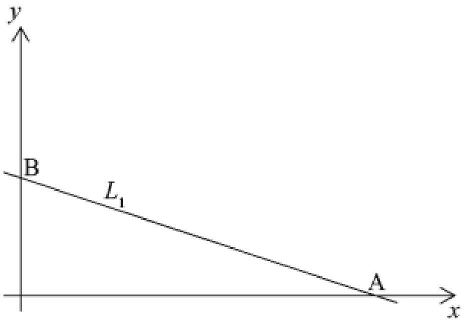
Topic 5 Part 6 *[213 marks]*

The diagram shows the straight line L_1, which intersects the x-axis at $\mathrm{A}(6,\ 0)$ and the y-axis at $\mathrm{B}(0,\ 2)$.



1a. Write down the coordinates of M, the midpoint of line segment AB. *[2 marks]*

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1b. Calculate the gradient of L_1. *[2 marks]*

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1c. The line L_2 is parallel to L_1 and passes through the point $(3,\ 2)$. *[2 marks]*

Find the equation of L_2. Give your answer in the form $y = mx + c$.

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Assume the Earth is a perfect sphere with radius 6371 km.

2a. Calculate the volume of the Earth in km^3. Give your answer in the form $a \times 10^k$, where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. *[3 marks]*

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2b. The volume of the Moon is $2.1958 \times 10^{10}\ \mathrm{km}^3$. *[3 marks]*

Calculate how many times greater in volume the Earth is compared to the Moon.

Give your answer correct to the nearest **integer**.

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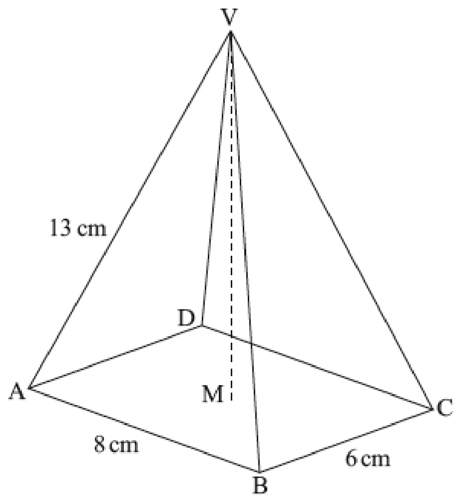
A right pyramid has apex V and rectangular base ABCD, with $\mathrm{AB} = 8\ \mathrm{cm}$, $\mathrm{BC} = 6\ \mathrm{cm}$ and $\mathrm{VA} = 13\ \mathrm{cm}$. The vertical height of the pyramid is VM.



3a. Calculate VM. *[4 marks]*

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3b. Calculate the volume of the pyramid. *[2 marks]*

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A cuboid has a rectangular base of width x cm and length 2
x cm . The height of the cuboid is h cm . The total length of the edges of the cuboid is 72 cm.

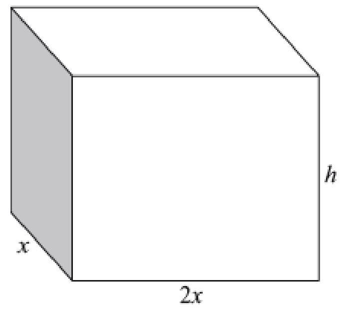

diagram not to scale

The volume, V, of the cuboid can be expressed as $V = ax^2 - 6x^3$.

4a. Find the value of a. *[3 marks]*

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4b. Find the value of x that makes the volume a maximum. *[3 marks]*

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The distance
d from a point $\mathrm{P}(x,\ y)$ to the point $\mathrm{A}(1,\ -2)$ is given by
$d=\sqrt{(x-1)^2+(y+2)^2}$

5a. Find the distance from $\mathrm{P}(100,\ 200)$ to A. Give your answer correct to two decimal places. *[3 marks]*

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5b. Write down your answer to **part (a)** correct to three significant figures. *[1 mark]*

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5c. Write down your answer to **part (b)** in the form $a\times 10^k$, where $1\leqslant a<10$ and $k\in\mathbb{Z}$. *[2 marks]*

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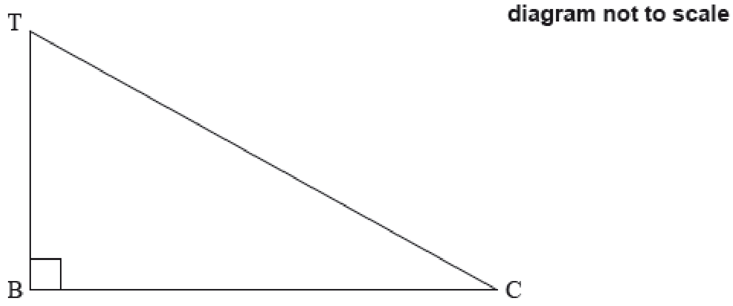
Fabián stands on top of a building, T, which is on a horizontal street.

He observes a car, C, on the street, at an angle of depression of 30°. The base of the building is at B. The height of the building is 80 metres.

The following diagram indicates the positions of T, B and C.



6a. Show, in the appropriate place on the diagram, **the values** of *[2 marks]*

(i) the height of the building;

(ii) the angle of depression.

6b. Find the distance, BC, from the base of the building to the car. *[2 marks]*

6c. Fabián estimates that the distance from the base of the building to the car is 150 metres. Calculate the percentage error of Fabián’s estimate. *[2 marks]*

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The equation of the line L_1 is $2x + y = 10$.

7a. Write down *[2 marks]*

(i) the gradient of L_1;

(ii) the y-intercept of L_1.

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7b. The line L_2 is parallel to L_1 and passes through the point $\mathrm{P}(0,\ 3)$. *[2 marks]*

Write down the equation of L_2.

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7c. The line L_2 is parallel to L_1 and passes through the point $\mathrm{P}(0,\ 3)$. *[2 marks]*

Find the x-coordinate of the point where L_2 crosses the x-axis.

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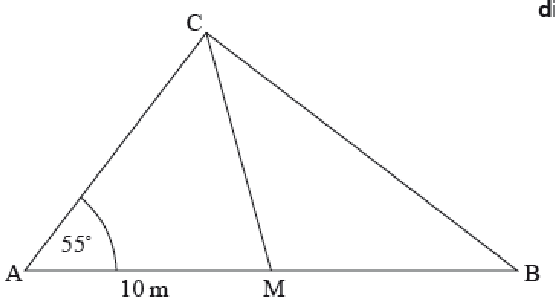
The diagram shows a triangle ABC. The size of angle $\mathrm{C\hat{A}B}$ is 55° and the length of AM is 10 m, where M is the midpoint of AB. Triangle CMB is isosceles with $\mathrm{CM} = \mathrm{MB}$.

diagram not to scale



8a. Write down the length of MB. *[1 mark]*

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8b. Find the size of angle $\mathrm{C\hat{M}B}$. *[2 marks]*

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8c. Find the length of CB. *[3 marks]*

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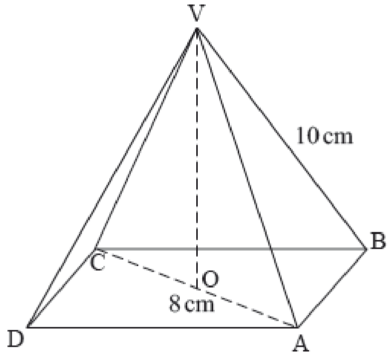
In the following diagram, ABCD is the square base of a right pyramid with vertex V. The centre of the base is O. The diagonal of the base, AC, is 8 cm long. The sloping edges are 10 cm long.

diagram not to scale



9a. Write down the length of AO. *[1 mark]*

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9b. Find the size of the angle that the sloping edge VA makes with the base of the pyramid. *[2 marks]*

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9c. Hence, or otherwise, find the area of the triangle CAV. *[3 marks]*

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A building company has many rectangular construction sites, of varying widths, along a road.

The area, A, of each site is given by the function

$$A(x) = x(200 - x)$$

where x is the **width** of the site in metres and $20 \leqslant x \leqslant 180$.

10a. Site S has a width of 20 m. Write down the area of S. *[1 mark]*

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10b. Site T has the same area as site S, but a different width. Find the width of T. *[2 marks]*

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10c. When the width of the construction site is b metres, the site has a maximum area. *[2 marks]*

(i) Write down the value of b.

(ii) Write down the maximum area.

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10d. The range of $A(x)$ is $m \leqslant A(x) \leqslant n$. *[1 mark]*

Hence write down the value of m and of n.

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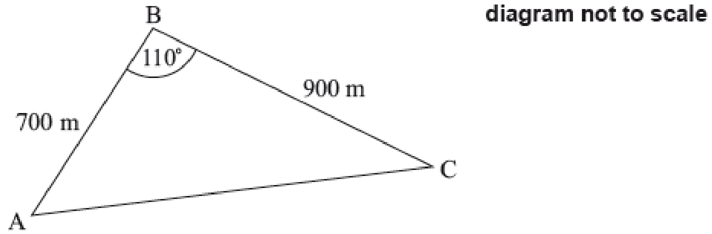
A boat race takes place around a triangular course, ABC, with $\mathrm{AB} = 700\ \mathrm{m}$, $\mathrm{BC} = 900\ \mathrm{m}$ and angle $\mathrm{ABC} = 110^{\circ}$. The race starts and finishes at point A.



11a. Calculate the total length of the course. *[4 marks]*

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11b. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. *[3 marks]*

Calculate an estimate of the winning time of the race. Give your answer to the nearest minute.

11c. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. *[3 marks]*

Find the size of angle ACB.

11d. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. *[3 marks]*

Calculate the area that must be kept clear of boats.

11e. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. *[3 marks]*

Determine, giving a reason, whether the course complies with the safety regulations.

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11f. The race is filmed from a helicopter, H, which is flying vertically above point A. *[2 marks]*

The angle of elevation of H from B is 15°.

Calculate the vertical height, AH, of the helicopter above A.

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11g. The race is filmed from a helicopter, H, which is flying vertically above point A. *[3 marks]*

The angle of elevation of H from B is 15°.

Calculate the maximum possible distance from the helicopter to a boat on the course.

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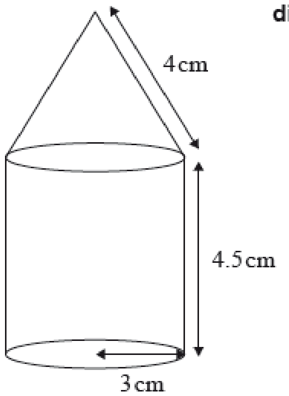
The following diagram shows a perfume bottle made up of a cylinder and a cone.



The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

12a. (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures. *[6 marks]*

(ii) Calculate the volume of the perfume bottle.

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12b. The bottle contains 125 cm^3 of perfume. The bottle is **not** full and all of the perfume is in the cylinder part. *[2 marks]*

Find the height of the perfume in the bottle.

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12c. Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle. *[4 marks]*

Find the **total** surface area of the perfume bottle.

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12d. Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of 7 cm^2. *[4 marks]*

Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places**.

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12e. Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is 1 EUR = 13.03 ZAR. *[2 marks]*

Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places**.

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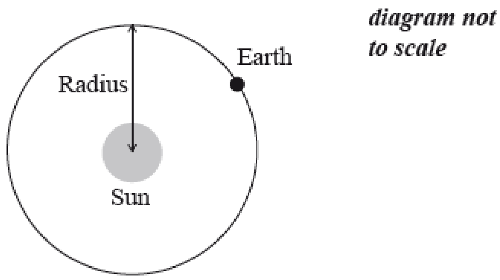
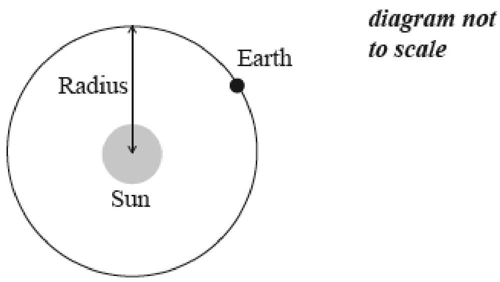
The average radius of the orbit of the Earth around the Sun is 150 million kilometres.



13a. Write down this radius, in kilometres, in the form $a \times 10^k$, where $1 \leqslant a < 10,\ k \in \mathbb{Z}$. *[2 marks]*

The average radius of the orbit of the Earth around the Sun is 150 million kilometres.



13b. The Earth travels around the Sun in one orbit. It takes one year for the Earth to complete one orbit. *[2 marks]*

Calculate the distance, in kilometres, the Earth travels around the Sun in one orbit, assuming that the orbit is a circle.

13c. Today is Anna's 17th birthday. *[2 marks]*

Calculate the total distance that Anna has travelled around the Sun, since she was born.

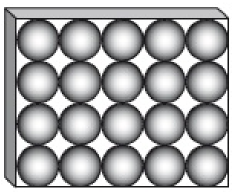
Chocolates in the shape of spheres are sold in boxes of 20.

Each chocolate has a radius of 1 cm.

14a. Find the volume of 1 chocolate. *[2 marks]*

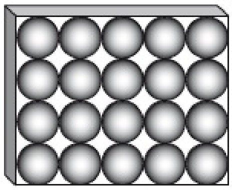
14b. Write down the volume of 20 chocolates. *[1 mark]*

14c. The diagram shows the chocolate box from above. The 20 chocolates fit perfectly in the box with each chocolate touching the ones around it or the sides of the box. *[2 marks]*



Calculate the volume of the box.

14d. The diagram shows the chocolate box from above. The 20 chocolates fit perfectly in the box with each chocolate touching the ones around it or the sides of the box. *[1 mark]*



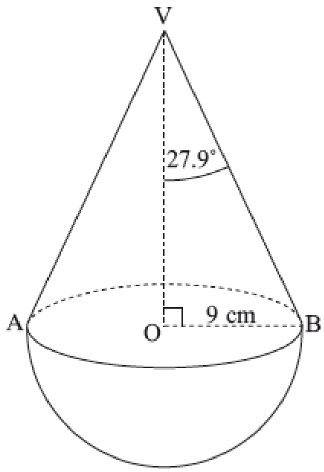
Calculate the volume of empty space in the box.

A child’s wooden toy consists of a hemisphere, of radius 9 cm , attached to a cone with the same base radius. O is the centre of the base of the cone and V is vertically above O.

Angle OVB is

27.9°.

Diagram not to scale.

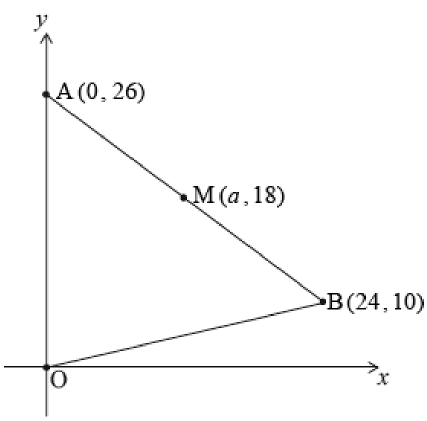


15a. Calculate OV, the height of the cone. *[2 marks]*

15b. Calculate the volume of wood used to make the toy. *[4 marks]*

The diagram shows the points M(a, 18) and B(24, 10) . The straight line BM intersects the y-axis at A(0, 26). M is the midpoint of the line segment AB.

diagram not to scale



16a. Write down the value of a. *[1 mark]*

16b. Find the gradient of the line AB. *[2 marks]*

16c. Decide whether triangle OAM is a right-angled triangle. Justify your answer. *[3 marks]*

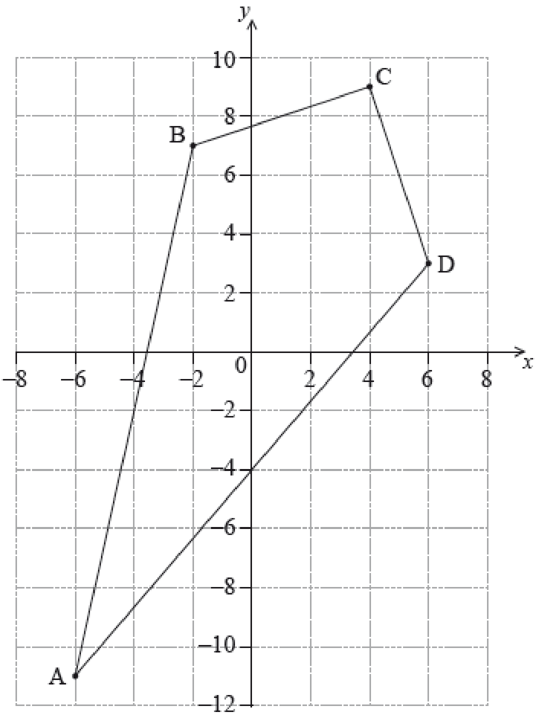
Let
$f(x) = x^4$.

17a. Write down *[1 mark]*
$f'(x)$.

17b. Point *[2 marks]*
$\mathrm{P}(2, 6)$ lies on the graph of
f.
Find the gradient of the tangent to the graph of
$y = f(x)$ at
P.

17c. Point *[3 marks]*
$\mathrm{P}(2, 16)$ lies on the graph of
f.
Find the equation of the normal to the graph at
P. Give your answer in the form
$ax + by + d = 0$, where
a,
b and
d are integers.

The four points A(−6, −11) , B(−2, 7) , C(4, 9) and D(6, 3) define the vertices of a kite.



18a. Calculate the distance between points B and D. *[2 marks]*

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18b. The distance between points A and C is $\sqrt{500}$. *[4 marks]*

Calculate the area of the kite ABCD.

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The number of apartments in a housing development has been increasing by a constant amount every year.

At the end of the first year the number of apartments was 150, and at the end of the sixth year the number of apartments was 600.

The number of apartments, y, can be determined by the equation $y = mt + n$, where t is the time, in years.

19a. Find the value of m. *[2 marks]*

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19b. State what m represents **in this context**. *[1 mark]*

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19c. Find the value of n. *[2 marks]*

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19d. State what n represents **in this context**. *[1 mark]*

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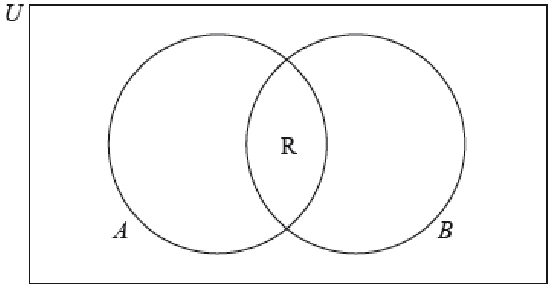
Tuti has the following polygons to classify: rectangle (R), rhombus (H), isosceles triangle (I), regular pentagon (P), and scalene triangle (T).

In the Venn diagram below, set A consists of the polygons that have at least one pair of parallel sides, and set B consists of the polygons that have at least one pair of equal sides.



20a. Complete the Venn diagram by placing the letter corresponding to each polygon in the appropriate region. For example, R has already been placed, and represents the rectangle. *[3 marks]*

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20b. State which polygons from Tuti's list are elements of *[3 marks]*

(i) $A \cap B$;

(ii) $(A \cup B)'$.

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A hotel has a rectangular swimming pool. Its length is x metres, its width is y metres and its perimeter is 44 metres.

21a. Write down an equation for x and y. *[1 mark]*

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21b. The area of the swimming pool is $112\mathrm{m}^2$. *[1 mark]*

Write down a second equation for x and y.

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21c. Use your graphic display calculator to find the value of x and the value of y. *[2 marks]*

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21d. An Olympic sized swimming pool is 50 m long and 25 m wide. *[2 marks]*

Determine the area of the hotel swimming pool as a percentage of the area of an Olympic sized swimming pool.

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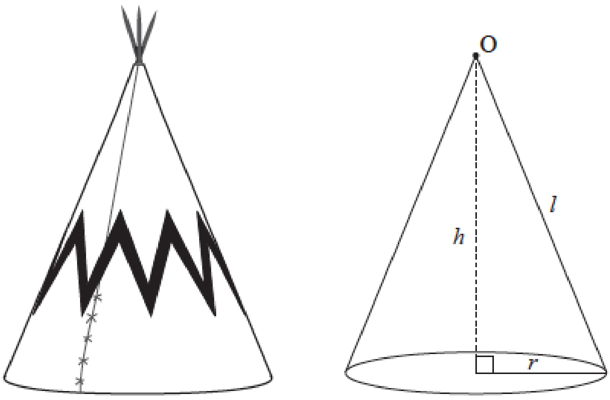
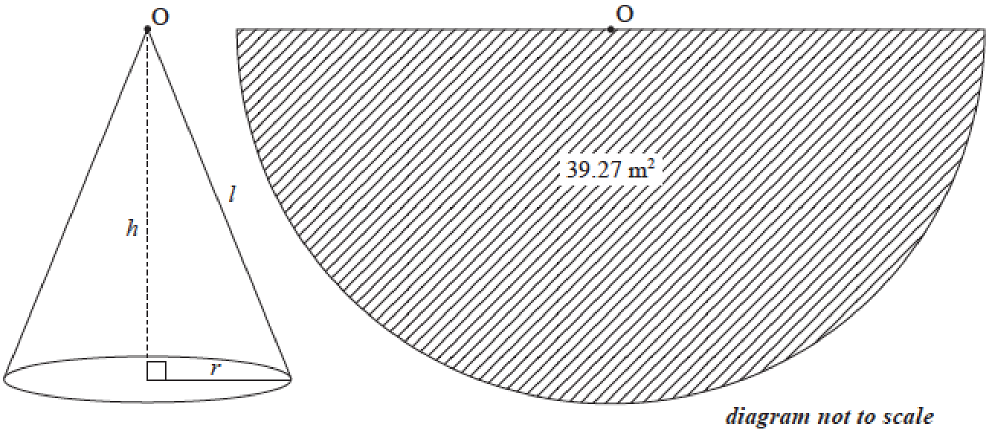
Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r, height, h, and slant height, l.



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is $39.27\ \mathrm{m}^2$, and has the shape of a semicircle, as shown in the following diagram.



22a. Show that the slant height, l, is 5 m, correct to the nearest metre. *[2 marks]*

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22b. (i) Find the circumference of the base of the cone. *[6 marks]*

(ii) Find the radius, r, of the base.

(iii) Find the height, h.

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22c. A company designs cone-shaped tents to resemble the traditional tepees. *[1 mark]*

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Write down an expression for the height, h, in terms of the radius, r, of these cone-shaped tents.

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22d. A company designs cone-shaped tents to resemble the traditional tepees. *[1 mark]*

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V, can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

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22e. A company designs cone-shaped tents to resemble the traditional tepees. *[2 marks]*

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

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22f. A company designs cone-shaped tents to resemble the traditional tepees. *[4 marks]*

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

(i) Determine the exact value of r for which the volume is a maximum.

(ii) Find the maximum volume.

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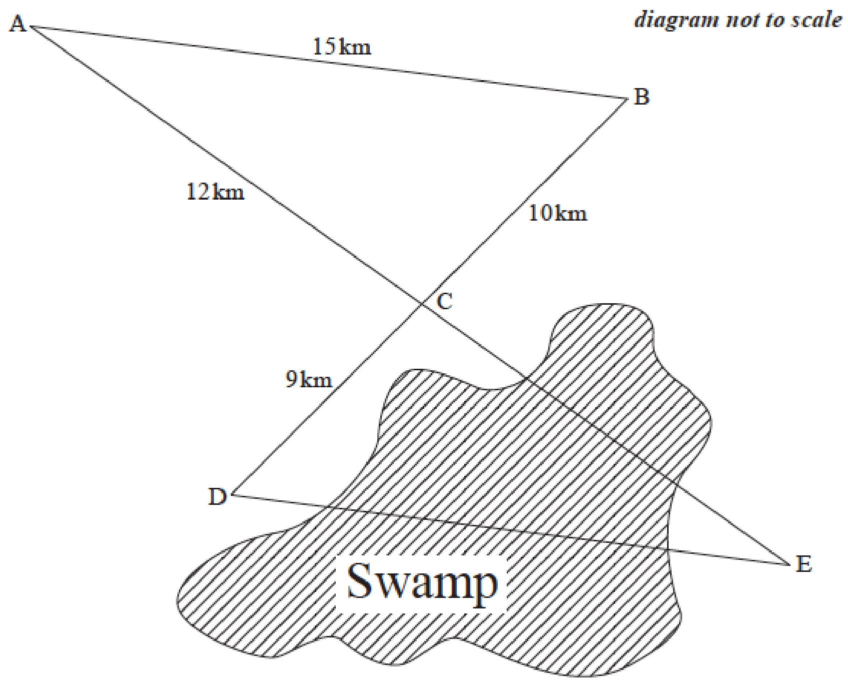
A surveyor has to calculate the area of a triangular piece of land, DCE.

The lengths of CE and DE cannot be directly measured because they go through a swamp.

AB, DE, BD and AE are straight paths. Paths AE and DB intersect at point C.

The length of AB is 15 km, BC is 10 km, AC is 12 km, and DC is 9 km.

The following diagram shows the surveyor's information.



23a. (i) Find the size of angle ACB. *[4 marks]*

(ii) Show that the size of angle DCE is 85.5°, correct to one decimal place.

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23b. The surveyor measures the size of angle CDE to be twice that of angle DEC. *[5 marks]*

(i) Using angle $\mathrm{DCE} = 85.5^{\circ}$, find the size of angle DEC.

(ii) Find the length of DE.

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23c. Calculate the area of triangle DEC. *[4 marks]*

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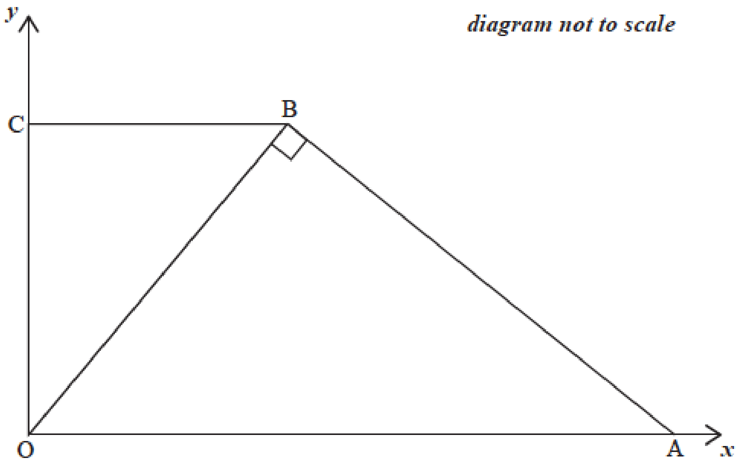
The following diagram shows two triangles, OBC and OBA, on a set of axes. Point C lies on the y-axis, and O is the origin.



24a. The equation of the line BC is $y = 4$. *[1 mark]*

Write down the coordinates of point C.

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24b. The x-coordinate of point B is a. *[2 marks]*

(i) Write down the coordinates of point B;

(ii) Write down the gradient of the line OB.

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24c. Point A lies on the x-axis and the line AB is perpendicular to line OB. *[4 marks]*

(i) Write down the gradient of line AB.

(ii) Show that the equation of the line AB is $4y + ax - a^2 - 16 = 0$.

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24d. The area of triangle AOB is **three times** the area of triangle OBC. *[3 marks]*

Find an expression, **in terms of *a***, for

(i) the area of triangle OBC;

(ii) the *x*-coordinate of point A.

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24e. Calculate the value of a. *[2 marks]*

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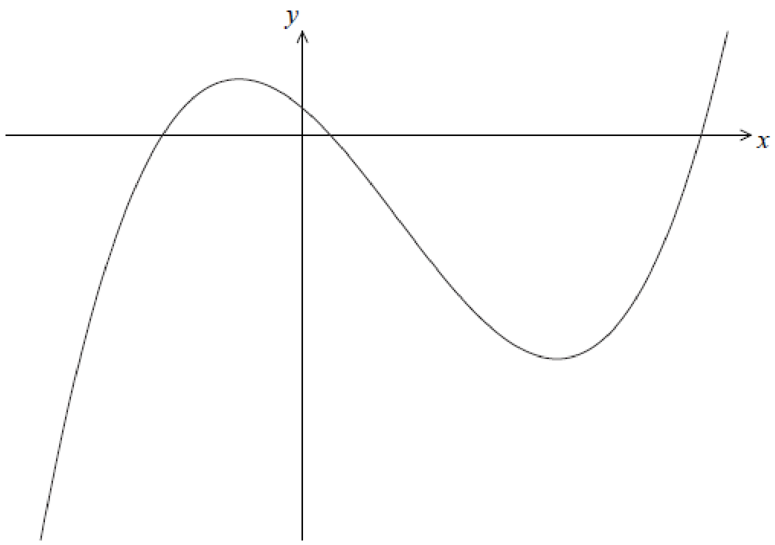
The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

diagram not to scale



25a. Write down the values of x where the graph of $f(x)$ intersects the x-axis. *[3 marks]*

25b. Write down $f'(x)$. *[3 marks]*

25c. Find the value of the local maximum of $y = f(x)$. *[4 marks]*

25d. Let P be the point where the graph of $f(x)$ intersects the y axis. *[1 mark]*

Write down the coordinates of P.

25e. Let P be the point where the graph of $f(x)$ intersects the y axis. *[2 marks]*

Find the gradient of the curve at P.

25f. The line, L, is the tangent to the graph of $f(x)$ at P. *[2 marks]*

Find the equation of L in the form $y = mx + c$.

25g. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. *[1 mark]*

Write down the gradient of the tangent at Q.

25h. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. *[3 marks]*

Calculate the x-coordinate of Q.