Topic 7 Part 4 *[73 marks]*

1a. *[3 marks]*

Markscheme

(i)

$f_1'(x) = 1$ ***(A1)***

(ii)

$f_2'(x) = -2x$ ***(A1)(A1)***

(A1) *for correct differentiation of each term.* ***(C3)***

[3 marks]

Examiners report

Most candidates were able to differentiate correctly, but only a third were able to calculate the value of x for which the gradients of the graphs were the same and a similar number did not attempt to. Some found the x-coordinate of the point of intersection.

1b. *[2 marks]*

Markscheme

$1 = -2x$ ***(M1)***

$x = -\frac{1}{2}$ ***(A1)*****(ft)** ***(C2)***

[2 marks]

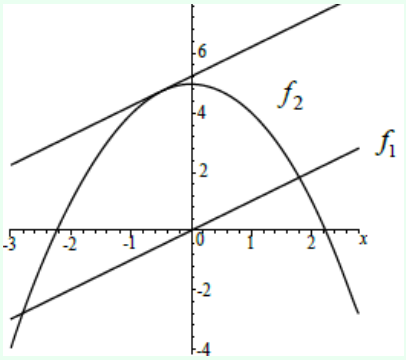
Examiners report

Most candidates were able to differentiate correctly, but only a third were able to calculate the value of x for which the gradients of the graphs were the same and a similar number did not attempt to. Some found the x-coordinate of the point of intersection.

1c. *[1 mark]*

Markscheme

(A1) is for the tangent drawn at $x = \frac{1}{2}$ *and reasonably parallel to the line* f_1 *as shown.*



(A1) ***(C1)***

[1 mark]

Examiners report

c) Very few candidates were able to draw the tangent correctly. Some tangents were drawn horizontally and some at the point of intersection. The line could have been drawn without any knowledge of calculus so the indication here was that many of the candidates misunderstood the question.

2a. *[1 mark]*

Markscheme

B, F ***(C1)***

Examiners report

[N/A]

2b. *[1 mark]*

Markscheme

H ***(C1)***

Examiners report

[N/A]

2c. *[1 mark]*

Markscheme

F ***(C1)***

Examiners report

[N/A]

2d. *[1 mark]*

Markscheme

A, E *(C1)*

Examiners report

[N/A]

2e. *[2 marks]*

Markscheme

C *(C2)*

Examiners report

[N/A]

3a. *[1 mark]*

Markscheme

$2x$ *(A1)* *(C1)*

Examiners report

[N/A]

3b. *[2 marks]*

Markscheme

2×3 *(M1)*

$= 6$ *(A1)* *(C2)*

Examiners report

[N/A]

3c. *[3 marks]*

Markscheme

$m(\text{perp}) = -\frac{1}{6}$ ***(A1)*****(ft)**

Note: Follow through from their answer to part (b).

Equation
$(y-9) = -\frac{1}{6}(x-3)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in any formula for equation of a line.

$y = -\frac{1}{6}x + 9\frac{1}{2}$ ***(A1)*****(ft)** ***(C3)***

Note: Follow through from correct substitution of their gradient of the normal.
Note: There are no extra marks awarded for rearranging the equation to the form $y = mx + c$.

Examiners report

[N/A]

4a. *[2 marks]*

Markscheme

$f'(x) = 15x^2 - 15x^4$ ***(A1)(A1)*** ***(C2)***

Note: Award a maximum of ***(A1)(A0)*** if extra terms seen.

Examiners report

[N/A]

4b. *[2 marks]*

Markscheme

$f'(1) = 0$ ***(M1)***

Note: Award ***(M1)*** for
$f'(x) = 0$.

$y = 3$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their answer to part (a).

Examiners report

[N/A]

4c. *[2 marks]*

Markscheme

$(-1.38, 3)$
$(-1.38481\ldots, 3)$ ***(A1)*****(ft)*****(A1)*****(ft)** ***(C2)***

Note: Follow through from their answer to parts (a) and (b).

Note: Accept
$x = -1.38$,
$y = 3$ (
$x = -1.38481\ldots$,
$y = 3$) .

Examiners report

[N/A]

5a. *[2 marks]*

Markscheme

$P(x) = I(x) - C(x)$ ***(M1)***
$= -x^2 + 150x - 2600$ ***(A1)*** ***(C2)***

Examiners report

[N/A]

5b. *[2 marks]*

Markscheme

$-2x + 150 = 0$ ***(M1)***

Note: Award ***(M1)*** for setting
$P'(x) = 0$.

OR

Award ***(M1)*** for sketch of
$P(x)$ and maximum point identified. ***(M1)***
$x = 75$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their answer to part (a).

Examiners report

[N/A]

5c. *[2 marks]*

Markscheme

$\frac{7875}{75}$ ***(M1)***

Note: Award ***(M1)*** for
7875 seen.

$= 105$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their answer to part (b).

Examiners report

[N/A]

6a. *[1 mark]*

Markscheme

equation of asymptote is $x = 0$ ***(A1)***

(Must be an equation.)

[1 mark]

Examiners report

(i) An attempt at part (a) was seen only rarely. If there was an attempt, it was often not a meaningful equation. If an equation was seen, sometimes it was for y, not x.

6b. *[2 marks]*

Markscheme

$f'(x) = 2x + 3x^{-2}$ *(or equivalent)* ***(A1)*** *for each term* ***(A1)(A1)***

[2 marks]

Examiners report

The derivative seemed manageable for many, though with the expected mis-handling of the negative power quite often. Parts (c) and (d) proved problematical. Marking of (d) was lenient and it was reaffirmed that testing of the concept in (d) will be done in a more straightforward context in future, when done at all.

6c. *[2 marks]*

Markscheme

stationary point (–1.14, 3.93) ***(G1)(G1)*(ft)**

(-1,4) or similar error is awarded ***(G0)(G1)*(ft)**. *Here and also as follow through in part (d) accept exact values*
$-\left(\frac{3}{2}\right)^{\frac{1}{3}}$ *for the x coordinate and*
$3\left(\frac{3}{2}\right)^{\frac{2}{3}}$ *for the y coordinate.*

OR
$2x + \frac{3}{x^2} = 0$ *or equivalent*

Correct coordinates as above ***(M1)***

Follow through from candidate's
$f'(x)$. ***(A1)*(ft)**

[2 marks]

Examiners report

The derivative seemed manageable for many, though with the expected mis-handling of the negative power quite often. Parts (c) and (d) proved problematical. Marking of (d) was lenient and it was reaffirmed that testing of the concept in (d) will be done in a more straightforward context in future, when done at all.

6d. *[3 marks]*

Markscheme

In all alternative answers for (d), follow through from candidate's x coordinate in part (c).

Alternative answers include:

$-1.14 \leq x < 0, \quad 0 < x < 5$ ***(A1)(A1)*(ft)*(A1)***

OR [–1.14,0), (0,5)

Accept alternative bracket notation for open interval] [. (Union of these sets is not correct, award ***(A2)*** *if all else is right in this case.)*

OR
$-1.14 \leqslant x < 5, x \neq 0$

In all versions 0 ***must*** *be excluded* ***(A1)****. -1.14 must be the left bound . 5 must be the right bound* ***(A1)****. For*
$x \geqslant -1.14$ *or*
$x > -1.14$ *alone, award* ***(A1)****. For*
$-1.4 \leqslant x < 0$ *together with*
$x > 0$ *award* ***(A2)****.*

[3 marks]

Examiners report

The derivative seemed manageable for many, though with the expected mis-handling of the negative power quite often. Parts (c) and (d) proved problematical. Marking of (d) was lenient and it was reaffirmed that testing of the concept in (d) will be done in a more straightforward context in future, when done at all.

6e. *[1 mark]*

Markscheme

a = 5.30 *(3sf)* *(Allow (5.30, 0) but 5.3 receives an* ***(AP)****.)* ***(A1)***

[1 mark]

Examiners report

(ii) Many candidates failed to recognise that extensive use of the GDC was intended for this question. An indicator of this was the choice of awkward coefficients. It is recognised that the context confused some candidates and that the horizontal shift was a bit disturbing for some.

Nevertheless, a lot of candidates could have earned more marks here if they had persevered. Many gave up on the graph, and elementary marks for scale and labels were lost unnecessarily.

As this was the first time for the unit penalty, we were lenient about the units left off the labels but this is likely to change in the future.

6f. *[2 marks]*

Markscheme

$\frac{dy}{dx} = -0.042x + 1.245$ ***(A1)*** *for each term.* ***(A1)(A1)***

[2 marks]

Examiners report

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As this was the first time for the unit penalty, we were lenient about the units left off the labels but this is likely to change in the future.

6g. *[4 marks]*

Markscheme

Unit penalty ***(UP)*** *is applicable where indicated in the left hand column.*

(i) Maximum value when
$f'(x) = 0$,
$-0.042x + 1.245 = 0$, ***(M1)***

(M1) *is for either of the above but at least one must be seen.*

(x = 29.6.)

Football has travelled 29.6 – 5.30 = 24.3 m (3sf) horizontally. ***(A1)*****(ft)**

For answer of 24.3 m with no working or for correct subtraction of 5.3 from candidate's x-coordinate at the maximum (if not 29.6), award ***(A1)****(d).*

(UP) (ii) Maximum vertical height, f (29.6) = 12.4 m ***(M1)(A1)*****(ft)*****(G2)***

(M1) *is for substitution into f of a value seen in part (c)(i). f(24.3) with or without evaluation is awarded* ***(M1)(A0)****. For any other value without working, award* ***(G0)****. If lines are seen on the graph in part (d) award* ***(M1)*** *and then* ***(A1)*** *for candidate's value* ± 0.5 *(3sf not required.)*

[4 marks]

Examiners report

(ii) Many candidates failed to recognise that extensive use of the GDC was intended for this question. An indicator of this was the choice of awkward coefficients. It is recognised that the context confused some candidates and that the horizontal shift was a bit disturbing for some.

Nevertheless, a lot of candidates could have earned more marks here if they had persevered. Many gave up on the graph, and elementary marks for scale and labels were lost unnecessarily.

As this was the first time for the unit penalty, we were lenient about the units left off the labels but this is likely to change in the future.

6h. *[4 marks]*

Markscheme

(not to scale)

***(A1)(A1)(A1)*(ft)*(A1)*(ft)**

*Award **(A1)** for labels (units not required) and scale, **(A1)**(ft) for max*(29.6,12.4), ***(A1)*(ft)** for *x-intercepts at 5.30 and 53.9, (all coordinates can be within 0.5), **(A1)** for well-drawn parabola ending at the x-intercepts.*

[4 marks]

Examiners report

(ii) Many candidates failed to recognise that extensive use of the GDC was intended for this question. An indicator of this was the choice of awkward coefficients. It is recognised that the context confused some candidates and that the horizontal shift was a bit disturbing for some.

Nevertheless, a lot of candidates could have earned more marks here if they had persevered. Many gave up on the graph, and elementary marks for scale and labels were lost unnecessarily.

As this was the first time for the unit penalty, we were lenient about the units left off the labels but this is likely to change in the future.

6i. *[2 marks]*

Markscheme

*Unit penalty **(UP)** is applicable where indicated in the left hand column.*

***(UP)** f* (40.3) = 10.1 m (3sf).

*Follow through from (a). If graph used, award **(M1)** for lines drawn and **(A1)** for candidate's value* ± 0.5. *(3sf not required).* ***(M1)(A1)*(ft)*(G2)***

[2 marks]

Examiners report

(ii) Many candidates failed to recognise that extensive use of the GDC was intended for this question. An indicator of this was the choice of awkward coefficients. It is recognised that the context confused some candidates and that the horizontal shift was a bit disturbing for some.

Nevertheless, a lot of candidates could have earned more marks here if they had persevered. Many gave up on the graph, and elementary marks for scale and labels were lost unnecessarily.

As this was the first time for the unit penalty, we were lenient about the units left off the labels but this is likely to change in the future.

7a. *[7 marks]*

Markscheme

(i)
$\text{Area} = \pi(5)^2$ ***(M1)***
$= 78.5\ (\text{cm}^2)$ (
$78.5398\ldots$) ***(A1)(G2)***

Note: Accept
25π .

(ii)
$8000 = 78.5398\ldots \times h$ ***(M1)***
$h = 102\ (\text{cm})$ (
$101.859\ldots$) ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answer to part (a)(i).

(iii)
$\text{Area} = \pi(5)^2 + 2\pi(5)(101.859\ldots)$ ***(M1)(M1)***

Note: Award ***(M1)*** for their substitution in curved surface area formula, ***(M1)*** for addition of their two areas.

$= 3280\ (\text{cm}^2)$ (
$3278.53\ldots$) ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answers to parts (a)(i) and (ii).

Examiners report

[N/A]

7b. *[2 marks]*

Markscheme

No, it is too tall/narrow. ***(A1)*****(ft)*****(R1)***

Note: Follow through from their value for
h.

Examiners report

[N/A]

7c. *[1 mark]*

Markscheme

$8000 = \pi r^2 h$ ***(A1)***

Examiners report

[N/A]

7d. *[2 marks]*

Markscheme

$A = \pi r^2 + 2\pi r\left(\frac{8000}{\pi r^2}\right)$ ***(A1)(M1)***

Note: Award ***(A1)*** for correct rearrangement of **their** part (c), ***(M1)*** for substitution of **their** rearrangement into area formula.

$= \pi r^2 + \frac{16000}{r}$ ***(AG)***

Examiners report

[N/A]

7e. *[3 marks]*

Markscheme

$\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r - 16000r^{-2}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for
$2\pi r$, ***(A1)*** for
-16000 ***(A1)*** for
r^{-2} . If an extra term is present award at most ***(A1)(A1)(A0)***.

Examiners report

[N/A]

7f. *[5 marks]*

Markscheme

(i)
$\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ ***(M1)***
$2\pi r^3 - 16000 = 0$ ***(M1)***
$r = 13.7$ cm (
$13.6556\ldots$) ***(A1)*****(ft)**

Note: Follow through from their part (e).

(ii)
$h = \frac{8000}{\pi(13.65\ldots)^2}$ ***(M1)***
$= 13.7$ cm (
$13.6556\ldots$) ***(A1)*****(ft)**

Note: Accept
13.6 if
13.7 used.

Examiners report

[N/A]

7g. *[2 marks]*

Markscheme

Yes or No, accompanied by a consistent and sensible reason. ***(A1)(R1)***

Note: Award ***(A0)(R0)*** if no reason is given.

Examiners report

[N/A]

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