Topic 7 Part 3 *[223 marks]*

Consider the function $f(x) = x^3 - 3x - 24x + 30$.

1a. Write down $f(0)$. *[1 mark]*

1b. Find $f'(x)$. *[3 marks]*

1c. Find the gradient of the graph of $f(x)$ at the point where $x = 1$. *[2 marks]*

1d. (i) Use $f'(x)$ to find the x-coordinate of M and of N. *[5 marks]*

(ii) Hence or otherwise write down the coordinates of M and of N.

1e. Sketch the graph of $f(x)$ for $-5 \leqslant x \leqslant 7$ and $-60 \leqslant y \leqslant 60$. Mark clearly M and N on your graph. *[4 marks]*

1f. Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$. *[6 marks]*

(i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B.

$$f(x) = \frac{1}{3}x^3 + 2x^2 - 12x + 3$$

2a. Find $f'(x)$. *[3 marks]*

2b. Find the interval of x for which $f(x)$ is decreasing. *[3 marks]*

Consider the function $g(x) = bx - 3 + \frac{1}{x^2},\ x \neq 0$.

3a. Write down the equation of the vertical asymptote of the graph of $y = g(x)$. *[2 marks]*

3b. Write down $g'(x)$. *[3 marks]*

3c. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. *[2 marks]*

Show that $b = 5$.

3d. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. *[3 marks]*

Find the equation of T.

3e. Using your graphic display calculator find the coordinates of the point where the graph of $y = g(x)$ intersects the x-axis. *[2 marks]*

3f. (i) Sketch the graph of $y = g(x)$ for $-2 \le x \le 5$ and $-15 \le y \le 25$, indicating clearly your answer to part (e). *[6 marks]*

(ii) Draw the line T on your sketch.

3g. Using your graphic display calculator find the coordinates of the local minimum point of $y = g(x)$. *[2 marks]*

3h. Write down the interval for which $g(x)$ is increasing in the domain $0 < x < 5$. *[2 marks]*

Consider the function
$f(x) = -\frac{1}{3}x^3 + \frac{5}{3}x^2 - x - 3.$

4a. Sketch the graph of $y = f(x)$ for $-3 \le x \le 6$ and $-10 \le y \le 10$ showing clearly the axes intercepts and local maximum and minimum points. Use a scale of 2 cm to represent 1 unit on the x-axis, and a scale of 1 cm to represent 1 unit on the y-axis. *[4 marks]*

4b. Find the value of $f(-1)$. *[2 marks]*

4c. Write down the coordinates of the y-intercept of the graph of $f(x)$. *[1 mark]*

4d. Find $f'(x)$. *[3 marks]*

4e. Show that
$f'(-1) = -\frac{16}{3}$. *[1 mark]*

4f. Explain what $f'(-1)$ represents. *[2 marks]*

4g. Find the equation of the tangent to the graph of $f(x)$ at the point where x is -1. *[2 marks]*

4h. Sketch the tangent to the graph of $f(x)$ at $x = -1$ on your diagram for (a). *[2 marks]*

4i. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. *[2 marks]*

Write down the value of

(i) a ;

(ii) b .

4j. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. *[1 mark]*

Describe the behaviour of $f(x)$ for $a < x < b$.

The equation of a curve is given as
$y = 2x^2 - 5x + 4.$

5a. Find
$\frac{dy}{dx}$. *[2 marks]*

5b. The equation of the line L is
$6x + 2y = -1.$ [4 marks]

Find the x-coordinate of the point on the curve
$y = 2x^2 - 5x + 4$ where the tangent is parallel to L.

A shipping container is to be made with six rectangular faces, as shown in the diagram.

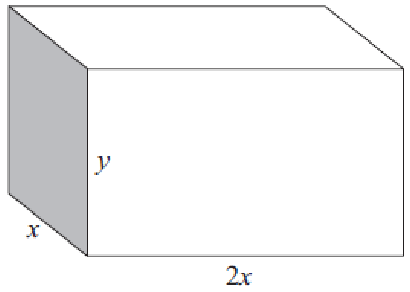


diagram not to scale

The dimensions of the container are

length $2x$
width x
height y.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

6a. Show that $y = 12 - 3x$. [3 marks]

6b. Show that the volume V m^3 of the container is given by [2 marks]

$$V = 24x^2 - 6x^3$$

6c. Find [2 marks]
$\frac{dV}{dx}$.

6d. Find the value of x for which V is a maximum. [3 marks]

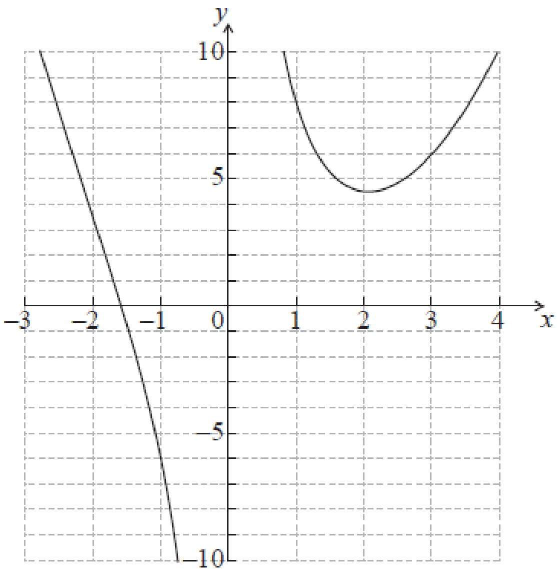
6e. Find the maximum volume of the container. [2 marks]

6f. Find the length and height of the container for which the volume is a maximum. [3 marks]

6g. The shipping container is to be painted. One litre of paint covers an area of 15 m^2 . Paint comes in tins containing four litres. [4 marks]

Calculate the number of tins required to paint the shipping container.

The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.



7a. Write down [5 marks]

(i) the equation of the vertical asymptote to the graph of $y = f(x)$;

(ii) the solution to the equation $f(x) = 0$;

(iii) the coordinates of the local minimum point.

7b. Find $f'(x)$. [4 marks]

7c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3 - 2x^2 - 9}{x^2}$. [2 marks]

7d. Find the gradient of the tangent to $y = f(x)$ at the point $\text{A}(1, 8)$. [2 marks]

7e. The line, L, passes through the point A and is perpendicular to the tangent at A. [1 mark]

Write down the gradient of L.

7f. The line, L, passes through the point A and is perpendicular to the tangent at A. [3 marks]

Find the equation of L. Give your answer in the form $y = mx + c$.

7g. The line, L, passes through the point A and is perpendicular to the tangent at A. [2 marks]

L also intersects the graph of $y = f(x)$ at points B and C. Write down the ***x*-coordinate** of B and of C.

Consider the function
$f(x) = x^3 + \frac{48}{x}, x \neq 0.$

8a. Calculate $f(2)$. *[2 marks]*

8b. Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-200 \leqslant y \leqslant 200$. *[4 marks]*

8c. Find $f'(x)$. *[3 marks]*

8d. Find $f'(2)$. *[2 marks]*

8e. Write down the coordinates of the local maximum point on the graph of f . *[2 marks]*

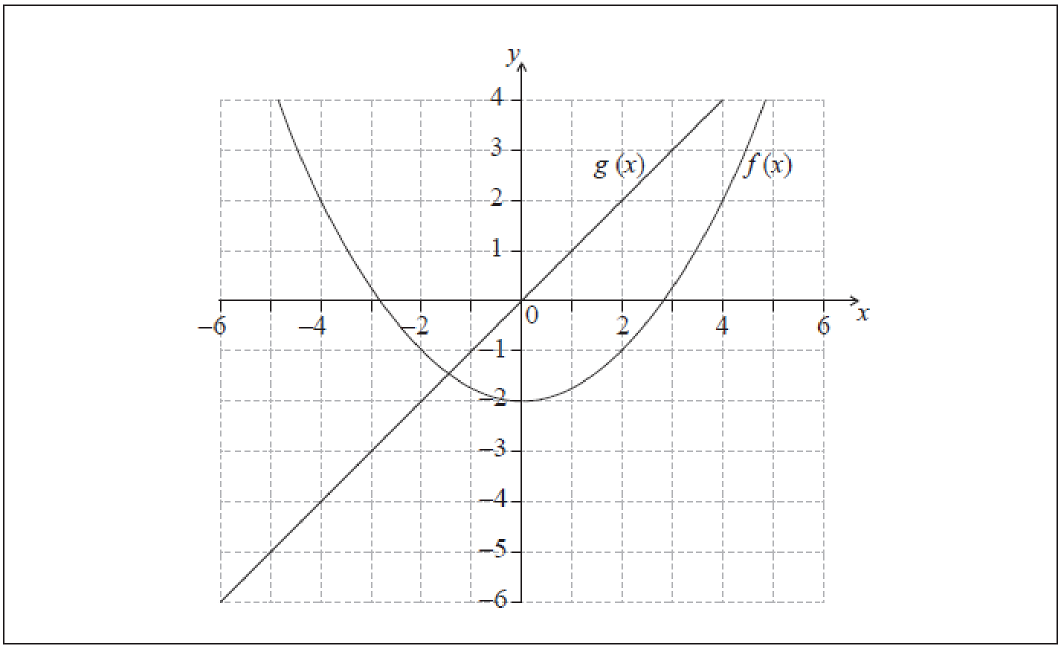
8f. Find the range of f . *[3 marks]*

8g. Find the gradient of the tangent to the graph of f at $x = 1.$ *[2 marks]*

8h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1.$ *[2 marks]*

Find the x-coordinate of this point.

The figure shows the graphs of the functions
$f(x) = \frac{1}{4}x^2 - 2$ and
$g(x) = x$.



9a. Differentiate $f(x)$ with respect to x . *[1 mark]*

9b. Differentiate $g(x)$ with respect to x . *[1 mark]*

9c. Calculate the value of x for which the gradients of the two graphs are the same. *[2 marks]*

9d. Draw the tangent to the parabola at the point with the value of x found in part (c). *[2 marks]*

The function $f(x)$ is defined by $f(x) = 1.5x + 4 + \frac{6}{x}, x \neq 0$.

10a. Write down the equation of the vertical asymptote. *[2 marks]*

10b. Find $f'(x)$. *[3 marks]*

10c. Find the gradient of the graph of the function at $x = -1$. *[2 marks]*

10d. Using your answer to part (c), decide whether the function $f(x)$ is increasing or decreasing at $x = -1$. Justify your answer. *[2 marks]*

10e. Sketch the graph of $f(x)$ for $-10 \leqslant x \leqslant 10$ and $-20 \leqslant y \leqslant 20$. *[4 marks]*

10f. P_1 is the local maximum point and P_2 is the local minimum point on the graph of $f(x)$. *[4 marks]*

Using your graphic display calculator, write down the coordinates of

(i) P_1 ;

(ii) P_2 .

10g. Using your sketch from (e), determine the range of the function $f(x)$ for $-10 \leqslant x \leqslant 10$. *[3 marks]*

Consider the function $f(x) = 3x + \frac{12}{x^2},\ x \neq 0.$

11a. Differentiate $f(x)$ with respect to x. *[3 marks]*

11b. Calculate $f'(x)$ when $x = 1$. *[2 marks]*

11c. Use your answer to part (b) to decide whether the function, f, is increasing or decreasing at $x = 1$. Justify your answer. *[2 marks]*

11d. Solve the equation $f'(x) = 0$. *[3 marks]*

11e. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. *[2 marks]*

Write down the coordinates of P.

11f. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. *[1 mark]*

Write down the gradient of *T*.

11g. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. *[2 marks]*

Write down the equation of *T*.

11h. Sketch the graph of the function *f*, for $-3 \le x \le 6$ and $-7 \le y \le 15$. Indicate clearly the point P and any intercepts of the curve with the axes. *[4 marks]*

11i. On your graph draw and label the tangent *T*. *[2 marks]*

11j. T intersects the graph of f at a second point. Write down the x-coordinate of this point of intersection. *[1 mark]*

A function is defined by
$f(x) = \frac{5}{x^2} + 3x + c,\ x \neq 0,\ c \in \mathbb{Z}$.

12a. Write down an expression for
$f'(x)$. *[4 marks]*

12b. Consider the graph of f. The graph of f passes through the point P(1, 4). *[2 marks]*

Find the value of c.

12c. There is a local minimum at the point Q. *[4 marks]*

Find the coordinates of Q.

12d. There is a local minimum at the point Q. *[3 marks]*

Find the set of values of x for which the function is decreasing.

12e. Let T be the tangent to the graph of f at P. *[2 marks]*

Show that the gradient of T is –7.

12f. Let T be the tangent to the graph of f at P. *[2 marks]*

Find the equation of T.

12g. T intersects the graph again at R. Use your graphic display calculator to find the coordinates of R. *[2 marks]*

Consider the curve
$y = x^3 + \frac{3}{2}x^2 - 6x - 2$.

13a. (i) Write down the value of
y when
x is
2. *[3 marks]*

(ii) Write down the coordinates of the point where the curve intercepts the
y-axis.

13b. Sketch the curve for
$-4 \leqslant x \leqslant 3$ and
$-10 \leqslant y \leqslant 10$. Indicate clearly the information found in (a). *[4 marks]*

13c. Find
$\frac{\mathrm{d}y}{\mathrm{d}x}$. *[3 marks]*

13d. Let L_1 be the tangent to the curve at $x = 2$. *[8 marks]*

Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

13e. It is known that $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ for $x < -2$ and $x > b$ where b is positive. *[5 marks]*

(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$.