

Topic 1 Part 8 [231 marks]

$$T = \frac{(\tan(2z)+1)(2 \cos(z)-1)}{y^2-x^2}, \text{ where } x = 9, y = 41 \text{ and } z = 30^\circ.$$

1a. Calculate the **exact** value of T .

[2 marks]

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1b. Give your answer to T correct to

[2 marks]

- (i) two significant figures;
- (ii) three decimal places.

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1c. Pyotr estimates the value of T to be 0.002.

[2 marks]

Calculate the percentage error in Pyotr's estimate.

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Assume the Earth is a perfect sphere with radius 6371 km.

- 2a. Calculate the volume of the Earth in km^3 . Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[3 marks]

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- 2b. The volume of the Moon is $2.1958 \times 10^{10} \text{ km}^3$.
Calculate how many times greater in volume the Earth is compared to the Moon.
Give your answer correct to the nearest **integer**.

[3 marks]

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Pietro arrives in Singapore and, at the airport, changes 800 euros (EUR) to Singapore dollars (SGD).

The bank rates quoted at the airport for exchanging EUR with SGD are given in the following table. Also given are the rates for exchanging SGD with British pounds (GBP) and US dollars (USD). There is no commission charged on exchanges.

Bank Buys	Bank Sells
1 EUR = 1.55 SGD	1 EUR = 1.75 SGD
1 GBP = 1.92 SGD	1 GBP = 2.05 SGD
1 USD = 1.15 SGD	1 USD = 1.28 SGD

3a. Calculate the number of SGD Pietro receives.

[2 marks]

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3b. Pietro also has 100 GBP that he wishes to change to USD for a trip to Cambodia.

[4 marks]

To perform this transaction, the GBP must first be converted to SGD and then to USD.

Calculate the number of USD Pietro receives.

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The second term of an arithmetic sequence is 30. The fifth term is 90.

4a. Calculate

[3 marks]

- (i) the common difference of the sequence;
- (ii) the first term of the sequence.

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4b. The first, second and fifth terms of this arithmetic sequence are the first three terms of a geometric sequence. [3 marks]
Calculate the seventh term of the **geometric** sequence.

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Pierre invests 5000 euros in a fixed deposit that pays a nominal annual interest rate of 4.5%, compounded **monthly**, for seven years.

5a. Calculate the value of Pierre's investment at the end of this time. Give your answer correct to two decimal places. [3 marks]

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5b. Carla has 7000 dollars to invest in a fixed deposit which is compounded **annually**.

[3 marks]

She aims to double her money after 10 years.

Calculate the minimum annual interest rate needed for Carla to achieve her aim.

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An iron bar is heated. Its length, L , in millimetres can be modelled by a linear function, $L = mT + c$, where T is the temperature measured in degrees Celsius ($^{\circ}\text{C}$).

6a. At 150°C the length of the iron bar is 180 mm.

[1 mark]

Write down an equation that shows this information.

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6b. At 210°C the length of the iron bar is 181.5 mm.

[1 mark]

Write down an equation that shows this second piece of information.

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6c. At 210°C the length of the iron bar is 181.5 mm.

[4 marks]

Hence, find the length of the iron bar at 40°C.

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The distance

d from a point $P(x, y)$ to the point $A(1, -2)$ is given by

$$d = \sqrt{(x - 1)^2 + (y + 2)^2}$$

7a. Find the distance from $P(100, 200)$ to A . Give your answer correct to two decimal places.

[3 marks]

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7b. Write down your answer to **part (a)** correct to three significant figures.

[1 mark]

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7c. Write down your answer to **part (b)** in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[2 marks]

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Fabián stands on top of a building, T, which is on a horizontal street.

He observes a car, C, on the street, at an angle of depression of 30° . The base of the building is at B. The height of the building is 80 metres.

The following diagram indicates the positions of T, B and C.

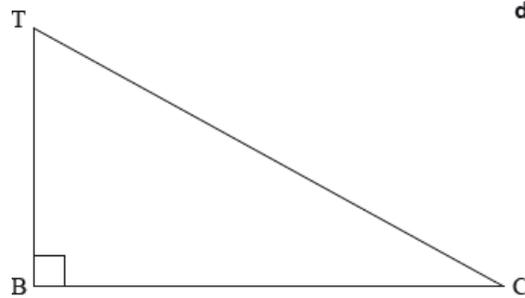


diagram not to scale

8a. Show, in the appropriate place on the diagram, **the values** of

[2 marks]

- (i) the height of the building;
- (ii) the angle of depression.

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8b. Find the distance, BC, from the base of the building to the car.

[2 marks]

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8c. Fabián estimates that the distance from the base of the building to the car is 150 metres. Calculate the percentage error of Fabián's estimate.

[2 marks]

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9a. Only one of the following four sequences is arithmetic and only one of them is geometric.

[2 marks]

$$a_n = 1, 2, 3, 5, \dots$$

$$b_n = 1, \frac{3}{2}, \frac{9}{4}, \frac{27}{8}, \dots$$

$$c_n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

$$d_n = 1, 0.95, 0.90, 0.85, \dots$$

State which sequence is

- (i) arithmetic;
- (ii) geometric.

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9b. For **another** geometric sequence $e_n = -6, -3, -\frac{3}{2}, -\frac{3}{4}, \dots$

[1 mark]

write down the common ratio;

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9c. For **another** geometric sequence $e_n = -6, -3, -\frac{3}{2}, -\frac{3}{4}, \dots$

[3 marks]

find the **exact** value of the tenth term. Give your answer as a fraction.

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Minta deposits 1000 euros in a bank account. The bank pays a nominal annual interest rate of 5%, **compounded quarterly**.

10a. Find the amount of money that Minta will have in the bank after 3 years. Give your answer correct to two decimal places.

[3 marks]

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10b. Minta will withdraw the money from her bank account when the interest earned is 300 euros.

[3 marks]

Find the time, in years, until Minta withdraws the money from her bank account.

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A building company has many rectangular construction sites, of varying widths, along a road.

The area, A , of each site is given by the function

$$A(x) = x(200 - x)$$

where x is the **width** of the site in metres and $20 \leq x \leq 180$.

11a. Site S has a width of 20 m. Write down the area of S.

[1 mark]

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11b. Site T has the same area as site S, but a different width. Find the width of T.

[2 marks]

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11c. When the width of the construction site is b metres, the site has a maximum area.

[2 marks]

- (i) Write down the value of b .
- (ii) Write down the maximum area.

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11d. The range of $A(x)$ is $m \leq A(x) \leq n$.

[1 mark]

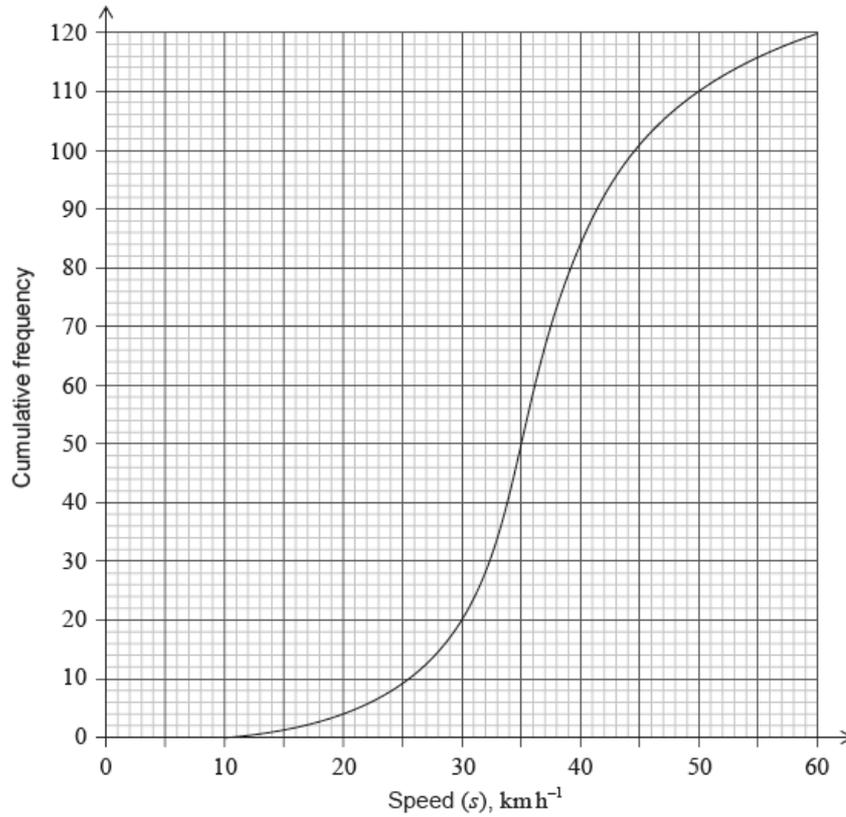
Hence write down the value of m and of n .

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The cumulative frequency graph shows the speed, s , in km h^{-1} , of 120 vehicles passing a hospital gate.



12a. Estimate the minimum possible speed of one of these vehicles passing the hospital gate. [1 mark]

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12b. Find the median speed of the vehicles. [2 marks]

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12c. Write down the 75th percentile.

[1 mark]

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12d. Calculate the interquartile range.

[2 marks]

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12e. The speed limit past the hospital gate is 50 km h^{-1} .

[2 marks]

Find the number of these vehicles that exceed the speed limit.

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12f. The table shows the speeds of these vehicles travelling past the hospital gate.

[2 marks]

Speed of Vehicles	Number of Vehicles
$0 < s \leq 10$	0
$10 < s \leq 20$	p
$20 < s \leq 30$	16
$30 < s \leq 40$	64
$40 < s \leq 50$	26
$50 < s \leq 60$	q

Find the value of p and of q .

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12g. The table shows the speeds of these vehicles travelling past the hospital gate.

[2 marks]

Speed of Vehicles	Number of Vehicles
$0 < s \leq 10$	0
$10 < s \leq 20$	p
$20 < s \leq 30$	16
$30 < s \leq 40$	64
$40 < s \leq 50$	26
$50 < s \leq 60$	q

- (i) Write down the modal class.
- (ii) Write down the mid-interval value for this class.

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12h. The table shows the speeds of these vehicles travelling past the hospital gate.

[3 marks]

Speed of Vehicles	Number of Vehicles
$0 < s \leq 10$	0
$10 < s \leq 20$	p
$20 < s \leq 30$	16
$30 < s \leq 40$	64
$40 < s \leq 50$	26
$50 < s \leq 60$	q

Use your graphic display calculator to calculate an estimate of

- (i) the mean speed of these vehicles;
- (ii) the standard deviation.

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12i. It is proposed that the speed limit past the hospital gate is reduced to 40 km h^{-1} from the current 50 km h^{-1} . [2 marks]

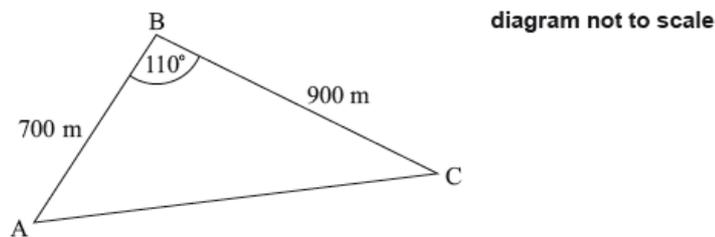
Find the percentage of these vehicles passing the hospital gate that **do not** exceed the current speed limit but **would** exceed the new speed limit.

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A boat race takes place around a triangular course, ABC , with $AB = 700$ m, $BC = 900$ m and angle $ABC = 110^\circ$. The race starts and finishes at point A .



13a. Calculate the total length of the course. [4 marks]

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13b. It is estimated that the fastest boat in the race can travel at an average speed of 1.5 m s^{-1} . [3 marks]
Calculate an estimate of the winning time of the race. Give your answer to the nearest minute.

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13c. It is estimated that the fastest boat in the race can travel at an average speed of 1.5 m s^{-1} . [3 marks]
Find the size of angle ACB .

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- 13d. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and [3 marks]
the shortest distance from B to AC must be greater than 375 metres.

Calculate the area that must be kept clear of boats.

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- 13e. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and [3 marks]
the shortest distance from B to AC must be greater than 375 metres.

Determine, giving a reason, whether the course complies with the safety regulations.

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- 13f. The race is filmed from a helicopter, H, which is flying vertically above point A. [2 marks]

The angle of elevation of H from B is 15° .

Calculate the vertical height, AH, of the helicopter above A.

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13g. The race is filmed from a helicopter, H , which is flying vertically above point A .

[3 marks]

The angle of elevation of H from B is 15° .

Calculate the maximum possible distance from the helicopter to a boat on the course.

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Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

14a. Write down $f'(x)$.

[3 marks]

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14b. The graph of $y = f(x)$ has a local minimum point at $x = 4$.

[2 marks]

Show that $k = 3$.

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14c. The graph of $y = f(x)$ has a local minimum point at $x = 4$.

[2 marks]

Find $f(2)$.

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14d. The graph of $y = f(x)$ has a local minimum point at $x = 4$.

[2 marks]

Find $f'(2)$.

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14e. The graph of $y = f(x)$ has a local minimum point at $x = 4$.

[3 marks]

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

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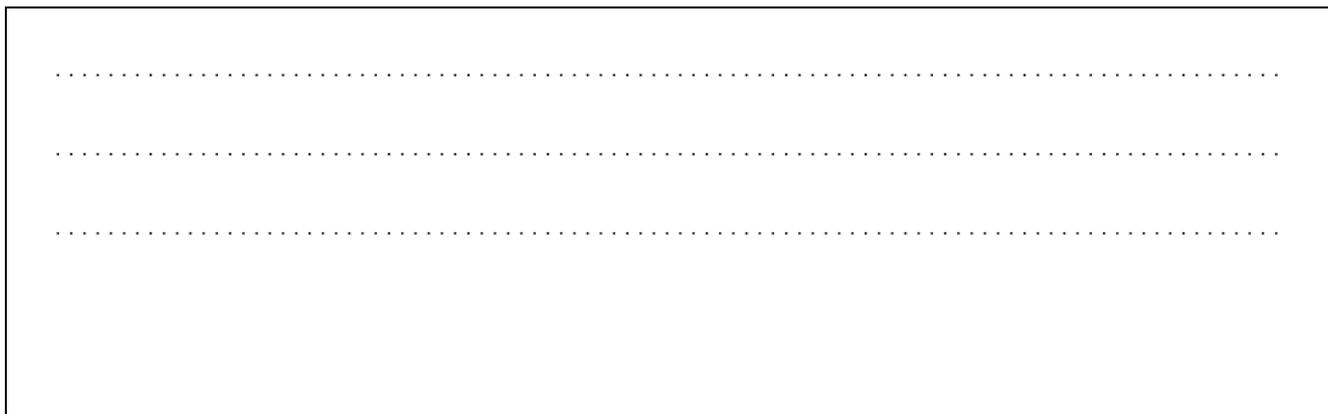
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14f. The graph of $y = f(x)$ has a local minimum point at $x = 4$.

[4 marks]

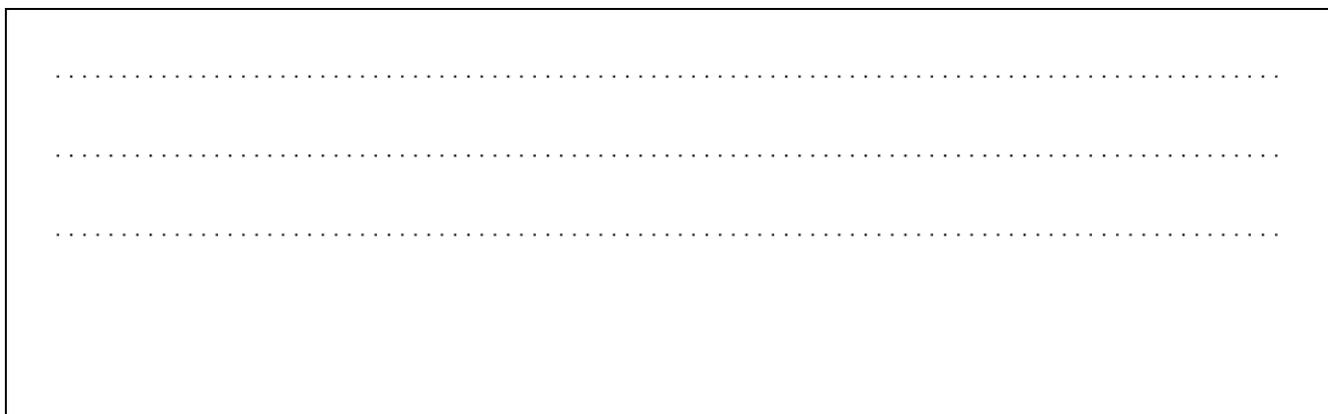
Sketch the graph of $y = f(x)$, for $-5 \leq x \leq 10$ and $-10 \leq y \leq 100$.



14g. The graph of $y = f(x)$ has a local minimum point at $x = 4$.

[2 marks]

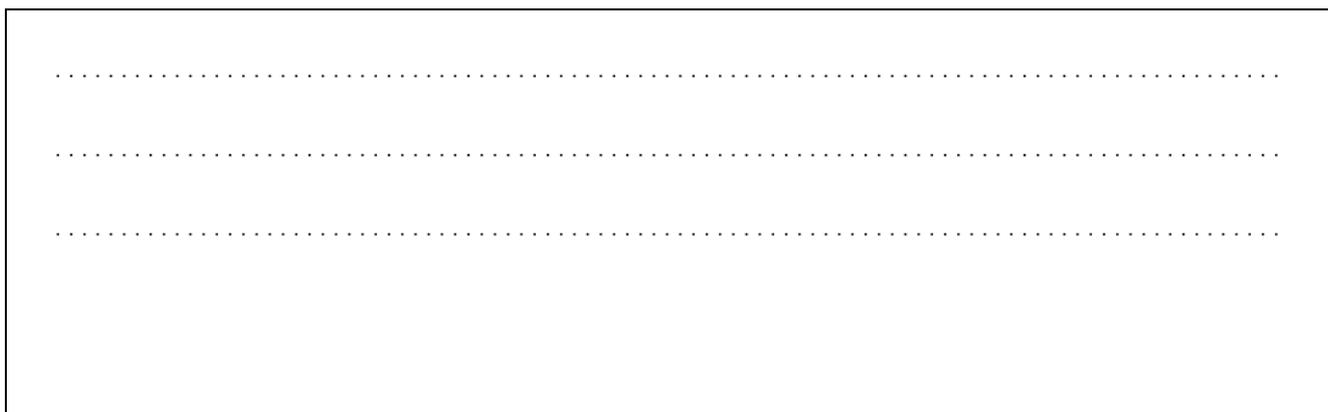
Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x -axis.



14h. The graph of $y = f(x)$ has a local minimum point at $x = 4$.

[2 marks]

State the values of x for which $f(x)$ is decreasing.

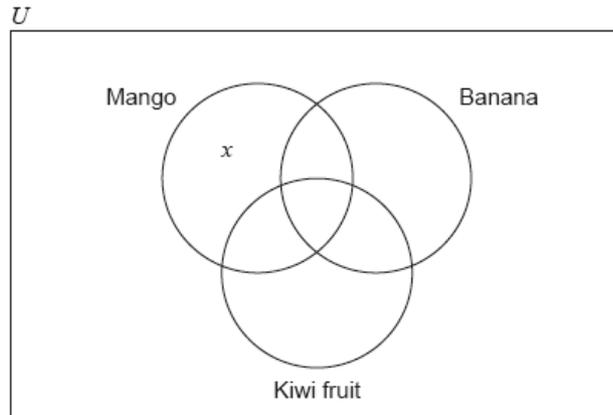


A group of 100 customers in a restaurant are asked which fruits they like from a choice of mangoes, bananas and kiwi fruits. The results are as follows.

- 15 like all three fruits
- 22 like mangoes and bananas
- 33 like mangoes and kiwi fruits
- 27 like bananas and kiwi fruits
- 8 like none of these three fruits
- x like **only** mangoes

15a. **Copy** the following Venn diagram and correctly insert all values from the above information.

[3 marks]



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15b. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. [2 marks]
 This number is half of the number of customers that like **only** bananas.

Complete your Venn diagram from part (a) with this additional information **in terms of x** .

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- 15c. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. [2 marks]
This number is half of the number of customers that like **only** bananas.

Find the value of x .

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- 15d. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. [2 marks]
This number is half of the number of customers that like **only** bananas.

Write down the number of customers who like

- (i) mangoes;
- (ii) mangoes or bananas.

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- 15e. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. [4 marks]
This number is half of the number of customers that like **only** bananas.

A customer is chosen at random from the 100 customers. Find the probability that this customer

- (i) likes none of the three fruits;
- (ii) likes only two of the fruits;
- (iii) likes all three fruits given that the customer likes mangoes and bananas.

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- 15f. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. [3 marks]
This number is half of the number of customers that like **only** bananas.

Two customers are chosen at random from the 100 customers. Find the probability that the two customers like none of the three fruits.

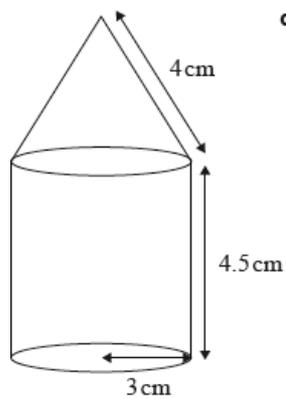
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The following diagram shows a perfume bottle made up of a cylinder and a cone.

diagram not to scale



The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

- 16a. (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures. [6 marks]
(ii) Calculate the volume of the perfume bottle.

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- 16b. The bottle contains 125 cm^3 of perfume. The bottle is **not** full and all of the perfume is in the cylinder part.

[2 marks]

Find the height of the perfume in the bottle.

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- 16c. Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle. [4 marks]

Find the **total** surface area of the perfume bottle.

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- 16d. Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of 7 cm^2 . [4 marks]

Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places.**

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- 16e. Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is 1 EUR = 13.03 ZAR. [2 marks]

Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places.**

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The sum of the first n terms of an arithmetic sequence is given by $S_n = 6n + n^2$.

- 17a. Write down the value of

[2 marks]

- (i) S_1 ;
(ii) S_2 .

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- 17b. The n^{th} term of the arithmetic sequence is given by u_n .

[1 mark]

Show that $u_2 = 9$.

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17c. The n^{th} term of the arithmetic sequence is given by u_n .

[2 marks]

Find the common difference of the sequence.

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17d. The n^{th} term of the arithmetic sequence is given by u_n .

[2 marks]

Find u_{10} .

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17e. The n^{th} term of the arithmetic sequence is given by u_n .

[3 marks]

Find the lowest value of n for which u_n is greater than 1000.

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17f. The n^{th} term of the arithmetic sequence is given by u_n .

[2 marks]

There is a value of n for which

$$u_1 + u_2 + \dots + u_n = 1512.$$

Find the value of n .

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Jenny has a circular cylinder with a lid. The cylinder has height 39 cm and diameter 65 mm.

18a. Calculate the volume of the cylinder in cm^3 . Give your answer correct to two decimal places.

[3 marks]

18b. The cylinder is used for storing tennis balls. Each ball has a radius of 3.25 cm.

[1 mark]

Calculate how many balls Jenny can fit in the cylinder if it is filled to the top.

18c. (i) Jenny fills the cylinder with the number of balls found in part (b) and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls.

[4 marks]

(ii) Convert your answer to (c) (i) into cubic metres.

An old tower (BT) leans at 10° away from the vertical (represented by line TG).

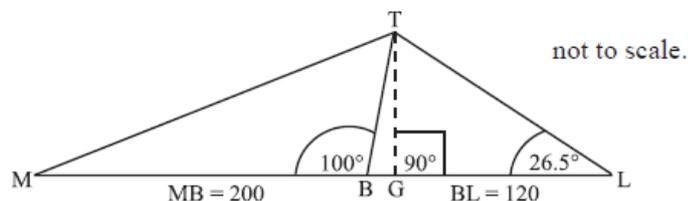
The base of the tower is at B so that

$$\hat{M}BT = 100^\circ.$$

Leonardo stands at L on flat ground 120 m away from B in the direction of the lean.

He measures the angle between the ground and the top of the tower T to be

$$\hat{B}LT = 26.5^\circ.$$



18d. (i) Find the value of angle $\hat{B}TL$.

[5 marks]

(ii) Use triangle BTL to calculate the sloping distance BT from the base, B to the top, T of the tower.

18e. Calculate the vertical height TG of the top of the tower.

[2 marks]

18f. Leonardo now walks to point M, a distance 200 m from B on the opposite side of the tower. Calculate the distance from M to the top of the tower at T.

[3 marks]

The natural numbers: 1, 2, 3, 4, 5... form an arithmetic sequence.

19a. State the values of u_1 and d for this sequence. [2 marks]

19b. Use an appropriate formula to show that the sum of the natural numbers from 1 to n is given by $\frac{1}{2}n(n+1)$. [2 marks]

19c. Calculate the sum of the natural numbers from 1 to 200. [2 marks]

A geometric progression G_1 has 1 as its first term and 3 as its common ratio.

19d. The sum of the first n terms of G_1 is 29 524. Find n . [3 marks]

19e. A second geometric progression G_2 has the form $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$ [1 mark]

19f. Calculate the sum of the first 10 terms of G_2 . [2 marks]

19g. Explain why the sum of the first 1000 terms of G_2 will give the same answer as the sum of the first 10 terms, when corrected to three significant figures. [1 mark]

19h. Using your results from parts (a) to (c), or otherwise, calculate the sum of the first 10 terms of the sequence $2, 3\frac{1}{3}, 9\frac{1}{9}, 27\frac{1}{27}, \dots$ [3 marks]

Give your answer **correct to one decimal place**.

The lengths (l) in centimetres of 100 copper pipes at a local building supplier were measured. The results are listed in the table below.

Length l (cm)	Frequency
17.5	12
32.5	26
47.5	32
62.5	21
77.5	9

20a. Write down the mode. [1 mark]

20b. Using your graphic display calculator, write down the value of
(i) the mean;
(ii) the standard deviation;
(iii) the median. [4 marks]

20c. Find the interquartile range. [2 marks]

- 20d. Draw a box and whisker diagram for this data, on graph paper, using a scale of 1 cm to represent 5 cm. [4 marks]

- 20e. Sam estimated the value of the mean of the measured lengths to be 43 cm. [2 marks]

Find the percentage error of Sam's estimated mean.

The Brahma chicken produces eggs with weights in grams that are normally distributed about a mean of 55 g with a standard deviation of 7 g. The eggs are classified as small, medium, large or extra large according to their weight, as shown in the table below.

Size	Weight (g)
Small	Weight < 53
Medium	$53 \leq \text{Weight} < 63$
Large	$63 \leq \text{Weight} < 73$
Extra Large	Weight ≥ 73

- 21a. Sketch a diagram of the distribution of the weight of Brahma chicken eggs. On your diagram, show clearly the boundaries for the classification of the eggs. [3 marks]

- 21b. An egg is chosen at random. Find the probability that the egg is [4 marks]
 (i) medium;
 (ii) extra large.

- 21c. There is a probability of 0.3 that a randomly chosen egg weighs more than w grams. [2 marks]

Find w .

- 21d. The probability that a Brahma chicken produces a large size egg is 0.121. Frank's Brahma chickens produce 2000 eggs each month. [2 marks]

Calculate an estimate of the number of large size eggs produced by Frank's chickens each month.

- 21e. The selling price, in US dollars (USD), of each size is shown in the table below. [3 marks]

Size	Selling price (USD)
Small	0.30
Medium	0.50
Large	0.65
Extra Large	0.80

The probability that a Brahma chicken produces a small size egg is 0.388.

Estimate the monthly income, in USD, earned by selling the 2000 eggs. Give your answer correct to two decimal places.