Topic 5 Part 3 *[209 marks]*

The straight line, L_1, has equation
$2y - 3x = 11$. The point A has coordinates $(6, 0)$.

1a. Give a reason why L_1 **does not** pass through A. *[1 mark]*

1b. Find the gradient of L_1. *[2 marks]*

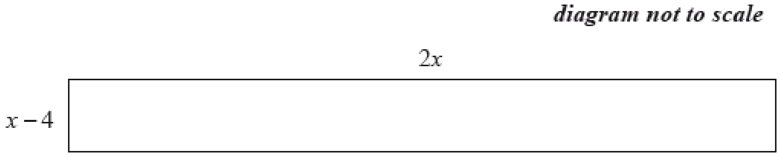
1c. L_2 is a line perpendicular to L_1. The equation of L_2 is $y = mx + c$. *[1 mark]*

Write down the value of m.

1d. L_2 **does** pass through A. *[2 marks]*

Find the value of c.

The surface of a red carpet is shown below. The dimensions of the carpet are in metres.

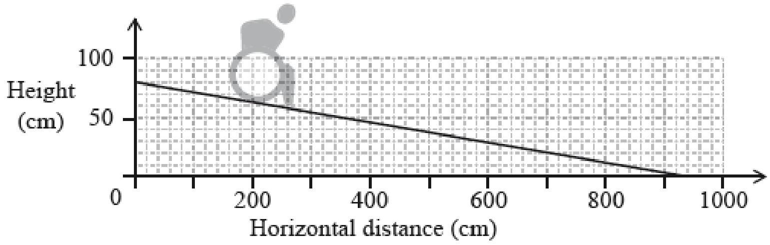


2a. Write down an expression for the area, A, in m^2, of the carpet. *[1 mark]*

2b. The area of the carpet is $10\ \text{m}^2$. Calculate the value of x. *[3 marks]*

2c. The area of the carpet is $10\ \text{m}^2$. *[2 marks]*
Hence, write down the value of the length and of the width of the carpet, in metres.

The diagram shows a wheelchair ramp, A, designed to descend from a height of 80 cm.

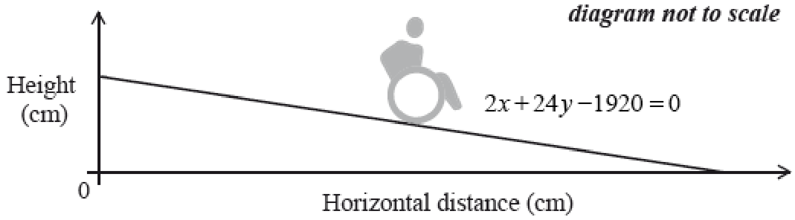


3a. Use the diagram above to calculate the gradient of the ramp. *[1 mark]*

3b. The gradient for a **safe** descending wheelchair ramp is $-\frac{1}{12}$. *[1 mark]*

Using your answer to part (a), comment on why wheelchair ramp A is **not safe**.

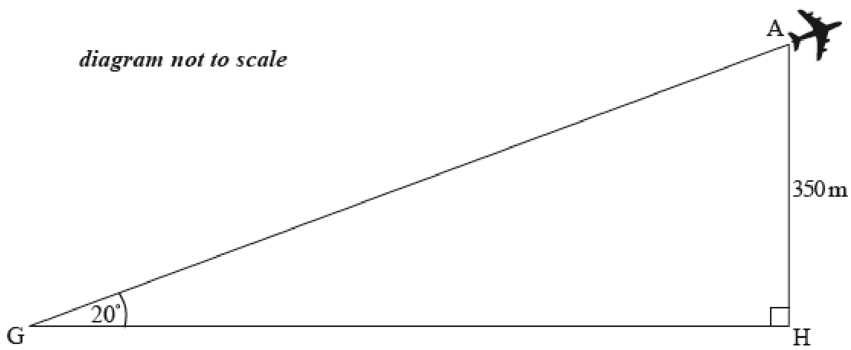
3c. The equation of a second wheelchair ramp, B, is $2x+24y-1920=0$. *[4 marks]*



(i) Determine whether wheelchair ramp B is safe or not. Justify your answer.

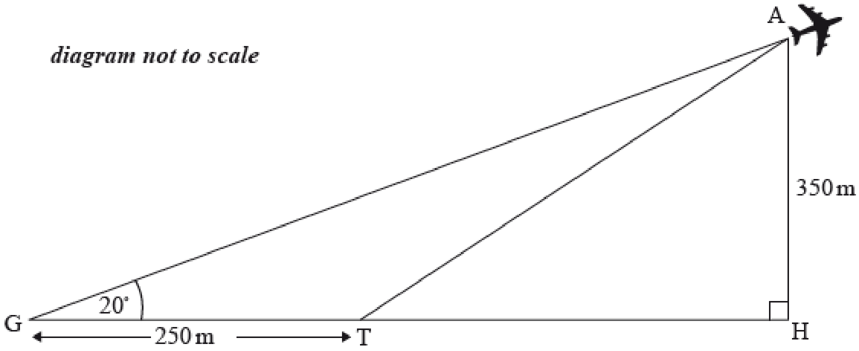
(ii) Find the horizontal distance of wheelchair ramp B.

Günter is at Berlin Tegel Airport watching the planes take off. He observes a plane that is at an angle of elevation of 20° from where he is standing at point G. The plane is at a height of 350 metres. This information is shown in the following diagram.



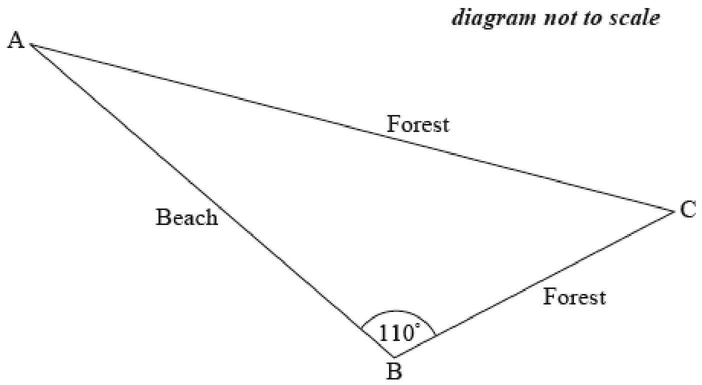
4a. Calculate the horizontal distance, GH, of the plane from Günter. **Give your answer to the nearest metre.** *[3 marks]*

4b. The plane took off from a point T, which is 250 metres from where Günter is standing, as shown in the following diagram. *[3 marks]*



Using your answer from part (a), calculate the angle ATH, the takeoff angle of the plane.

A cross-country running course consists of a beach section and a forest section. Competitors run from
A to
B, then from
B to
C and from
C back to
A.
The running course from
A to
B is along the beach, while the course from
B, through
C and back to
A, is through the forest.
The course is shown on the following diagram.



Angle
ABC is
$110°$.
It takes Sarah
5 minutes and
20 seconds to run from
A to
B at a speed of
$3.8\ \text{ms}^{-1}$.

5a. Using '*distance* = *speed* *[2 marks]*
$\times$ *time*', show that the distance from
A to
B is
1220 metres correct to 3 significant figures.

5b. The distance from *[1 mark]*
B to
C is
850 metres. Running this part of the course takes Sarah
5 minutes and
3 seconds.
Calculate the speed, in
ms^{-1}, that Sarah runs from
B to
C.

5c. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3 marks]

Calculate the distance, in metres, from C **to** A.

5d. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [2 marks]

Calculate the total distance, in metres, of the cross-country running course.

5e. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3 marks]

Find the size of angle BCA.

5f. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3 marks]

Calculate the area of the cross-country course bounded by the lines AB, BC and CA.

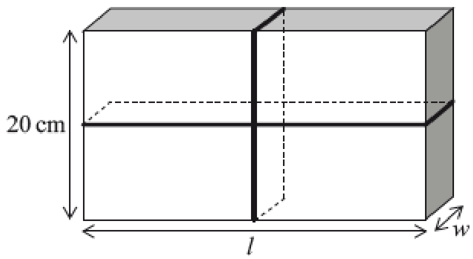
A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm.

The total volume of the parcel is $3000\ \text{cm}^3$.

6a. Express the volume of the parcel in terms of l and w. [1 mark]

6b. Show that $l = \frac{150}{w}$. [2 marks]

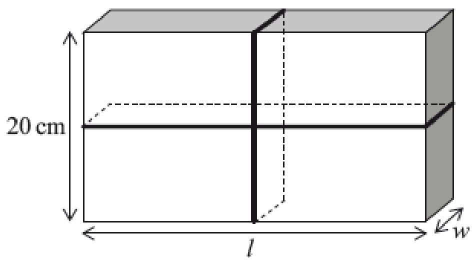
6c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



Show that the length of string,
S cm, required to tie up the parcel can be written as

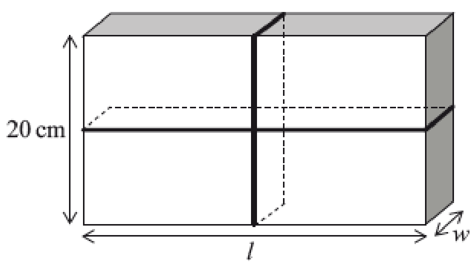
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

6d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



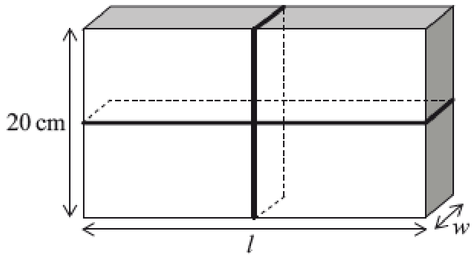
Draw the graph of
S for
$0 < w \leqslant 20$ and
$0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of
2 cm to represent
5 units on the horizontal axis
w (cm), and a scale of
2 cm to represent
100 units on the vertical axis
S (cm).

6e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[3 marks]*



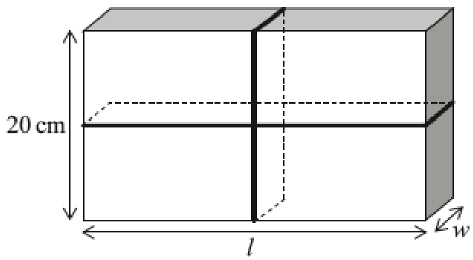
Find
$\frac{\mathrm{d}S}{\mathrm{d}w}$.

6f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



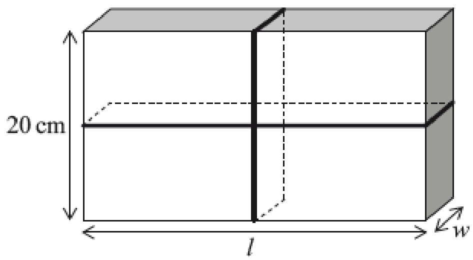
Find the value of
w for which
S is a minimum.

6g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[1 mark]*



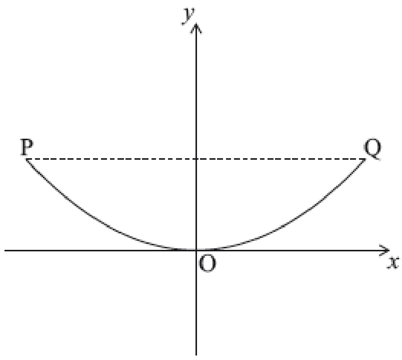
Write down the value,
l, of the parcel for which the length of string is a minimum.

6h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



Find the minimum length of string required to tie up the parcel.

The front view of the edge of a water tank is drawn on a set of axes shown below.
The edge is modelled by
$y = ax^2 + c$.



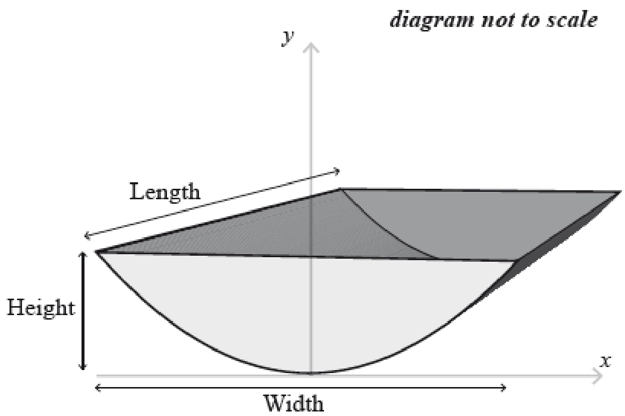
Point
P has coordinates
$(-3, 1.8)$, point
O has coordinates
$(0, 0)$ and point
Q has coordinates
$(3, 1.8)$.

7a. Write down the value of
c. *[1 mark]*

7b. Find the value of
a. *[2 marks]*

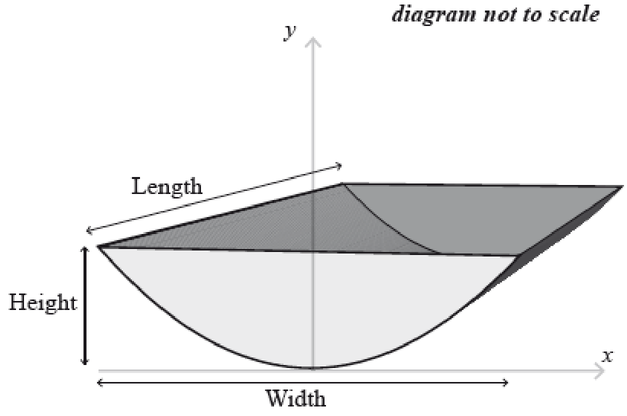
7c. Hence write down the equation of the quadratic function which models the edge of the water tank. *[1 mark]*

7d. The water tank is shown below. It is partially filled with water. *[2 marks]*



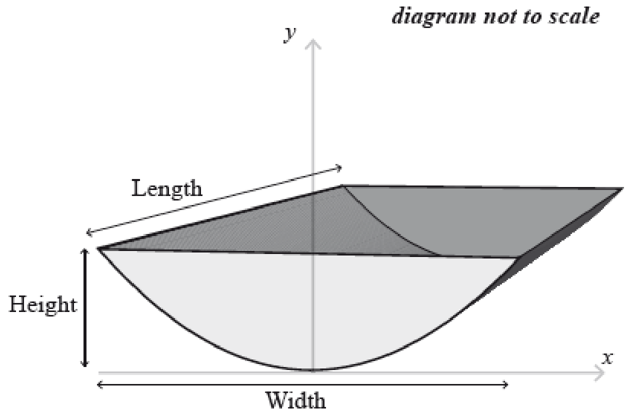
Calculate the value of y when
$x = 2.4$ m.

7e. The water tank is shown below. It is partially filled with water. *[2 marks]*



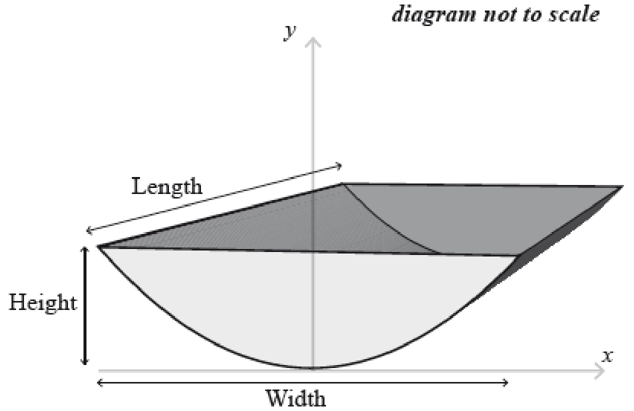
State what the value of
x and the value of
y represent for this water tank.

7f. The water tank is shown below. It is partially filled with water. *[2 marks]*



Find the value of
x when the height of water in the tank is
$0.9\,\text{m}$.

7g. The water tank is shown below. It is partially filled with water. *[2 marks]*



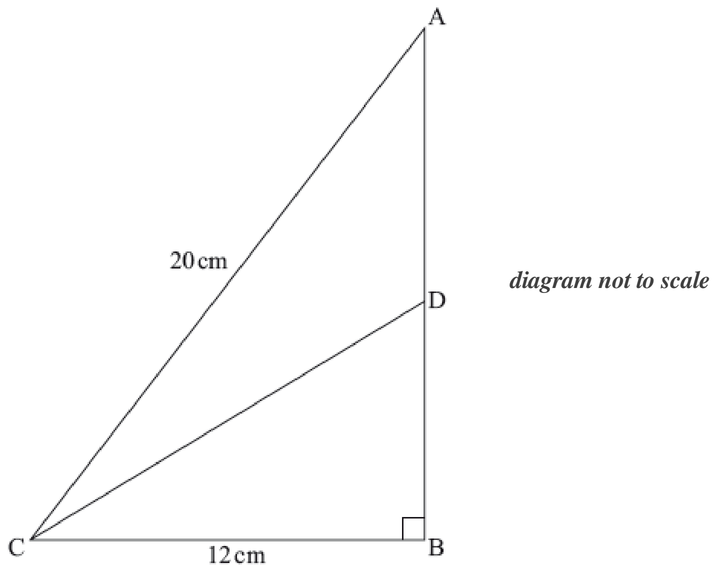
When the water tank is filled to a height of 0.9 m, the front cross-sectional area of the water is 2.55 m^2.

(i) Calculate the volume of water in the tank.

The total volume of the tank is 36 m^3.

(ii) Calculate the percentage of water in the tank.

In triangle ABC, $\text{AC} = 20$ cm, $\text{BC} = 12$ cm and $\text{A}\hat{\text{B}}\text{C} = 90^\circ$.

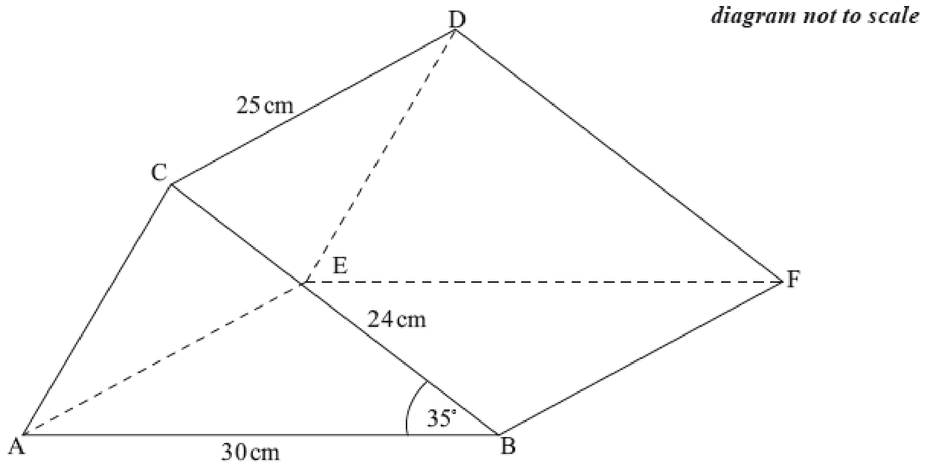


8a. Find the length of AB. *[2 marks]*

8b. D is the point on AB such that $\tan(\text{D}\hat{\text{C}}\text{B}) = 0.6$. Find the length of DB. *[2 marks]*

8c. D is the point on AB such that $\tan(\mathrm{D\hat{C}B}) = 0.6$. Find the area of triangle ADC. *[2 marks]*

A manufacturer has a contract to make 2600 solid blocks of wood. Each block is in the shape of a right triangular prism, ABCDEF, as shown in the diagram.
$\mathrm{AB} = 30\ \mathrm{cm},\ \mathrm{BC} = 24\ \mathrm{cm},\ \mathrm{CD} = 25\ \mathrm{cm}$ and angle $\mathrm{A\hat{B}C} = 35^\circ$.



9a. Calculate the length of AC. *[3 marks]*

9b. Calculate the area of triangle ABC. *[3 marks]*

9c. Assuming that no wood is wasted, show that the volume of wood required to make all 2600 blocks is $13\,400\,000\ \mathrm{cm}^3$, correct to three significant figures. *[2 marks]*

9d. Write $13\,400\,000$ in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. *[2 marks]*

9e. Show that the total surface area of one block is $2190\ \mathrm{cm}^2$, correct to three significant figures. *[3 marks]*

9f. The blocks are to be painted. One litre of paint will cover $22\ \mathrm{m}^2$. Calculate the number of litres required to paint all 2600 blocks. *[3 marks]*

Consider the function
$f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20.$

10a. Find [2 marks]
$f(-2)$.

10b. Find [3 marks]
$f'(x)$.

10c. The graph of the function [5 marks]
$f(x)$ has a local minimum at the point where
$x = -2$.
Using your answer to part (b), show that there is a second local minimum at
$x = 3$.

10d. The graph of the function [4 marks]
$f(x)$ has a local minimum at the point where
$x = -2$.
Sketch the graph of the function
$f(x)$ for
$-5 \leqslant x \leqslant 5$ and
$-40 \leqslant y \leqslant 50$. Indicate on your
sketch the coordinates of the
y-intercept.

10e. The graph of the function [2 marks]
$f(x)$ has a local minimum at the point where
$x = -2$.
Write down the coordinates of the local maximum.

10f. Let [2 marks]
T be the tangent to the graph of the function
$f(x)$ at the point
$(2, -12)$.
Find the gradient of
T.

10g. The line L passes through the point $(2, -12)$ and is perpendicular to T.
L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$.

[5 marks]

Find

(i) the gradient of L;

(ii) the value of b and the value of c.

ABC is a triangular field on horizontal ground. The lengths of AB and AC are 70 m and 50 m respectively. The size of angle BCA is 78°.

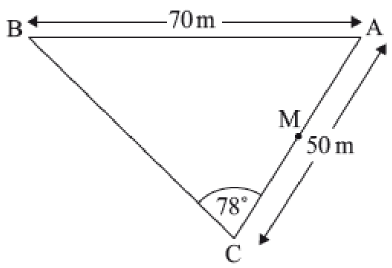


diagram not to scale

11a. Find the size of angle ABC.

[3 marks]

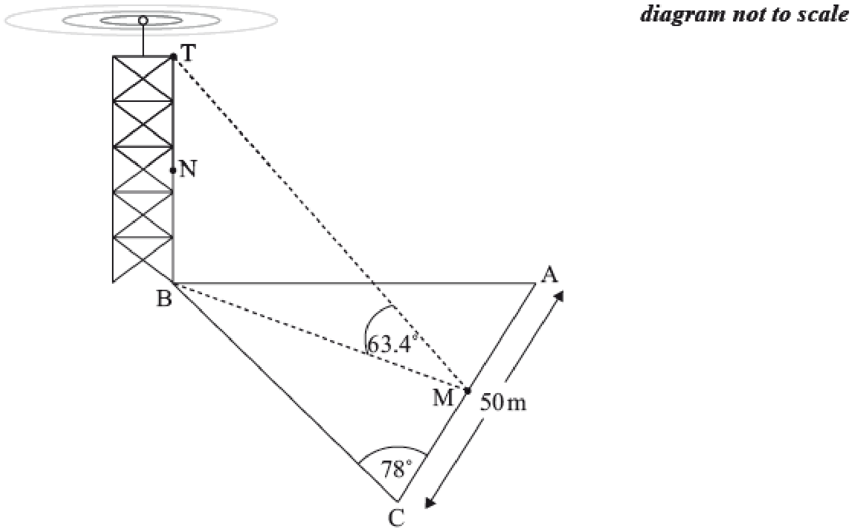
11b. Find the area of the triangular field.

[4 marks]

11c. M is the midpoint of AC.
Find the length of BM.

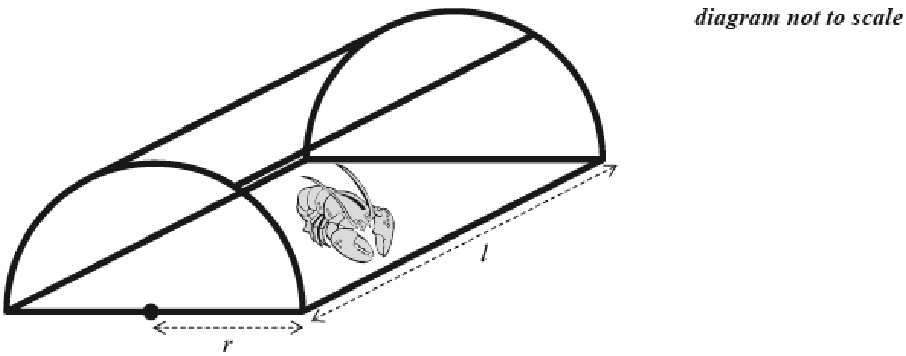
[3 marks]

11d. A vertical mobile phone mast, TB, is built next to the field with its base at B. The angle of elevation of T from M is 63.4°. N is the midpoint of the mast. *[5 marks]*



Calculate the angle of elevation of N from M.

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.



The semicircular ends each have radius r and the support rods each have length l. Let T be the total length of steel used in the frame of the lobster trap.

12a. Write down an expression for T in terms of r, l and π. *[3 marks]*

12b. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3 marks]

Write down an equation for the volume of the lobster trap in terms of

r,

l and

π.

12c. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [2 marks]

Show that

$T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

12d. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3 marks]

Find

$\frac{\mathrm{d}T}{\mathrm{d}r}$.

12e. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2 marks]

Show that the value of

r for which

T is a minimum is

$0.719\ \mathrm{m}$, correct to three significant figures.

12f. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2 marks]

Calculate the value of

l for which

T is a minimum.

12g. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2 marks]

Calculate the minimum value of

T.

A shipping container is a cuboid with dimensions
$16\ \mathrm{m}$,
$1\frac{3}{4}\ \mathrm{m}$ and
$2\frac{2}{3}\ \mathrm{m}$.

13a. Calculate the **exact** volume of the container. Give your answer as a fraction. [3 marks]

13b. Jim estimates the dimensions of the container as 15 m, 2 m and 3 m and uses these to estimate the volume of the container. [3 marks]

Calculate the percentage error in Jim's estimated volume of the container.

The volume of a sphere is
$V = \sqrt{\frac{S^3}{36\pi}}$, where
S is its surface area.

The surface area of a sphere is $500\ \mathrm{cm}^2$.

14a. Calculate the volume of the sphere. Give your answer correct to **two decimal places**. [3 marks]

14b. Write down your answer to (a) correct to the nearest integer. *[1 mark]*

14c. Write down your answer to (b) in the form *[2 marks]*
$a \times 10^n$, where
$1 \leqslant a < 10$ and
$n \in \mathbb{Z}$.

The diagram shows a triangle ABC in which AC = 17 cm. M is the midpoint of AC.
Triangle ABM is equilateral.

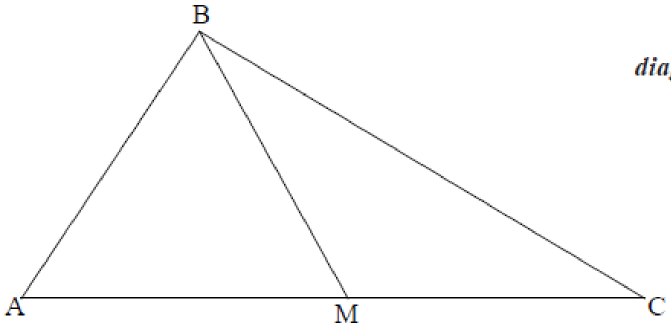


diagram not to scale

15a. Write down the size of angle MCB. *[1 mark]*

15b. Write down the length of BM in cm. *[1 mark]*

15c. Write down the size of angle BMC. *[1 mark]*

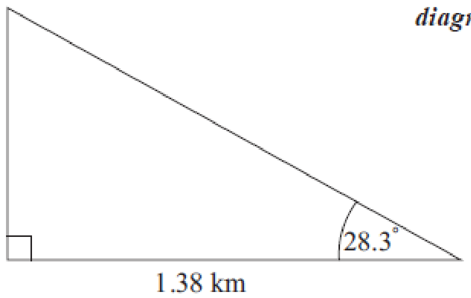
15d. Calculate the length of BC in cm. *[3 marks]*

José stands 1.38 kilometres from a vertical cliff.

16a. Express this distance in metres. *[1 mark]*

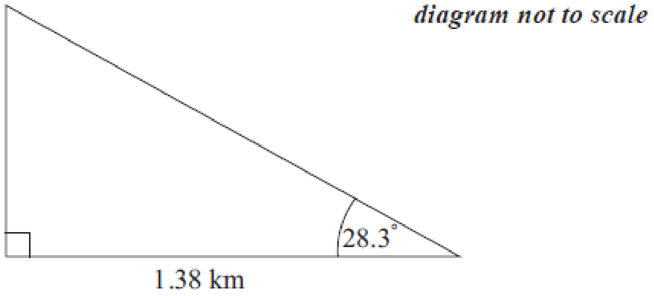
16b. José estimates the angle between the horizontal and the top of the cliff as 28.3° and uses it to find the height of the cliff. *[3 marks]*

diagram not to scale



Find the height of the cliff according to José’s calculation. **Express your answer in metres, to the nearest whole metre.**

16c. José estimates the angle between the horizontal and the top of the cliff as 28.3° and uses it to find the height of the cliff. *[2 marks]*



The actual height of the cliff is 718 metres. Calculate the percentage error made by José when calculating the height of the cliff.

The straight line, L_1, has equation
$y = -\frac{1}{2}x - 2$.

17a. Write down the y intercept of L_1. *[1 mark]*

17b. Write down the gradient of L_1. *[1 mark]*

17c. The line L_2 is perpendicular to L_1 and passes through the point (3, 7). *[1 mark]*

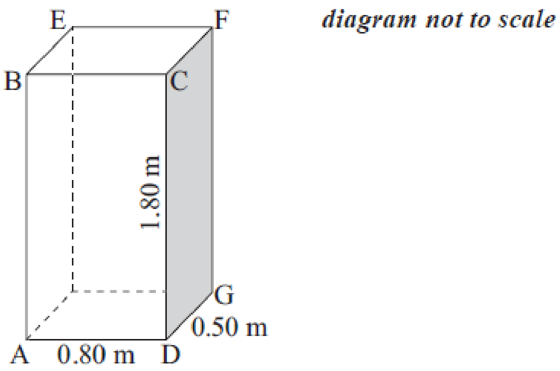
Write down the gradient of the line L_2.

17d. The line L_2 is perpendicular to L_1 and passes through the point (3, 7). *[3 marks]*

Find the equation of L_2. Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

A rectangular cuboid has the following dimensions.

Length	0.80 metres	(AD)
Width	0.50 metres	(DG)
Height	1.80 metres	(DC)



18a. Calculate the length of AG. *[2 marks]*

18b. Calculate the length of AF. *[2 marks]*

18c. Find the size of the angle between AF and AG. *[2 marks]*

The diagram shows triangle ABC. Point C has coordinates (4, 7) and the equation of the line AB is $x + 2y = 8$.

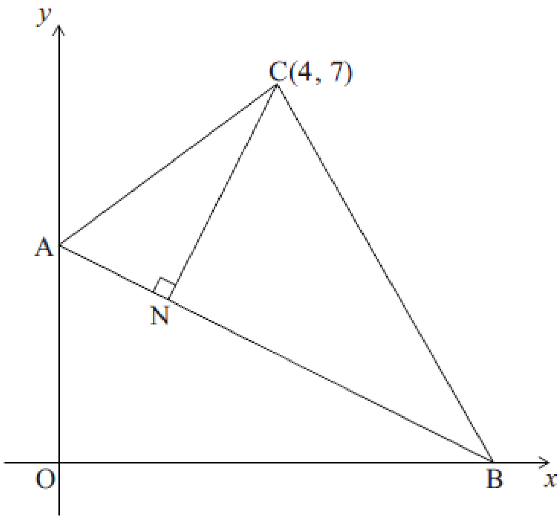


diagram not to scale

19a. Find the coordinates of A. *[1 mark]*

19b. Find the coordinates of B. *[1 mark]*

19c. Show that the distance between A and B is 8.94 correct to 3 significant figures. *[2 marks]*

19d. N lies on the line AB. The line CN is perpendicular to the line AB. *[3 marks]*

Find the gradient of CN.

19e. N lies on the line AB. The line CN is perpendicular to the line AB. *[2 marks]*

Find the equation of CN.

19f. N lies on the line AB. The line CN is perpendicular to the line AB. *[3 marks]*

Calculate the coordinates of N.

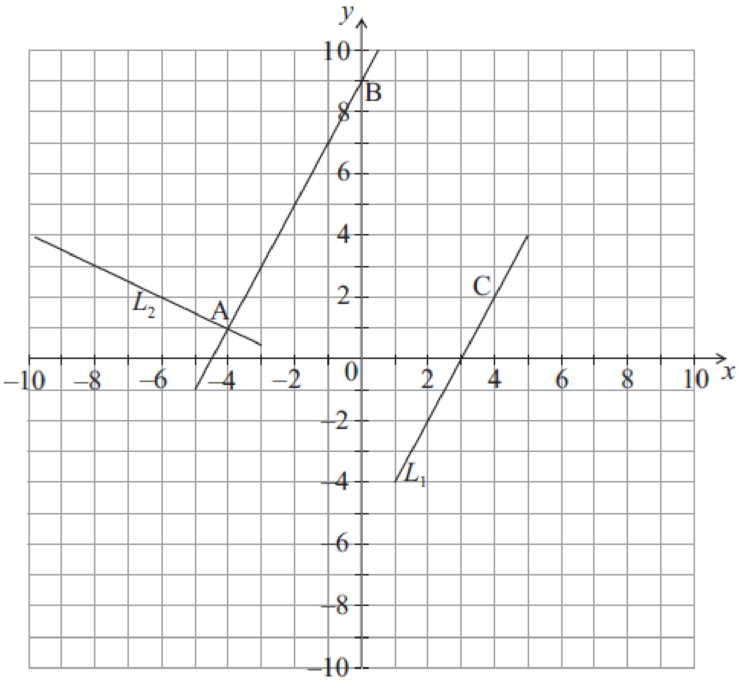
19g. It is known that AC = 5 and BC = 8.06. *[3 marks]*

Calculate the size of angle ACB.

19h. It is known that AC = 5 and BC = 8.06. *[3 marks]*

Calculate the area of triangle ACB.

The points A (–4, 1), B (0, 9) and C (4, 2) are plotted on the diagram below. The diagram also shows the lines AB, L_1 and L_2.



20a. Find the gradient of AB. *[2 marks]*

20b. L_1 passes through C and is parallel to AB. *[1 mark]*

Write down the y-intercept of L_1.

20c. L_2 passes through A and is perpendicular to AB. *[3 marks]*

Write down the equation of L_2. Give your answer in the form $\boldsymbol{ax + by + d = 0}$ where a, b and d $\in \mathbb{Z}$.

20d. Write down the coordinates of the point D, the intersection of L_1 and L_2. *[1 mark]*

20e. There is a point R on L_1 such that ABRD is a rectangle. *[2 marks]*

Write down the coordinates of R.

20f. The distance between A and D is $\sqrt{45}$. *[4 marks]*

(i) Find the distance between D and R .

(ii) Find the area of the triangle BDR .