

Topic 3 Part 1 [234 marks]

1a. Complete the following truth table.

[2 marks]

p	q	$p \Rightarrow \neg q$
T	T	F
T	F	T
F	T	F
F	F	T

1b. Consider the propositions

[2 marks]

p : Cristina understands logic

q : Cristina will do well on the logic test.

Write down the following compound proposition in symbolic form.

“If Cristina understands logic then she will do well on the logic test”

1c. Write down in words the contrapositive of the proposition given in part (b).

[2 marks]

A survey was carried out in a group of 200 people. They were asked whether they smoke or not. The collected information was organized in the following table.

	Smoker	Non-smoker
Male	60	40
Female	30	70

One person from this group is chosen at random.

2a. Write down the probability that this person is a smoker.

[2 marks]

2b. Write down the probability that this person is male given that they are a smoker.

[2 marks]

2c. Find the probability that this person is a smoker or is male.

[2 marks]

Consider the universal set

$U = \{x \in \mathbb{N} | 3 < x < 13\}$, and the subsets

$A = \{\text{multiples of 3}\}$ and

$B = \{4, 6, 12\}$.

3a. List the elements of the following set.

[1 mark]

A

3b. List the elements of the following set.

[1 mark]

$A \cap B'$

3c. Write down one element of $(A \cup B)'$. [2 marks]

3d. One of the statements in the table below is false. Indicate with an **X** which statement is false. Give a reason for your answer. [2 marks]

$n(A \cup B) = 4$	
$15 \in A'$	
$A \subset A \cup B$	

Part A

100 students are asked what they had for breakfast on a particular morning. There were three choices: cereal (X), bread (Y) and fruit (Z). It is found that

- 10 students had all three
- 17 students had bread and fruit only
- 15 students had cereal and fruit only
- 12 students had cereal and bread only
- 13 students had only bread
- 8 students had only cereal
- 9 students had only fruit

4a. Represent this information on a Venn diagram. [4 marks]

4b. Find the number of students who had none of the three choices for breakfast. [2 marks]

4c. Write down the percentage of students who had fruit for breakfast. [2 marks]

4d. Describe in words what the students in the set $X \cap Y'$ had for breakfast. [2 marks]

4e. Find the probability that a student had **at least** two of the three choices for breakfast. [2 marks]

4f. Two students are chosen at random. Find the probability that both students had all three choices for breakfast. [3 marks]

Part B

The same 100 students are also asked how many meals on average they have per day. The data collected is organized in the following table.

	3 or fewer meals per day	4 or 5 meals per day	More than 5 meals per day	Total
Male	15	25	15	55
Female	12	20	13	45
Total	27	45	28	100

A

χ^2 test is carried out at the 5 % level of significance.

- 4g. Write down the null hypothesis, H_0 , for this test. [1 mark]
- 4h. Write down the number of degrees of freedom for this test. [1 mark]
- 4i. Write down the critical value for this test. [1 mark]
- 4j. Show that the expected number of females that have more than 5 meals per day is 13, correct to the nearest integer. [2 marks]
- 4k. Use your graphic display calculator to find the χ^2_{calc} for this data. [2 marks]
- 4l. Decide whether H_0 must be accepted. Justify your answer. [2 marks]

One day the numbers of customers at three cafés, “Alan’s Diner” (A), “Sarah’s Snackbar” (S) and “Pete’s Eats” (P), were recorded and are given below.

17 were customers of Pete’s Eats only
27 were customers of Sarah’s Snackbar only
15 were customers of Alan’s Diner only
10 were customers of Pete’s Eats **and** Sarah’s Snackbar **but not** Alan’s Diner
8 were customers of Pete’s Eats **and** Alan’s Diner **but not** Sarah’s Snackbar

- 5a. Draw a Venn Diagram, using sets labelled A , S and P , that shows this information. [3 marks]
- 5b. There were 48 customers of Pete’s Eats that day. Calculate the number of people who were customers of all three cafés. [2 marks]
- 5c. There were 50 customers of Sarah’s Snackbar that day. Calculate the total number of people who were customers of Alan’s Diner. [3 marks]
- 5d. Write down the number of customers of Alan’s Diner that were also customers of Pete’s Eats. [1 mark]

5e. Find $n[(S \cup P) \cap A']$.

[2 marks]

Some of the customers in each café were given survey forms to complete to find out if they were satisfied with the standard of service they received.

	Pete's Eats	Alan's Diner	Sarah's Snackbar	Total
Dissatisfied	16	8	16	40
Satisfied	26	20	34	80
Total	42	28	50	120

5f. One of the survey forms was chosen at random, find the probability that the form showed "Dissatisfied";

[2 marks]

5g. One of the survey forms was chosen at random, find the probability that the form showed "Satisfied" and was completed at Sarah's Snackbar;

[2 marks]

5h. One of the survey forms was chosen at random, find the probability that the form showed "Dissatisfied", given that it was completed at Alan's Diner.

[2 marks]

5i. A χ^2 test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés.

[1 mark]

Write down the null hypothesis,

H_0 , for the

χ^2 test.

5j. A χ^2 test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés.

[1 mark]

Write down the number of degrees of freedom for the test.

5k. A χ^2 test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés.

[2 marks]

Using your graphic display calculator, find

χ^2_{calc} .

5l. A χ^2 test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés.

[2 marks]

State, giving a reason, the conclusion to the test.

U is the set of all the **positive** integers less than or equal to 12.
 A ,
 B and
 C are subsets of
 U .

$$A = \{1, 2, 3, 4, 6, 12\}$$

$$B = \{\text{odd integers}\}$$

$$C = \{5, 6, 8\}$$

6a. Write down the number of elements in $A \cap C$.

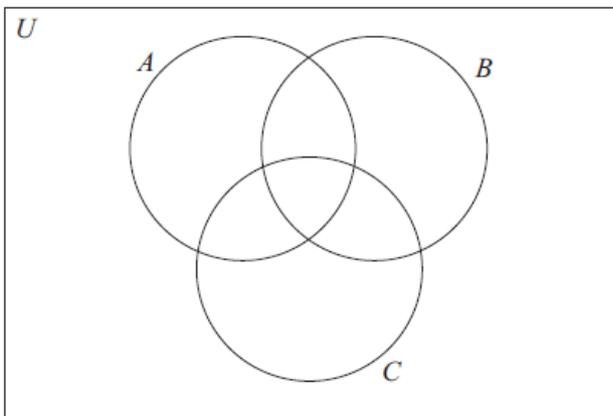
[1 mark]

6b. List the elements of B .

[1 mark]

6c. Complete the following Venn diagram with **all** the elements of U .

[4 marks]



In a particular school, students must choose at least one of three optional subjects: art, psychology or history.
 Consider the following propositions

a : I choose art,
 p : I choose psychology,
 h : I choose history.

7a. Write, in words, the compound proposition

[3 marks]

$$\neg h \Rightarrow (p \vee a)$$

7b. Complete the truth table for $\neg a \Rightarrow p$.

[1 mark]

a	p	$\neg a$	$\neg a \Rightarrow p$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

- 7c. State whether $\neg a \Rightarrow p$ is a tautology, a contradiction or neither. Justify your answer.

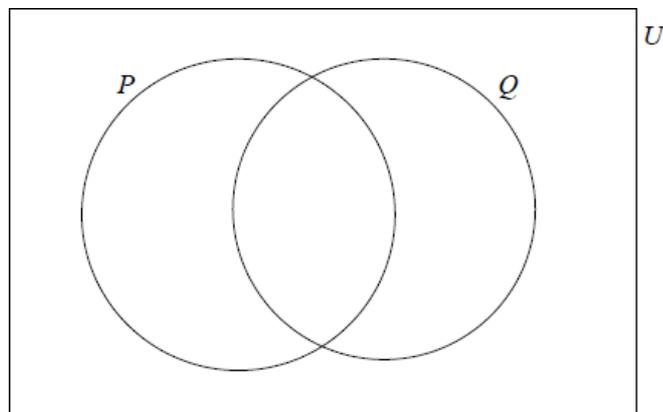
[2 marks]

A survey was carried out at an international airport. A number of travellers were interviewed and asked for their flight destinations. The results are shown in the table below.

Destination	America	Africa	Asia
Number of males	45	62	37
Number of females	35	46	25

- 8a. One traveller is to be chosen at random from all those interviewed. Find the probability that this traveller was going to Africa. [2 marks]
- 8b. One female traveller is to be chosen at random from all those interviewed. Find the probability that this female traveller was going to Asia. [2 marks]
- 8c. One traveller is to be chosen at random from those **not** going to America. Find the probability that the chosen traveller is female. [2 marks]
9. The sets P , Q and U are defined as [6 marks]

$U = \{\text{Real Numbers}\}$, $P = \{\text{Positive Numbers}\}$ and $Q = \{\text{Rational Numbers}\}$.



Write down in the correct region on the Venn diagram the numbers

$$\frac{22}{7},$$

$$5 \times 10^{-2},$$

$$\sin(60^\circ),$$

$$0,$$

$$\sqrt[3]{-8},$$

$$-\pi.$$

Consider two propositions p and q .

10a. Complete the truth table below.

[4 marks]

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg p$	$\neg p \Rightarrow q$
T	T				
T	F				
F	T				
F	F				

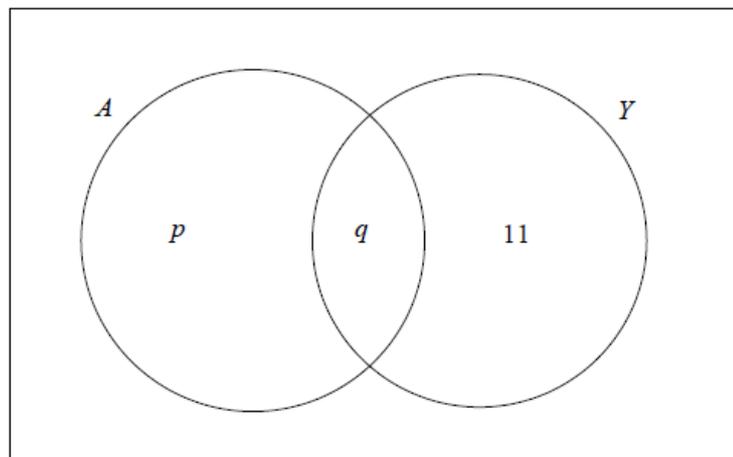
10b. Decide whether the compound proposition

[2 marks]

$$(p \Rightarrow \neg q) \Leftrightarrow (\neg p \Rightarrow q)$$

is a tautology. State the reason for your decision.

A fitness club has 60 members. 35 of the members attend the club's aerobics course (A) and 28 members attend the club's yoga course (Y). 17 members attend both courses. A Venn diagram is used to illustrate this situation.



11a. Write down the value of q .

[1 mark]

11b. Find the value of p .

[2 marks]

11c. Calculate the number of members of the fitness club who attend neither the aerobics course (A) nor the yoga course (Y).

[2 marks]

11d. Shade, on your Venn diagram, $A' \cap Y$.

[1 mark]

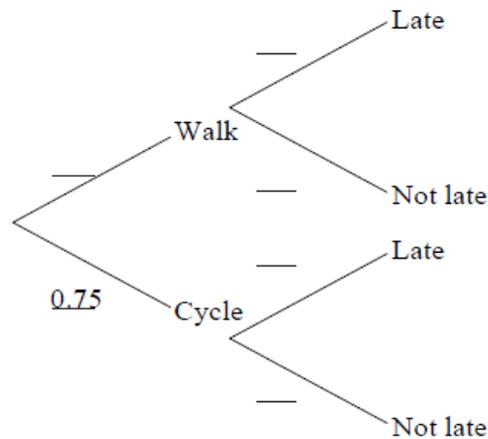
Maria travels to school either by walking or by bicycle. The probability she cycles to school is 0.75.

If she walks, the probability that she is late for school is 0.1.

If she cycles, the probability that she is late for school is 0.05.

12a. Complete the tree diagram below, showing the appropriate probabilities.

[3 marks]



12b. Find the probability that Maria is late for school.

[3 marks]

In a research project on the relation between the gender of 150 science students at college and their degree subject, the following set of data is collected.

		Degree Subject		
		Biology	Physics	Chemistry
Gender	Male	40	16	35
	Female	15	24	20

13a. Find the probability that a student chosen at random is male.

[2 marks]

13b. Find the probability that a student chosen at random is either male or studies Chemistry.

[2 marks]

13c. Find the probability that a student chosen at random studies Physics, given that the student is male.

[2 marks]

14a. Complete the truth table shown below.

[3 marks]

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$(p \vee (p \wedge q)) \Rightarrow p$
T	T			
T	F			
F	T			
F	F			

14b. State whether the compound proposition $(p \vee (p \wedge q)) \Rightarrow p$ is a contradiction, a tautology or neither.

[1 mark]

14c. Consider the following propositions.

[2 marks]

p : Feng finishes his homework

q : Feng goes to the football match

Write in symbolic form the following proposition.

If Feng does not go to the football match then Feng finishes his homework.

A group of 30 students were asked about their favourite topping for toast.

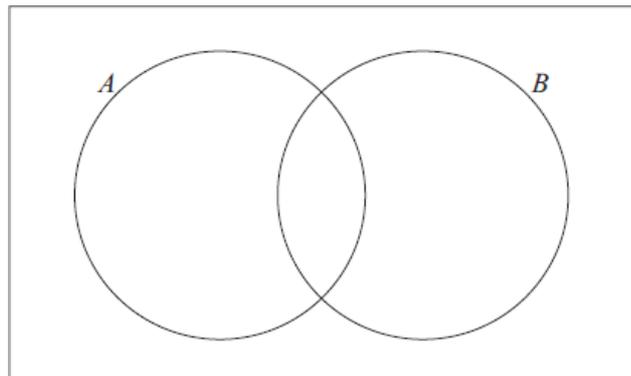
18 liked peanut butter (A)

10 liked jam (B)

6 liked neither

15a. Show this information on the Venn diagram below.

[2 marks]



15b. Find the number of students who like both peanut butter and jam.

[2 marks]

15c. Find the probability that a randomly chosen student from the group likes peanut butter, given that they like jam.

[2 marks]

Let

$$P(A) = 0.5,$$

$$P(B) = 0.6 \text{ and}$$

$$P(A \cup B) = 0.8.$$

16a. Find
 $P(A \cap B)$.

[2 marks]

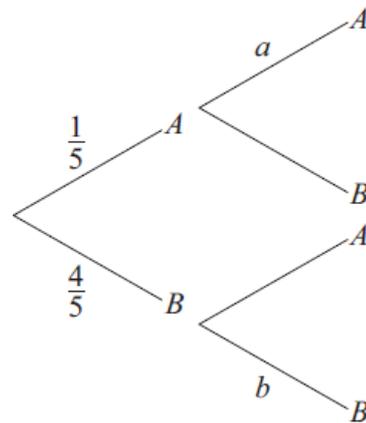
16b. Find
 $P(A|B)$.

[2 marks]

16c. Decide whether A and B are independent events. Give a reason for your answer.

[2 marks]

Phoebe chooses a biscuit from a blue tin on a shelf. The tin contains one chocolate biscuit and four plain biscuits. She eats the biscuit and chooses another one from the tin. The tree diagram below represents the situation with the four possible outcomes where A stands for chocolate biscuit and B for plain biscuit.



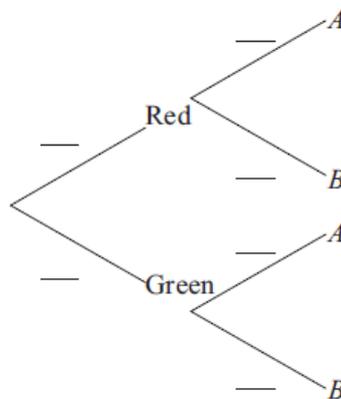
17a. Write down the value of a . [1 mark]

17b. Write down the value of b . [2 marks]

17c. Find the probability that both biscuits are plain. [3 marks]

On another shelf there are two tins, one red and one green. The red tin contains three chocolate biscuits and seven plain biscuits and the green tin contains one chocolate biscuit and four plain biscuits. Andrew randomly chooses either the red or the green tin and randomly selects a biscuit.

17d. **Copy and complete** the tree diagram below. [3 marks]



17e. Find the probability that he chooses a chocolate biscuit. [3 marks]

17f. Find the probability that he chooses a biscuit from the red tin given that it is a chocolate biscuit. [3 marks]

18. The Venn diagram shows the number sets

[6 marks]

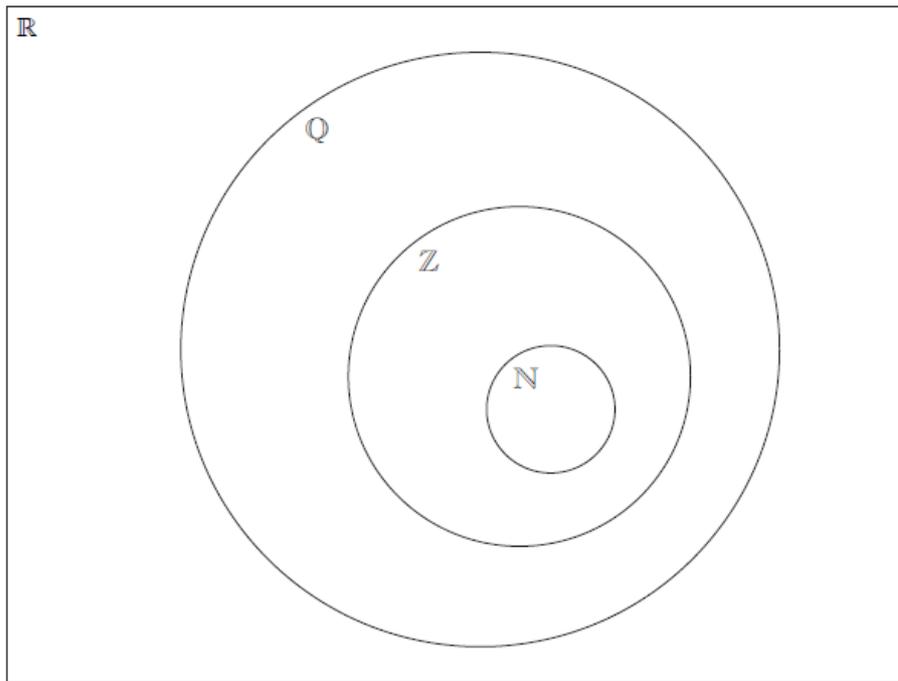
\mathbb{N} ,

\mathbb{Z} ,

\mathbb{Q} and

\mathbb{R} . Place each of the following numbers in the appropriate region of the Venn diagram.

$$\frac{1}{4}, -3, \pi, \cos 120^\circ, 2.7 \times 10^3, 3.4 \times 10^{-2}$$



Consider the statement p :

“If a quadrilateral is a square then the four sides of the quadrilateral are equal”.

19a. Write down the inverse of statement p in words.

[2 marks]

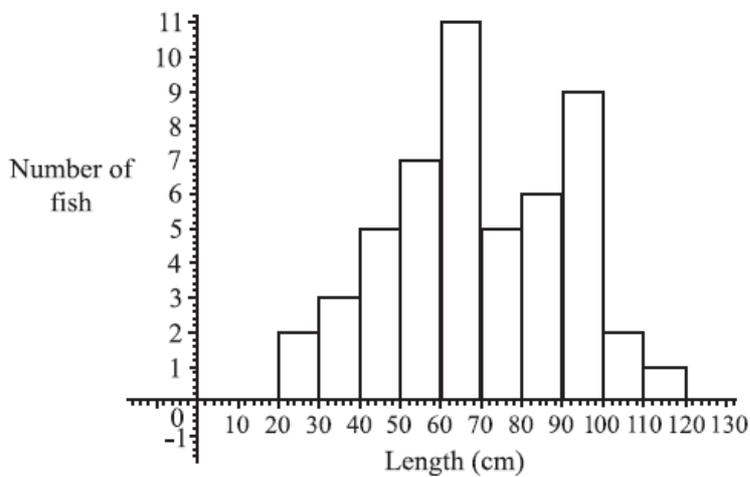
19b. Write down the converse of statement p in words.

[2 marks]

19c. Determine whether the converse of statement p is always true. Give an example to justify your answer.

[2 marks]

The figure below shows the lengths in centimetres of fish found in the net of a small trawler.



20a. Find the total number of fish in the net.

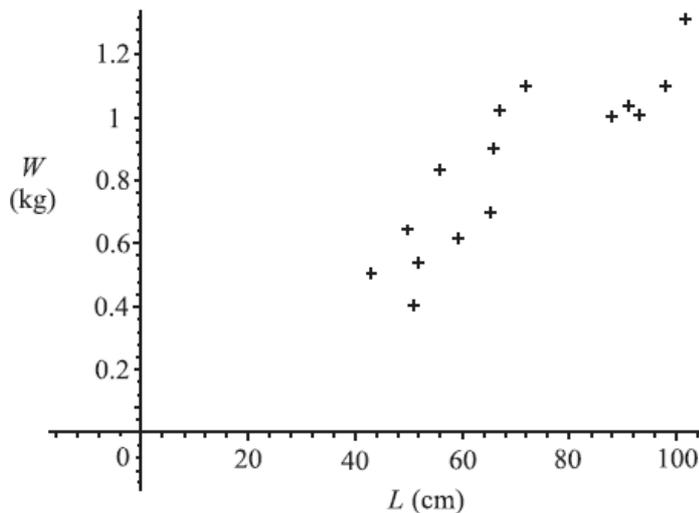
[2 marks]

- 20b. Find (i) the modal length interval, [5 marks]
(ii) the interval containing the median length,
(iii) an estimate of the mean length.

- 20c. (i) Write down an estimate for the standard deviation of the lengths. [3 marks]
(ii) How many fish (if any) have length **greater than** three standard deviations **above** the mean?

- 20d. The fishing company must pay a fine if more than 10% of the catch have lengths less than 40cm. [2 marks]
Do a calculation to decide whether the company is fined.

- 20e. A sample of 15 of the fish was weighed. The weight, W was plotted against length, L as shown below. [2 marks]

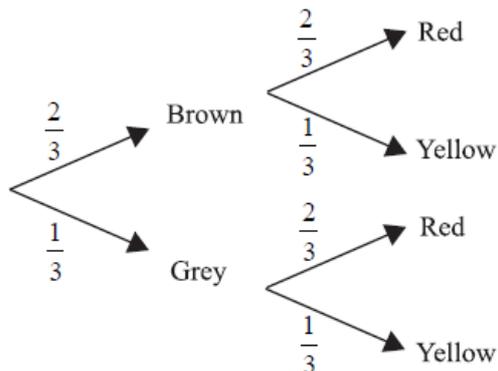


Exactly **two** of the following statements about the plot could be correct. Identify the two correct statements.

Note: You do **not** need to enter data in a GDC **or** to calculate r exactly.

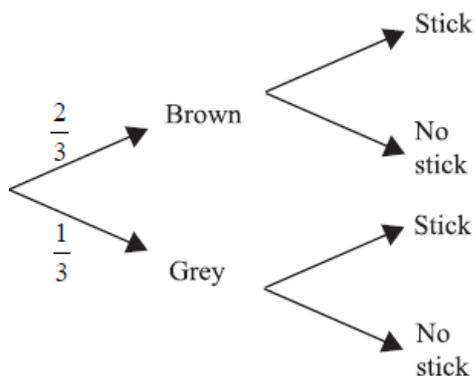
- (i) The value of r , the correlation coefficient, is approximately 0.871.
- (ii) There is an exact linear relation between W and L .
- (iii) The line of regression of W on L has equation $W = 0.012L + 0.008$.
- (iv) There is negative correlation between the length and weight.
- (v) The value of r , the correlation coefficient, is approximately 0.998.
- (vi) The line of regression of W on L has equation $W = 63.5L + 16.5$.

Neil has three dogs. Two are brown and one is grey. When he feeds the dogs, Neil uses three bowls and gives them out randomly. There are two red bowls and one yellow bowl. This information is shown on the tree diagram below.



- 21a. One of the dogs is chosen at random. [3 marks]
(i) Find P (the dog is grey and has the yellow bowl).
(ii) Find P (the dog does not get the yellow bowl).

- 21b. Neil often takes the dogs to the park after they have eaten. He has noticed that the grey dog plays with a stick for a quarter of [9 marks] the time and both brown dogs play with sticks for half of the time. This information is shown on the tree diagram below.



- (i) Copy the tree diagram and add the four missing probability values on the branches that refer to playing with a stick.
 During a trip to the park, one of the dogs is chosen at random.
- (ii) Find P (the dog is grey or is playing with a stick, but not both).
- (iii) Find P (the dog is grey given that the dog is playing with a stick).
- (iv) Find P (the dog is grey and was fed from the yellow bowl and is not playing with a stick).

There are 49 mice in a pet shop.

30 mice are white.

27 mice are male.

18 mice have short tails.

8 mice are white and have short tails.

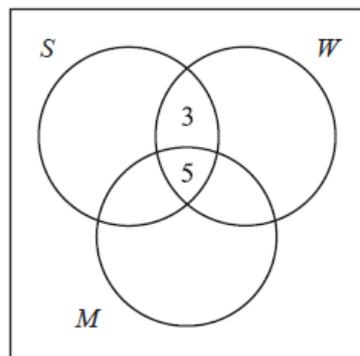
11 mice are male and have short tails.

7 mice are male but neither white nor short-tailed.

5 mice have all three characteristics and

2 have none.

Copy the diagram below to your examination script.



U W represents white mice.
 M represents male mice.
 S represents short-tailed mice.

- 21c. Complete the diagram, using the information given in the question. [4 marks]

- 21d. Find (i) $n(M \cap W)$ [3 marks]

(ii) $n(M' \cup S)$

- 21e. Two mice are chosen without replacement. [2 marks]
 Find P (both mice are short-tailed).

The truth table below shows the truth-values for the proposition

$$p \vee q \Rightarrow \neg p \vee \neg q$$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg p \vee \neg q$	$p \vee q \Rightarrow \neg p \vee \neg q$
T	T	F	F		F	
T	F	F		T	T	T
F	T	T	F	T	T	T
F	F	T	T	F		T

22a. Explain the distinction between the compound propositions,

[1 mark]

$$p \vee q \text{ and}$$

$$p \vee q.$$

22b. Fill in the four missing truth-values on the table.

[4 marks]

22c. State whether the proposition

[1 mark]

$p \vee q \Rightarrow \neg p \vee \neg q$ is a tautology, a contradiction or neither.

B and C are subsets of a universal set U such that

$$U = \{x : x \in \mathbb{Z}, 0 \leq x < 10\}, B = \{\text{prime numbers} < 10\}, C = \{x : x \in \mathbb{Z}, 1 < x \leq 6\}.$$

23a. List the members of sets

[4 marks]

(i)

$$B$$

(ii)

$$C \cap B$$

(iii)

$$B \cup C'$$

23b. Consider the propositions:

[2 marks]

p : x is a prime number less than 10.

q : x is a positive integer between 1 and 7.

Write down, in words, the contrapositive of the statement, "If x is a prime number less than 10, then x is a positive integer between 1 and 7."

Consider each of the following statements

p : Alex is from Uruguay

q : Alex is a scientist

r : Alex plays the flute

24a. Write the following argument in words

[3 marks]

$$\neg r \Rightarrow (q \vee p)$$

24b. Complete the truth table for the argument in part (a) using the values below for

[2 marks]

p ,
 q ,
 r and
 $\neg r$.

p	q	r	$\neg r$	$q \vee p$	$\neg r \Rightarrow (q \vee p)$
T	T	T	F		
T	T	F	T		
T	F	T	F		
T	F	F	T		
F	T	T	F		
F	T	F	T		
F	F	T	F		
F	F	F	T		

24c. The argument
 $\neg r \Rightarrow (q \vee p)$ is invalid. State the reason for this.

[1 mark]

Consider the following statements about the quadrilateral ABCD

q : ABCD has four equal sides

s : ABCD is a square

25a. Express in words the statement,
 $s \Rightarrow q$.

[2 marks]

25b. Write down in words, the inverse of the statement,
 $s \Rightarrow q$.

[2 marks]

25c. Determine the validity of the argument in (b). Give a reason for your decision.

[2 marks]

50 students at Rambling High School were asked how they travelled to school yesterday. All of the students travelled by bus, by car or walked.

12 students travelled by car only

7 students travelled by bus only

5 students travelled by car and walked, but did not use a bus

10 students travelled by bus and walked, but did not use a car

3 students used all three forms of travel.

26a. Represent this information on a Venn Diagram.

[4 marks]

26b. There were
28 students who used a bus to travel to school. Calculate the number of students

[4 marks]

- who travelled by car and by bus but did not walk;
- who travelled by car.

- 26c. Tomoko used a bus to travel to school yesterday.
Find the probability that she also walked.

[2 marks]

- 26d. Two students are chosen at random from all
50 students.

[7 marks]

Find the probability that

- (i) both students walked;
- (ii) only one of the students walked.