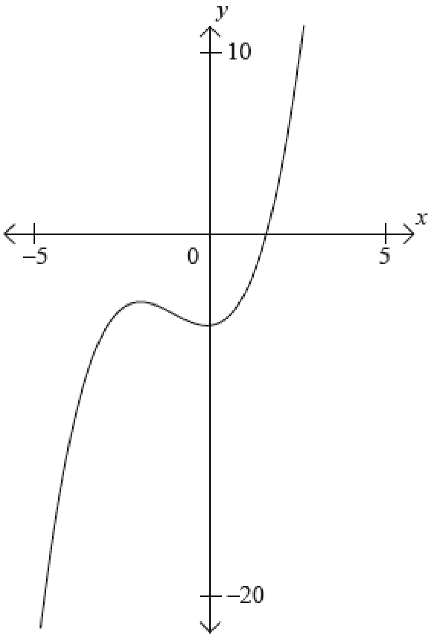
Topic 7 Part 2 *[228 marks]*

Consider the graph of the function
$f(x) = x^3 + 2x^2 - 5$.



1a. Label the local maximum as A on the graph. *[1 mark]*

1b. Label the local minimum as B on the graph. *[1 mark]*

1c. Write down the interval where $f'(x) < 0$. *[1 mark]*

1d. Draw the tangent to the curve at $x = 1$ on the graph. *[1 mark]*

1e. Write down the equation of the tangent at $x = 1$. *[2 marks]*

A function is given as
$f(x) = 2x^3 - 5x + \frac{4}{x} + 3,\ -5 \leqslant x \leqslant 10,\ x \neq 0$.

2a. Write down the derivative of the function. *[4 marks]*

2b. Use your graphic display calculator to find the coordinates of the local minimum point of $f(x)$ in the given domain. *[2 marks]*

Let
$f(x) = x^4$.

3a. Write down $f'(x)$. *[1 mark]*

3b. Point $\mathrm{P}(2,6)$ lies on the graph of f. *[2 marks]*

Find the gradient of the tangent to the graph of $y = f(x)$ at P.

3c. Point $\mathrm{P}(2,16)$ lies on the graph of f. *[3 marks]*

Find the equation of the normal to the graph at P. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers.

Consider the curve
$y = x^3 + kx$.

4a. Write down $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[1 mark]*

4b. The curve has a local minimum at the point where $x = 2$. *[3 marks]*

Find the value of k.

4c. The curve has a local minimum at the point where $x = 2$. *[2 marks]*

Find the value of y at this local minimum.

Consider the curve $y = x^2 + \frac{a}{x} - 1,\ x \neq 0$.

5a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[3 marks]*

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5b. The gradient of the tangent to the curve is -14 when $x = 1$. *[3 marks]*

Find the value of a.

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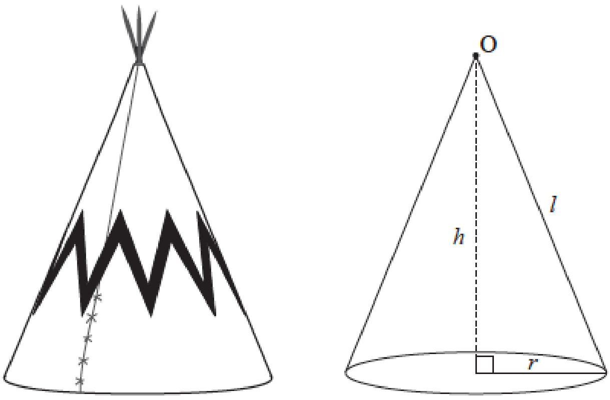
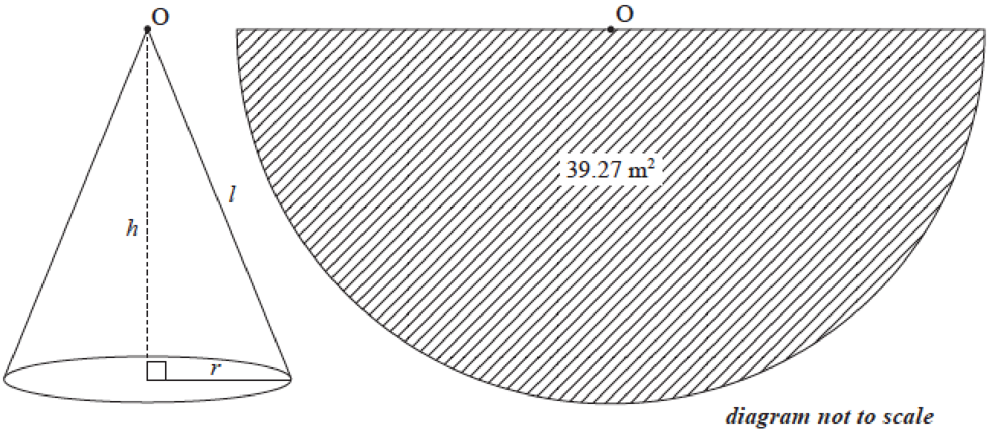
Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r, height, h, and slant height, l.



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is $39.27\ \mathrm{m}^2$, and has the shape of a semicircle, as shown in the following diagram.



6a. Show that the slant height, l, is 5 m, correct to the nearest metre. *[2 marks]*

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6b. (i) Find the circumference of the base of the cone. *[6 marks]*

(ii) Find the radius, r, of the base.

(iii) Find the height, h.

6c. A company designs cone-shaped tents to resemble the traditional tepees. *[1 mark]*

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Write down an expression for the height, h, in terms of the radius, r, of these cone-shaped tents.

6d. A company designs cone-shaped tents to resemble the traditional tepees. *[1 mark]*

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V, can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

6e. A company designs cone-shaped tents to resemble the traditional tepees. *[2 marks]*

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

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6f. A company designs cone-shaped tents to resemble the traditional tepees. *[4 marks]*

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

(i) Determine the exact value of r for which the volume is a maximum.

(ii) Find the maximum volume.

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A cuboid has a rectangular base of width x cm and length 2
x cm . The height of the cuboid is h cm . The total length of the edges of the cuboid is 72 cm.

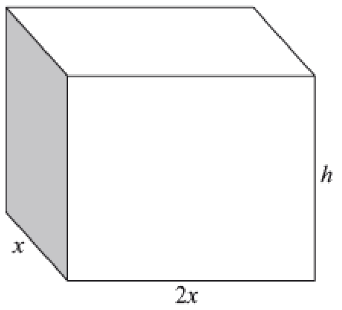

diagram not to scale

The volume, V, of the cuboid can be expressed as $V = ax^2 - 6x^3$.

7a. Find the value of a. *[3 marks]*

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7b. Find the value of x that makes the volume a maximum. *[3 marks]*

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Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

8a. Write down $f'(x)$. *[3 marks]*

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8b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

Show that $k = 3$.

8c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

Find $f(2)$.

8d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

Find $f'(2)$

8e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[3 marks]*

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

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8f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[4 marks]*

Sketch the graph of
$y = f(x)$, for $-5 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 100$.

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8g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis.

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8h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

State the values of x for which $f(x)$ is decreasing.

Consider the function $f(x) = 0.5x^2 - \frac{8}{x},\ x \neq 0$.

9a. Find $f(-2)$. *[2 marks]*

9b. Find $f'(x)$. *[3 marks]*

9c. Find the gradient of the graph of f at $x = -2$. *[2 marks]*

9d. Let T be the tangent to the graph of f at $x = -2$. *[2 marks]*

Write down the equation of T.

9e. Let T be the tangent to the graph of f at $x = -2$. *[4 marks]*

Sketch the graph of f for $-5 \leqslant x \leqslant 5$ and $-20 \leqslant y \leqslant 20$.

9f. Let T be the tangent to the graph of f at $x = -2$. *[2 marks]*

Draw T on your sketch.

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9g. The tangent, T, intersects the graph of f at a second point, P. *[2 marks]*

Use your graphic display calculator to find the coordinates of P.

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$f(x) = 5x^3 - 4x^2 + x$

10a. Find $f'(x)$. *[3 marks]*

10b. Find using your answer to part (a) the x-coordinate of *[3 marks]*

(i) the local maximum point;

(ii) the local minimum point.

Consider the function

$g(x) = bx - 3 + \frac{1}{x^2},\ x \neq 0.$

11a. Write down the equation of the vertical asymptote of the graph of $y = g(x)$. *[2 marks]*

11b. Write down $g'(x)$. *[3 marks]*

11c. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. *[2 marks]*

Show that $b = 5$.

11d. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. *[3 marks]*

Find the equation of T.

11e. Using your graphic display calculator find the coordinates of the point where the graph of $y = g(x)$ intersects the x-axis. *[2 marks]*

11f. (i) Sketch the graph of $y = g(x)$ for $-2 \le x \le 5$ and $-15 \le y \le 25$, indicating clearly your answer to part (e). *[6 marks]*

(ii) Draw the line T on your sketch.

11g. Using your graphic display calculator find the coordinates of the local minimum point of $y = g(x)$. *[2 marks]*

11h. Write down the interval for which $g(x)$ is increasing in the domain $0 < x < 5$. *[2 marks]*

Consider the function
$f(x) = ax^3 - 3x + 5$, where
$a \neq 0$.

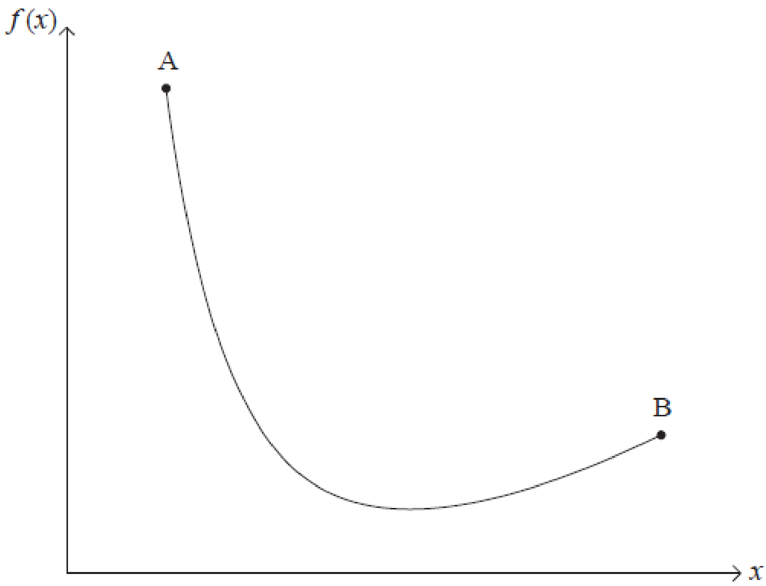
12a. Find
$f'(x)$. *[2 marks]*

12b. Write down the value of
$f'(0)$. *[1 mark]*

12c. The function has a local maximum at $x = -2$. *[3 marks]*

Calculate the value of a.

The graph of the function
$f(x) = \frac{14}{x} + x - 6$, for $1 \le x \le 7$ is given below.



13a. Calculate
$f(1)$. *[2 marks]*

13b. Find
$f'(x)$. *[3 marks]*

13c. **Use your answer to part (b)** to show that the x-coordinate of the local minimum point of the graph of f is 3.7 correct to 2 significant figures. *[3 marks]*

13d. Find the range of f. *[3 marks]*

13e. Points A and B lie on the graph of f. The x-coordinates of A and B are 1 and 7 respectively. *[1 mark]*

Write down the y-coordinate of B.

13f. Points A and B lie on the graph of f . The x-coordinates of A and B are 1 and 7 respectively. *[2 marks]*

Find the gradient of the straight line passing through A and B.

13g. M is the midpoint of the line segment AB. *[2 marks]*

Write down the coordinates of M.

13h. L is the tangent to the graph of the function $y = f(x)$, at the point on the graph with the same x-coordinate as M. *[2 marks]*

Find the gradient of L.

13i. Find the equation of L. Give your answer in the form $y = mx + c$. *[3 marks]*

A curve is described by the function $f(x) = 3x - \frac{2}{x^2}$, $x \neq 0$.

14a. Find $f'(x)$. *[3 marks]*

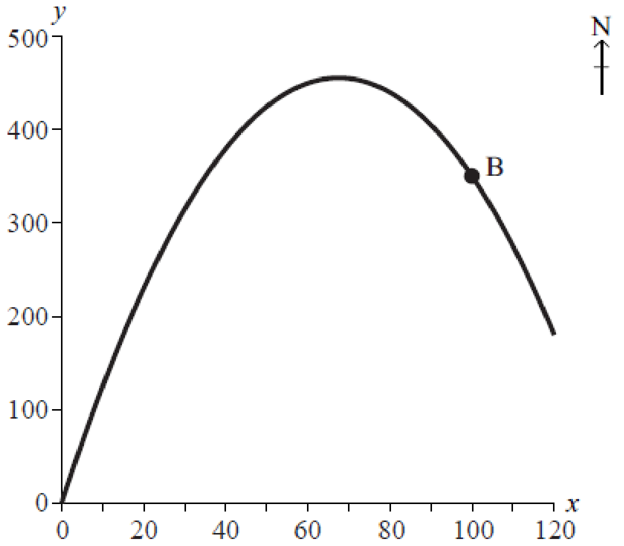
14b. The gradient of the curve at point A is 35. *[3 marks]*

Find the x-coordinate of point A.

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$y = \frac{-x^2}{10} + \frac{27}{2}x$, where
$x \geqslant 0,\ y \geqslant 0$

$(x\ ,\ y)$ are the coordinates of a point x metres east and y metres north of O , where O is the origin (0, 0) . B is a point on the bicycle track with coordinates (100, 350) .



15a. The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer. *[3 marks]*

15b. Find the derivative of $y = \frac{-x^2}{10} + \frac{27}{2}x$. *[2 marks]*

15c. Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between O and B. Give a reason for your answer. *[4 marks]*

15d. (i) Write down the midpoint of the line segment OB. *[3 marks]*

(ii) Find the gradient of the line segment OB.

15e. Scott starts from a point C(0,150) . He hikes along a straight road towards the bicycle track, parallel to the line segment OB. *[3 marks]*

Find the equation of Scott's road. Express your answer in the form
$ax + by = c$, where
a, b and $c \in \mathbb{R}$.

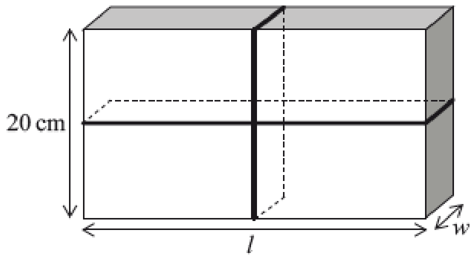
15f. Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track. *[2 marks]*

A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length
l cm, width
w cm and height of
20 cm.
The total volume of the parcel is
$3000\ \text{cm}^3$.

16a. Express the volume of the parcel in terms of
l and
w. *[1 mark]*

16b. Show that $l = \frac{150}{w}$. *[2 marks]*

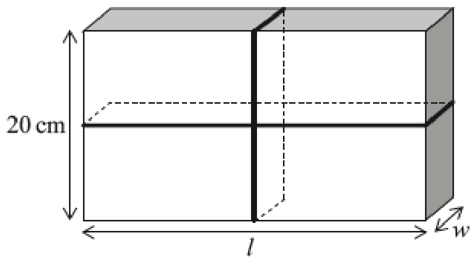
16c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



Show that the length of string, S cm, required to tie up the parcel can be written as

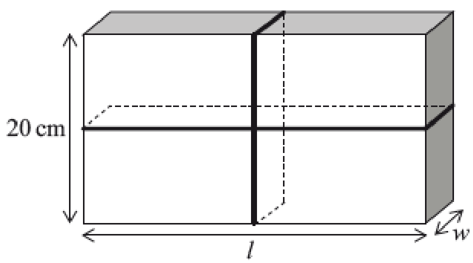
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

16d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



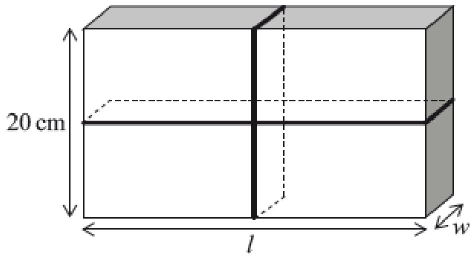
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

16e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[3 marks]*



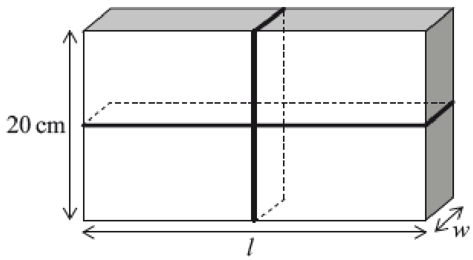
Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

16f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



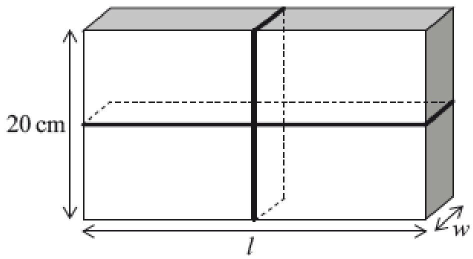
Find the value of w for which S is a minimum.

16g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[1 mark]*



Write down the value, l, of the parcel for which the length of string is a minimum.

16h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



Find the minimum length of string required to tie up the parcel.

17a. Expand the expression $x(2x^3 - 1)$. *[2 marks]*

17b. Differentiate $f(x) = x(2x^3 - 1)$. *[2 marks]*

17c. Find the x-coordinate of the local minimum of the curve $y = f(x)$. *[2 marks]*

Consider the function
$f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20.$

18a. Find
$f(-2).$

[2 marks]

18b. Find
$f'(x).$

[3 marks]

18c. The graph of the function
$f(x)$ has a local minimum at the point where
$x = -2.$
Using your answer to part (b), show that there is a second local minimum at
$x = 3.$

[5 marks]

18d. The graph of the function
$f(x)$ has a local minimum at the point where
$x = -2.$
Sketch the graph of the function
$f(x)$ for
$-5 \leqslant x \leqslant 5$ and
$-40 \leqslant y \leqslant 50.$ Indicate on your
sketch the coordinates of the
y-intercept.

[4 marks]

18e. The graph of the function
$f(x)$ has a local minimum at the point where
$x = -2.$
Write down the coordinates of the local maximum.

[2 marks]

18f. Let
T be the tangent to the graph of the function
$f(x)$ at the point
$(2, -12).$
Find the gradient of
$T.$

[2 marks]

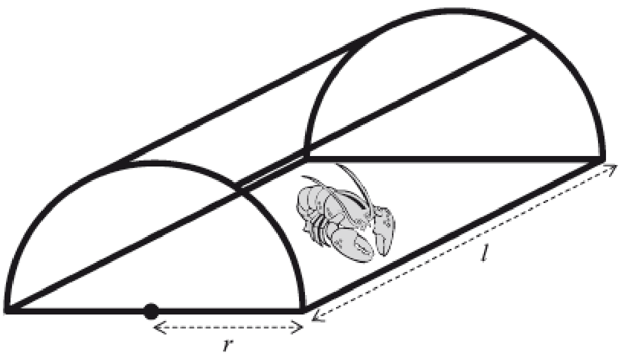
18g. The line L passes through the point $(2, -12)$ and is perpendicular to T. L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$. [5 marks]

Find

(i) the gradient of L;

(ii) the value of b and the value of c.

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.



The semicircular ends each have radius r and the support rods each have length l. Let T be the total length of steel used in the frame of the lobster trap.

19a. Write down an expression for T in terms of r, l and π. [3 marks]

19b. The volume of the lobster trap is 0.75 m^3. [3 marks]

Write down an equation for the volume of the lobster trap in terms of r, l and π.

19c. The volume of the lobster trap is 0.75 m^3. [2 marks]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

19d. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. *[3 marks]*

Find $\frac{\mathrm{d}T}{\mathrm{d}r}$.

19e. The lobster trap is designed so that the length of steel used in its frame is a minimum. *[2 marks]*

Show that the value of r for which T is a minimum is $0.719\ \mathrm{m}$, correct to three significant figures.

19f. The lobster trap is designed so that the length of steel used in its frame is a minimum. *[2 marks]*

Calculate the value of l for which T is a minimum.

19g. The lobster trap is designed so that the length of steel used in its frame is a minimum. *[2 marks]*

Calculate the minimum value of T.