Topic 6 Part 3 *[209 marks]*

A hotel has a rectangular swimming pool. Its length is x metres, its width is y metres and its perimeter is 44 metres.

1a. Write down an equation for x and y. *[1 mark]*

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1b. The area of the swimming pool is $112\mathrm{m}^2$. *[1 mark]*

Write down a second equation for x and y.

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1c. Use your graphic display calculator to find the value of x and the value of y. *[2 marks]*

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1d. An Olympic sized swimming pool is 50 m long and 25 m wide. *[2 marks]*

Determine the area of the hotel swimming pool as a percentage of the area of an Olympic sized swimming pool.

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Consider the functions $f(x) = x + 1$ and $g(x) = 3^x - 2$.

2a. Write down *[2 marks]*

(i) the x-intercept of the graph of $y = f(x)$;

(ii) the y-intercept of the graph of $y = g(x)$.

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2b. Solve $f(x) = g(x)$. *[2 marks]*

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2c. Write down the interval for the values of x for which $f(x) > g(x)$. *[2 marks]*

A potato is placed in an oven heated to a temperature of 200°C.

The temperature of the potato, in °C, is modelled by the function $p(t) = 200 - 190(0.97)^t$, where t is the time, in minutes, that the potato has been in the oven.

3a. Write down the temperature of the potato at the moment it is placed in the oven. *[2 marks]*

3b. Find the temperature of the potato half an hour after it has been placed in the oven. *[2 marks]*

3c. After the potato has been in the oven for k minutes, its temperature is 40°C. *[2 marks]*

Find the value of k.

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The axis of symmetry of the graph of a quadratic function has the equation $x = -\frac{1}{2}$

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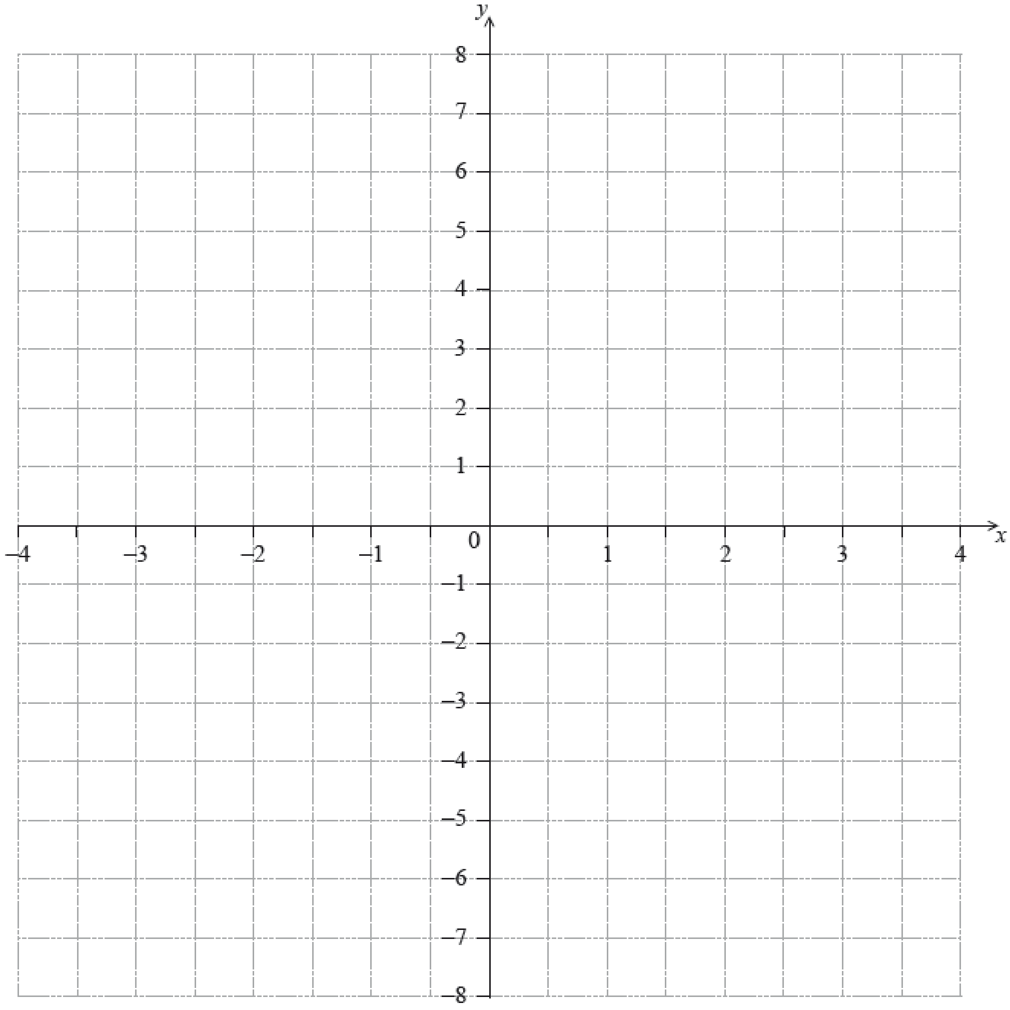
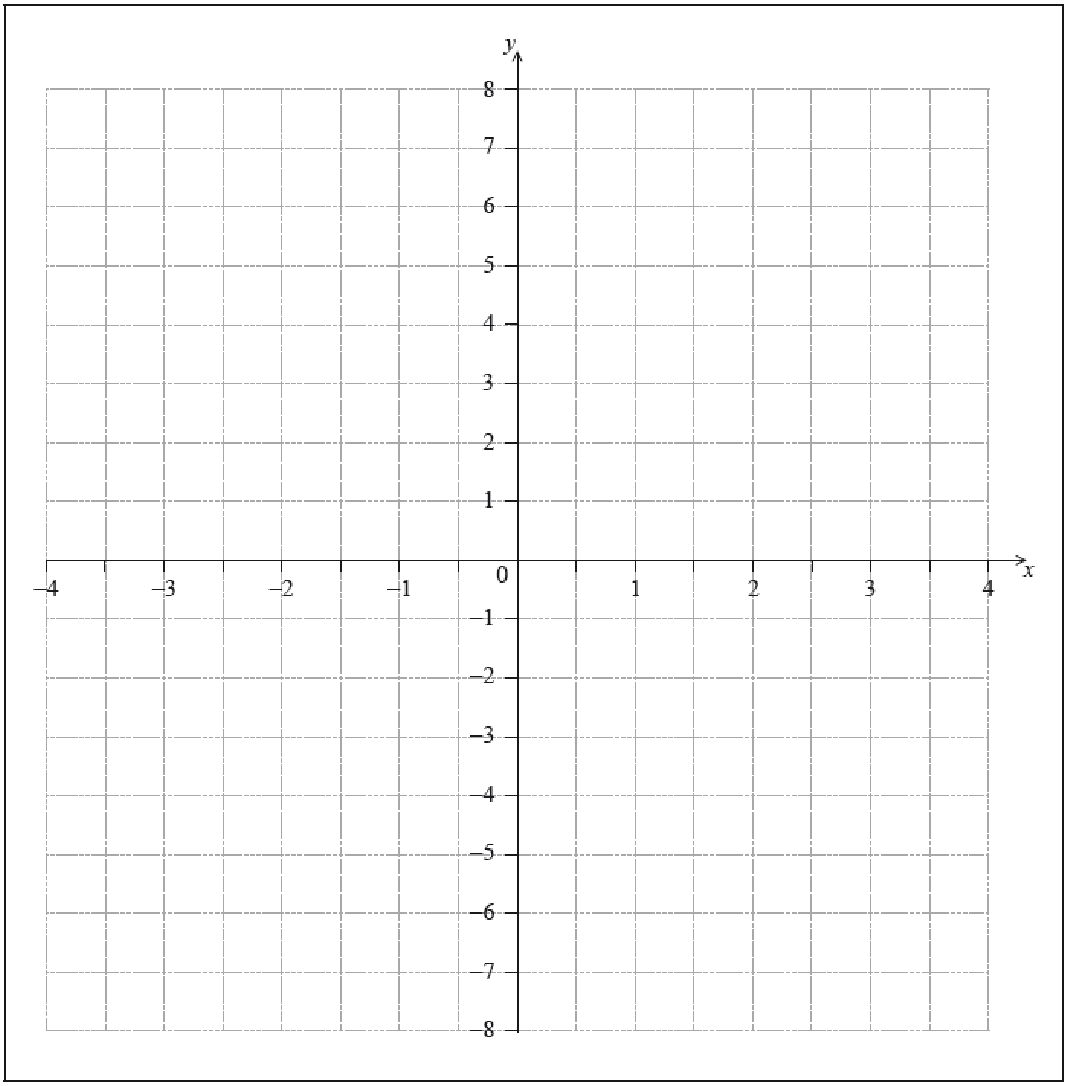
4a. Draw the axis of symmetry on the following axes. *[1 mark]*



The graph of the quadratic function intersects the x-axis at the point N(2, 0) . There is a second point, M, at which the graph of the quadratic function intersects the x-axis.

4b. Draw the axis of symmetry on the following axes. *[1 mark]*



4c. The graph of the quadratic function intersects the x-axis at the point $\mathrm{N}(2, 0)$. There is a second point, M, at which the graph of the quadratic function intersects the x-axis. *[1 mark]*

Clearly mark and label point M on the axes.

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4d. (i) Find the value of b and the value of c. *[4 marks]*

(ii) Draw the graph of the function on the axes.

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An iron bar is heated. Its length, L, in millimetres can be modelled by a linear function, $L = mT + c$, where T is the temperature measured in degrees Celsius (°C).

5a. At 150°C the length of the iron bar is 180 mm. *[1 mark]*

Write down an equation that shows this information.

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5b. At 210°C the length of the iron bar is 181.5 mm. *[1 mark]*

Write down an equation that shows this second piece of information.

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5c. At 210°C the length of the iron bar is 181.5 mm. *[4 marks]*

Hence, find the length of the iron bar at 40°C.

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Consider the quadratic function, $f(x) = px(q - x)$, where p and q are positive integers.

The graph of
$y = f(x)$ passes through the point $(6,\ 0)$.

6a. Calculate the value of q. *[2 marks]*

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6b. The vertex of the function is $(3,\ 27)$. *[2 marks]*

Find the value of p.

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6c. The vertex of the function is $(3,\ 27)$. *[2 marks]*

Write down the range of f.

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The number of fish, N, in a pond is decreasing according to the model

$$N(t) = ab^{-t} + 40, \quad t \geqslant 0$$

where
a and
b are positive constants, and
t is the time in months since the number of fish in the pond was first counted.

At the beginning 840 fish were counted.

7a. Find the value of a. *[2 marks]*

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7b. After 4 months 90 fish were counted. *[3 marks]*

Find the value of b.

7c. The number of fish in the pond will **not** decrease below p. *[1 mark]*

Write down the value of p.

A building company has many rectangular construction sites, of varying widths, along a road.

The area, A, of each site is given by the function

$$A(x) = x(200 - x)$$

where x is the **width** of the site in metres and $20 \leqslant x \leqslant 180$.

8a. Site S has a width of 20 m. Write down the area of S. *[1 mark]*

8b. Site T has the same area as site S, but a different width. Find the width of T. *[2 marks]*

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8c. When the width of the construction site is b metres, the site has a maximum area. *[2 marks]*

(i) Write down the value of b.

(ii) Write down the maximum area.

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8d. The range of $A(x)$ is $m \leqslant A(x) \leqslant n$. *[1 mark]*

Hence write down the value of m and of n.

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Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

9a. Write down $f'(x)$. *[3 marks]*

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9b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

Show that $k = 3$.

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9c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

Find $f(2)$.

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9d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

Find $f'(2)$

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9e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[3 marks]*

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

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9f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[4 marks]*

Sketch the graph of
$y = f(x)$, for $-5 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 100$.

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9g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis.

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9h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. *[2 marks]*

State the values of x for which $f(x)$ is decreasing.

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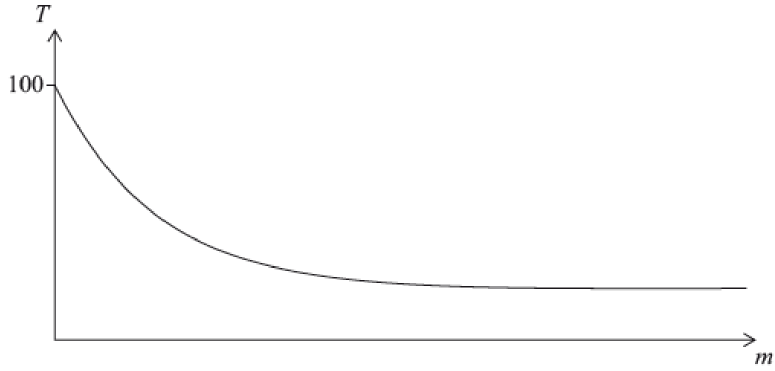
A cup of boiling water is placed in a room to cool. The temperature of the room is 20°C . This situation can be modelled by the exponential function $T = a + b(k^{-m})$, where T is the temperature of the water, in °C , and m is the number of minutes for which the cup has been placed in the room. A sketch of the situation is given as follows.



10a. Explain why $a = 20$. *[2 marks]*

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10b. Initially, at $m = 0$, the temperature of the water is 100°C. *[2 marks]*

Find the value of b.

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10c. After being placed in the room for one minute, the temperature of the water is 84°C. *[2 marks]*

Show that $k = 1.25$.

10d. After being placed in the room for one minute, the temperature of the water is 84°C. *[2 marks]*

Find the temperature of the water three minutes after it has been placed in the room.

10e. After being placed in the room for one minute, the temperature of the water is 84°C. *[2 marks]*

Find the total time needed for the water to reach a temperature of 35°C. Give your answer in minutes and seconds, correct to the nearest second.

Consider the function $f(x) = 0.5x^2 - \frac{8}{x},\ x \neq 0$.

11a. Find $f(-2)$. *[2 marks]*

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11b. Find $f'(x)$. *[3 marks]*

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11c. Find the gradient of the graph of f at $x = -2$. *[2 marks]*

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11d. Let T be the tangent to the graph of f at $x = -2$. *[2 marks]*

Write down the equation of T.

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11e. Let T be the tangent to the graph of f at $x = -2$. *[4 marks]*

Sketch the graph of f for $-5 \leqslant x \leqslant 5$ and $-20 \leqslant y \leqslant 20$.

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11f. Let T be the tangent to the graph of f at $x = -2$. *[2 marks]*

Draw T on your sketch.

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11g. The tangent, T, intersects the graph of f at a second point, P. *[2 marks]*

Use your graphic display calculator to find the coordinates of P.

The following table shows the number of bicycles, x, produced daily by a factory and their total production cost, y, in US dollars (USD). The table shows data recorded over seven days.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Number of bicycles, x	12	15	14	17	20	18	21
Production cost, y	3900	4600	4100	5300	6000	5400	6000

12a. (i) Write down the Pearson's product–moment correlation coefficient, r, for these data. *[4 marks]*

(ii) Hence comment on the result.

12b. Write down the equation of the regression line y on x for these data, in the form $y = ax + b$. *[2 marks]*

12c. Estimate the total cost, **to the nearest USD**, of producing 13 bicycles on a particular day. *[3 marks]*

12d. All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**. *[2 marks]*

Explain why the factory does **not** make a profit when producing 13 bicycles on a particular day.

12e. All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**. *[5 marks]*

(i) Write down an expression for the total selling price of x bicycles.

(ii) Write down an expression for the **profit** the factory makes when producing x bicycles on a particular day.

(iii) Find the least number of bicycles that the factory should produce, on a particular day, in order to make a profit.

13a. Sketch the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$. Indicate clearly where the curve intersects the y-axis. *[3 marks]*

13b. Write down the equation of the asymptote of the graph of $y = 2^x$. *[2 marks]*

13c. On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. *[3 marks]*

13d. Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. *[2 marks]*

13e. Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. *[1 mark]*

13f. Use Differential Calculus to verify that your answer to (e) is correct. *[5 marks]*

13g. The curve $y = px^2 + qx - 4$ passes through the point (2, –10). *[2 marks]*

Use the above information to write down an equation in p and q.

13h. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. *[2 marks]*

Find $\frac{dy}{dx}$.

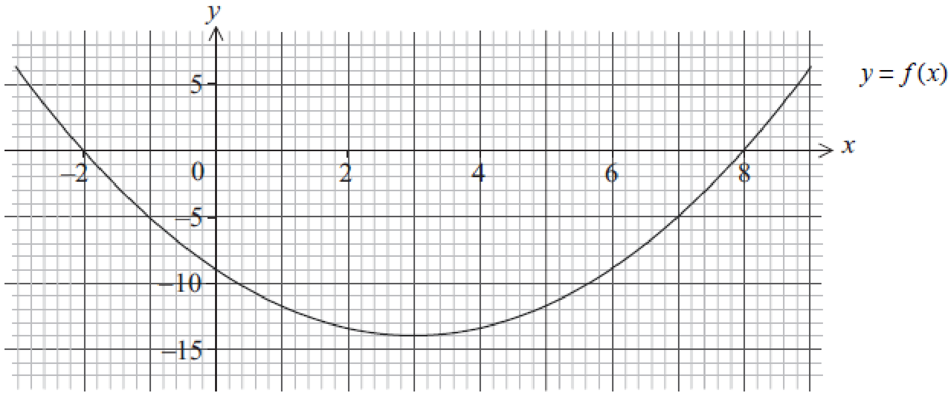
13i. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. *[1 mark]*

Hence, find a second equation in p and q.

13j. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. *[3 marks]*

Solve the equations to find the value of p and of q.

The graph of a quadratic function $y = f(x)$ is given below.



14a. Write down the equation of the axis of symmetry. *[2 marks]*

14b. Write down the coordinates of the minimum point. *[2 marks]*

14c. Write down the range of $f(x)$. *[2 marks]*

The number of cells, C, in a culture is given by the equation $C = p \times 2^{0.5t} + q$, where t is the time in hours measured from 12:00 on Monday and p and q are constants.

The number of cells in the culture at 12:00 on Monday is 47.

The number of cells in the culture at 16:00 on Monday is 53.

15a. Use the above information to write down two equations in p and q ; *[2 marks]*

15b. Use the above information to calculate the value of p and of q ; *[2 marks]*

15c. Use the above information to find the number of cells in the culture at 22:00 on Monday. *[2 marks]*

The straight line, L, has equation
$2y - 27x - 9 = 0$.

16a. Find the gradient of L. *[2 marks]*

16b. Sarah wishes to draw the tangent to
$f(x) = x^4$ parallel to L. *[1 mark]*

Write down
$f'(x)$.

16c. Find the x coordinate of the point at which the tangent must be drawn. *[2 marks]*

16d. Write down the value of
$f(x)$ at this point. *[1 mark]*

Consider the function
$f(x) = 3x + \frac{12}{x^2}$, $x \neq 0$.

17a. Differentiate
$f(x)$ with respect to
x. *[3 marks]*

17b. Calculate
$f'(x)$ when
$x = 1$. *[2 marks]*

17c. Use your answer to part (b) to decide whether the function,
f, is increasing or decreasing at
$x = 1$. Justify your answer. *[2 marks]*

17d. Solve the equation
$f'(x) = 0$. *[3 marks]*

17e. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. *[2 marks]*

Write down the coordinates of P.

17f. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. *[1 mark]*

Write down the gradient of T.

17g. The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P. *[2 marks]*

Write down the equation of T.

17h. Sketch the graph of the function f, for $-3 \leq x \leq 6$ and $-7 \leq y \leq 15$. Indicate clearly the point P and any intercepts of the curve with the axes. *[4 marks]*

17i. On your graph draw and label the tangent T. *[2 marks]*

17j. T intersects the graph of f at a second point. Write down the x-coordinate of this point of intersection. *[1 mark]*

A manufacturer claims that fertilizer has an effect on the height of rice plants. He measures the height of fertilized and unfertilized plants. The results are given in the following table.

Plant height	Fertilized plants	Unfertilized plants
>75 cm	115	80
$50-75$ cm	45	65
<50 cm	20	35

A chi-squared test is performed to decide if the manufacturer's claim is justified at the **1 %** level of significance.

18a. Write down the null and alternative hypotheses for this test. *[2 marks]*

18b. For the number of fertilized plants with height greater than 75 cm, show that the expected value is 97.5. *[3 marks]*

18c. Write down the value of χ^2_{calc}. *[2 marks]*

18d. Write down the number of degrees of freedom. *[1 mark]*

18e. Is the manufacturer's claim justified? Give a reason for your answer. *[2 marks]*

The population of fleas on a dog after t days, is modelled by

$$N = 4 \times (2)^{\frac{t}{4}}\,,\ t \geqslant 0$$

Some values of N are shown in the table below.

t	0	4	8	12	16	20
N	p	8	16	32	q	128

18f. Write down the value of p. *[1 mark]*

18g. Write down the value of q. *[2 marks]*

18h. Using the values in the table above, draw the graph of N for $0 \le t \le 20$. Use 1 cm to represent 2 days on the horizontal axis and 1 cm to represent 10 fleas on the vertical axis. *[6 marks]*

18i. **Use your graph** to estimate the number of days for the population of fleas to reach 55. *[2 marks]*

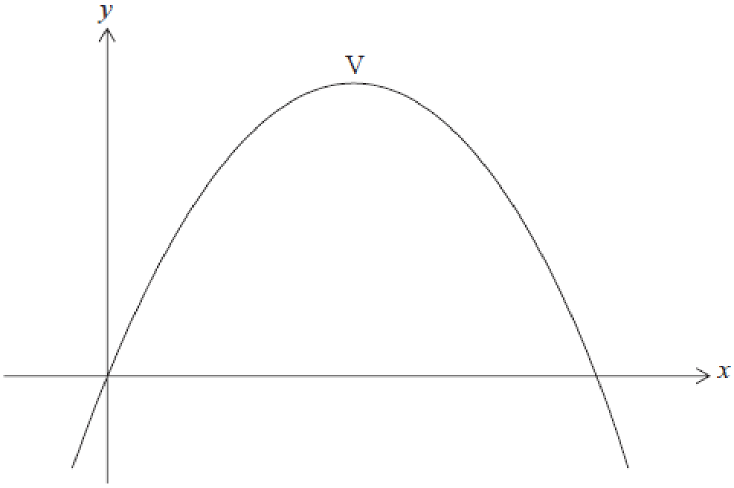
A plumber in Australia charges 90 AUD per hour for work, plus a fixed cost. His total charge is represented by the cost function $C = 60 + 90t$, where t is in hours.

19a. Write down the fixed cost. *[1 mark]*

19b. It takes $3\frac{1}{2}$ hours to complete a job for Paula. Find the total cost. *[2 marks]*

19c. Steve received a bill for 510 AUD. Calculate the time it took the plumber to complete the job. *[3 marks]*

A quadratic curve with equation $y = ax\,(x - b)$ is shown in the following diagram.



The x-intercepts are at (0, 0) and (6, 0), and the vertex V is at (h, 8).

20a. Find the value of h. *[2 marks]*

20b. Find the equation of the curve. *[4 marks]*