Topic 1 Part 9 *[177 marks]*

$U = \{x | x \text{ is an integer}, 2 < x < 10\}$

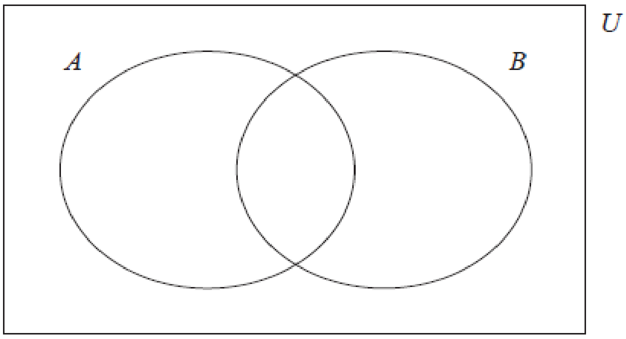
A and B are subsets of U such that $A = \{\text{multiples of } 3\}$, $B = \{\text{factors of } 24\}$.

1a. List the elements of *[2 marks]*

(i) U ;

(ii) B .

1b. Write down the elements of U on the Venn diagram. *[3 marks]*



1c. Write down *[1 mark]*
$n(A \cap B)$.

2a. Complete the truth table. *[2 marks]*

p	q	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

2b. Consider the propositions p and q: *[2 marks]*

p: x is a number less than 10.

q: x2 is a number greater than 100.

Write in words the compound proposition
$\neg p \vee q$.

2c. Using part (a), determine whether *[1 mark]*
$\neg p \vee q$ is true or false, for the case where
x is a number less than 10 and
x^2 is a number greater than 100.

2d. Write down a value of *[1 mark]*
x for which
$\neg p \vee q$ is false.

$z = \frac{17x^2}{a-b}$.

3a. Find the value of z when $x = 12.5$, $a = 0.572$ and $b = 0.447$. Write down your full calculator display. *[2 marks]*

3b. Write down your answer to part (a) *[2 marks]*

(i) correct to the nearest 1000 ;

(ii) correct to three significant figures.

3c. Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \le a < 10$, $k \in \mathbb{Z}$. *[2 marks]*

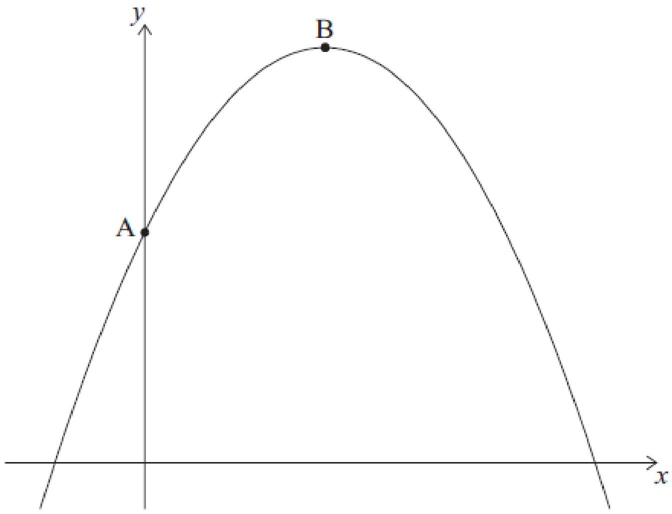
The fourth term, u_4, of a geometric sequence is 135. The fifth term, u_5, is 101.25 .

4a. Find the common ratio of the sequence. *[2 marks]*

4b. Find u_1, the first term of the sequence. *[2 marks]*

4c. Calculate the sum of the first 10 terms of the sequence. *[2 marks]*

The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the y-axis at point A(0, 5) and has its vertex at point B(2, 9).



5a. Write down the value of c. *[1 mark]*

5b. Find the value of b. *[2 marks]*

5c. Find the x-intercepts of the graph of f . *[2 marks]*

5d. Write down $f(x)$ in the form $f(x) = -(x-p)(x+q)$. *[1 mark]*

Neung is going home to Vietnam after working in Singapore.

She has 5000 Singapore dollars (SGD) and changes these into American dollars (USD) to take home.

The exchange rate between Singapore dollars (SGD) and American dollars (USD) is

1 USD = 1.2945 SGD.

There is also a 2.4 % commission on the exchange.

6a. Calculate the amount of commission on the exchange **in SGD**. *[2 marks]*

6b. Calculate the number of American dollars (USD) Neung takes home. **Give your answer correct to 2 decimal places.** *[2 marks]*

6c. At the airport in Vietnam, Neung changes 150 USD into Vietnamese dong (VND) to pay for her transport home. *[2 marks]*

The exchange rate between American dollars (USD) and Vietnamese dong (VND) is

1 USD = 19 495 VND.

There is no commission.

Calculate the number of Vietnamese dong that Neung receives. **Give your answer correct to the nearest thousand dong.**

Give your answers to parts (a) to (e) to the nearest dollar.

On Hugh's 18th birthday his parents gave him options of how he might receive his monthly allowance for the next two years.

Option A

$60 each month for two years

Option B

$10 in the first month,

$15 in the second month,

$20 in the third month, increasing by

$5 each month for two years

Option C

$15 in the first month and increasing by

10% each month for two years

Option D Investing

$1500 at a bank at the beginning of the first year, with an interest rate of

6% per annum, **compounded monthly**.

Hugh does not spend any of his allowance during the two year period.

7a. If Hugh chooses **Option A**, calculate the total value of his allowance at the end of the two year period. *[2 marks]*

7b. If Hugh chooses **Option B**, calculate *[5 marks]*

(i) the amount of money he will receive in the 17th month;

(ii) the total value of his allowance at the end of the two year period.

7c. If Hugh chooses **Option C**, calculate *[5 marks]*

(i) the amount of money Hugh would receive in the 13th month;

(ii) the total value of his allowance at the end of the two year period.

7d. If Hugh chooses **Option D**, calculate the total value of his allowance at the end of the two year period. *[3 marks]*

7e. State which of the options, A, B, C or D, Hugh should choose to give him the greatest total value of his allowance at the end of the two year period. *[1 mark]*

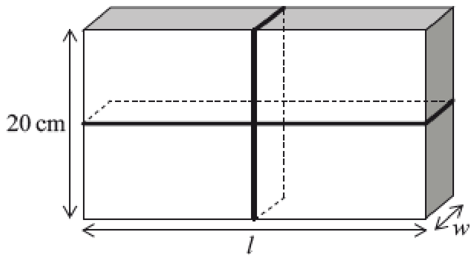
7f. Another bank guarantees Hugh an amount of $1750 after two years of investment if he invests $1500 at this bank. The interest is **compounded annually**. Calculate the interest rate per annum offered by the bank. [3 marks]

A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm. The total volume of the parcel is 3000 cm^3.

8a. Express the volume of the parcel in terms of l and w. [1 mark]

8b. Show that $l = \frac{150}{w}$. [2 marks]

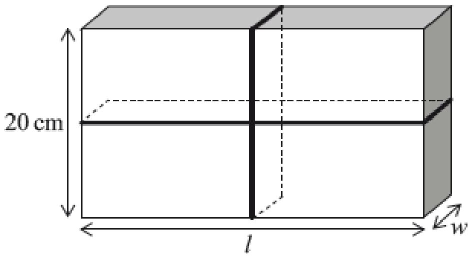
8c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2 marks]



Show that the length of string, S cm, required to tie up the parcel can be written as

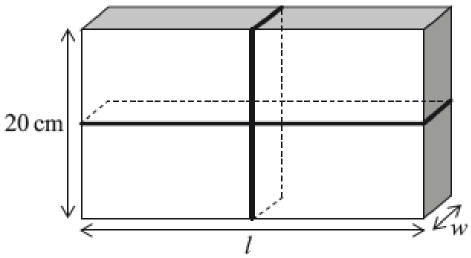
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

8d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



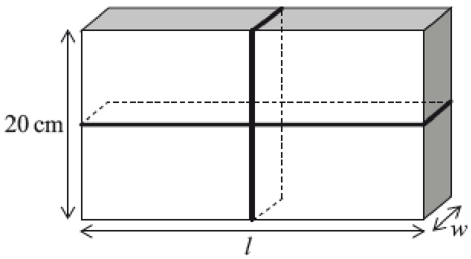
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

8e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[3 marks]*



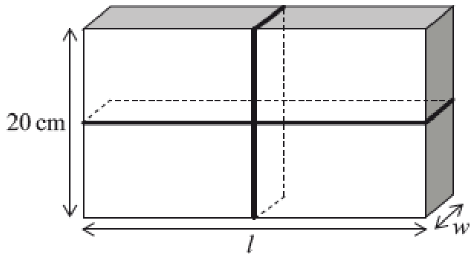
Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

8f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



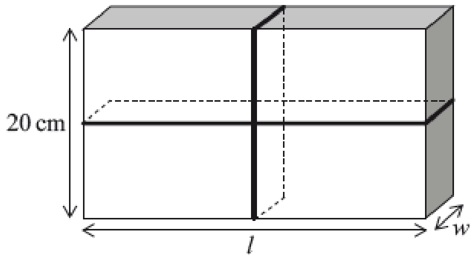
Find the value of w for which S is a minimum.

8g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[1 mark]*



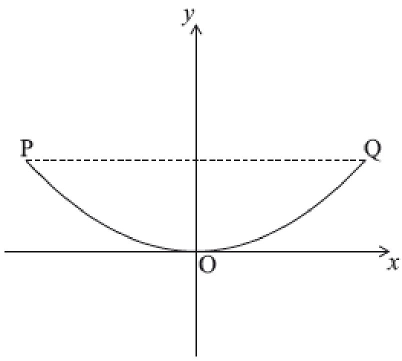
Write down the value,
l, of the parcel for which the length of string is a minimum.

8h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



Find the minimum length of string required to tie up the parcel.

The front view of the edge of a water tank is drawn on a set of axes shown below.
The edge is modelled by
$y = ax^2 + c$.



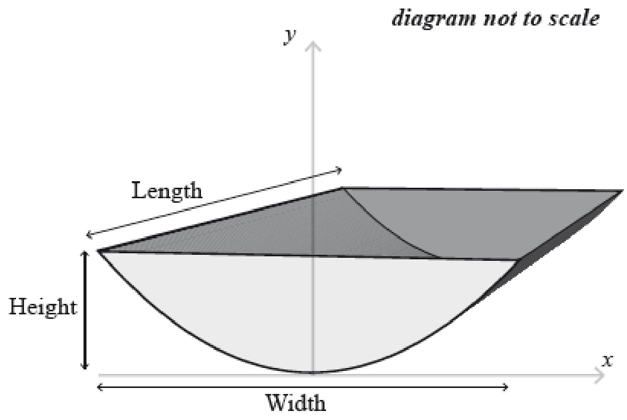
Point
P has coordinates
$(-3, 1.8)$, point
O has coordinates
$(0, 0)$ and point
Q has coordinates
$(3, 1.8)$.

9a. Write down the value of
c. *[1 mark]*

9b. Find the value of
a. *[2 marks]*

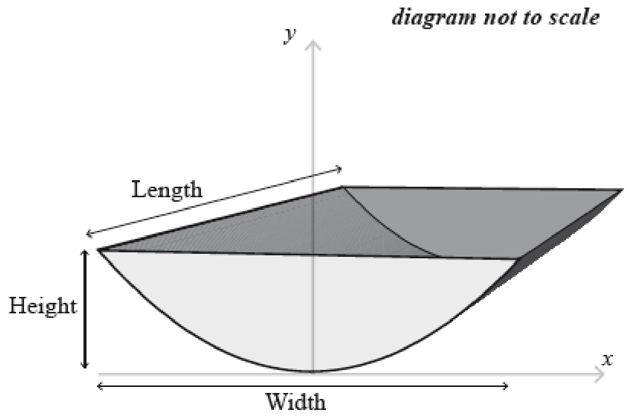
9c. Hence write down the equation of the quadratic function which models the edge of the water tank. *[1 mark]*

9d. The water tank is shown below. It is partially filled with water. *[2 marks]*



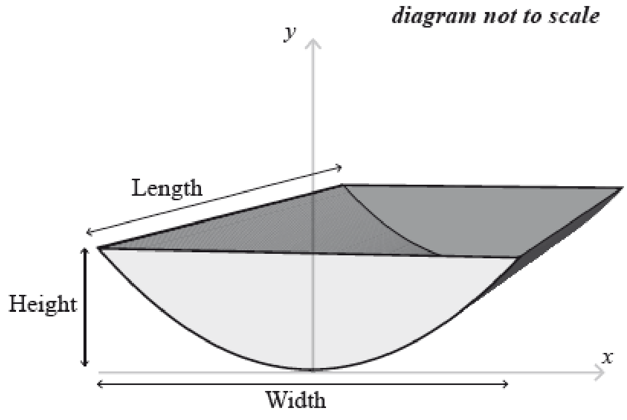
Calculate the value of y when
$x = 2.4$ m.

9e. The water tank is shown below. It is partially filled with water. *[2 marks]*



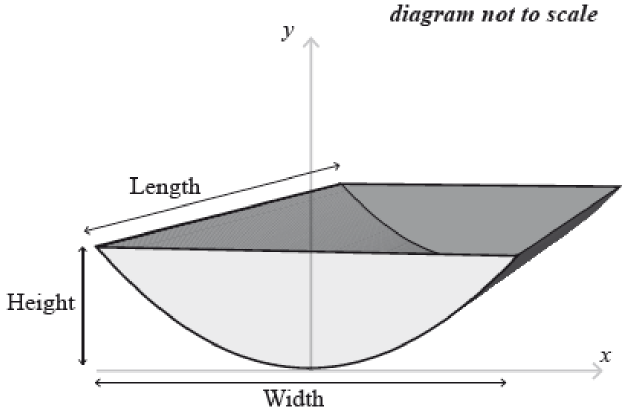
State what the value of
x and the value of
y represent for this water tank.

9f. The water tank is shown below. It is partially filled with water. *[2 marks]*



Find the value of
x when the height of water in the tank is
0.9 m.

9g. The water tank is shown below. It is partially filled with water. *[2 marks]*



When the water tank is filled to a height of 0.9 m, the front cross-sectional area of the water is 2.55 m^2.

(i) Calculate the volume of water in the tank.

The total volume of the tank is 36 m^3.

(ii) Calculate the percentage of water in the tank.

U is the set of **positive** integers less than or equal to 10. A, B and C are subsets of U.

$A = \{\text{even integers}\}$

$B = \{\text{multiples of } 3\}$

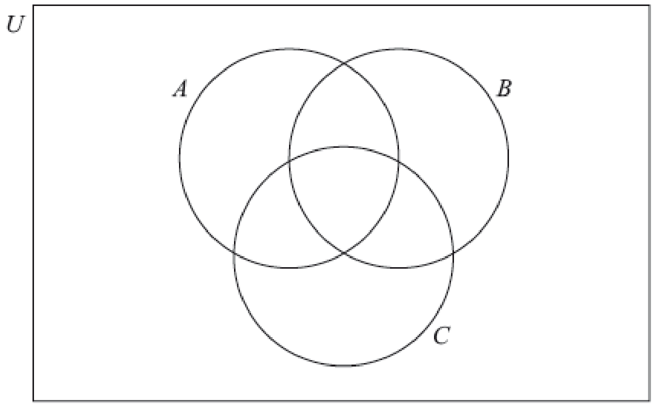
$C = \{6,\ 7,\ 8,\ 9\}$

10a. List the elements of A. *[1 mark]*

10b. List the elements of B. *[1 mark]*

10c. Complete the Venn diagram with **all** the elements of U.

[4 marks]



The first term, u_1, of an arithmetic sequence is 145. The fifth term, u_5, of the sequence is 113.

11a. Find the common difference of the sequence.

[2 marks]

11b. The n^{th} term, u_n, of the sequence is -7.
Find the value of n.

[2 marks]

11c. The n^{th} term, u_n, of the sequence is -7.
Find S_{20}, the sum of the first twenty terms of the sequence.

[2 marks]

12a. Expand the expression $x(2x^3 - 1)$.

[2 marks]

12b. Differentiate $f(x) = x(2x^3 - 1)$.

[2 marks]

12c. Find the x-coordinate of the local minimum of the curve $y = f(x)$.

[2 marks]

512 competitors enter round 1 of a tennis tournament, in which each competitor plays a match against one other competitor.

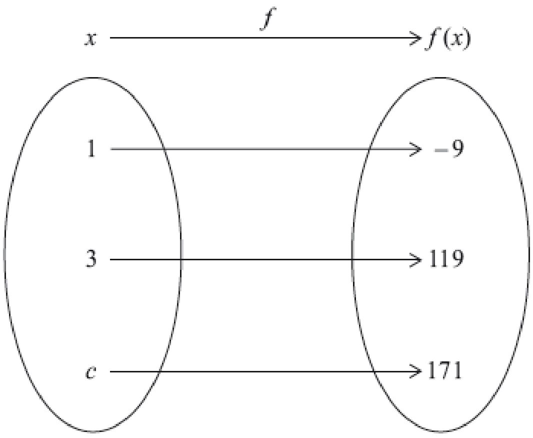
The winning competitor progresses to the next round (round 2); the losing competitor leaves the tournament.

The tournament continues in this manner until there is a winner.

13a. Find the number of competitors who play in round 6 of the tournament. *[3 marks]*

13b. Find the total number of matches played in the tournament. *[3 marks]*

A quadratic function
$f : x \mapsto ax^2 + b$, where
a and
$b \in \mathbb{R}$ and
$x \geqslant 0$, is represented by the mapping diagram.



14a. Using the mapping diagram, write down two equations in terms of
a and
b. *[2 marks]*

14b. Solve the equations to find the value of *[2 marks]*
(i)
a;
(ii)
b.

14c. Find the value of
c. *[2 marks]*

The table shows the distance, in km, of eight regional railway stations from a city centre terminus and the price, in \$, of a return ticket from each regional station to the terminus.

Distance in km (x)	3	15	23	42	56	62	74	93
Price in \$ (y)	5	24	43	56	68	74	86	100

15a. Draw a scatter diagram for the above data. Use a scale of 1 cm to represent 10 km on the x-axis and 1 cm to represent \$$10$ on the y-axis. *[4 marks]*

15b. Use your graphic display calculator to find *[2 marks]*

(i) $\bar{x}$, the mean of the distances;

(ii) $\bar{y}$, the mean of the prices.

15c. Plot and label the point M $(\bar{x}, \bar{y})$ on your scatter diagram. *[1 mark]*

15d. Use your graphic display calculator to find *[3 marks]*

(i) the product–moment correlation coefficient, r;

(ii) the equation of the regression line y on x.

15e. Draw the regression line y on x on your scatter diagram. *[2 marks]*

15f. A ninth regional station is 76 km from the city centre terminus. *[3 marks]*

Use the equation of the regression line to estimate the price of a return ticket to the city centre terminus from this regional station. **Give your answer correct to the nearest** \$.

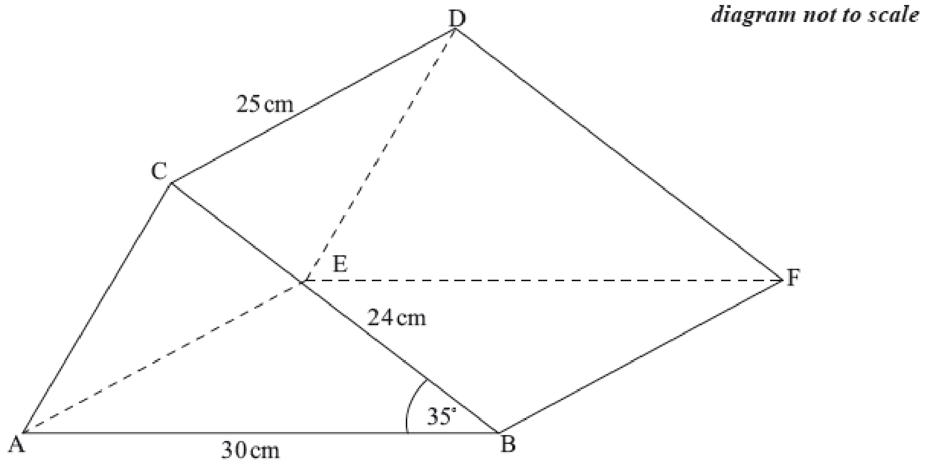
15g. Give a reason why it is valid to use your regression line to estimate the price of this return ticket. *[1 mark]*

15h. The actual price of the return ticket is \$$80$. *[2 marks]*

Using your answer to part (f), calculate the percentage error in the estimated price of the ticket.

A manufacturer has a contract to make 2600 solid blocks of wood. Each block is in the shape of a right triangular prism, ABCDEF, as shown in the diagram.

$\text{AB} = 30\text{ cm},\ \text{BC} = 24\text{ cm},\ \text{CD} = 25\text{ cm}$ and angle $\text{A}\hat{\text{B}}\text{C} = 35^\circ$.



16a. Calculate the length of AC. *[3 marks]*

16b. Calculate the area of triangle ABC. *[3 marks]*

16c. Assuming that no wood is wasted, show that the volume of wood required to make all 2600 blocks is $13\,400\,000\text{ cm}^3$, correct to three significant figures. *[2 marks]*

16d. Write $13\,400\,000$ in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. *[2 marks]*

16e. Show that the total surface area of one block is 2190 cm^2, correct to three significant figures. *[3 marks]*

16f. The blocks are to be painted. One litre of paint will cover 22 m^2.
Calculate the number of litres required to paint all 2600 blocks. *[3 marks]*

Give all answers in this question correct to two decimal places.
Arthur lives in London. On 1^{st} August 2008 Arthur paid $37\,500$ euros (EUR) for a new car from Germany. The price of the same car in London was $34\,075$ British pounds (GBP).
The exchange rate on 1^{st} August 2008 was $1\,\text{EUR} = 0.7234\,\text{GBP}$.

17a. Calculate, **in GBP**, the price that Arthur paid for the car. *[2 marks]*

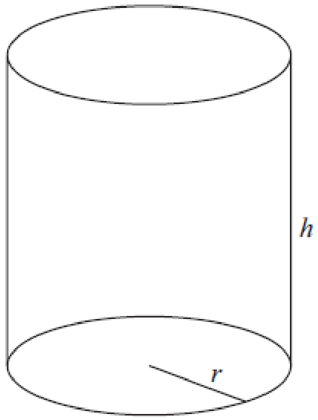
17b. Write down, in GBP, the amount of money Arthur saved by buying the car in Germany. *[1 mark]*

17c. Between 1^{st} August 2008 and 1^{st} August 2012 Arthur's car depreciated at an annual rate of 9% of its current value. *[3 marks]*
Calculate the value, in GBP, of Arthur's car on 1^{st} August **2009**.

17d. Between 1^{st} August 2008 and 1^{st} August 2012 Arthur's car depreciated at an annual rate of 9% of its current value. *[3 marks]*
Show that the value of Arthur's car on 1^{st} August **2012** was $18\,600\,\text{GBP}$, correct to the nearest $100\,\text{GBP}$.

Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3.

diagram not to scale



Nadia decides to make the radius, r, of the bin 5 cm.

18a. Calculate *[7 marks]*

(i) the area of the base of the wastepaper bin;

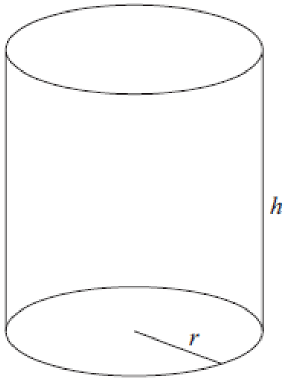
(ii) the height, h, of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin.

18b. State whether Nadia's design is practical. Give a reason. *[2 marks]*

Merryn also designs a cylindrical wastepaper bin with a volume of 8000 cm^3. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

diagram not to scale



Let the radius of the base of Merryn's wastepaper bin be r, and let its height be h.

18c. Write down an equation in h and r, using the given volume of the bin. *[1 mark]*

18d. Show that the total external surface area, A, of the bin is $A = \pi r^2 + \frac{16000}{r}$. *[2 marks]*

18e. Write down $\frac{\mathrm{d}A}{\mathrm{d}r}$. *[3 marks]*

18f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin. *[5 marks]*

(ii) Calculate the value of h corresponding to this value of r .

18g. Determine whether Merryn’s design is an improvement upon Nadia’s. Give a reason. *[2 marks]*
