

Topic 3 Part 3 [215 marks]

Consider the following logic statements.

p : Carlos is playing the guitar

q : Carlos is studying for his IB exams

- 1a. Write in words the compound statement $\neg p \wedge q$. [2 marks]
- 1b. Write the following statement in symbolic form. [1 mark]
“Either Carlos is playing the guitar or he is studying for his IB exams but not both.”
- 1c. Write the **converse** of the following statement in **symbolic form**. [3 marks]
“If Carlos is playing the guitar then he is not studying for his IB exams.”

In a college 450 students were surveyed with the following results

150 have a television

205 have a computer

220 have an iPhone

75 have an iPhone and a computer

60 have a television and a computer

70 have a television and an iPhone

40 have all three.

- 2a. Draw a Venn diagram to show this information. Use T to represent the set of students who have a television, C the set of students who have a computer and I the set of students who have an iPhone. [4 marks]
- 2b. Write down the number of students that [2 marks]
(i) have a computer only;
(ii) have an iPhone and a computer but no television.
- 2c. Write down [1 mark]
 $n[T \cap (C \cup I)']$.
- 2d. Calculate the number of students who have none of the three. [2 marks]
- 2e. Two students are chosen at random from the 450 students. Calculate the probability that [6 marks]
(i) neither student has an iPhone;
(ii) only one of the students has an iPhone.

- 2f. The students are asked to collect money for charity. In the first month, the students collect x dollars and the students collect y dollars in each subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x + 5y = 7650$. [3 marks]
- In the first 10 months they collect 13 050 dollars.
- (i) Write down a second equation in x and y to represent this information.
- (ii) Write down the value of x and of y .

- 2g. The students are asked to collect money for charity. In the first month, the students collect x dollars and the students collect y dollars in each subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x + 5y = 7650$. [3 marks]
- In the first 10 months they collect 13 050 dollars.
- Calculate the number of months that it will take the students to collect 49 500 dollars.

A store recorded their sales of televisions during the 2010 football World Cup. They looked at the numbers of televisions bought by gender and the size of the television screens.

This information is shown in the table below; S represents the size of the television screen in inches.

| | $S \leq 22$ | $22 < S \leq 32$ | $32 < S \leq 46$ | $S > 46$ | Total |
|--------|-------------|------------------|------------------|----------|-------|
| Female | 65 | 100 | 40 | 15 | 220 |
| Male | 20 | 65 | 140 | 55 | 280 |
| Total | 85 | 165 | 180 | 70 | 500 |

The store wants to use this information to predict the probability of selling these sizes of televisions for the 2014 football World Cup.

- 3a. Use the table to find the probability that [6 marks]
- (i) a television will be bought by a female;
- (ii) a television with a screen size of $32 < S \leq 46$ will be bought;
- (iii) a television with a screen size of $32 < S \leq 46$ will be bought by a female;
- (iv) a television with a screen size greater than 46 inches will be bought, given that it is bought by a male.
- 3b. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [1 mark]
- Write down the null hypothesis.
- 3c. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [2 marks]
- Show that the expected frequency for females who bought a screen size of $32 < S \leq 46$, is 79, correct to the nearest integer.
- 3d. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [1 mark]
- Write down the number of degrees of freedom.
- 3e. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [2 marks]
- Write down the χ^2 calculated value.
- 3f. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % significance level. [1 mark]
- Write down the critical value for this test.

- 3g. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed [2 marks]
at the 1 % significance level.

Determine if the null hypothesis should be accepted. Give a reason for your answer.

Consider the statements

p : The numbers x and y are both even.

q : The sum of x and y is an even number.

- 4a. Write down, in words, the statement $p \Rightarrow q$. [2 marks]

- 4b. Write down, in words, the inverse of the statement $p \Rightarrow q$. [2 marks]

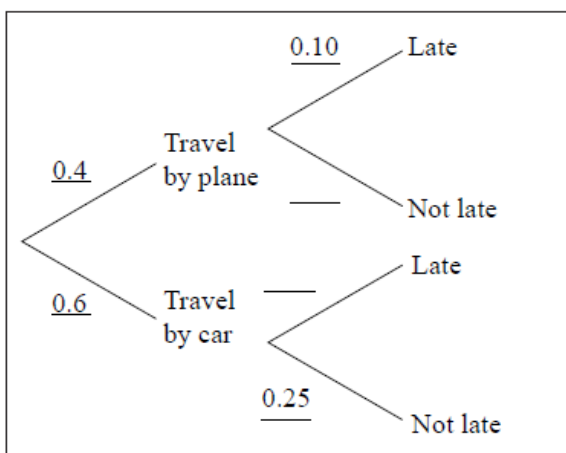
- 4c. State whether the inverse of the statement $p \Rightarrow q$ is always true. Justify your answer. [2 marks]

Merryn plans to travel to a concert tomorrow. Due to bad weather, there is a 60 % chance that all flights will be cancelled tomorrow. If the flights are cancelled Merryn will travel by car.

If she travels by plane the probability that she **will be late** for the concert is 10 %.

If she travels by car, the probability that she **will not be late** for the concert is 25 %.

- 5a. Complete the tree diagram below. [1 mark]



- 5b. Find the probability that Merryn will not be late for the concert. [3 marks]

- 5c. Merryn was not late for the concert the next day. [2 marks]
Given that, find the probability that she travelled to the concert by car.

Beartown has three local newspapers: *The Art Journal*, *The Beartown News*, and *The Currier*.

A survey shows that

- 32 % of the town's population read *The Art Journal*,
- 46 % read *The Beartown News*,
- 54 % read *The Currier*,
- 3 % read *The Art Journal* and *The Beartown News* **only**,
- 8 % read *The Art Journal* and *The Currier* **only**,
- 12 % read *The Beartown News* and *The Currier* **only**, and
- 5 % of the population reads **all** three newspapers.

- 6a. Draw a Venn diagram to represent this information. Label A the set that represents *The Art Journal* readers, B the set that represents *The Beartown News* readers, and C the set that represents *The Currier* readers. [4 marks]
- 6b. Find the percentage of the population that does not read any of the three newspapers. [2 marks]
- 6c. Find the percentage of the population that reads exactly one newspaper. [2 marks]
- 6d. Find the percentage of the population that reads *The Art Journal* or *The Beartown News* but not *The Currier*. [2 marks]
- 6e. A local radio station states that 83 % of the population reads either *The Beartown News* or *The Currier*. [2 marks]
Use your Venn diagram to decide whether the statement is true. Justify your answer.
- 6f. The population of Beartown is 120 000. The local radio station claimed that 34 000 of the town's citizens read at least two of the local newspapers. [4 marks]
Find the percentage error in this claim.

Consider the propositions p and q .

p : I take swimming lessons

q : I can swim 50 metres

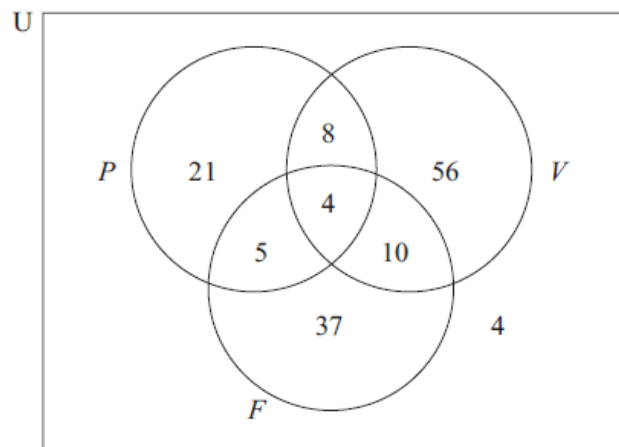
- 7a. Complete the truth table below. [2 marks]

| p | q | $\neg q$ | $p \vee \neg q$ |
|-----|-----|----------|-----------------|
| T | T | | |
| T | F | | |
| F | T | | |
| F | F | | |

- 7b. Write the following compound proposition in symbolic form. [2 marks]
"I cannot swim 50 metres and I take swimming lessons."
- 7c. Write the following compound proposition in words. [2 marks]
 $q \Rightarrow \neg q$

Music lessons in Piano (P), Violin (V) and Flute (F) are offered to students at a school.

The Venn diagram shows the number of students who learn each kind of instrument.



8a. Write down the total number of students in the school. [1 mark]

8b. Write down the number of students who [3 marks]

- (i) learn violin only;
- (ii) learn piano or flute or both;
- (iii) do not learn flute.

8c. Explain, in words, the meaning of the part of the diagram that represents the set $P \cap F'$. [2 marks]

100 students at IB College were asked whether they study Music (M), Chemistry (C), or Economics (E) with the following results.

- 10 study all three
- 15 study Music and Chemistry
- 17 study Music and Economics
- 12 study Chemistry and Economics
- 11 study Music **only**
- 6 study Chemistry **only**

9a. Draw a Venn diagram to represent the information above. [4 marks]

9b. Write down the number of students who study Music but not Economics. [1 mark]

9c. There are 22 Economics students **in total**. [4 marks]

- (i) Calculate the number of students who study Economics only.
- (ii) Find the number of students who study none of these three subjects.

9d. A student is chosen at random from the 100 that were asked above. [7 marks]

Find the probability that this student

- (i) studies Economics;
- (ii) studies Music and Chemistry but not Economics;
- (iii) does not study either Music or Economics;
- (iv) does not study Music given that the student does not study Economics.

Forty families were surveyed about the places they went to on the weekend. The places were the circus (C), the museum (M) and the park (P).

16 families went to the circus

22 families went to the museum

14 families went to the park

4 families went to all three places

7 families went to both the circus and the museum, but not the park

3 families went to both the circus and the park, but not the museum

1 family went to the park only

- 10a. Draw a Venn diagram to represent the given information using sets labelled C , M and P . Complete the diagram to include the number of families represented in each region. [4 marks]

- 10b. Find the number of families who [4 marks]

(i) went to the circus only;

(ii) went to the museum and the park but not the circus;

(iii) did not go to any of the three places on the weekend.

- 10c. A family is chosen at random from the group of 40 families. Find the probability that the family went to [8 marks]

(i) the circus;

(ii) two or more places;

(iii) the park or the circus, but not the museum;

(iv) the museum, given that they also went to the circus.

- 10d. Two families are chosen at random from the group of 40 families. [3 marks]

Find the probability that both families went to the circus.

- 11a. Complete the truth table below. [4 marks]

| p | q | $\neg p$ | $(p \wedge q)$ | $(\neg p \vee q)$ | $(p \wedge q) \Rightarrow (\neg p \vee q)$ |
|-----|-----|----------|----------------|-------------------|--|
| T | T | | | | |
| T | F | | | | |
| F | T | | | | |
| F | F | | | | |

- 11b. State whether the statement [1 mark]

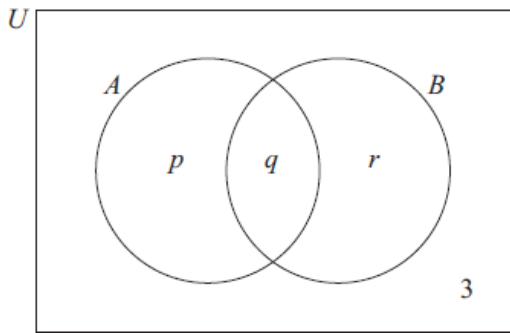
$(p \wedge q) \Rightarrow (\neg p \vee q)$ is a logical contradiction, a tautology or neither.

- 11c. Give a reason for your answer to part (b)(i). [1 mark]

A group of 33 people was asked about the passports they have. 21 have Australian passports, 15 have British passports and 3 have neither.

- 12a. Find the number that have both Australian and British passports. [2 marks]

- 12b. In the Venn diagram below, set A represents the people in the group with Australian passports and set B those with British passports. [2 marks]

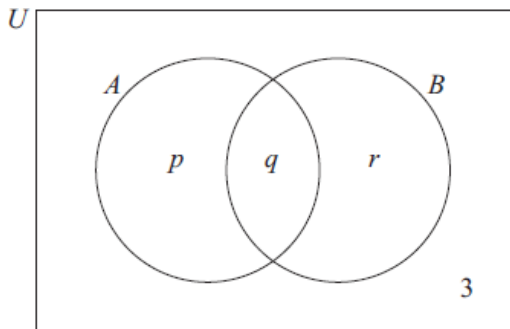


Write down the value of

- (i) q ;
- (ii) p and of r .

A group of 33 people was asked about the passports they have. 21 have Australian passports, 15 have British passports and 3 have neither.

- 12c. In the Venn diagram below, set A represents the people in the group with Australian passports and set B those with British passports. [2 marks]



Find
 $n(A \cup B)$.

A bag contains 7 red discs and 4 blue discs. Ju Shen chooses a disc at random from the bag and removes it. Ramón then chooses a disc from those left in the bag.

- 13a. Write down the probability that [3 marks]

- (i) Ju Shen chooses a red disc from the bag;
- (ii) Ramón chooses a blue disc from the bag, given that Ju Shen has chosen a red disc;
- (iii) Ju Shen chooses a red disc and Ramón chooses a blue disc from the bag.

- 13b. Find the probability that Ju Shen and Ramón choose different coloured discs from the bag. [3 marks]

The seniors from Gulf High School are required to participate in exactly one after-school sport. Data were gathered from a sample of 120 students regarding their choice of sport. The following data were recorded.

| | Sport | | | |
|--------|----------|--------|------------|-------|
| Gender | Football | Tennis | Basketball | Total |
| Male | 17 | 8 | 10 | 35 |
| Female | 31 | 17 | 37 | 85 |
| Total | 48 | 25 | 47 | 120 |

A

χ^2 test was carried out at the 5 % significance level to analyse the relationship between gender and choice of after-school sport.

- 14a. Write down the null hypothesis, H_0 , for this test. [1 mark]
- 14b. Find the expected value of female footballers. [2 marks]
- 14c. Write down the number of degrees of freedom. [1 mark]
- 14d. Write down the critical value of χ^2 , at the 5 % level of significance. [1 mark]
- 14e. Use your graphic display calculator to determine the χ^2_{calc} value. [2 marks]
- 14f. Determine whether H_0 should be accepted. Justify your answer. [2 marks]
- 14g. One student is chosen at random from the 120 students. [2 marks]
Find the probability that this student
(i) is male;
(ii) plays tennis.
- 14h. Two students are chosen at random from the 120 students. [5 marks]
Find the probability that
(i) both play football;
(ii) neither play basketball.

Leanne goes fishing at her favourite pond. The pond contains four different types of fish: bream, flathead, whiting and salmon. The fish are either undersized or normal. This information is shown in the table below.

| Size / Type of fish | Bream | Flathead | Whiting | Salmon | Total |
|---------------------|-------|----------|---------|--------|-------|
| Undersized | 3 | 12 | 18 | 9 | 42 |
| Normal | 0 | 11 | 24 | 13 | 48 |
| Total | 3 | 23 | 42 | 22 | |

- 15a. Write down the total number of fish in the pond. [1 mark]

15b. Leanne catches a fish.

[7 marks]

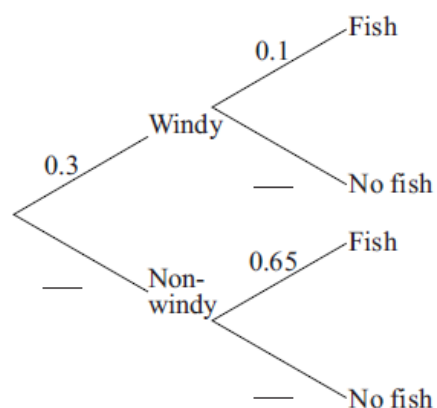
Find the probability that she

- (i) catches an undersized bream;
- (ii) catches either a flathead or an undersized fish or both;
- (iii) does not catch an undersized whiting;
- (iv) catches a whiting given that the fish was normal.

15c. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. The probability that it will be windy on a particular day is 0.3.

[3 marks]

Copy and complete the probability tree diagram below.



15d. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. The probability that it will be windy on a particular day is 0.3.

[2 marks]

Calculate the probability that it is windy and Leanne catches a fish on a particular day.

15e. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. The probability that it will be windy on a particular day is 0.3.

[3 marks]

Calculate the probability that Leanne catches a fish on a particular day.

15f. Use your answer to part (e) to calculate the probability that Leanne catches a fish on two consecutive days.

[2 marks]

15g. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. The probability that it will be windy on a particular day is 0.3.

[3 marks]

Given that Leanne catches a fish on a particular day, calculate the probability that the day was windy.

Pam has collected data from a group of 400 IB Diploma students about the Mathematics course they studied and the language in which they were examined (English, Spanish or French). The summary of her data is given below.

| | Mathematics HL | Mathematics SL | Mathematical Studies SL | Total |
|---------|----------------|----------------|-------------------------|-------|
| English | 50 | 70 | 80 | 200 |
| Spanish | 30 | 50 | 30 | 110 |
| French | 20 | 30 | 40 | 90 |
| Total | 100 | 150 | 150 | 400 |

- 16a. A student is chosen at random from the group. Find the probability that the student [8 marks]
- studied Mathematics HL;
 - was examined in French;
 - studied Mathematics HL and was examined in French;
 - did not study Mathematics SL and was not examined in English;
 - studied Mathematical Studies SL given that the student was examined in Spanish.

- 16b. Pam believes that the Mathematics course a student chooses is independent of the language in which the student is examined. [2 marks]
- Using your answers to parts (a) (i), (ii) and (iii) above, state whether there is any evidence for Pam's belief. Give a reason for your answer.

- 16c. Pam decides to test her belief using a Chi-squared test at the 5% level of significance. [3 marks]
- State the null hypothesis for this test.
 - Show that the expected number of Mathematical Studies SL students who took the examination in Spanish is 41.3, correct to 3 significant figures.

- 16d. Write down [4 marks]
- the Chi-squared calculated value;
 - the number of degrees of freedom;
 - the Chi-squared critical value.

- 16e. State, giving a reason, whether there is sufficient evidence at the 5% level of significance that Pam's belief is correct. [2 marks]

Given the set
 $A = \{x - 4 \leq x \leq 2, x \text{ is an integer}\}.$

- 17a. List the elements of the set A . [1 mark]

- 17b. A number is chosen at random from set A . Write down the probability that the number chosen is a negative integer. [2 marks]

- 17c. A number is chosen at random from set A . Write down the probability that the number chosen is a positive even integer. [1 mark]

- 17d. A number is chosen at random from set [2 marks]
 A. Write down the probability that the number chosen is an odd integer less than -1 .

Police in a town are investigating the theft of mobile phones one evening from three cafés, “Alan’s Diner”, “Sarah’s Snackbar” and “Pete’s Eats”.

They interviewed two suspects, Matthew and Anna, about that evening.

Matthew said:

“I visited Pete’s Eats and visited Alan’s Diner and I did not visit Sarah’s Snackbar.”

Let

p ,

q and

r be the statements:

p : I visited Alan’s Diner

q : I visited Sarah’s Snackbar

r : I visited Pete’s Eats

- 18a. Write down Matthew’s statement in symbolic logic form. [3 marks]

- 18b. What Anna said was lost by the police, but in symbolic form it was [3 marks]

$$(q \vee r) \Rightarrow \neg p$$

Write down, in words, what Anna said.

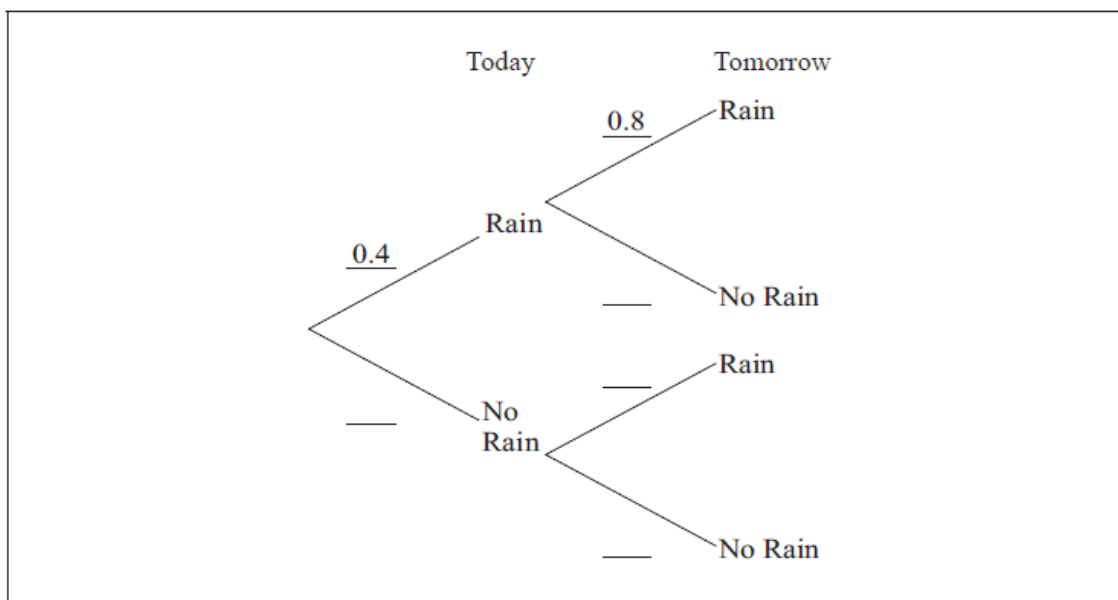
The probability that it rains today is

0.4. If it rains today, the probability that it will rain tomorrow is

0.8. If it does not rain today, the probability that it will rain tomorrow is

0.7.

- 19a. Complete the tree diagram below. [3 marks]



- 19b. Calculate the probability of rain tomorrow. [3 marks]

U is the set of all the **positive** integers less than or equal to 12.
 A ,
 B and
 C are subsets of
 U .

$$A = \{1, 2, 3, 4, 6, 12\}$$

$$B = \{\text{odd integers}\}$$

$$C = \{5, 6, 8\}$$

- 20a. Write down the number of elements in $A \cap C$.

[1 mark]

- 20b. List the elements of B .

[1 mark]

- 20c. Complete the following Venn diagram with **all** the elements of U .

[4 marks]

