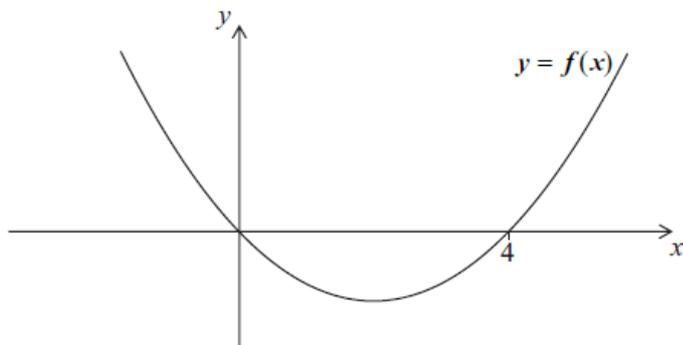


Topic 6 Part 2 [235 marks]

The following is the graph of the quadratic function $y = f(x)$.



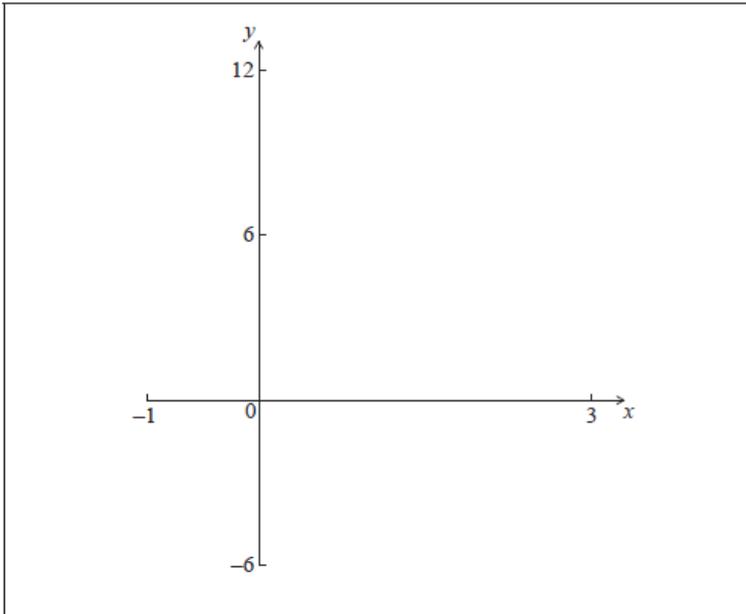
- 1a. Write down the solutions to the equation $f(x) = 0$. [2 marks]
- 1b. Write down the equation of the axis of symmetry of the graph of $f(x)$. [2 marks]
- 1c. The equation $f(x) = 12$ has two solutions. One of these solutions is $x = 6$. Use the symmetry of the graph to find the other solution. [1 mark]
- 1d. The minimum value for y is -4 . Write down the range of $f(x)$. [1 mark]

The
 x -coordinate of the minimum point of the quadratic function
 $f(x) = 2x^2 + kx + 4$ is
 $x = 1.25$.

- 2a. (i) Find the value of k . [4 marks]
- (ii) Calculate the y -coordinate of this minimum point.

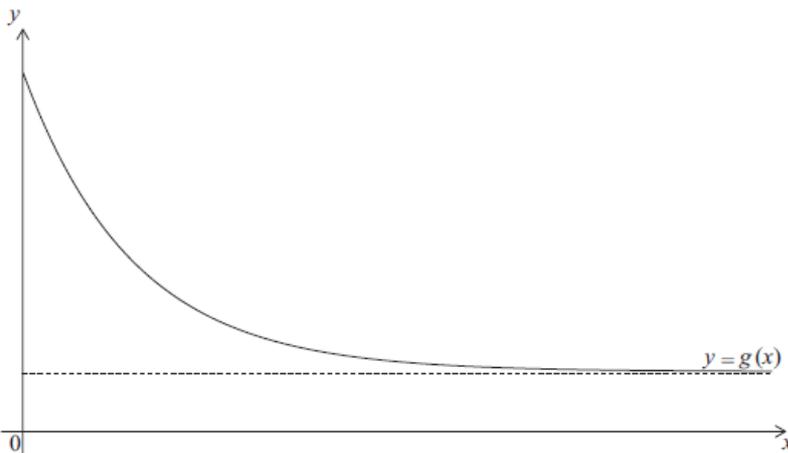
- 2b. Sketch the graph of $y = f(x)$ for the domain $-1 \leq x \leq 3$.

[2 marks]



The function $g(x)$ is defined as $g(x) = 16 + k(c^{-x})$ where $c > 0$.

The graph of the function g is drawn below on the domain $x \geq 0$.



The graph of g intersects the y-axis at $(0, 80)$.

- 3a. Find the value of k .

[2 marks]

- 3b. The graph passes through the point $(2, 48)$.
Find the value of c .

[2 marks]

- 3c. The graph passes through the point $(2, 48)$.
Write down the equation of the horizontal asymptote to the graph of $y = g(x)$.

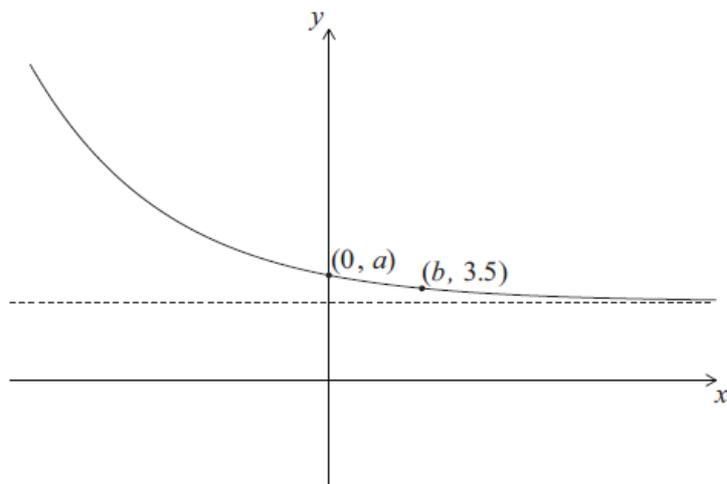
[2 marks]

Consider the function

$$f(x) = -\frac{1}{3}x^3 + \frac{5}{3}x^2 - x - 3.$$

- 4a. Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 6$ and $-10 \leq y \leq 10$ showing clearly the axes intercepts and local maximum and minimum points. Use a scale of 2 cm to represent 1 unit on the x -axis, and a scale of 1 cm to represent 1 unit on the y -axis. [4 marks]
- 4b. Find the value of $f(-1)$. [2 marks]
- 4c. Write down the coordinates of the y -intercept of the graph of $f(x)$. [1 mark]
- 4d. Find $f'(x)$. [3 marks]
- 4e. Show that $f'(-1) = -\frac{16}{3}$. [1 mark]
- 4f. Explain what $f'(-1)$ represents. [2 marks]
- 4g. Find the equation of the tangent to the graph of $f(x)$ at the point where x is -1 . [2 marks]
- 4h. Sketch the tangent to the graph of $f(x)$ at $x = -1$ on your diagram for (a). [2 marks]
- 4i. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x -coordinate of P is a , and the x -coordinate of Q is b , $b > a$. [2 marks]
- Write down the value of
- (i) a ;
- (ii) b .
- 4j. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x -coordinate of P is a , and the x -coordinate of Q is b , $b > a$. [1 mark]
- Describe the behaviour of $f(x)$ for $a < x < b$.

The diagram shows part of the graph of $y = 2^{-x} + 3$, and its horizontal asymptote. The graph passes through the points $(0, a)$ and $(b, 3.5)$.



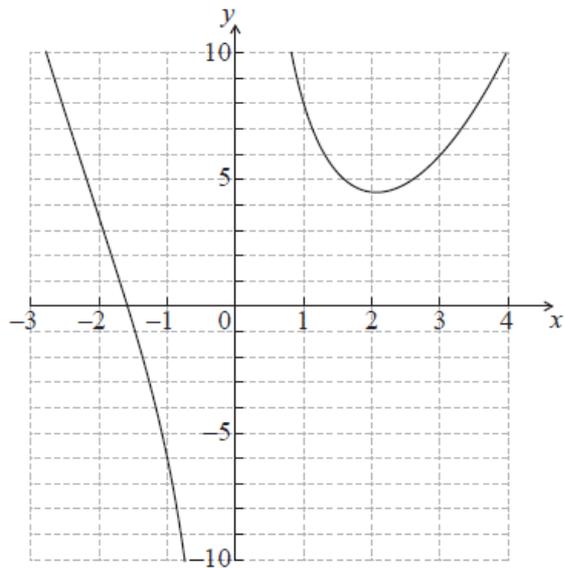
5a. Find the value of [4 marks]

- (i) a ;
- (ii) b .

5b. Write down the equation of the horizontal asymptote to this graph. [2 marks]

The diagram shows part of the graph of

$$f(x) = x^2 - 2x + \frac{9}{x}, \text{ where } x \neq 0 .$$



6a. Write down [5 marks]

- (i) the equation of the vertical asymptote to the graph of $y = f(x)$;
- (ii) the solution to the equation $f(x) = 0$;
- (iii) the coordinates of the local minimum point.

6b. Find $f'(x)$. [4 marks]

- 6c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3 - 2x^2 - 9}{x^2}$. [2 marks]
- 6d. Find the gradient of the tangent to $y = f(x)$ at the point $A(1, 8)$. [2 marks]
- 6e. The line, L , passes through the point A and is perpendicular to the tangent at A . Write down the gradient of L . [1 mark]
- 6f. The line, L , passes through the point A and is perpendicular to the tangent at A . Find the equation of L . Give your answer in the form $y = mx + c$. [3 marks]
- 6g. The line, L , passes through the point A and is perpendicular to the tangent at A . L also intersects the graph of $y = f(x)$ at points B and C . Write down the x -coordinate of B and of C . [2 marks]

The diagram shows a Ferris wheel that moves with constant speed and completes a rotation every 40 seconds. The wheel has a radius of 12 m and its lowest point is 2 m above the ground.

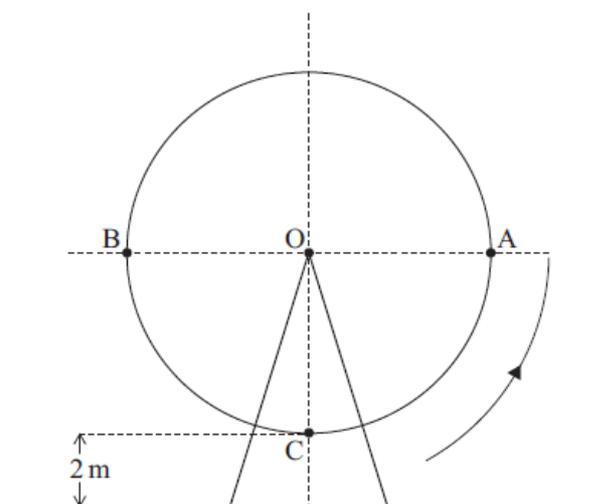
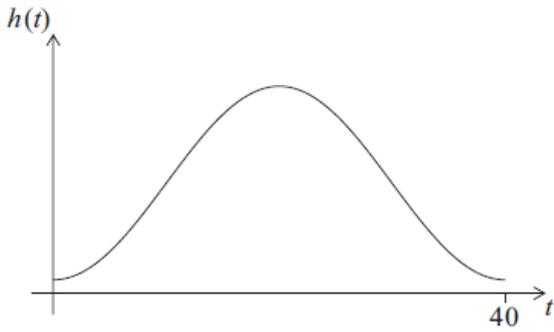


diagram not to scale

- 7a. Initially, a seat C is vertically below the centre of the wheel, O . It then rotates in an anticlockwise (counterclockwise) direction. [2 marks]
Write down
(i) the height of O above the ground;
(ii) the maximum height above the ground reached by C .
- 7b. In a revolution, C reaches points A and B , which are at the same height above the ground as the centre of the wheel. Write down the number of seconds taken for C to first reach A and then B . [2 marks]

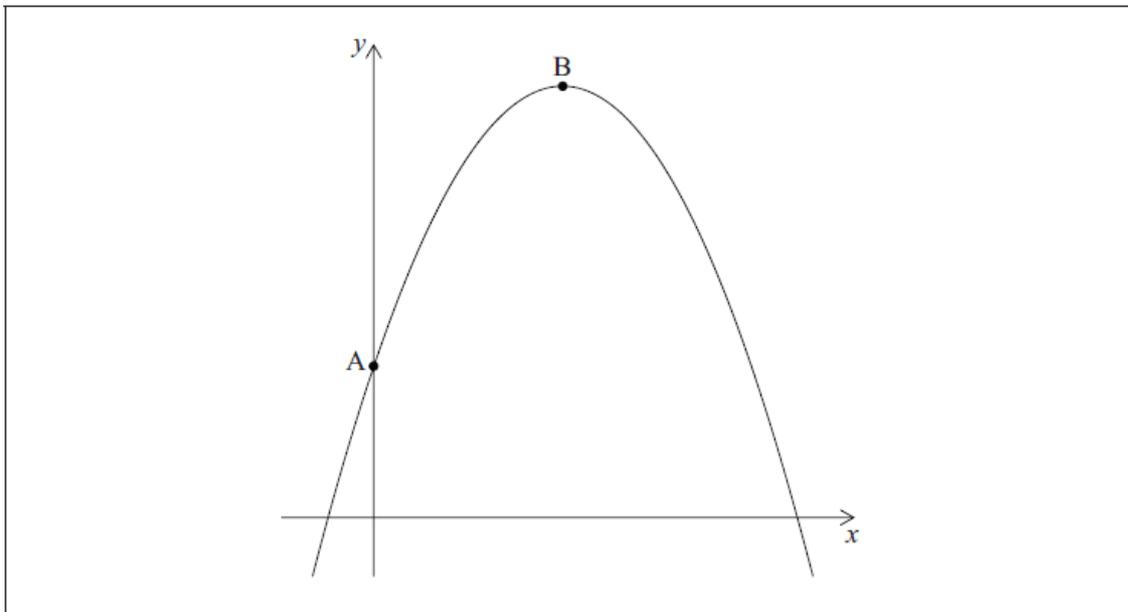
- 7c. The sketch below shows the graph of the function, $h(t)$, for the height above ground of C, where h is measured in metres and t is the time in seconds, $0 \leq t \leq 40$.

[4 marks]



Copy the sketch and show the results of part (a) and part (b) on your diagram. Label the points clearly with their coordinates.

The graph of the quadratic function $f(x) = 3 + 4x - x^2$ intersects the y -axis at point A and has its vertex at point B.



- 8a. Find the coordinates of B.

[3 marks]

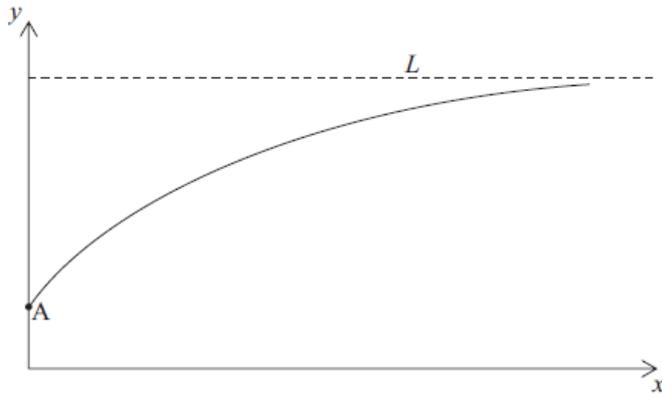
- 8b. Another point, C, which lies on the graph of $y = f(x)$ has the same y -coordinate as A.
- Plot and label C on the graph above.
 - Find the x -coordinate of C.

[3 marks]

Consider the function

$f(x) = 1.25 - a^{-x}$, where a is a positive constant and $x \geq 0$. The diagram shows a sketch of the graph of

f . The graph intersects the y -axis at point A and the line L is its horizontal asymptote.



9a. Find the y -coordinate of A . [2 marks]

9b. The point $(2, 1)$ lies on the graph of $y = f(x)$. Calculate the value of a . [2 marks]

9c. The point $(2, 1)$ lies on the graph of $y = f(x)$. Write down the equation of L . [2 marks]

Consider the function

$$f(x) = x^3 + \frac{48}{x}, x \neq 0.$$

10a. Calculate $f(2)$. [2 marks]

10b. Sketch the graph of the function $y = f(x)$ for $-5 \leq x \leq 5$ and $-200 \leq y \leq 200$. [4 marks]

10c. Find $f'(x)$. [3 marks]

10d. Find $f'(2)$. [2 marks]

10e. Write down the coordinates of the local maximum point on the graph of f . [2 marks]

10f. Find the range of f . [3 marks]

10g. Find the gradient of the tangent to the graph of f at $x = 1$. [2 marks]

10h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. [2 marks]

Find the x -coordinate of this point.

Shiyun bought a car in 1999. The value of the car V , in USD, is depreciating according to the exponential model

$$V = 25000 \times 1.5^{-0.2t}, t \geq 0$$

where t is the time, in years, that Shiyun has owned the car.

11a. Write down the value of the car when Shiyun bought it. [1 mark]

11b. Calculate the value of the car three years after Shiyun bought it. Give your answer correct to **two decimal places**. [2 marks]

11c. Calculate the time for the car to depreciate to half of its value since Shiyun bought it. [3 marks]

The function $f(x)$ is defined by $f(x) = 1.5x + 4 + \frac{6}{x}, x \neq 0$.

12a. Write down the equation of the vertical asymptote. [2 marks]

12b. Find $f'(x)$. [3 marks]

12c. Find the gradient of the graph of the function at $x = -1$. [2 marks]

12d. Using your answer to part (c), decide whether the function $f(x)$ is increasing or decreasing at $x = -1$. Justify your answer. [2 marks]

12e. Sketch the graph of $f(x)$ for $-10 \leq x \leq 10$ and $-20 \leq y \leq 20$. [4 marks]

- 12f. P_1 is the local maximum point and
 P_2 is the local minimum point on the graph of
 $f(x)$.

[4 marks]

Using your graphic display calculator, write down the coordinates of

- (i)
 P_1 ;
(ii)
 P_2 .

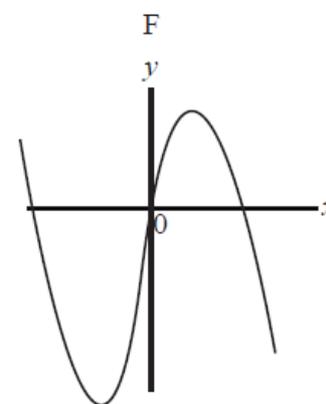
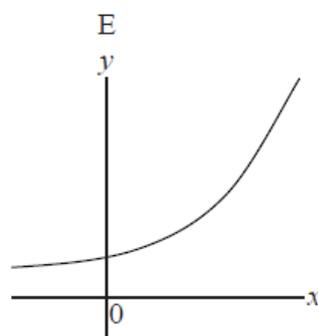
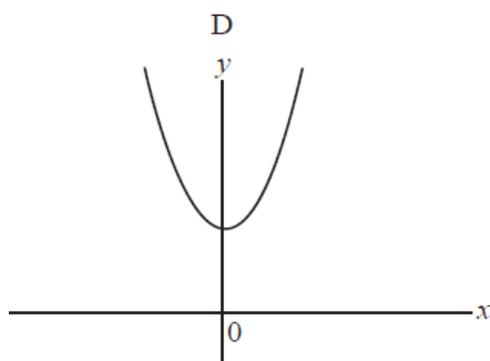
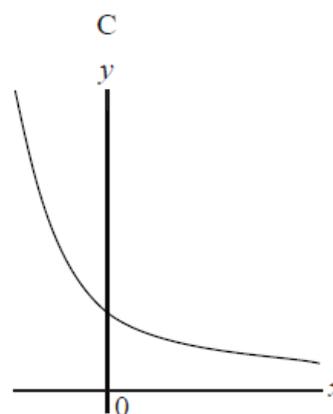
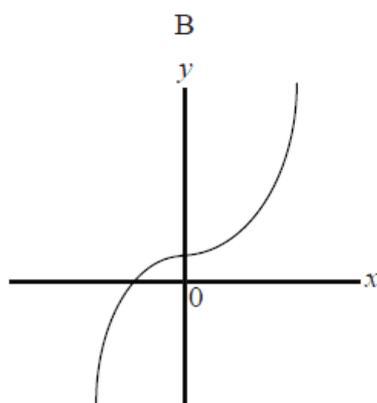
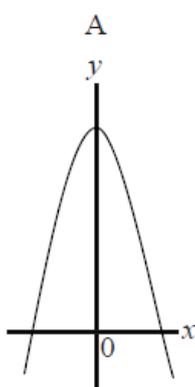
- 12g. Using your sketch from (e), determine the range of the function
 $f(x)$ for
 $-10 \leq x \leq 10$.

[3 marks]

13. The following curves are sketches of the graphs of the functions given below, but in a different order. Using your graphic display calculator, match the equations to the curves, writing your answers in the table below.

[6 marks]

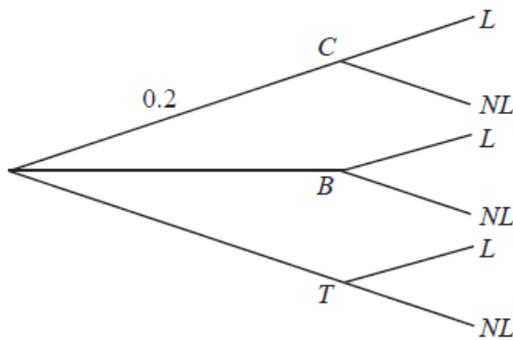
(the diagrams are not to scale)



	Function	Graph label
(i)	$y = x^3 + 1$	
(ii)	$y = x^2 + 3$	
(iii)	$y = 4 - x^2$	
(iv)	$y = 2^x + 1$	
(v)	$y = 3^{-x} + 1$	
(vi)	$y = 8x - 2x^2 - x^3$	

When Geraldine travels to work she can travel either by car (C), bus (B) or train (T). She travels by car on one day in five. She uses the bus 50 % of the time. The probabilities of her being late (L) when travelling by car, bus or train are 0.05, 0.12 and 0.08 respectively.

- 14a. Copy the tree diagram below and fill in all the probabilities, where NL represents not late, to represent this information. [5 marks]



- 14b. Find the probability that Geraldine travels by bus and is late. [1 mark]

- 14c. Find the probability that Geraldine is late. [3 marks]

- 14d. Find the probability that Geraldine travelled by train, given that she is late. [3 marks]

It is not necessary to use graph paper for this question.

- 14e. Sketch the curve of the function $f(x) = x^3 - 2x^2 + x - 3$ for values of x from -2 to 4 , giving the intercepts with both axes. [3 marks]

- 14f. On the same diagram, sketch the line $y = 7 - 2x$ and find the coordinates of the point of intersection of the line with the curve. [3 marks]

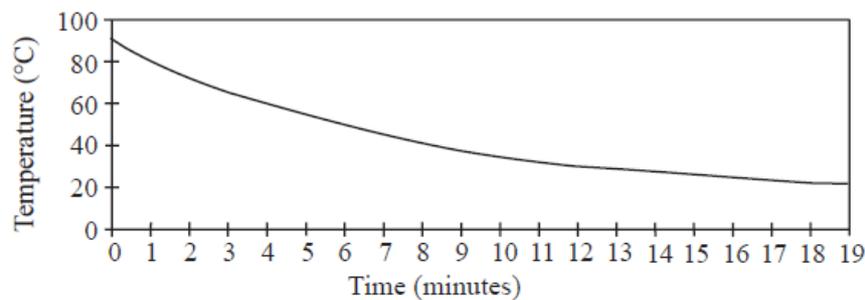
- 14g. Find the value of the gradient of the curve where $x = 1.7$. [2 marks]

The following graph shows the temperature in degrees Celsius of Robert's cup of coffee, t minutes after pouring it out. The equation of the cooling graph is

$$f(t) = 16 + 74 \times 2.8^{-0.2t} \text{ where}$$

$f(t)$ is the temperature and

t is the time in minutes after pouring the coffee out.



- 15a. Find the initial temperature of the coffee. [1 mark]

- 15b. Write down the equation of the horizontal asymptote. [1 mark]

- 15c. Find the room temperature. [1 mark]
- 15d. Find the temperature of the coffee after 10 minutes. [1 mark]
- 15e. Find the temperature of Robert's coffee after being heated in the microwave for 30 **seconds** after it has reached the temperature in part (d). [3 marks]
- 15f. Calculate the length of time it would take a similar cup of coffee, initially at 20°C, to be heated in the microwave to reach 100°C. [4 marks]

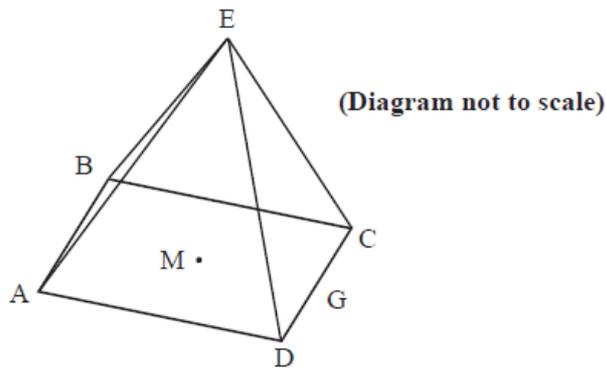
Robert, who lives in the UK, travels to Belgium. The exchange rate is 1.37 euros to one British Pound (GBP) with a commission of 3 GBP, which is subtracted before the exchange takes place. Robert gives the bank 120 GBP.

- 15g. Calculate **correct to 2 decimal places** the amount of euros he receives. [3 marks]
- 15h. He buys 1 kilogram of Belgian chocolates at 1.35 euros per 100 g. [3 marks]
Calculate the cost of his chocolates in GBP **correct to 2 decimal places**.

Mal is shopping for a school trip. He buys
50 tins of beans and
20 packets of cereal. The total cost is
260 Australian dollars (
AUD).

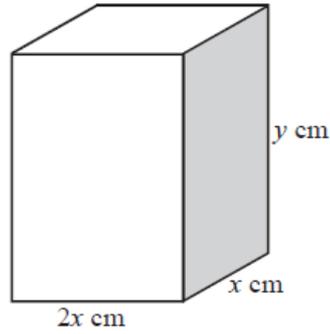
- 16a. Write down an equation showing this information, taking b to be the cost of one tin of beans and c to be the cost of one packet of cereal in AUD. [1 mark]
- 16b. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays 66 AUD. [1 mark]
Write down another equation to represent this information.
- 16c. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays 66 AUD. [2 marks]
Find the cost of one tin of beans.
- 16d. (i) Sketch the graphs of the two equations from parts (a) and (b). [4 marks]
(ii) Write down the coordinates of the point of intersection of the two graphs.

The triangular faces of a square based pyramid, ABCDE, are all inclined at 70° to the base. The edges of the base ABCD are all 10 cm and M is the centre. G is the mid-point of CD.



- 16e. Using the letters on the diagram draw a triangle showing the position of a 70° angle. [1 mark]
- 16f. Show that the height of the pyramid is 13.7 cm, to 3 significant figures. [2 marks]
- 16g. Calculate [4 marks]
- (i) the length of EG;
 - (ii) the size of angle DEC.
- 16h. Find the total surface area of the pyramid. [2 marks]
- 16i. Find the volume of the pyramid. [2 marks]
- 17a. Factorise $3x^2 + 13x - 10$. [2 marks]
- 17b. Solve the equation $3x^2 + 13x - 10 = 0$. [2 marks]
- 17c. Consider a function $f(x) = 3x^2 + 13x - 10$. [2 marks]
Find the equation of the axis of symmetry on the graph of this function.
- 17d. Consider a function $f(x) = 3x^2 + 13x - 10$. [2 marks]
Calculate the minimum value of this function.

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



17e. Show that [2 marks]
 $4x^2 + 6xy = 300$.

17f. Find an expression for [2 marks]
 y in terms of
 x .

17g. Hence show that the volume [2 marks]
 V of the box is given by
 $V = 100x - \frac{4}{3}x^3$.

17h. Find [2 marks]
 $\frac{dV}{dx}$.

17i. (i) Hence find the value of [5 marks]
 x and of
 y required to make the volume of the box a maximum.
(ii) Calculate the maximum volume.

Consider the function

$$f : x \mapsto \frac{kx}{2^x}.$$

18a. Given that [2 marks]
 $f(1) = 2$, show that
 $k = 4$.

18b. Write down the values of [2 marks]
 q and
 r for the following table.

x	-1	0	1	2	4	8
$f(x)$	-8	0	2	q	1	r

- 18c. As [4 marks]
 x increases from
 -1 , the graph of
 $y = f(x)$ reaches a maximum value and then decreases, behaving asymptotically.
Draw the graph of
 $y = f(x)$ for
 $-1 \leq x \leq 8$. Use a scale of
1 cm to represent 1 unit on both axes. The position of the maximum, (M) , the
 y -intercept and the asymptotic behaviour should be clearly shown.
- 18d. Using your graphic display calculator, find the coordinates of [2 marks]
 M , the maximum point on the graph of
 $y = f(x)$.
- 18e. Write down the equation of the horizontal asymptote to the graph of [2 marks]
 $y = f(x)$.
- 18f. (i) Draw and label the line [4 marks]
 $y = 1$ on your graph.
(ii) The equation
 $f(x) = 1$ has two solutions. One of the solutions is
 $x = 4$. Use your **graph** to find the other solution.

The cost per person, in euros, when
 x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x}$$

- 18g. Find [3 marks]
 $C'(x)$.
- 18h. Show that the cost per person is a minimum when [2 marks]
10 people are invited to the party.
- 18i. Calculate the minimum cost per person. [2 marks]