

## Topic 4 Part 3 [159 marks]

George leaves a cup of hot coffee to cool and measures its temperature every minute. His results are shown in the table below.

Time, $t$ (minutes)	0	1	2	3	4	5	6
Temperature, $y$ ( $^{\circ}\text{C}$ )	94	54	34	24	$k$	16.5	15.25

- 1a. Write down the decrease in the temperature of the coffee [3 marks]  
(i) during the first minute (between  $t = 0$  and  $t = 1$ ) ;  
(ii) during the second minute;  
(iii) during the third minute.
- 1b. Assuming the pattern in the answers to part (a) continues, show that [2 marks]  
 $k = 19$ .
- 1c. Use the **seven** results in the table to draw a graph that shows how the temperature of the coffee changes during the first six minutes. [4 marks]  
Use a scale of 2 cm to represent 1 minute on the horizontal axis and 1 cm to represent  $10^{\circ}\text{C}$  on the vertical axis.
- 1d. The function that models the change in temperature of the coffee is  $y = p(2^{-t}) + q$ . [2 marks]  
(i) Use the values  $t = 0$  and  $y = 94$  to form an equation in  $p$  and  $q$ .  
(ii) Use the values  $t = 1$  and  $y = 54$  to form a second equation in  $p$  and  $q$ .
- 1e. Solve the equations found in part (d) to find the value of  $p$  and the value of  $q$ . [2 marks]
- 1f. The graph of this function has a horizontal asymptote. [2 marks]  
Write down the equation of this asymptote.
- 1g. George decides to model the change in temperature of the coffee with a linear function using correlation and linear regression. [4 marks]  
Use the **seven** results in the table to write down  
(i) the correlation coefficient;  
(ii) the equation of the regression line  $y$  on  $t$ .
- 1h. Use the equation of the regression line to estimate the temperature of the coffee at  $t = 3$ . [2 marks]
- 1i. Find the percentage error in this estimate of the temperature of the coffee at  $t = 3$ . [2 marks]

Francesca is a chef in a restaurant. She cooks eight chickens and records their masses and cooking times. The mass  $m$  of each chicken, in kg, and its cooking time  $t$ , in minutes, are shown in the following table.

Mass $m$ (kg)	Cooking time $t$ (minutes)
1.5	62
1.6	75
1.8	82
1.9	83
2.0	86
2.1	87
2.1	91
2.3	98

- 2a. Draw a scatter diagram to show the relationship between the mass of a chicken and its cooking time. Use 2 cm to represent 0.5 kg on the horizontal axis and 1 cm to represent 10 minutes on the vertical axis. [4 marks]
- 2b. Write down for this set of data [2 marks]
- (i) the mean mass,  
 $\bar{m}$  ;
- (ii) the mean cooking time,  
 $\bar{t}$  .
- 2c. Label the point  $M(\bar{m}, \bar{t})$  on the scatter diagram. [1 mark]
- 2d. Draw the line of best fit on the scatter diagram. [2 marks]
- 2e. Using your line of best fit, estimate the cooking time, in minutes, for a 1.7 kg chicken. [2 marks]
- 2f. Write down the Pearson's product-moment correlation coefficient,  $r$  . [2 marks]
- 2g. Using your value for  $r$  , comment on the correlation. [2 marks]
- 2h. The cooking time of an additional 2.0 kg chicken is recorded. If the mass and cooking time of this chicken is included in the data, the correlation is weak. [2 marks]
- (i) Explain how the cooking time of this additional chicken might differ from that of the other eight chickens.
- (ii) Explain how a new line of best fit might differ from that drawn in part (d).

3a. State the null hypothesis,  $H_0$ , for the test. [2 marks]

3b. Write down the number of degrees of freedom. [1 mark]

3c. Write down the critical value for the test. [1 mark]

3d. Show that the expected number of Medium Yield crops using Fertilizer C is 17, correct to the nearest integer. [2 marks]

3e. Use your graphic display calculator to find for the data [3 marks]

- (i) the  
 $\chi^2$  calculated value,  
 $\chi^2_{calc}$ ;  
(ii) the  $p$ -value.

3f. State the conclusion of the test. Give a reason for your decision. [2 marks]

A market researcher surveyed men and women about their preferred holiday destination. The holiday destinations were Antigua, Barbados, Cuba, Guadeloupe and Jamaica. A  $\chi^2$  test for independence was conducted at the 5 % significance level.  
The  $\chi^2$  calculated value was found to be 8.73.

4a. Write down the null hypothesis. [1 mark]

4b. Find the number of degrees of freedom for this test. [2 marks]

4c. Write down the critical value for this test. [1 mark]

4d. State the conclusion of this test. Give a reason for your decision. [2 marks]

180 spectators at a swimming championship were asked which, of four swimming styles, was the one they preferred to watch. The results of their responses are shown in the table.

Swimming style	Male	Female
Freestyle	20	15
Butterfly	20	30
Backstroke	10	35
Breaststroke	10	40

A  $\chi^2$  test was conducted at the 5% significance level.

5a. Write down the null hypothesis for this test. [1 mark]

5b. Write down the number of degrees of freedom. [1 mark]

5c. Write down the value of  $\chi^2_{calc}$ . [2 marks]

5d. The critical value, at the 5% significance level, is 7.815. [2 marks]  
State, giving a reason, the conclusion to the test.

The table shows the distance, in km, of eight regional railway stations from a city centre terminus and the price, in \$, of a return ticket from each regional station to the terminus.

Distance in km ( $x$ )	3	15	23	42	56	62	74	93
Price in \$ ( $y$ )	5	24	43	56	68	74	86	100

6a. Draw a scatter diagram for the above data. Use a scale of 1 cm to represent 10 km on the  $x$ -axis and 1 cm to represent \$10 on the  $y$ -axis. [4 marks]

6b. Use your graphic display calculator to find [2 marks]  
(i)  $\bar{x}$ , the mean of the distances;  
(ii)  $\bar{y}$ , the mean of the prices.

6c. Plot and label the point M ( $\bar{x}$ ,  $\bar{y}$ ) on your scatter diagram. [1 mark]

6d. Use your graphic display calculator to find [3 marks]  
(i) the product-moment correlation coefficient,  $r$ ;  
(ii) the equation of the regression line  $y$  on  $x$ .

6e. Draw the regression line  $y$  on  $x$  on your scatter diagram. [2 marks]

6f. A ninth regional station is 76 km from the city centre terminus. [3 marks]  
Use the equation of the regression line to estimate the price of a return ticket to the city centre terminus from this regional station. **Give your answer correct to the nearest \$.**

6g. Give a reason why it is valid to use your regression line to estimate the price of this return ticket. [1 mark]

- 6h. The actual price of the return ticket is \$80. [2 marks]
- Using your answer to part (f), calculate the percentage error in the estimated price of the ticket.
- A group of candidates sat a Chemistry examination and a Physics examination. The candidates' marks in the Chemistry examination are normally distributed with a mean of 60 and a standard deviation of 12.
- 7a. Draw a diagram that shows this information. [2 marks]
- 7b. Write down the probability that a randomly chosen candidate who sat the Chemistry examination scored at most 60 marks. [1 mark]
- 7c. Hee Jin scored 80 marks in the Chemistry examination. [2 marks]
- Find the probability that a randomly chosen candidate who sat the Chemistry examination scored **more** than Hee Jin.
- 7d. The candidates' marks in the Physics examination are normally distributed with a mean of 63 and a standard deviation of 10. Hee Jin also scored 80 marks in the Physics examination. [2 marks]
- Find the probability that a randomly chosen candidate who sat the Physics examination scored **less** than Hee Jin.
- 7e. The candidates' marks in the Physics examination are normally distributed with a mean of 63 and a standard deviation of 10. Hee Jin also scored 80 marks in the Physics examination. [2 marks]
- Determine whether Hee Jin's Physics mark, **compared to the other candidates**, is better than her mark in Chemistry. Give a reason for your answer.
- 7f. To obtain a "grade A" a candidate must be in the top 10% of the candidates who sat the Physics examination. [3 marks]
- Find the minimum possible mark to obtain a "grade A". Give your answer correct to the nearest integer.
- The scores obtained by five candidates in Mathematics and Physics examinations are given below.
- |                     |    |    |    |    |    |
|---------------------|----|----|----|----|----|
| Mathematics ( $x$ ) | 62 | 84 | 47 | 55 | 32 |
| Physics ( $y$ )     | 80 | 91 | 44 | 48 | 53 |
- 8a. Write down the correlation coefficient,  $r$ , for the examination scores. [2 marks]
- 8b. Write down the equation of the regression line,  $y$  on  $x$ , for the examination scores of the five candidates. [2 marks]

- 8c. A sixth candidate scored 72 in the Mathematics examination. Use the regression line,  $y$  on  $x$ , to estimate his score on the Physics examination. [2 marks]

The table below shows the scores for 12 golfers for their first two rounds in a local golf tournament.

Round 1 ( $x$ )	71	79	66	73	69	76	68	75	82	67	69	74
Round 2 ( $y$ )	73	81	68	75	70	79	69	77	83	68	72	76

- 9a. (i) Write down the mean score in Round 1. [5 marks]  
(ii) Write down the standard deviation in Round 1.  
(iii) Find the number of these golfers that had a score of more than one standard deviation above the mean in Round 1.
- 9b. Write down the correlation coefficient,  $r$ . [2 marks]
- 9c. Write down the equation of the regression line of  $y$  on  $x$ . [2 marks]
- 9d. Another golfer scored 70 in Round 1. [2 marks]  
Calculate an estimate of his score in Round 2.
- 9e. Another golfer scored 89 in Round 1. [2 marks]  
Determine whether you can use the equation of the regression line to estimate his score in Round 2. Give a reason for your answer.

A store recorded their sales of televisions during the 2010 football World Cup. They looked at the numbers of televisions bought by gender and the size of the television screens.

This information is shown in the table below;  $S$  represents the size of the television screen in inches.

	$S \leq 22$	$22 < S \leq 32$	$32 < S \leq 46$	$S > 46$	Total
Female	65	100	40	15	220
Male	20	65	140	55	280
Total	85	165	180	70	500

The store wants to use this information to predict the probability of selling these sizes of televisions for the 2014 football World Cup.

- 10a. Use the table to find the probability that [6 marks]  
(i) a television will be bought by a female;  
(ii) a television with a screen size of  $32 < S \leq 46$  will be bought;  
(iii) a television with a screen size of  $32 < S \leq 46$  will be bought by a female;  
(iv) a television with a screen size greater than 46 inches will be bought, given that it is bought by a male.
- 10b. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed [1 mark]  
at the 1 % significance level.  
Write down the null hypothesis.
- 10c. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [2 marks]  
Show that the expected frequency for females who bought a screen size of  $32 < S \leq 46$ , is 79, correct to the nearest integer.

- 10d. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed [1 mark]  
at the 1 % significance level.

Write down the number of degrees of freedom.

- 10e. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [2 marks]

Write down the  $\chi^2$  calculated value.

- 10f. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [1 mark]

Write down the critical value for this test.

- 10g. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [2 marks]

Determine if the null hypothesis should be accepted. Give a reason for your answer.

The number of calories a person burns during a walk depends on the time they spend walking. The table below shows the number of calories,  $y$ , burned by a person in relation to the time they spend walking,  $x$ , in minutes.

Time spent walking ( $x$ ) (minutes)	10	15	20	25	30
Calories ( $y$ )	90	125	200	300	375

- 11a. Use your graphic display calculator to write down the equation of the regression line for  $y$  on  $x$  in the form  $y = ax + b$ . [2 marks]

- 11b. Use your equation to estimate the number of calories that a person will burn during a 17 minute walk. [2 marks]

- 11c. State whether your answer to part (b) is reliable. Give a reason for your answer. [2 marks]

The seniors from Gulf High School are required to participate in exactly one after-school sport. Data were gathered from a sample of 120 students regarding their choice of sport. The following data were recorded.

	Sport			
Gender	Football	Tennis	Basketball	Total
Male	17	8	10	35
Female	31	17	37	85
Total	48	25	47	120

A

$\chi^2$  test was carried out at the 5 % significance level to analyse the relationship between gender and choice of after-school sport.

- 12a. Write down the null hypothesis,  $H_0$ , for this test. [1 mark]

- 12b. Find the expected value of female footballers. [2 marks]

- 12c. Write down the number of degrees of freedom. [1 mark]

12d. Write down the critical value of  $\chi^2$ , at the 5 % level of significance. [1 mark]

12e. Use your graphic display calculator to determine the  $\chi^2_{calc}$  value. [2 marks]

12f. Determine whether  $H_0$  should be accepted. Justify your answer. [2 marks]

12g. One student is chosen at random from the 120 students. [2 marks]

Find the probability that this student

(i) is male;

(ii) plays tennis.

12h. Two students are chosen at random from the 120 students. [5 marks]

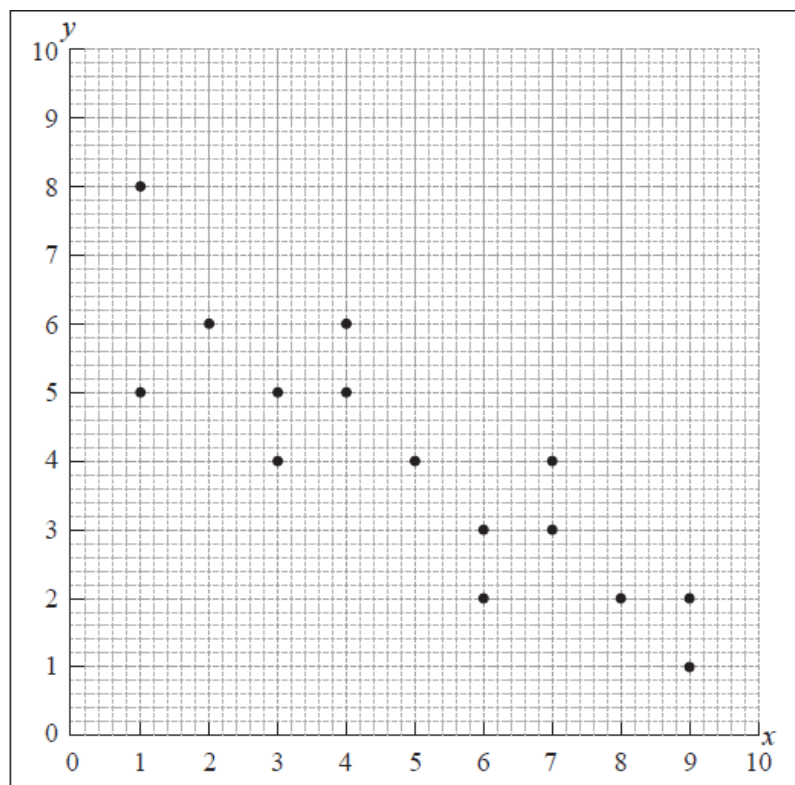
Find the probability that

(i) both play football;

(ii) neither play basketball.

Consider the following values of  $x$  and  $y$  and the scatter diagram which represents the information given in the table.

$x$	1	1	2	3	3	4	4	$b$	6	6	7	7	8	9	9
$y$	5	$a$	6	4	5	5	6	4	2	3	3	4	2	1	2



13a. Write down the value of [2 marks]

(i)  $a$  ;

(ii)  $b$  .



13b. The mean of the  $x$  values is 5 and the mean of the  $y$  values is 4. Draw the line of best fit on the scatter diagram above. [2 marks]

13c. Use your line of best fit to estimate the value of  $y$  when  $x = 6.5$ . [2 marks]

A researcher consulted 500 men and women to see if the colour of the car they drove was independent of gender. A  $\chi^2$  test for independence was carried out.

14a. Write down the null hypothesis. [1 mark]

14b. The colours of the cars were red, green, blue, black and silver. [2 marks]  
Find the number of degrees of freedom for this test.

14c. At the 5 % significance level the  $\chi^2_{calc}$  was found to be 8.73. [1 mark]  
Write down the critical value,  $\chi^2_{crit}$ , for this test.