Topic 5 Part 4 *[241 marks]*

A straight line, L_1, has equation $x + 4y + 34 = 0$.

1a. Find the gradient of L_1. *[2 marks]*

1b. The equation of line L_2 is $y = mx$. L_2 is perpendicular to L_1. *[2 marks]*

Find the value of m.

1c. The equation of line L_2 is $y = mx$. L_2 is perpendicular to L_1. *[2 marks]*

Find the coordinates of the point of intersection of the lines L_1 and L_2.

The diagram shows a pyramid VABCD which has a square base of length 10 cm and edges of length 13 cm. M is the midpoint of the side BC.

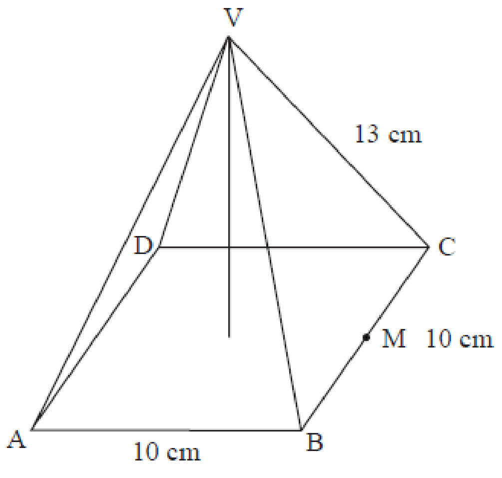


diagram not to scale

2a. Calculate the length of VM. *[2 marks]*

2b. Calculate the vertical height of the pyramid. *[2 marks]*

The quadrilateral ABCD shown below represents a sandbox. AB and BC have the same length. AD is 9 m long and CD is 4.2 m long. Angles ADC and ABC are 95° and 130° respectively.

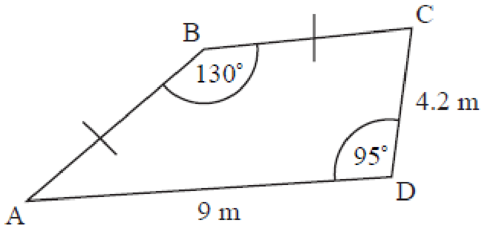


diagram not to scale

3a. Find the length of AC. *[3 marks]*

3b. (i) Write down the size of angle BCA. *[4 marks]*

(ii) Calculate the length of AB.

3c. Show that the area of the sandbox is 31.1 m^2 correct to 3 s.f. *[4 marks]*

3d. The sandbox is a prism. Its edges are 40 cm high. The sand occupies one third of the volume of the sandbox. Calculate the volume of sand in the sandbox. *[3 marks]*

Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$.

4a. (i) Write down the value of y when x is 2. *[3 marks]*

(ii) Write down the coordinates of the point where the curve intercepts the y-axis.

4b. Sketch the curve for $-4 \leqslant x \leqslant 3$ and $-10 \leqslant y \leqslant 10$. Indicate clearly the information found in (a). *[4 marks]*

4c. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[3 marks]*

4d. Let L_1 be the tangent to the curve at $x = 2$. *[8 marks]*

Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

4e. It is known that $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ for $x < -2$ and $x > b$ where b is positive. *[5 marks]*

(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$.

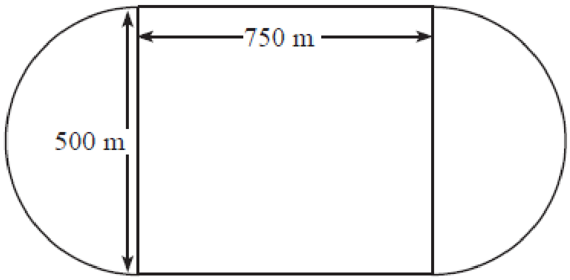
Triangle ABC is such that AC is 7 cm, angle ABC is $65°$ and angle ACB is $30°$.

5a. Sketch the triangle writing in the side length and angles. *[1 mark]*

5b. Calculate the length of AB. *[2 marks]*

5c. Find the area of triangle ABC. *[3 marks]*

A race track is made up of a rectangular shape
750 m by
500 m with semi-circles at each end as shown in the diagram.



Michael drives around the track once at an average speed of
$140\ \text{kmh}^{-1}$.

6a. Calculate the distance that Michael travels. *[2 marks]*

6b. Calculate how long Michael takes in **seconds**. *[4 marks]*

Consider the functions
$f(x) = \frac{2x+3}{x+4}$ and
$g(x) = x + 0.5$.

7a. Sketch the graph of the function
$f(x)$, for
$-10 \leqslant x \leqslant 10$. Indicating clearly the axis intercepts and any asymptotes. *[6 marks]*

7b. Write down the equation of the vertical asymptote. *[2 marks]*

7c. On the same diagram as part (a) sketch the graph of
$g(x) = x + 0.5$. *[2 marks]*

7d. Using your graphical display calculator write down the coordinates of **one** of the points of intersection on the graphs of
f and
g, **giving your answer correct to five decimal places**. *[3 marks]*

7e. Write down the gradient of the line
$g(x) = x + 0.5$. *[1 mark]*

7f. The line
L passes through the point with coordinates
$(-2,\ -3)$ and is perpendicular to the line
$g(x)$. Find the equation of
L. *[3 marks]*

Mal is shopping for a school trip. He buys
50 tins of beans and
20 packets of cereal. The total cost is
260 Australian dollars (
AUD).

8a. Write down an equation showing this information, taking
b to be the cost of one tin of beans and
c to be the cost of one packet of cereal in
AUD. *[1 mark]*

8b. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays 66 AUD. *[1 mark]*

Write down another equation to represent this information.

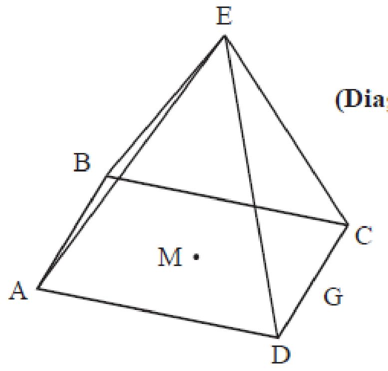
8c. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays 66 AUD. *[2 marks]*

Find the cost of one tin of beans.

8d. (i) Sketch the graphs of the two equations from parts (a) and (b). *[4 marks]*

(ii) Write down the coordinates of the point of intersection of the two graphs.

The triangular faces of a square based pyramid, ABCDE, are all inclined at 70° to the base. The edges of the base ABCD are all 10 cm and M is the centre. G is the mid-point of CD.



(Diagram not to scale)

8e. Using the letters on the diagram draw a triangle showing the position of a 70° angle. *[1 mark]*

8f. Show that the height of the pyramid is 13.7 cm, to 3 significant figures. *[2 marks]*

8g. Calculate *[4 marks]*

(i) the length of EG;

(ii) the size of angle DEC.

8h. Find the total surface area of the pyramid. *[2 marks]*

8i. Find the volume of the pyramid. *[2 marks]*

9a. Factorise $3x^2 + 13x - 10$. *[2 marks]*

9b. Solve the equation $3x^2 + 13x - 10 = 0$. *[2 marks]*

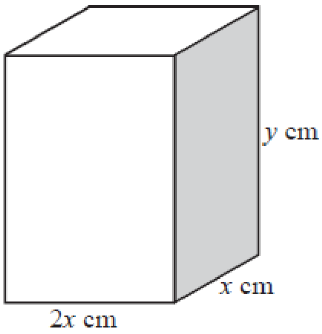
9c. Consider a function $f(x) = 3x^2 + 13x - 10$. *[2 marks]*

Find the equation of the axis of symmetry on the graph of this function.

9d. Consider a function $f(x) = 3x^2 + 13x - 10$. *[2 marks]*

Calculate the minimum value of this function.

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of $300\ \text{cm}^2$.



9e. Show that $4x^2 + 6xy = 300$. *[2 marks]*

9f. Find an expression for y in terms of x. *[2 marks]*

9g. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. *[2 marks]*

9h. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. *[2 marks]*

9i. (i) Hence find the value of x and of y required to make the volume of the box a maximum. *[5 marks]*

(ii) Calculate the maximum volume.

Triangle ABC is drawn such that angle ABC is 90°, angle ACB is 60° and AB is 7.3 cm.

10a. (i) Sketch a diagram to illustrate this information. Label the points A, B, C. Show the angles 90°, 60° and the length 7.3 cm on your diagram. *[3 marks]*

(ii) Find the length of BC.

10b. Point D is on the straight line AC extended and is such that angle CDB is 20°. *[3 marks]*

(i) Show the point D and the angle 20° on your diagram.

(ii) Find the size of angle CBD.

11a. Write down the gradient of the line $y = 3x + 4$. *[1 mark]*

11b. Find the gradient of the line which is perpendicular to the line $y = 3x + 4$. *[1 mark]*

11c. Find the equation of the line which is perpendicular to $y = 3x + 4$ and which passes through the point $(6, 7)$. *[2 marks]*

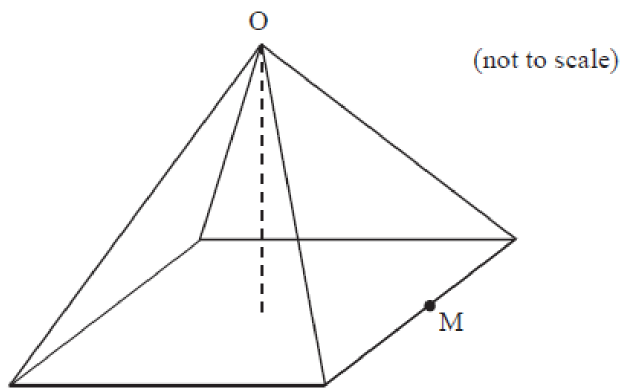
11d. Find the coordinates of the point of intersection of these two lines. *[2 marks]*

Amir needs to construct an isosceles triangle ABC whose area is 100 cm^2. The equal sides, AB and BC, are 20 cm long.

12a. Angle ABC is acute. Show that the angle ABC is 30°. *[2 marks]*

12b. Find the length of AC. *[3 marks]*

Sylvia is making a square-based pyramid. Each triangle has a base of length 12 cm and a height of 10 cm.



12c. Show that the **height** of the pyramid is 8 cm. *[2 marks]*

12d. M is the midpoint of the base of one of the triangles and O is the apex of the pyramid. *[3 marks]*

Find the angle that the line MO makes with the base of the pyramid.

12e. Calculate the volume of the pyramid. *[2 marks]*

12f. Daniel wants to make a rectangular prism with the same volume as that of Sylvia's pyramid. The base of his prism is to be a square of side 10 cm. Calculate the height of the prism. *[2 marks]*

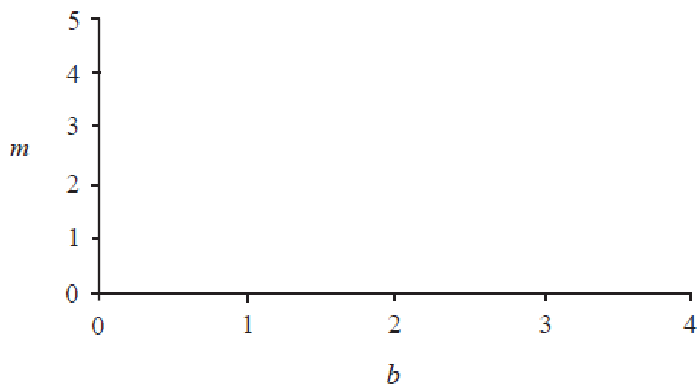
A store sells bread and milk. On Tuesday, 8 loaves of bread and 5 litres of milk were sold for $21.40. On Thursday, 6 loaves of bread and 9 litres of milk were sold for $23.40.

If $b =$ the price of a loaf of bread and $m =$ the price of one litre of milk, Tuesday's sales can be written as $8b + 5m = 21.40$.

13a. Using simplest terms, write an equation in b and m for Thursday's sales. *[2 marks]*

13b. Find b and m. *[2 marks]*

13c. Draw a sketch, in the space provided, to show how the prices can be found graphically. *[2 marks]*

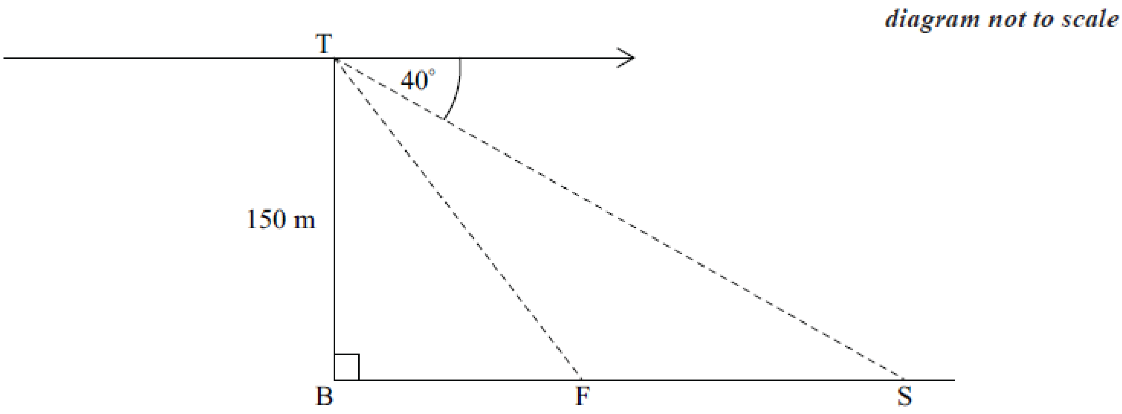


The mid-point, M, of the line joining A(s , 8) to B(-2, t) has coordinates M(2, 3).

14a. Calculate the values of s and t. *[2 marks]*

14b. Find the equation of the straight line perpendicular to AB, passing through the point M. *[4 marks]*

Tom stands at the top, T , of a vertical cliff
150 m high and sees a fishing boat, F , and a ship, S . B represents a point at the bottom of the cliff directly below T . The angle of depression of the ship is
40° and the angle of depression of the fishing boat is
55° .



15a. Calculate, SB, the distance between the ship and the bottom of the cliff. *[2 marks]*

15b. Calculate, SF, the distance between the ship and the fishing boat. Give your answer correct to the nearest metre. *[4 marks]*

P (4, 1) and Q (0, –5) are points on the coordinate plane.

16a. Determine the *[4 marks]*

(i) coordinates of M, the midpoint of P and Q.

(ii) gradient of the line drawn through P and Q.

(iii) gradient of the line drawn through M, perpendicular to PQ.

16b. The perpendicular line drawn through M meets the y-axis at R (0, k). *[2 marks]*

Find k.

17a. The length of one side of a rectangle is 2 cm longer than its width. *[1 mark]*

If the smaller side is x cm, find the perimeter of the rectangle in terms of x.

17b. The length of one side of a rectangle is 2 cm longer than its width. *[1 mark]*

The perimeter of a square is equal to the perimeter of the rectangle in part (a).

Determine the length of each side of the square in terms of x.

17c. The length of one side of a rectangle is 2 cm longer than its width. *[4 marks]*

The perimeter of a square is equal to the perimeter of the rectangle in part (a).

The sum of the areas of the rectangle and the square is
$2x^2 + 4x + 1$ (cm^2).

(i) Given that this sum is 49 cm^2, find x.

(ii) Find the area of the square.

Jenny has a circular cylinder with a lid. The cylinder has height 39 **cm** and diameter 65 **mm**.

18a. Calculate the volume of the cylinder **in cm^3**. Give your answer correct to **two** decimal places. *[3 marks]*

18b. The cylinder is used for storing tennis balls. Each ball has a **radius** of 3.25 cm. *[1 mark]*

Calculate how many balls Jenny can fit in the cylinder if it is filled to the top.

18c. (i) Jenny fills the cylinder with the number of balls found in part (b) and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls. *[4 marks]*

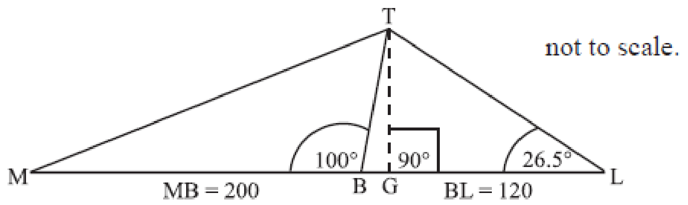
(ii) Convert your answer to (c) (i) into cubic metres.

An old tower (BT) leans at 10° away from the vertical (represented by line TG).

The base of the tower is at B so that
$\text{M}\hat{\text{B}}\text{T} = 100°$.

Leonardo stands at L on flat ground 120 m away from B in the direction of the lean.

He measures the angle between the ground and the top of the tower T to be
$\text{B}\hat{\text{L}}\text{T} = 26.5°$.



18d. (i) Find the value of angle
$\text{B}\hat{\text{T}}\text{L}$. *[5 marks]*

(ii) Use triangle BTL to calculate the sloping distance BT from the base, B to the top, T of the tower.

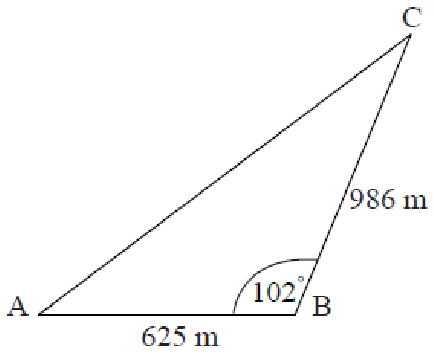
18e. Calculate the vertical height TG of the top of the tower. *[2 marks]*

18f. Leonardo now walks to point M, a distance 200 m from B on the opposite side of the tower. Calculate the distance from M to the top of the tower at T. *[3 marks]*

On a map three schools A, B and C are situated as shown in the diagram.

Schools A and B are 625 metres apart.

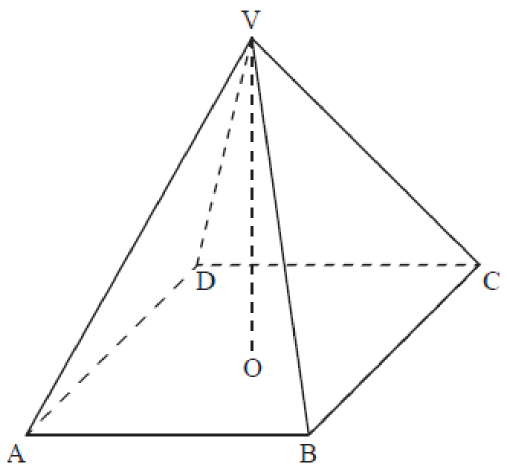
Angle ABC = 102° and BC = 986 metres.



19a. Find the distance between A and C. *[3 marks]*

19b. Find the size of angle BAC. *[3 marks]*

ABCDV is a solid glass pyramid. The base of the pyramid is a square of side 3.2 cm. The vertical height is 2.8 cm. The vertex V is directly above the centre O of the base.



20a. Calculate the volume of the pyramid. *[2 marks]*

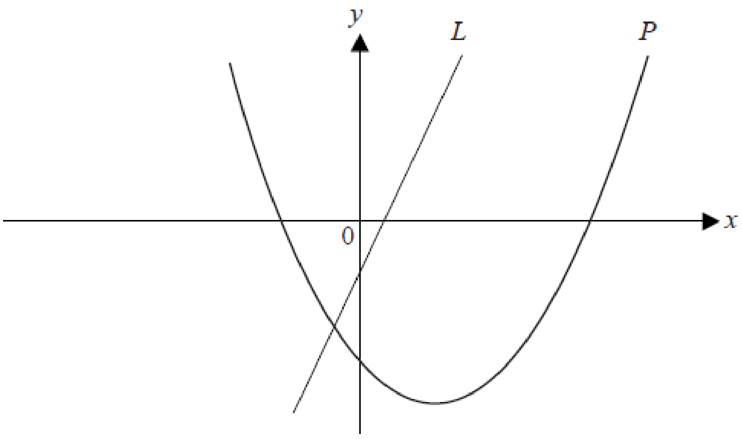
20b. The glass weighs 9.3 grams per cm^3. Calculate the weight of the pyramid. *[2 marks]*

20c. Show that the length of the sloping edge VC of the pyramid is 3.6 cm. *[4 marks]*

20d. Calculate the angle at the vertex, $B\hat{V}C$. *[3 marks]*

20e. Calculate the total surface area of the pyramid. *[4 marks]*

The diagram below shows the graph of a line
L passing through (1, 1) and (2 , 3) and the graph
P of the function
$f(x) = x^2 - 3x - 4$



21a. Find the gradient of the line L. *[2 marks]*

21b. Differentiate $f(x)$. *[2 marks]*

21c. Find the coordinates of the point where the tangent to P is parallel to the line L. *[3 marks]*

21d. Find the coordinates of the point where the tangent to P is perpendicular to the line L. *[4 marks]*

21e. Find *[3 marks]*

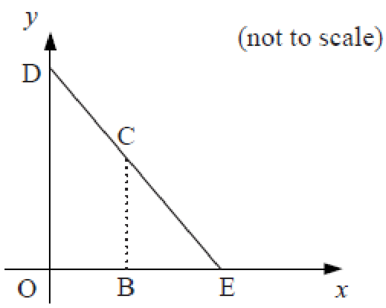
(i) the gradient of the tangent to P at the point with coordinates (2, – 6).

(ii) the equation of the tangent to P at this point.

21f. State the equation of the axis of symmetry of P. *[1 mark]*

21g. Find the coordinates of the vertex of P and state the gradient of the curve at this point. *[3 marks]*

On the coordinate axes below,
D is a point on the
y-axis and
E is a point on the
x-axis.
O is the origin. The equation of the line
DE is
$y + \frac{1}{2}x = 4$.



22a. Write down the coordinates of point E. *[2 marks]*

22b. C is a point on the line DE. *[2 marks]*
B is a point on the
x-axis such that
BC is parallel to the
y-axis. The
x-coordinate of
C is
t.

Show that the
y-coordinate of
C is
$4 - \frac{1}{2}t$.

22c. OBCD is a trapezium. The y-coordinate of point D is 4. *[3 marks]*

Show that the area of
OBCD is
$4t - \frac{1}{4}t^2$.

22d. The area of OBCD is 9.75 square units. Write down a quadratic equation that expresses this information. *[1 mark]*

22e. (i) Using your graphic display calculator, or otherwise, find the two solutions to the quadratic equation written in part (d). *[4 marks]*

(ii) Hence find the correct value for
t. Give a reason for your answer.