Topic 6 Part 3 *[209 marks]*

1a. *[1 mark]*

Markscheme

$2x + 2y = 44$ ***(A1)*** ***(C1)***

Note: Accept equivalent forms.

Examiners report

[N/A]

1b. *[1 mark]*

Markscheme

$xy = 112$ ***(A1)*** ***(C1)***

Examiners report

[N/A]

1c. *[2 marks]*

Markscheme

$8, 14$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Notes: Accept $x = 8, y = 14$ **OR** $x = 14, y = 8$

Follow through from their answers to parts (a) and (b) only if both values are positive.

Examiners report

[N/A]

1d. *[2 marks]*

Markscheme

$\frac{112}{1250} \times 100$ ***(M1)***

Note: Award ***(M1)*** for 112 divided by 1250.

$= 8.96$ ***(A1)*** ***(C2)***

Note: Do not penalize if percentage sign seen.

Examiners report

[N/A]

2a. *[2 marks]*

Markscheme

(i) $(-1, 0)$ ***(A1)***

Note: Accept -1.

(ii) $(0, -1)$ ***(A1)*** ***(C2)***

Note: Accept -1.

Examiners report

[N/A]

2b. *[2 marks]*

Markscheme

$(x =) -2.96 (-2.96135\ldots)$ ***(A1)***

$(x =) 1.34 (1.33508\ldots)$ ***(A1)*** ***(C2)***

Examiners report

[N/A]

2c. *[2 marks]*

Markscheme

$-2.96 < x < 1.34$ OR $]-2.96, 1.34[$ OR $(-2.96, 1.34)$ ***(A1)(ft)(A1)*** ***(C2)***

Notes: Award ***(A1)(ft)*** for both correct endpoints of the interval, ***(A1)*** for correct strict inequalities (or correct open interval notation).

Follow through from part (b).

Examiners report

[N/A]

3a. *[2 marks]*

Markscheme

For parts (a) and (b) only, the first time a correct answer has incorrect or missing units, the final (A1) is not awarded.

$200 - 190(0.97)^0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$= 10\,^\circ\text{C}$ ***(A1)*** ***(C2)***

Note: Units are required.

Examiners report

[N/A]

3b. *[2 marks]*

Markscheme

For parts (a) and (b) only, the first time a correct answer has incorrect or missing units, the final (A1) is not awarded.

$200 - 190(0.97)^{30}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$= 124^{\circ}\text{C}\ (123.808\ldots^{\circ}\text{C})$ ***(A1)*** ***(C2)***

Note: Units are required, unless already omitted in part (a).

Examiners report

[N/A]

3c. *[2 marks]*

Markscheme

$200 - 190(0.97)^{k} = 40$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$k = 5.64\ (\text{minutes})\ (5.64198\ldots)$ ***(A1)*** ***(C2)***

Examiners report

[N/A]

4a. *[1 mark]*

Markscheme

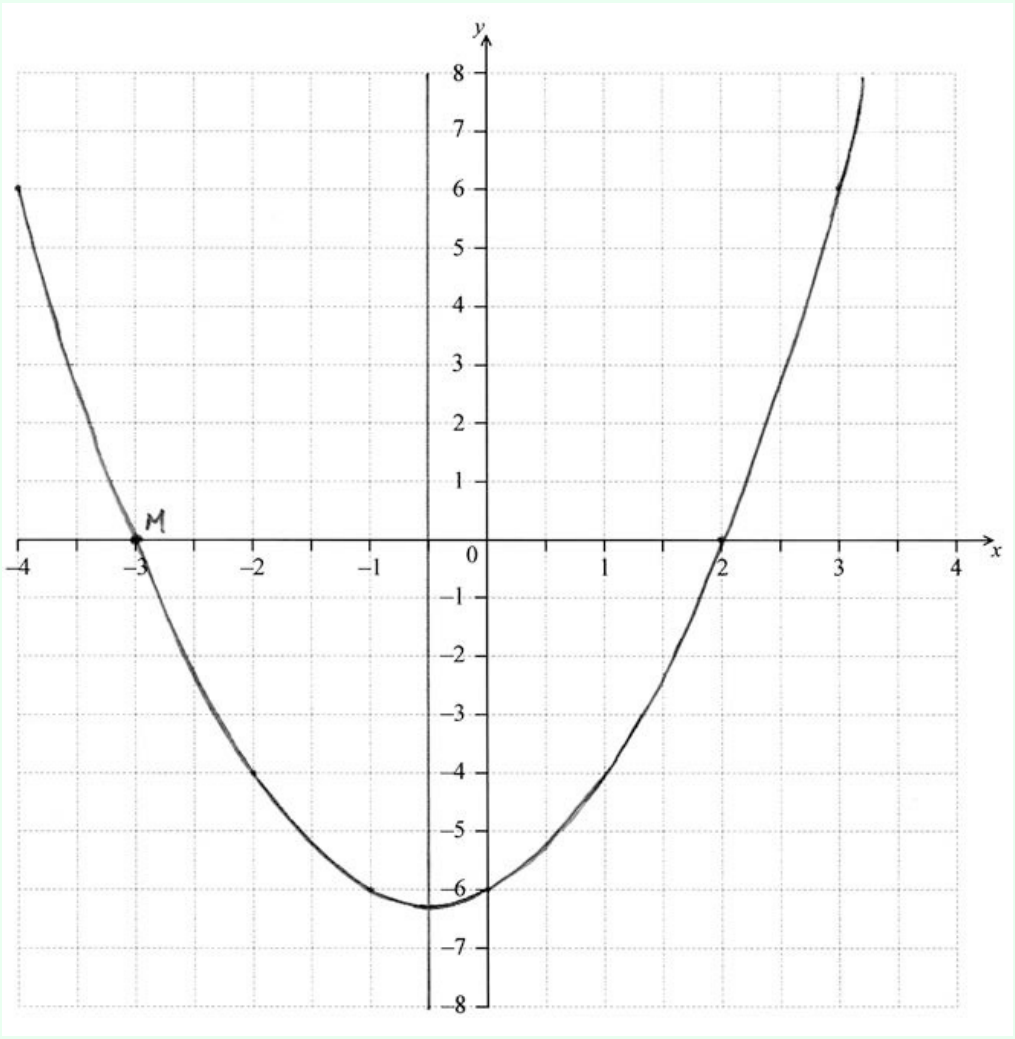
vertical straight line which may be dotted passing through $\left(-\frac{1}{2},\ 0\right)$ ***(A1)*** ***(C1)***

Examiners report

[N/A]

4b. *[1 mark]*

Markscheme



vertical straight line which may be dotted passing through $\left(-\frac{1}{2},\ 0\right)$ ***(A1)*** ***(C1)***

Examiners report

[N/A]

4c. *[1 mark]*

Markscheme

point $\mathrm{M}\ (-3,\ 0)$ correctly marked on the x-axis ***(A1)(ft)*** ***(C1)***

Note: Follow through from part (a).

Examiners report

[N/A]

4d. *[4 marks]*

Markscheme

(i)

$b = 1, c = -6$ ***(A1)*****(ft)*****(A1)*****(ft)***

Notes: Follow through from (b).

(ii) smooth parabola passing through M and N ***(A1)*****(ft)**

Note: Follow through from their point M from part (b).

parabola passing through $(0, \ -6)$ and symmetrical about $x = -0.5$ ***(A1)*****(ft)** ***(C4)***

Note: Follow through from part (c)(i).

If parabola is not smooth and not concave up award at most ***(A1)(A0)***.

Examiners report

[N/A]

5a. *[1 mark]*

Markscheme

$180 = 150m + c$ (or equivalent) ***(A1)*** ***(C1)***

Examiners report

The equations in part (a) and (b) were given correctly by the vast majority of the candidates.

5b. *[1 mark]*

Markscheme

$181.5 = 210m + c$ (or equivalent) ***(A1)*** ***(C1)***

Examiners report

The equations in part (a) and (b) were given correctly by the vast majority of the candidates.

5c. *[4 marks]*

Markscheme

$m = 0.25, \ c = 176.25$ ***(A1)(A1)*****(ft)**

Note: Follow through from part (a) and part (b), irrespective of working shown.

$L = 0.025(4) + 176.25$ ***(M1)***

Note: Award ***(M1)*** for substitution of their m, their c and 40 into equation.

$L = 177 \quad (177.25) \ (\mathrm{mm})$ ***(A1)*****(ft)** ***(C4)***

Note: Follow through, within **part (c)**, from their m and c only if working shown.

Examiners report

Part (c) was in most cases either completely correct or awarded no marks at all. Only few were able to find the values of *m* and *c*, and therefore the length at 40°C. Part (c) was often left open or answered incorrectly. A common answer was $L = 40m + c$. Very few partial correct responses were given. Some candidates managed a correct 3 sf answer by intelligent guessing. As the question was not structured asking for the m and c values explicitly, not many candidates made an attempt to find those values. Very few seemed to realize they could find those values using their GDC. An attempt to use simultaneous equations was the most common approach.

6a. *[2 marks]*

Markscheme

$0 = p(6)(q - 6)$ ***(M1)***

$q = 6$ ***(A1)***

OR

$f(x) = -px^2 + pqx$

$3 = \frac{-pq}{-2p}$ ***(M1)***

$q = 6$ ***(A1)***

OR

$f(x) = -px^2 + pqx$

$f'(x) = pq - 2px$ ***(M1)***

$pq - 2p(3) = 0$

$q = 6$ ***(A1)*** ***(C2)***

Examiners report

This question was left unanswered by many candidates. For candidates who attempted the question, one method mark was often awarded for a correct equation resulting from substitution of the point (6, 0). Many were unable to find the value of *q*, and therefore did not continue to find the value of *p*.

6b. *[2 marks]*

Markscheme

$27 = p(3)(6 - 3)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of the vertex $(3,\ 27)$ **and** *their q* into or equivalent $f(x) = px(q - x)$ or equivalent.

$p = 3$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a).

Examiners report

Many were unable to find the value of *q*, and therefore did not continue to find the value of *p*.

6c. *[2 marks]*

Markscheme

$y \leqslant 27 \quad (f(x) \leqslant 27)$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)*** for $y \leqslant \ (\text{or } f(x) \leqslant)$, ***(A1)*** for 27 as part of an inequality.

Accept alternative notation: $(-\infty, 27],]-\infty, 27]$.

Award ***(A0)(A1)*** for $[27, \ -\infty)$.

Award ***(A0)(A0)*** for $(-\infty, \infty)$.

Examiners report

Part (c) was poorly attempted, although the range was independent of the values of *p* and *q*. The most common error was confusion between domain and range, resulting in an answer of (-∞, ∞).

7a. *[2 marks]*

Markscheme

$ab^0 + 40 = 840$ ***(M1)***

Note: Award ***(M1)*** for substituting $t = 0$ and equating to 840.

$a = 800$ ***(A1)(C2)***

Examiners report

[N/A]

7b. *[3 marks]*

Markscheme

$800b^{-4} + 40 = 90$ ***(M1)***

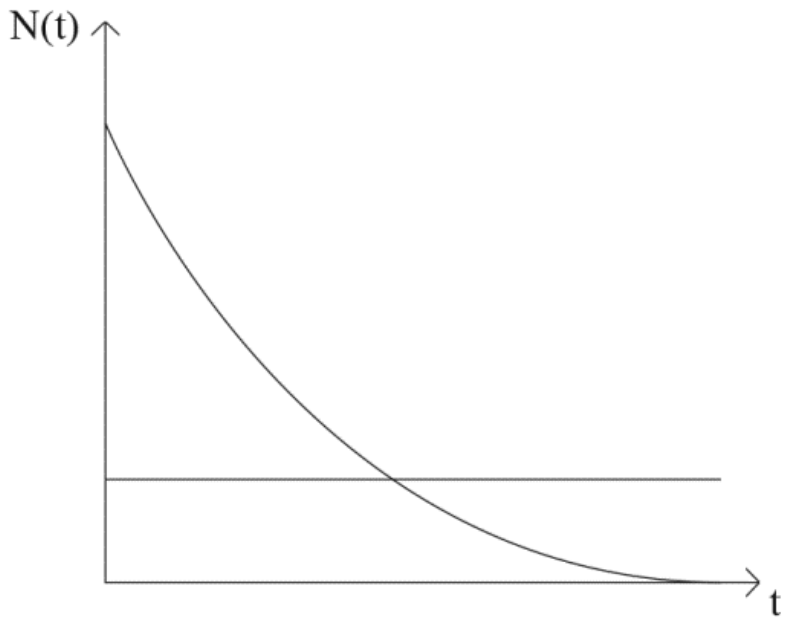
Note: Award ***(M1)*** for correct substitution of their a (from part (a)) **and** 4 in the formula of the function and equating to 90.

$b^4 = 16$ **OR** $\frac{1}{b^4} = \frac{1}{16}$ **OR** $b = \sqrt[4]{16}$ **OR** $b = 16^{\frac{1}{4}}$ ***(M1)***

Notes: Award second ***(M1)*** for correctly rearranging their equation and eliminating the negative index (see above examples).

Accept $\frac{800}{50}$ in place of 16.

OR



(M1)(M1)

Notes: Award ***(M1)*** for a decreasing exponential and a horizontal line that are both in the first quadrant, and ***(M1)*** for their graphs intersecting.

For graphs drawn in both first and second quadrants award at most ***(M1)(M0)***.

$b = 2$ ***(A1)*(ft)** ***(C3)***

Note: Follow through from their answer to part (a) only if a is positive.

Examiners report

[N/A]

7c. *[1 mark]*

Markscheme

40 ***(A1)*** ***(C1)***

Examiners report

[N/A]

8a. *[1 mark]*

Markscheme

$3600\ (\text{m}^2)$ ***(A1)(C1)***

Examiners report

[N/A]

8b. *[2 marks]*

Markscheme

$x(200 - x) = 3600$ ***(M1)***

Note: Award ***(M1)*** for setting up an equation, equating to their 3600.

$180\ (\text{m})$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to part (a).

Examiners report

[N/A]

8c. *[2 marks]*

Markscheme

(i) $100\ (\text{m})$ ***(A1)*** ***(C1)***

(ii) $10\,000\ (\text{m}^2)$ ***(A1)*(ft)*(C1)***

Note: Follow through from their answer to part (c)(i).

Examiners report

[N/A]

8d. *[1 mark]*

Markscheme

$m = 3600$ **and** $n = 10\,000$ ***(A1)*(ft)** ***(C1)***

Notes: Follow through from part (a) and part (c)(ii), but only if their m is less than their n. Accept the answer $3600 \leqslant A \leqslant 10\,000$.

Examiners report

[N/A]

9a. *[3 marks]*

$\frac{-192}{x^3} + k$

-192 x^{-3} k

Examiners report

Differentiation of terms including negative indices remains a testing process; it will continue to be tested. There was, however, a noticeable improvement in responses compared to previous years. The parameter k was problematic for a number of candidates.

9b. *[2 marks]*

Markscheme

at local minimum $f'(x) = 0$ ***(M1)***

Note: Award ***(M1)*** for seeing $f'(x) = 0$ (may be implicit in their working).

$\frac{-192}{4^3} + k = 0$ ***(A1)***

$k = 3$ ***(AG)***

Note: Award ***(A1)*** for substituting $x = 4$ in their $f'(x) = 0$, provided it leads to $k = 3$. The conclusion $k = 3$ must be seen for the ***(A1)*** to be awarded.

Examiners report

In part (b), the manipulation of the derivative to find the local minimum point caused difficulties for all but the most able; note that a GDC approach is not accepted in such questions and that candidates are expected to be able to apply the theory of the calculus as appropriate. Further, once a parameter is given, candidates are expected to use this value in subsequent parts.

9c. *[2 marks]*

Markscheme

$\frac{96}{2^2} + 3(2)$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = 2$ and $k = 3$ in $f(x)$.

$= 30$ ***(A1)(G2)***

Examiners report

Parts (c) and (d) were accessible and all but the weakest candidates scored well.

9d. *[2 marks]*

Markscheme

$\frac{-192}{2^3} + 3$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = 2$ and $k = 3$ in their $f'(x)$.

$= -21$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a).

Examiners report

Parts (c) and (d) were accessible and all but the weakest candidates scored well.

9e. *[3 marks]*

Markscheme

$y - 30 = \frac{1}{21}(x - 2)$ ***(A1)*****(ft)*****(M1)***

Notes: Award ***(A1)*****(ft)** for their $\frac{1}{21}$ seen, ***(M1)*** for the correct substitution of their point and their normal gradient in equation of a line.

Follow through from part (c) and part (d).

OR

gradient of normal $= \frac{1}{21}$ ***(A1)*****(ft)**

$30 = \frac{1}{21} \times 2 + c$ ***(M1)***

$c = 29\frac{19}{21}$

$y = \frac{1}{21}x + 29\frac{19}{21} \quad (y = 0.0476x + 29.904)$

$x - 21y + 628 = 0$ ***(A1)*****(ft)*****(G2)***

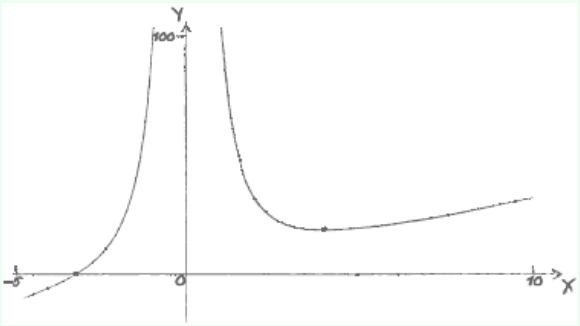
Notes: Accept equivalent answers.

Examiners report

Part (e) discriminated at the highest level; the gradient of the normal often was not used, the form of the answer not given correctly.

9f. *[4 marks]*

Markscheme



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for correct window (at least one value, other than zero, labelled on each axis), the axes must also be labelled; ***(A1)*** for a smooth curve with the correct shape (graph should not touch y-axis and should not curve away from the y-axis), on the given domain; ***(A1)*** for axis intercept in approximately the correct position (nearer -5 than zero); ***(A1)*** for local minimum in approximately the correct position (first quadrant, nearer the y-axis than $x = 10$).

If there is no scale, award a maximum of ***(A0)(A1)(A0)(A1)*** – the final ***(A1)*** being awarded for the zero and local minimum in approximately correct positions relative to each other.

Examiners report

Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).

9g. *[2 marks]*

Markscheme

$(-3.17,\ 0)$ $((-3.17480\ldots,\ 0))$ ***(G1)(G1)***

Notes: If parentheses are omitted award ***(G0)(G1)*****(ft)**.

Accept $x = -3.17,\ y = 0$. Award ***(G1)*** for -3.17 seen.

Examiners report

Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).

9h. *[2 marks]*

Markscheme

$0 < x \leqslant 4$ or $0 < x < 4$ ***(A1)(A1)***

Notes: Award ***(A1)*** for correct end points of interval, ***(A1)*** for correct notation (note: lower inequality must be strict).

Award a maximum of ***(A1)(A0)*** if y or $f(x)$ used in place of x.

Examiners report

Part (h) was not well attempted in the main; decreasing (and increasing) functions is a testing concept for the majority.

10a. *[2 marks]*

Markscheme

the temperature of the water cannot fall below room temperature ***(R1)***

an (informal) explanation that as $m \to \infty,\ k^{-m} \to 0$ ***R1)***

OR

recognition that there is a horizontal asymptote at $y = a$ ***(R1)***

Note: Award ***(R1)*** for a contextual reason involving room temperature.

Award ***(R1)*** for a mathematical reason similar to one of the two alternatives.

Examiners report

Comments on some of the G2 forms indicated that teachers felt the presence of three parameters in the formula was inconsistent with the aims of Mathematical Studies; however, one parameter was given and a justification required, a second required knowledge only of $k^0=1$ and the third was also given. Candidates should have been exposed to graphs of this type in their classwork.

The response of the candidature indicated that most were able to make some progress with the question, though for many the “show that” part was not attempted. As in previous sessions, comments were made about the use (or not) of logarithms in the final part – logarithms are not part of the Mathematical Studies SL syllabus (however, their correct use is never penalized), but efficient use of the GDC is very much part of a candidate’s “toolbox”. Questions of this nature – essentially requiring the use of the GDC as part of a problem solving exercise – will continue to be set.

10b. *[2 marks]*

Markscheme

$100 = 20 + b(k^0)$ ***(M1)***

Note: Award ***(M1)*** for substituting 100, 20 and 0.

$b = 80$ ***(A1)(G2)***

Note: The ***(A1)*** is awarded only if all working seen is consistent with the final answer of 80.

Examiners report

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10c. *[2 marks]*

Markscheme

$84 = 20 + 80k^{-1}$ ***(M1)***

Note: Substituting $k = 1.25$ at any stage is an invalid method and is awarded ***(M0)(M0)***. Award ***(M1)*** for correctly substituting 84, 20 and their 80.

$\frac{64}{80} = k^{-1}$ ***(M1)***

$k = 1.25$ ***(AG)***

Note: Award ***(M1)*** for correct rearrangement that isolates k; $k = 1.25$ must be consistent with their working **and** the conclusion $k = 1.25$ must be seen.

Examiners report

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10d. *[2 marks]*

Markscheme

$T = 20 + 80(1.25^{-3})$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions into T. Follow through from part (b) and $k = 1.25$.

$T = 61.0 \quad (60.96)$ ***(A1)(ft)(G2)***

Examiners report

Comments on some of the G2 forms indicated that teachers felt the presence of three parameters in the formula was inconsistent with the aims of Mathematical Studies; however, one parameter was given and a justification required, a second required knowledge only of $k^0=1$ and the third was also given. Candidates should have been exposed to graphs of this type in their classwork.

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10e. *[2 marks]*

Markscheme

$35 = 20 + 80(1.25^{-m})$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions into T. Follow through from part (b). Accept graphical solutions. Award ***(M1)*** for sketch of function.

$(m =) \ 7.50 \ (\text{minutes}) \quad (7.50179\ldots)$ ***(A1)*****(ft)*****(G2)***

7 minutes and 30 seconds ***(A1)***

Note: Award the final ***(A1)*** for correct conversion of **their** m in minutes to minutes and seconds, but only if answer in minutes is explicitly shown.

Examiners report

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11a. *[2 marks]*

Markscheme

$0.5 \times (-2)^2 - \frac{8}{-2}$ ***(M1)***

Note: Award ***(M1)*** for substitution of $x = -2$ into the formula of the function.

6 ***(A1)(G2)***

Examiners report

[N/A]

11b. *[3 marks]*

Markscheme

$f'(x) = x + 8x^{-2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for x, ***(A1)*** for 8, ***(A1)*** for x^{-2} or $\frac{1}{x^2}$ (each term must have correct sign). Award at most ***(A1)(A1)(A0)*** if there are additional terms present or further incorrect simplifications are seen.

Examiners report

[N/A]

11c. *[2 marks]*

Markscheme

$f'(-2) = -2 + 8(-2)^{-2}$ ***(M1)***

Note: Award ***(M1)*** for $x = -2$ substituted into their $f'(x)$ from part (b).

$= 0$ ***(A1)(ft)(G2)***

Note: Follow through from their derivative function.

Examiners report

[N/A]

11d. *[2 marks]*

Markscheme

$y = 6$ **OR** $y = 0x + 6$ **OR** $y - 6 = 0(x + 2)$ ***(A1)(ft)(A1)(ft)(G2)***

Notes: Award ***(A1)*(ft)** for their gradient from part (c), ***(A1)*(ft)** for their answer from part (a). Answer must be an equation.

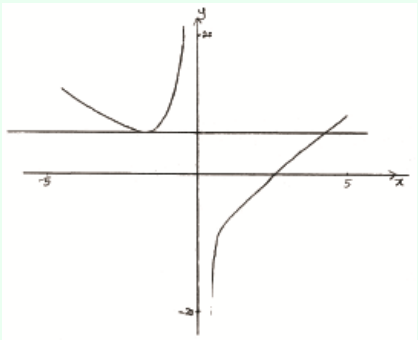
Award ***(A0)(A0)*** for $x = 6$.

Examiners report

[N/A]

11e. *[4 marks]*

Markscheme



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for labels and some indication of scales in the stated window. The point $(-2,\ 6)$ correctly labelled, or an x-value and a y-value on their axes in approximately the correct position, are acceptable indication of scales.

Award ***(A1)*** for correct general shape (curve must be smooth and must not cross the y-axis).

Award ***(A1)*** for x-intercept in approximately the correct position.

Award ***(A1)*** for local minimum in the second quadrant.

Examiners report

[N/A]

11f. *[2 marks]*

Markscheme

Tangent to graph drawn approximately at $x = -2$ ***(A1)*****(ft)*****(A1)*****(ft)**

Notes: Award ***(A1)*****(ft)** for straight line tangent to curve at approximately $x = -2$, with approximately correct gradient. Tangent must be straight for the ***(A1)*****(ft)** to be awarded.

Award ***(A1)*****(ft)** for (extended) line passing through approximately their y-intercept from (d). Follow through from their gradient in part (c) and their equation in part (d).

Examiners report

[N/A]

11g. *[2 marks]*

Markscheme

$(4,\ 6)$ **OR** $x = 4,\ y = 6$ ***(G1)*****(ft)*****(G1)*****(ft)**

Notes: Follow through from their tangent from part (d). If brackets are missing then award ***(G0)(G1)*****(ft).**

If line intersects their graph at more than one point (apart from $(-2,\ 6)$), follow through from the first point of intersection (to the right of -2).

Award ***(G0)(G0)*** for $(-2,\ 6)$.

Examiners report

[N/A]

12a. *[4 marks]*

Markscheme

(i) $r = 0.985 \quad (0.984905\ldots)$ ***(G2)***

Notes: If unrounded answer is not seen, award ***(G1)(G0)*** for 0.99 or 0.984. Award ***(G2)*** for 0.98.

(ii) strong, positive ***(A1)(A1)***

Examiners report

[N/A]

12b. *[2 marks]*

Markscheme

$y = 259.909\ldots x + 698.648\ldots \quad (y = 260x + 699)$ ***(G1)(G1)***

Notes: Award ***(G1)*** for $260x$ and ***(G1)*** for 699. If the answer is not an equation award a maximum of ***(G1)(G0)***.

Examiners report

[N/A]

12c. *[3 marks]*

Markscheme

$y = 259.909\ldots \times 13 + 698.648\ldots$ ***(M1)***

Note: Award ***(M1)*** for substitution of 13 into their regression line equation from part (b).

$y = 4077.47\ldots$ ***(A1)*****(ft)*****(G2)***

$y = 4077\ (\text{USD})$ ***(A1)*****(ft)**

Notes: Follow through from their answer to part (b). If rounded values from part (b) used, answer is 4079. Award the final ***(A1)*****(ft)** for a correct rounding to the nearest USD of their answer. The unrounded answer may not be seen.

If answer is 4077 and no working is seen, award ***(G2)***.

Examiners report

[N/A]

12d. *[2 marks]*

Markscheme

$13 \times 304 - (4077.47) = -125.477\ldots \quad (-125)$ **OR**

$4077.47 - (13 \times 304) = 125.477\ldots \quad (125)$ ***(M1)***

Notes: Award ***(M1)*** for calculating the difference between 13×304 and their answer to part (c).

If rounded values are used in equation, answer is -127.

profit is negative **OR** $\text{cost} > \text{sales}$ ***(A1)***

OR

$13 \times 304 = 3952$ ***(M1)***

Note: Award ***(M1)*** for calculating the price of 13 bikes.

$3952 < 4077.47$ ***(A1)*****(ft)**

Note: Award ***(A1)*** for showing 3952 is less than their part (c). This may be communicated in words. Follow through from part (c), but only if value is greater than 3952.

OR

$\frac{4077}{13} = 313.62$ ***(M1)***

Note: Award ***(M1)*** for calculating the cost of 1 bicycle.

$313.62 > 304$ ***(A1)*****(ft)**

Note: Award ***(A1)*** for showing 313.62 is greater than 304. This may be communicated in words. Follow through from part (c), but only if value is greater than 304.

OR

$\frac{4077}{304} = 13.41$ ***(M1)***

Note: Award ***(M1)*** for calculating the number of bicycles that should have been be sold to cover total cost.

$13.41 > 13$ ***(A1)*****(ft)**

Note: Award ***(A1)*** for showing 13.41 is greater than 13. This may be communicated in words. Follow through from part (c), but only if value is greater than 13.

Examiners report

[N/A]

12e. *[5 marks]*

Markscheme

(i) $304x$ ***(A1)***

(ii) $304x - (259.909\ldots x + 698.648\ldots)$ ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Award ***(A1)*****(ft)** for difference between their answers to parts (b) and (e)(i), ***(A1)*****(ft)** for correct expression.

(iii) $304x - (259.909\ldots x + 698.648\ldots) > 0$ ***(M1)***

Notes: Award ***(M1)*** for comparing their expression in part (e)(ii) to 0. Accept an equation. Accept $3040x - y > 0$ or equivalent.

$x = 16$ bicycles ***(A1)*****(ft)*****(G2)***

Notes: Follow through from their answer to part (b). Answer must be a positive integer greater than 13 for the ***(A1)*****(ft)** to be awarded.

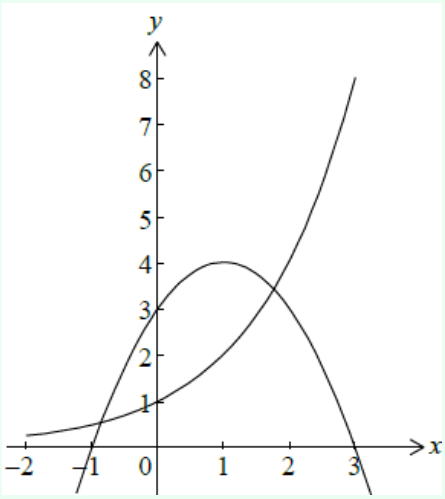
Award ***(G1)*** for an answer of 15.84.

Examiners report

[N/A]

13a. *[3 marks]*

Markscheme



(A1)(A1)(A1)

Note: Award *(A1)* for correct domain, *(A1)* for smooth curve, *(A1)* for y-intercept clearly indicated.

[3 marks]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The most common error was using the incorrect domain.

13b. *[2 marks]*

Markscheme

$y = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for y = constant, ***(A1)*** for 0.

[2 marks]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

Many had little idea of asymptotes. Others did not write their answer as an equation.

13c. *[3 marks]*

Markscheme

Note: Award ***(A1)*** for smooth parabola,

(A1) for vertex (maximum) in correct quadrant.

(A1) for all clearly indicated intercepts $x = -1$, $x = 3$ and $y = 3$.

The final mark is to be applied very strictly. ***(A1)(A1)(A1)***

[3 marks]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The intercepts being inexact or unlabelled was the most frequent cause of loss of marks.

13d. *[2 marks]*

Markscheme

$x = -0.857$ $x = 1.77$ ***(G1)(G1)***

Note: Award a maximum of ***(G1)*** if x and y coordinates are both given.

[2 marks]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

Often, only one solution to the equation was given. Elsewhere, a lack of appreciation that the solutions were the x coordinates was a common mistake.

13e. *[1 mark]*

Markscheme

4 ***(G1)***

Note: Award ***(G0)*** for (1, 4).

[1 mark]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The maximum is the *y* coordinate only; again a common misapprehension was the answer "(1, 4)".

13f. *[5 marks]*

Markscheme

$f'(x) = 2 - 2x$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

Award at most ***(A1)(A0)*** if any extra terms seen.

$2 - 2x = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their gradient function to zero.

$x = 1$ ***(A1)*****(ft)**

$f(1) = 3 + 2(1) - (1)^2 = 4$ ***(A1)***

Note: The final ***(A1)*** is for substitution of $x = 1$ into
$f(x)$ and subsequent correct answer. Working must be seen for final ***(A1)*** to be awarded.

[5 marks]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This was a major discriminator in the paper. Many candidates were unable to follow the analytic approach to finding a maximum point.

13g. *[2 marks]*

Markscheme

$2^2 \times p + 2q - 4 = -10$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the equation.

$4p + 2q = -6$ or $2p + q = -3$ ***(A1)***

Note: Accept equivalent simplified forms.

[2 marks]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

13h. *[2 marks]*

Markscheme

$\frac{dy}{dx} = 2px + q$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

Award at most ***(A1)(A0)*** if any extra terms seen.

[2 marks]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

13i. *[1 mark]*

Markscheme

$4p + q = 1$ ***(A1)*****(ft)**

[1 mark]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

13j. *[3 marks]*

Markscheme

$4p + 2q = -6$

$4p + q = 1$ ***(M1)***

Note: Award ***(M1)*** for sensible attempt to solve the equations.

$p = 2, q = -7$ ***(A1)(A1)*****(ft)*****(G3)***

[3 marks]

Examiners report

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

14a. *[2 marks]*

Markscheme

$x = 3$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)*** for "$x =$" ***(A1)*** for 3.

The mark for $x =$ is not awarded unless a constant is seen on the other side of the equation.

[2 marks]

Examiners report

The lack of the answer, "$x = 3$", expressed as an equation was a common fault.

14b. *[2 marks]*

Markscheme

$(3, -14)$ (Accept $x = 3$, $y = -14$) ***(A1)*****(ft)*****(A1)*** ***(C2)***

Note: Award ***(A1)(A0)*** for missing coordinate brackets.

[2 marks]

Examiners report

Misreading the y coordinate was the common error.

14c. *[2 marks]*

Markscheme

$y \geq -14$ ***(A1)(A1)*****(ft)** ***(C2)***

Notes: Award ***(A1)*** for y ≥ , ***(A1)*****(ft)** for –14.

Accept alternative notation for intervals.

[2 marks]

Examiners report

This part proved challenging for many; there was confusion between domain and range for many, the incorrect inequality also was a common error. It is the accepted practice in examinations that if a domain is not specified, then it is taken as the real numbers.

15a. *[2 marks]*

Markscheme

$p + q = 47$ ***(A1)***

$4p + q = 53$ ***(A1)*** ***(C2)***

[2 marks]

Examiners report

Concern was expressed about the wording of the question; answers were given great leeway by examiners and suggestions for wording are welcome. The 24 hour clock method of describing time is the norm in IB examinations. It should be recognised that the purpose of this question was to discriminate at the grade 6/7 level.

The concept of the zero index was not understood by many.

15b. *[2 marks]*

Markscheme

Reasonable attempt to solve their equations ***(M1)***

$p = 2, q = 45$ ***(A1)*** ***(C2)***

Note: Accept only the answers $p = 2$, $q = 45$.

[2 marks]

Examiners report

Concern was expressed about the wording of the question; answers were given great leeway by examiners and suggestions for wording are welcome. The 24 hour clock method of describing time is the norm in IB examinations. It should be recognised that the purpose of this question was to discriminate at the grade 6/7 level.

The use of the GDC was (as always) expected in solving the simultaneous equations.

15c. *[2 marks]*

Markscheme

$C = 2 \times 2^{0.5(10)} + 45$ ***(M1)***

$C = 109$ ***(A1)*****(ft)** ***(C2)***

Note: Award ***(M1)*** for substitution of 10 into the formula with their values of p and q.

[2 marks]

Examiners report

Concern was expressed about the wording of the question; answers were given great leeway by examiners and suggestions for wording are welcome. The 24 hour clock method of describing time is the norm in IB examinations. It should be recognised that the purpose of this question was to discriminate at the grade 6/7 level.

Working was required for follow through in this part.

16a. *[2 marks]*

Markscheme

$y = 13.5x + 4.5$ ***(M1)***

Note: Award ***(M1)*** for $13.5x$ seen.

gradient = 13.5 ***(A1)*** ***(C2)***

[2 marks]

Examiners report

The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Most were successful in this part.

16b. *[1 mark]*

Markscheme

$4x^3$ ***(A1)*** ***(C1)***

[1 mark]

Examiners report

The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

This part was usually well attempted.

16c. *[2 marks]*

Markscheme

$4x^3 = 13.5$ ***(M1)***

Note: Award ***(M1)*** for equating their answers to (a) and (b).

$x = 1.5$ ***(A1)*(ft)**

[2 marks]

Examiners report

The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Only the best candidates succeeded in this part.

16d. *[1 mark]*

Markscheme

$\frac{81}{16}$ (5.0625, 5.06) ***(A1)*(ft)** ***(C3)***

Note: Award ***(A1)*(ft)** for substitution of their (c)(i) into $x4$ with working seen.

[1 mark]

Examiners report

The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Only the best candidates succeeded in this part.

17a. *[3 marks]*

Markscheme

$f'(x) = 3 - \frac{24}{x^3}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for 3, ***(A1)*** for –24, ***(A1)*** for x^3 (or x^{-3}). If extra terms present award at most ***(A1)(A1)(A0)***.

[3 marks]

Examiners report

Many students did not know the term “differentiate” and did not answer part (a).

17b. *[2 marks]*

Markscheme

$f'(1) = -21$ ***(M1)(A1)*(ft)(G2)**

Note: **(ft)** from their derivative only if working seen.

[2 marks]

Examiners report

However, the derivative was seen in (b) when finding the gradient at $x = 1$. The negative index of the formula did cause problems for many when finding the derivative. The meaning of the derivative was not clear for a number of students.

17c. *[2 marks]*

Markscheme

Derivative (gradient, slope) is negative. Decreasing. ***(R1)(A1)*****(ft)**

Note: Do not award ***(R0)(A1)***.

[2 marks]

Examiners report

[N/A]

17d. *[3 marks]*

Markscheme

$3 - \frac{24}{x^3} = 0$ ***(M1)***

$x^3 = 8$ ***(A1)***

$x = 2$ ***(A1)*****(ft)*****(G2)***

[3 marks]

Examiners report

Part (d) was handled well by some but many substituted $x = 0$ into $f'(x)$.

17e. *[2 marks]*

Markscheme

(2, 9) (Accept $x = 2$, $y = 9$) ***(A1)(A1)(G2)***

Notes: **(ft)** from their answer in (d).

Award ***(A1)(A0)*** if brackets not included and not previously penalized.

[2 marks]

Examiners report

It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

17f. *[1 mark]*

Markscheme

0 ***(A1)***

[1 mark]

Examiners report

It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

17g. *[2 marks]*

Markscheme

$y = 9$ ***(A1)(A1)*****(ft)*****(G2)***

Notes: Award ***(A1)*** for y = constant, ***(A1)*** for 9.

Award ***(A1)*****(ft)** for their value of y in (e)(i).

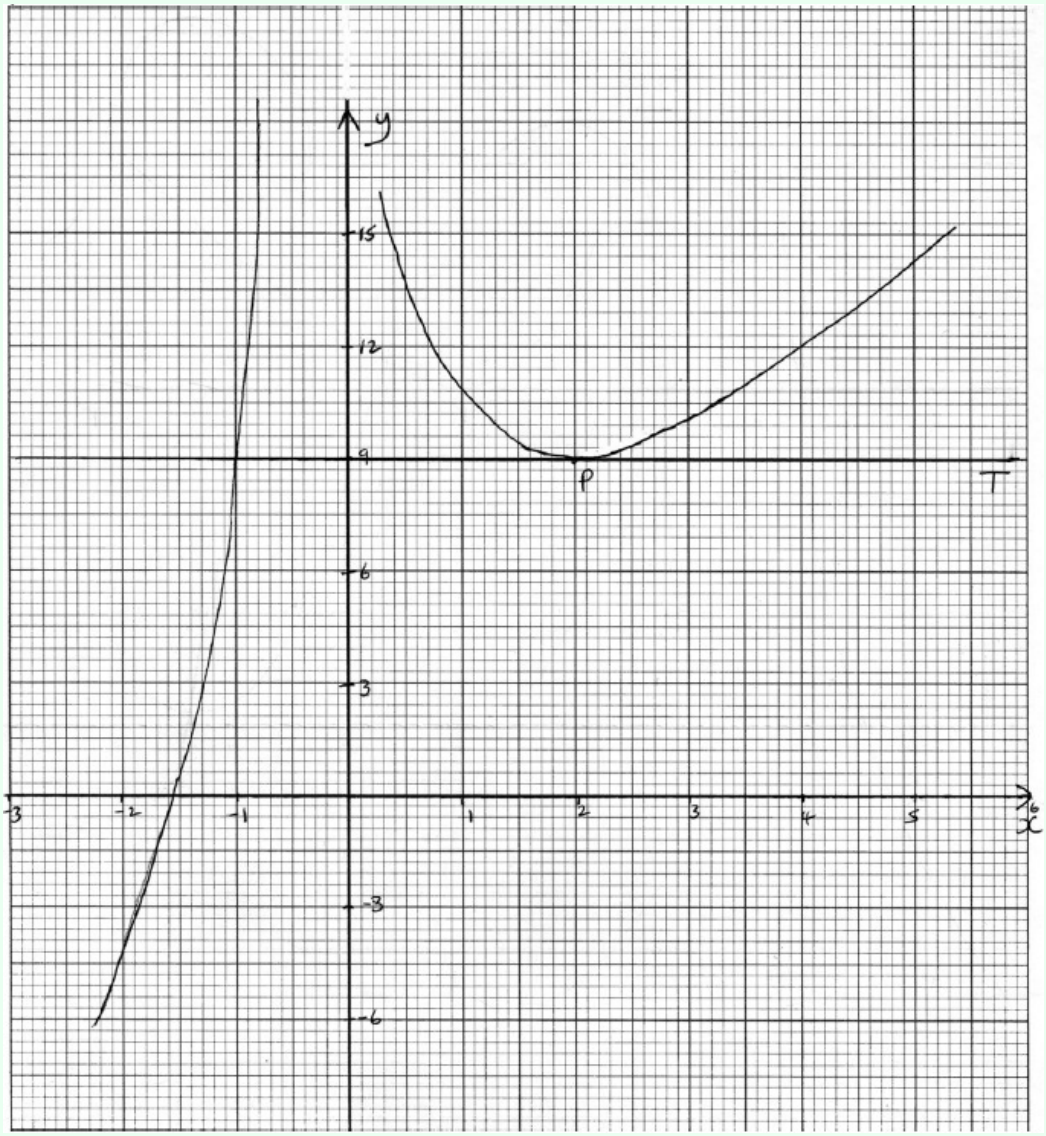
[2 marks]

Examiners report

It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

17h. *[4 marks]*

Markscheme



(A4)

Notes: Award ***(A1)*** for labels and some indication of scale in the stated window.

Award ***(A1)*** for correct general shape (curve must be smooth and must not cross the *y*-axis).

Award ***(A1)*** for *x*-intercept seen in roughly the correct position.

Award ***(A1)*** for minimum (P).

[4 marks]

Examiners report

There were good answers to the sketch though setting out axes and a scale seemed not to have had enough practise.

17i. *[2 marks]*

Markscheme

Tangent drawn at P (line must be a tangent and horizontal). ***(A1)***

Tangent labeled *T*. ***(A1)***

Note: (ft) from their tangent equation only if tangent is drawn and answer is consistent with graph.

[2 marks]

Examiners report

Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.

17j. *[1 mark]*

Markscheme

$x = -1$ ***(G1)*****(ft)**

[1 mark]

Examiners report

Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.

18a. *[2 marks]*

Markscheme

H_0: The height of the rice plants is independent of the use of a fertilizer. ***(A1)***

Notes: For independent accept "not associated", can accept "the use of a fertilizer has no effect on the height of the plants".

Do not accept "not correlated".

H_1: The height of the rice plants is not independent (dependent) of the use of fertilizer. ***(A1)*****(ft)**

Note: If H_0 and H_1 are reversed award ***(A0)(A1)*****(ft)**.

[2 marks]

Examiners report

It was clear that the candidates who performed poorly in part (i) lacked the basic knowledge of chi-squared analysis. Some mixed up the null and alternate hypotheses and also were not able to correctly demonstrate the way of finding the expected value. There were many errors in finding the critical value of
χ^2 at the 1% level of significance.

18b. *[3 marks]*

Markscheme

$\frac{180\times195}{360}$ **or**
$\frac{180}{360}\times\frac{195}{360}\times360$ ***(A1)(A1)(M1)***

$= 97.5$ ***(AG)***

Notes: Award ***(A1)*** for numerator, ***(A1)*** for denominator ***(M1)*** for division.

If final 97.5 is not seen award at most ***(A1)(A0)(M1)***.

[3 marks]

Examiners report

It was clear that the candidates who performed poorly in part (i) lacked the basic knowledge of chi-squared analysis. Some mixed up the null and alternate hypotheses and also were not able to correctly demonstrate the way of finding the expected value. There were many errors in finding the critical value of
χ^2 at the 1% level of significance.

18c. *[2 marks]*

Markscheme

$\chi^2_{calc} = 14.01(14.0,14)$ ***(G2)***

OR

If worked out by hand award ***(M1)*** for correct substituted formula with correct values, ***(A1)*** for correct answer. ***(M1)(A1)***

[2 marks]

Examiners report

It was clear that the candidates who performed poorly in part (i) lacked the basic knowledge of chi-squared analysis. Some mixed up the null and alternate hypotheses and also were not able to correctly demonstrate the way of finding the expected value. There were many errors in finding the critical value of
χ^2 at the 1% level of significance.

18d. *[1 mark]*

Markscheme

2 ***(A1)***

[1 mark]

Examiners report

It was clear that the candidates who performed poorly in part (i) lacked the basic knowledge of chi-squared analysis. Some mixed up the null and alternate hypotheses and also were not able to correctly demonstrate the way of finding the expected value. There were many errors in finding the critical value of
χ^2 at the 1% level of significance.

18e. *[2 marks]*

Markscheme

$\chi^2_{calc} > \chi^2_{crit}$ ***(R1)***

The manufacturer's claim is justified. (or equivalent statement) ***(A1)***

Note: Do not accept ***(R0)(A1)***.

[2 marks]

Examiners report

It was clear that the candidates who performed poorly in part (i) lacked the basic knowledge of chi-squared analysis. Some mixed up the null and alternate hypotheses and also were not able to correctly demonstrate the way of finding the expected value. There were many errors in finding the critical value of
χ^2 at the 1% level of significance.

18f. *[1 mark]*

Markscheme

$p = 4$ ***(G1)***

[1 mark]

Examiners report

Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

18g. *[2 marks]*

Markscheme

$q = 4(2)^{\frac{16}{4}}$ ***(M1)***

$= 64$ ***(A1)(G2)***

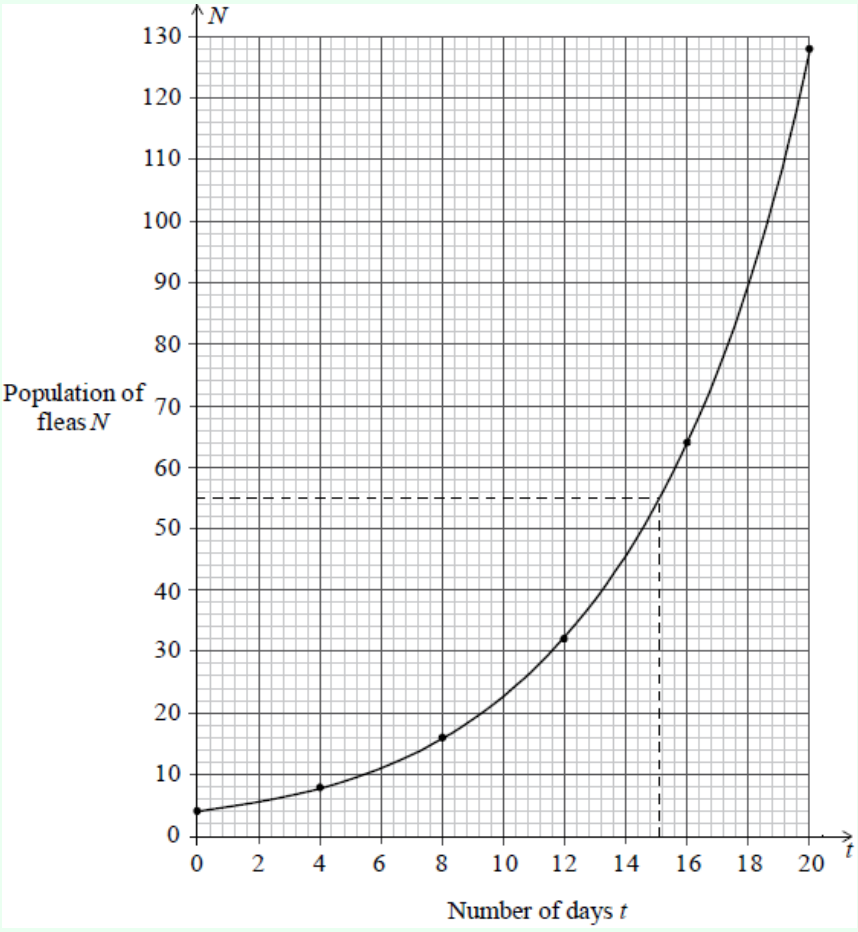
[2 marks]

Examiners report

Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

18h. *[6 marks]*

Markscheme



(A1)(A1)(A1) *(A3)*

Notes: Award *(A1)* for x axis with correct scale and label, *(A1)* for y axis with correct scale and label.

Accept x and y for labels.

If x and y axis reversed award at most *(A0)(A1)***(ft)**.

(A1) for smooth curve.

Award *(A3)* for all 6 points correct, *(A2)* for 4 or 5 points correct, *(A1)* for 2 or 3 points correct, *(A0)* otherwise.

[6 marks]

Examiners report

Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

18i. *[2 marks]*

Markscheme

15 (±0.8) *(M1)(A1)***(ft)***(G2)*

Note: Award *(M1)* for line drawn shown on graph, *(A1)***(ft)** from candidate's graph.

[2 marks]

Examiners report

Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

19a. *[1 mark]*

Markscheme

AUD 60 ***(A1)*** ***(C1)***

[1 mark]

Examiners report

The majority of candidates answered this question correctly. Some gave an answer of 150 AUD.

19b. *[2 marks]*

Markscheme

$C = 60 + 90(3.5) = \text{AUD } 375$ ***(M1)(A1)*** ***(C2)***

Note: Award ***(M1)*** for correct substitution of 3.5.

[2 marks]

Examiners report

The majority of candidates answered this question correctly.

19c. *[3 marks]*

Markscheme

Note: Unit penalty (UP) applies in this part

$510 = 60 + 90t$ ***(M1)(A1)***

(UP) $t = 5\text{h}$ (hours, hrs) ***(A1)*** ***(C3)***

Note: Award ***(M1)*** for setting formula = to any number.

(A1) for 510 seen.

[3 marks]

Examiners report

The majority of candidates answered this question correctly. Very few lost a unit penalty mark in this part.

20a. *[2 marks]*

Markscheme

$\frac{0+6}{2} = 3$
$h = 3$ ***(M1)(A1)*** ***(C2)***

Note: Award ***(M1)*** for any correct method.

[2 marks]

Examiners report

Most candidates successfully found h but very few could find the equation of the curve.

20b. *[4 marks]*

Markscheme

$y = ax(x-6)$ ***(A1)***

$8 = 3a(-3)$ ***(A1)*(ft)**

$a = -\frac{8}{9}$ ***(A1)*(ft)**

$y = -\frac{8}{9}x(x-6)$ ***(A1)*(ft)**

Notes: Award ***(A1)*** for correct substitution of
$b = 6$ into equation.

Award ***(A1)*(ft)** for substitution of their point V into the equation.

OR

$y = a(x-3)^2 + 8$ ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** for correct substitution of their h into the equation.

$0 = a(6-3)^2 + 8$ ***OR***
$0 = a(0-3)^2 + 8$ ***(A1)***

Note: Award ***(A1)*** for correct substitution of an x intercept.

$a = -\frac{8}{9}$ ***(A1)*(ft)**

$y = -\frac{8}{9}(x-3)^2 + 8$ ***(A1)*(ft)** ***(C4)***

[4 marks]

Examiners report

This question appeared to be the most difficult question on the paper.