Topic 3 Part 1 *[234 marks]*

1a. *[2 marks]*

Markscheme

p	q	$\ldots \neg q \ldots$	$p \Rightarrow \neg q$
T	T	F	$\ldots F \ldots$
T	F	T	$\ldots T \ldots$
F	T	F	$\ldots T \ldots$
F	F	T	$\ldots T \ldots$

(A1)(A1) (C2)

Note: Award *(A1)* for $\neg q$, *(A1)* for last column.

[2 marks]

Examiners report

This question was well answered with most candidates able to complete the truth table correctly in part a) and write the correct compound proposition in symbolic form in part b). A significant number of candidates could not write the correct contrapositive, although most were awarded one mark for writing an implication.

1b. *[2 marks]*

Markscheme

$p \Rightarrow q$ *(A1)(A1)   (C2)*

Note: Award *(A1)* for
$\Rightarrow$, *(A1)* for p and q in the correct order.

[2 marks]

Examiners report

This question was well answered with most candidates able to complete the truth table correctly in part a) and write the correct compound proposition in symbolic form in part b). A significant number of candidates could not write the correct contrapositive, although most were awarded one mark for writing an implication.

1c. *[2 marks]*

Markscheme

If Cristina does not do well on the logic test then she does not understand logic.   *(A1)(A1)   (C2)*

Note: Award *(A1)* for If…(then), must be an implication, *(A1)* for the correct propositions in the correct order.

[2 marks]

2a.

[2 marks]

Markscheme

$\frac{90}{200}$ (0.45, 45 %)   *(A1)(A1)*   *(C2)*

Note: Award *(A1)* for numerator, *(A1)* for denominator.

[2 marks]

Examiners report

This question was generally well answered by many of the candidates. Many found the conditional probability in part b) easier compared to previous sessions, since they were able to write it down directly from the table. A number of candidates found the final part difficult with a significant number unable to use the combined events probability formula correctly.

2b.

[2 marks]

Markscheme

$\frac{60}{90}$ (0.$\bar{6}$, 0.667, 66.$\bar{6}$ %, 66.6… %, 66.7 %)   *(A1)(A1)*(ft)   *(C2)*

Notes: Award *(A1)* for numerator, *(A1)*(ft) for denominator, follow through from their numerator in part (a). Last mark is lost if answer is not a probability.

[2 marks]

Examiners report

This question was generally well answered by many of the candidates. Many found the conditional probability in part b) easier compared to previous sessions, since they were able to write it down directly from the table. A number of candidates found the final part difficult with a significant number unable to use the combined events probability formula correctly.

2c.

Markscheme

$\frac{90}{200} + \frac{100}{200} - \frac{60}{200}$ *(M1)*

Note: Award *(M1)* for correct substitution in the combined events formula. Follow through from their answer to part (a).

$= \frac{130}{200}(0.65,\ 65\ \%)$ *(A1)*(ft)

OR

$\frac{60}{200} + \frac{40}{200} + \frac{30}{200}$ *(M1)*

Note: Award *(M1)* for adding the correct fractions.

$= \frac{130}{200}(0.65,\ 65\ \%)$ *(A1)*

OR

$1 - \frac{70}{200}$ *(M1)*

Note: Award *(M1)* for subtraction of correct fraction from 1.

$= \frac{130}{200}(0.65,\ 65\ \%)$ *(A1)*   *(C2)*

[2 marks]

Examiners report

This question was generally well answered by many of the candidates. Many found the conditional probability in part b) easier compared to previous sessions, since they were able to write it down directly from the table. A number of candidates found the final part difficult with a significant number unable to use the combined events probability formula correctly.

3a.

Markscheme

6, 9, 12   *(A1)*   *(C1)*

[1 mark]

Examiners report

The question was not well answered by the majority of the candidates. Many did not identify the universal set correctly and so took 3 to be a member of this set. This affected their answers in a)(i) and a)(ii).

3b.

Markscheme

9   *(A1)*(ft)   *(C1)*

Note: Follow through from their part (a)(i).

[1 mark]

3c.
[2 marks]

Markscheme

any element from $\{5, 7, 8, 10, 11\}$ *(A1)(A1)(ft)*   *(C2)*

Note: Award *(A1)(ft)* for finding $(A \cup B)$, follow through from their A.

Award full marks if all correct elements of $(A \cup B)'$ are listed.

[2 marks]

3d.
[2 marks]

Markscheme

$n(A \cup B) = 4$	
$15 \in A'$	**X**
$A \subset A \cup B$	

$15 \notin U$ *(R1)(A1)*   *(C2)*

Notes: Accept correct reason in words.

If the reason is incorrect, both marks are lost.
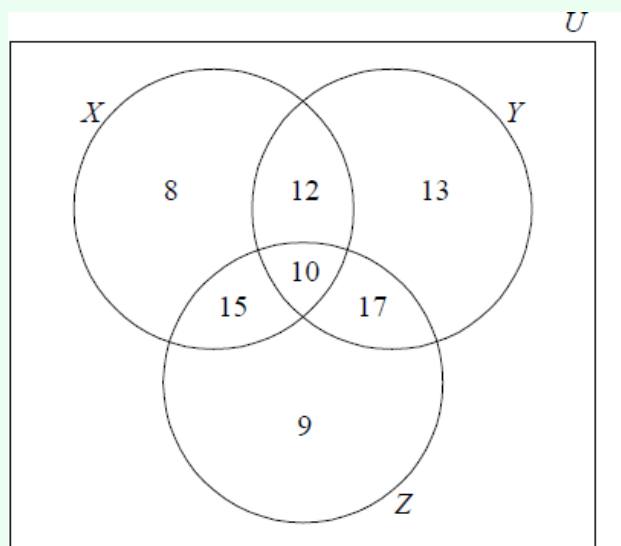
Do not award *(R0)(A1)*.

[2 marks]

4a.

Markscheme

(A1) for rectangle and three intersecting circles

(A1) for 10, *(A1)* for 8, 13 and 9, *(A1)* for 12, 15 and 17   *(A4)*

[4 marks]

Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in

$X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what Y was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were

$\frac{10}{100}$,

$\frac{10}{100} \times \frac{10}{100}$ or

$\frac{10}{100} + \frac{9}{99}$.

4b.

Markscheme

$100 - (9 + 12 + 13 + 15 + 10 + 17 + 8) = 16$ *(M1)(A1)(ft)(G2)*

Note: Follow through from their diagram.

[2 marks]

Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in

$X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what Y was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were

$\frac{10}{100}$,

$\frac{10}{100} \times \frac{10}{100}$ or

$\frac{10}{100} + \frac{9}{99}$.

Markscheme

$\frac{51}{100}$ (0.51)   *(A1)(ft)*

$= 51\%$ *(A1)(ft)(G2)*

Note: Follow through from their diagram.

[2 marks]

Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in $X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what Y was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were $\frac{10}{100}$, $\frac{10}{100} \times \frac{10}{100}$ or $\frac{10}{100} + \frac{9}{99}$.

Markscheme

Note: The following statements are correct. Please note that the connectives are important. It is not the same (had cereal) <u>and</u> (not bread) and (had cereal) <u>or</u> (not bread). The parentheses are not needed but are there to facilitate the understanding of the propositions.

(had cereal) and (did not have bread)

(had cereal only) or (had cereal and fruit only)

(had either cereal or (fruit and cereal)) and (did not have bread)   *(A1)(A1)*

Notes: If the statements are correct but the connectives are wrong then award at most *(A1)(A0)*. For the statement (had only cereal) and (cereal and fruit) award *(A1)(A0)*. For the statement had cereal and fruit award *(A0)(A0)*.

[2 marks]

Examiners report

4e.

Markscheme

$\frac{54}{100}$ (0.54, 54 %)   *(A1)(ft)(A1)(ft)(G2)*

Note: Award *(A1)(ft)* for numerator, follow through from their diagram, *(A1)(ft)* for denominator. Follow through from total or denominator used in part (c).

[2 marks]

Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in
$X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what Y was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were
$\frac{10}{100}$,
$\frac{10}{100} \times \frac{10}{100}$ or
$\frac{10}{100} + \frac{9}{99}$.

4f.

Markscheme

$\frac{10}{100} \times \frac{9}{99} = \frac{1}{110}$ (0.00909, 0.909 %)   *(A1)(ft)(M1)(A1)(ft)(G2)*

Notes: Award *(A1)(ft)* for their correct fractions, *(M1)* for multiplying two fractions, *(A1)(ft)* for their correct answer. Answer 0.009 with no working receives no marks. Follow through from denominator in parts (c) and (e) and from their diagram.

[3 marks]

Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in
$X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what Y was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were
$\frac{10}{100}$,
$\frac{10}{100} \times \frac{10}{100}$ or
$\frac{10}{100} + \frac{9}{99}$.

4g.

Markscheme

H_0 : The (average) number of meals per day a student has and gender are independent   *(A1)*

Note: For "independent" accept "not associated" but do not accept "not related" or "not correlated".

[1 mark]

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
- In (e) surprisingly many candidates found the χ^2_{calc} through the use of the formula. Unfortunately this led to some incorrect answers and also to a bad use of time. The question clearly says "use your graphic display calculator" and it is worth 2 marks therefore a student should not spend more than 2 minutes to answer this part question. Time management is essential in this type of examinations and the IB rule is one minute – one mark.

4h.   *[1 mark]*

Markscheme

2   *(A1)*

[1 mark]

Examiners report

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4i.   *[1 mark]*

Markscheme

5.99 (accept 5.991)   *(A1)*(**ft**)

Note: Follow through from their part (b).

[1 mark]

Examiners report

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4j.

Markscheme

$\frac{28 \times 45}{100} = 12.6 = 13$ or

$\frac{28}{100} \times \frac{25}{100} \times 100 = 12.6 = 13$ *(M1)(A1)(AG)*

Notes: Award *(M1)* for correct formula and *(A1)* for correct substitution. Unrounded answer must be seen for the *(A1)* to be awarded.

[2 marks]

Examiners report

In general this part question was well answered. The major concerns of the examining team were the following:

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4k.

Markscheme

0.0321   *(G2)*

Note: For 0.032 award *(G1)(G1)(AP)*. For 0.03 with no working award *(G0)*.
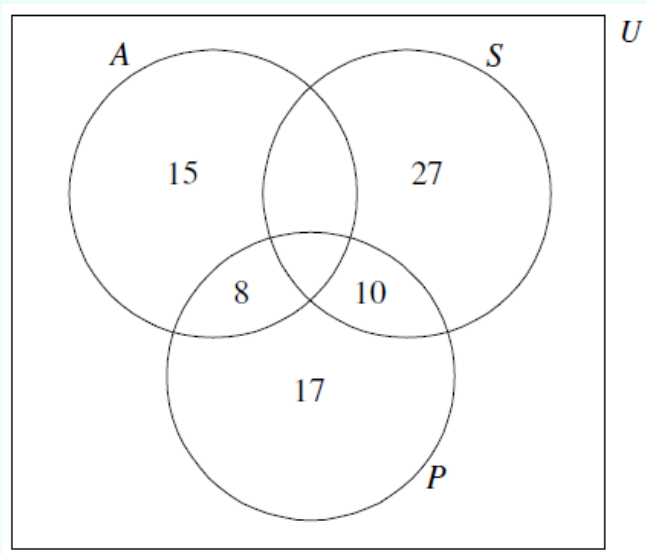
[2 marks]

Examiners report

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
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4l.

Markscheme

$0.0321 < 5.99$ or $0.984 > 0.05$ *(R1)*

accept H_0 *(A1)(ft)*

Note: If reason is incorrect both marks are lost, do not award *(R0)(A1)*.

[2 marks]

Examiners report

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
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5a.   [3 marks]

Markscheme

(A1) for rectangle and three labelled intersecting circles
(A1) for
15,
27 and
17
(A1) for
10 and
8   *(A3)*

[3 marks]

Examiners report

Part A

This part was successfully attempted by the great majority. A common mistake was the failure to intersect all three sets.

5b.   [2 marks]

Markscheme

$48 - (8 + 10 + 17)$ *or equivalent*   *(M1)*

$= 13$ *(A1)(ft)(G2)*

[2 marks]

5c.

Markscheme

$50 - (27 + 10 + 13)$ *(M1)*

Note: Award *(M1)* for working seen.

$= 0$ *(A1)*
number of elements in A
$= 36$ *(A1)(ft)(G3)*

Note: Follow through from (b).

[3 marks]

5d.

Markscheme

21   *(A1)(ft)*

Note: Follow through from (b) even if no working seen.

[1 mark]

5e.

Markscheme

54   *(M1)(A1)(ft)(G2)*

Note: Award *(M1)* for
17,
10,
27 seen. Follow through from (a).

[2 marks]

5f.   *[2 marks]*

Markscheme

$\frac{40}{120}$ $\left(\frac{1}{3}, 0.333, 33.3\%\right)$ *(A1)(A1)(G2)*

Note: Award *(A1)* for numerator, *(A1)* for denominator.

[2 marks]

Examiners report

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

5g.   *[2 marks]*

Markscheme

$\frac{34}{120}$ $\left(\frac{17}{60}, 0.283, 28.3\%\right)$ *(A1)(A1)(G2)*

Note: Award *(A1)* for numerator, *(A1)* for denominator.

[2 marks]

Examiners report

Part B

5h.   *[2 marks]*

Markscheme

$\frac{8}{28}$ $\left(\frac{2}{7}, 0.286, 28.6\%\right)$ *(A1)(A1)(G2)*

Note: Award *(A1)* for numerator, *(A1)* for denominator.

[2 marks]

5i. *[1 mark]*

Markscheme

customer satisfaction is **independent** of café   *(A1)*

Note: Accept "customer satisfaction is **not associated with** the café".

[1 mark]

Examiners report

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

The chi-squared test was well done by the great majority, however, it was clear that a number of centres do not teach this subject, since there were a number of scripts which either were left blank or showed no understanding in the responses seen.

5j. *[1 mark]*

Markscheme

2   *(A1)*

[1 mark]

Examiners report

Part B

5k. *[2 marks]*

Markscheme

0.754   *(G2)*

Note: Award *(G1)(G1)(AP)* for
0.75 or for correct answer incorrectly rounded to 3 s.f. or more, *(G0)* for
0.7.

[2 marks]

5l.   *[2 marks]*

Markscheme

since $\chi^2_{calc} < \chi^2_{crit}$ 5.991 accept (or Do not reject) H_0 *(R1)(A1)*(ft)

Note: Follow through from their value in (e).

OR

Accept (or Do not reject) H_0 as
p-value
$(0.686) > 0.05$ *(R1)(A1)*(ft)

Notes: Do not award *(A1)(R0)*. Award the *(R1)* for comparison of appropriate values.

[2 marks]

Examiners report

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

The chi-squared test was well done by the great majority, however, it was clear that a number of centres do not teach this subject, since there were a number of scripts which either were left blank or showed no understanding in the responses seen.

6a.   *[1 mark]*

Markscheme

1 (one)   *(A1)*   *(C1)*

Note:
6,
{6} or
{1} earns no marks.

[1 mark]

Examiners report

There was much confusion amongst candidates as to the understanding of the words *number of elements*. Many candidates simply wrote down
6 or
{6} and consequently lost the first mark.

6b.

Markscheme

1,
3,
5,
7,
9,
11   *(A1)*   *(C1)*

Note: Do not penalise if braces, parentheses or brackets are seen.

[1 mark]

Examiners report

There was much confusion amongst candidates as to the understanding of the words *number of elements*. Many candidates simply wrote down
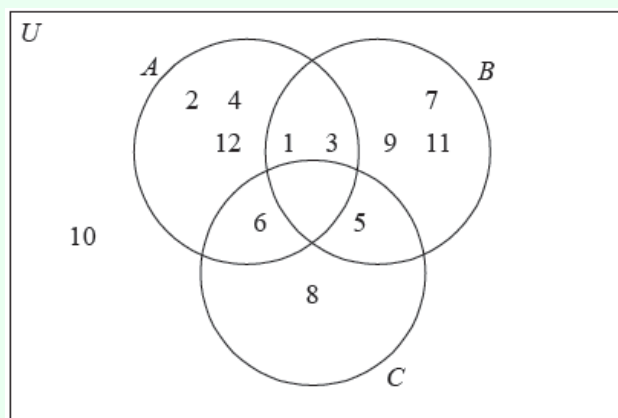6 or
{6} and consequently lost the first mark. Part (b) was done well and many successful attempts were made at completing the Venn diagram in part (c). The most common error in the last part of the question was the omission of the element
10.

6c.

Markscheme

(A1)(A1)(ft)(A1)(ft)(A1)(ft)   *(C4)*

Notes: Award *(A1)* for the empty set
$A \cap B \cap C$.

Award *(A1)*(ft) for the correct placement of
6,
5,
1 and
3.

Award *(A1)*(ft) for the correct placement of
2,
4,
12,
7,
9,
11,
8.

Award *(A1)*(ft) for the correct placement of
10.

Follow through from part (b).

[4 marks]

7a.   *[3 marks]*

Markscheme

If I do not choose history then I choose either psychology or I choose art   *(A1)(A1)(A1)*   *(C3)*

Notes: Award *(A1)* for 'if… (then)…'

Award *(A1)* for 'not choose history.'

Award *(A1)* for 'choose (either) psychology or art (or both).'

If the order of the statements is wrong award at most *(A1)(A1)(A0)*.

[3 marks]

7b.   *[1 mark]*

Markscheme

a	p	$\neg a$	$\neg a \Rightarrow p$
T	T	F	**T**
T	F	F	**T**
F	T	T	**T**
F	F	T	**F**

(A1)   *(C1)*

[1 mark]

7c.   *[2 marks]*

Markscheme

Neither, because not all the entries in the last column are the same.   *(A1)(ft)(R1)*   *(C2)*

Notes: Do not award *(R0)(A1)*. Follow through from their answer to part (b). Reasoning must be consistent with their answer to part (b).

[2 marks]

Examiners report

Many correct answers of 'neither' were seen in part (c) but the justification was sometimes lacking definitive reasoning. Without sufficient reasoning, the answer mark was not awarded.

8a.

[2 marks]

Markscheme

$\frac{108}{250}$ $\left(\frac{54}{125}, 0.432, 43.2\%\right)$ *(A1)(A1)* *(C2)*

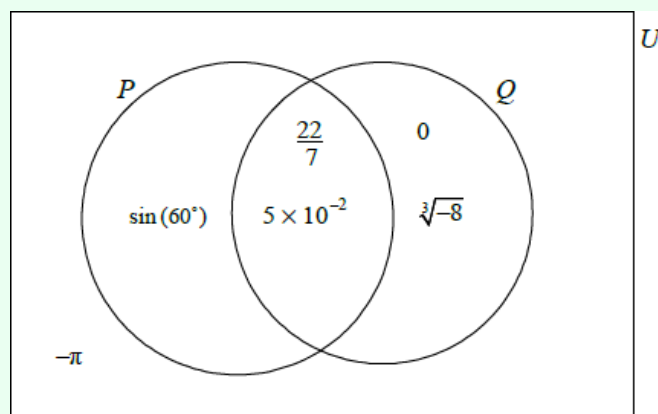**Note:** Award *(A1)* for numerator, *(A1)* for denominator.

[2 marks]

Examiners report

A reasonably well attempted question with parts (a) and (c) proving to provide many correct answers. A correct answer for part (b) however proved to be a little more elusive as, despite a correct numerator of
25 seen on many scripts, the total sample space was not reduced and a denominator of
250 lost the final mark in this part of the question. On a minority of scripts candidates simply wrote down decimal answers. Where these were correct, both marks for each part were earned. However, incorrect answers earned no marks – candidates would be well advised to at least write down the fraction answer first so that any part marks can be awarded. A case in question here was a predominance of incorrect answers of
0.10 or
10% for part (b). This, on its own earns no marks whereas
25/250 earned A1, A0.

8b.

[2 marks]

Markscheme

$\frac{25}{106}$ $(0.236, 23.6\%)$ *(A1)(A1)* *(C2)*

Note: Award *(A1)* for numerator, *(A1)* for denominator.

[2 marks]

Examiners report

A reasonably well attempted question with parts (a) and (c) proving to provide many correct answers. A correct answer for part (b) however proved to be a little more elusive as, despite a correct numerator of
25 seen on many scripts, the total sample space was not reduced and a denominator of
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0.10 or
10% for part (b). This, on its own earns no marks whereas
25/250 earned A1, A0.

8c.

[2 marks]

Markscheme

$\frac{71}{170}$ $(0.418, 41.8\%)$ *(A1)(A1)* *(C2)*

Note: Award *(A1)* for numerator, *(A1)* for denominator.

[2 marks]

9.

Markscheme

$(A1)(A1)(A1)(A1)(A1)(A1)$ $(C6)$

Note: Award $(A1)$ for each number placed once in the correct region. Accept equivalent forms for numbers.

[6 marks]

10a.

Markscheme

p	q	$\neg q$	$p \Rightarrow \neg q$	$\neg p$	$\neg p \Rightarrow q$
T	T	F	F	F	T
T	F	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	F

$(A1)(A1)(\text{ft})(A1)(A1)(\text{ft})$ $(C4)$

Note: Award $(A1)$ for each correct column (second column **(ft)** from first, fourth **(ft)** from third). Follow through from second column to fourth column for a consistent mistake in implication.

[4 marks]

10b.

[2 marks]

Markscheme

Since second and fourth columns are not identical   *(R1)*(**ft**)

⇒ Not a tautology   *(A1)*(**ft**)   *(C2)*

Note: *(R0)(A1)* may **not** be awarded.

[2 marks]

Examiners report

The truth table was well done by the majority of candidates but significantly fewer could give the correct reason for whether the compound proposition was a tautology, so many lost 2 marks in this part of the question.

11a.

[1 mark]

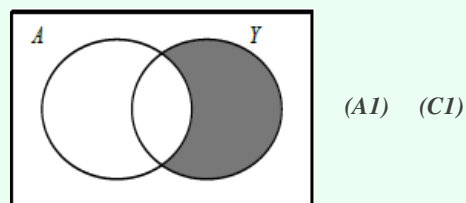## Markscheme

17   *(A1)*   *(C1)*

[1 mark]

Examiners report

This was probably the question that most candidates found the easiest. Nearly all candidates gained either 5 or 6 marks with the mark lost in shading the region on the Venn diagram.

11b.

[2 marks]

Markscheme

35 – 17   *(M1)*

= 18   *(A1)*   *(C2)*

Note: Award *(A1)* for correct answer only.

[2 marks]

Examiners report

This was probably the question that most candidates found the easiest. Nearly all candidates gained either 5 or 6 marks with the mark lost in shading the region on the Venn diagram.

Markscheme

$60 - (35 - 17) - (28 - 17) - 17$ *(M1)*

$= 14$ *(A1)(ft)* *(C2)*

Note: Follow through from (a) and (b).

[2 marks]

Examiners report

This was probably the question that most candidates found the easiest. Nearly all candidates gained either 5 or 6 marks with the mark lost in shading the region on the Venn diagram.

Markscheme

(A1) *(C1)*

[1 mark]

Examiners report

This was probably the question that most candidates found the easiest. Nearly all candidates gained either 5 or 6 marks with the mark lost in shading the region on the Venn diagram.

Markscheme

(A1)(A1)(A1) *(C3)*
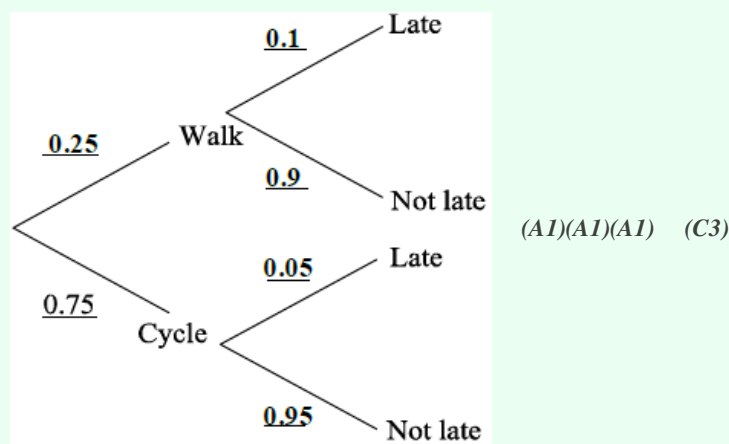
Note: Award *(A1)* for 0.25, *(A1)* for 0.1 and 0.9, *(A1)* for 0.05 and 0.95

[3 marks]

12b. [3 marks]

Markscheme

$P(\text{late}) = 0.25 \times 0.1 + 0.75 \times 0.05$ *(A1)(ft)(M1)*

Note: Award *(A1)(ft)* for two correct products from their diagram and award *(M1)* for addition of their two products.

$= 0.0625 \left(\frac{1}{16}, 6.25\%\right)$ *(A1)(ft)* *(C3)*

[3 marks]

Examiners report

Part (a) of this question was very well answered with many candidates gaining the maximum marks. Many candidates were less successful in part (b) and it seemed as if many of them either gained 3 marks or 0 marks. This shows that students who knew how to approach part (b) were also able to correctly substitute in the formula they used and reach the correct answer. Very few of those students lost the last mark for wrong rounding.

13a. [2 marks]

Markscheme

$= \frac{91}{150}(0.607, 60.6\%, 60.7\%)$ *(A1)(A1)* *(C2)*

Note: Award *(A1)* for numerator, *(A1)* for denominator.

[2 marks]

Examiners report

Parts (a) and (b) were well answered with many candidates gaining 4 marks there. The conditional probability in part (c) proved to be more challenging. Nearly all candidates attempted this question showing that time was not a factor in this paper. Many candidates gave their answers as incorrectly rounded decimals, which incurred an accuracy penalty and prevented them from gaining the maximum marks.

13b. [2 marks]

Markscheme

$= \frac{111}{150}\left(\frac{37}{50}, 0.74, 74\%\right)$ *(A1)(ft)(A1)* *(C2)*

Note: Award *(A1)(ft)* for their numerator in (a) +20 provided the final answer is not greater than 1. *(A1)* for denominator.

[2 marks]

13c.

Markscheme

$\frac{16}{91}$ (0.176, 17.6%)   *(A1)(A1)*(ft)   *(C2)*

Note: Award *(A1)* for numerator and *(A1)*(ft) for denominator. Follow through from their numerator in (a) provided answer is not greater than 1.

[2 marks]

Examiners report

14a.

Markscheme

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$(p \vee (p \wedge q)) \Rightarrow p$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

(A1)(A1)(ft)*(A1)*(ft)   *(C3)*

Note: Award *(A1)* for each correct column.

[3 marks]

Examiners report

The truth table was very well answered and where the table was incorrect a follow through mark could be given for part (b) for a correct answer resulting from their final column. Some candidates appeared unsure of the concept of a tautology.

14b.

Markscheme

tautology   *(A1)*(ft)   *(C1)*

Note: Follow through from their last column.

[1 mark]

14c.   *[2 marks]*

Markscheme

$\neg q \Rightarrow p$ *(A1)(A1)*   *(C2)*

Note: Award *(A1)* for
$\neg q$ and p in correct order, *(A1)* for
$\Rightarrow$ sign.

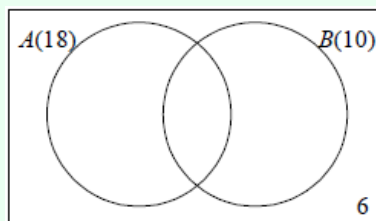*[2 marks]*

Examiners report
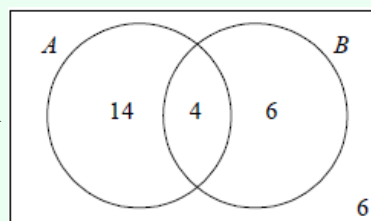
Nearly all candidates could write the proposition in part (c) in symbolic form.

15a.   *[2 marks]*

Markscheme

Note: Award *(A2)* for 3 correctly placed values, and no extras (4 need not be seen), *(A1)* for 2 correctly placed values, *(A0)* for 1 or no correctly placed values.

[2 marks]

Examiners report

The first two parts of this question were well answered with most candidates completing the Venn diagram correctly and finding the number in the intersection. The final part, requiring a conditional probability to be found, proved more difficult as many candidates tried to use the formula, when all that was required was to look at the values in the Venn diagram. Follow through marks were awarded in part (c) for values correctly used from parts (a) and (b).

15b.   *[2 marks]*

Markscheme

$18 + 10 + 6 = 30$ *(M1)*
$= 4$ *(A1)*   *(C2)*

[2 marks]

15c.   *[2 marks]*

Markscheme

$P(A|B) = \frac{4}{10} \left(\frac{2}{5},\ 0.4,\ 40\ \%\right)$ *(A1)(ft)(A1)*   *(C2)*

Note: Award *(A1)(ft)* for their numerator from part (b), *(A1)* for denominator.

[2 marks]

16a.   *[2 marks]*

Markscheme

$0.8 = 0.5 + 0.6 - P(A \cap B)$ *(M1)*
$P(A \cap B) = 0.3$ *(A1)*   *(C2)*

Note: Award *(M1)* for correct substitution, *(A1)* for correct answer.

[2 marks]

16b.   *[2 marks]*

Markscheme

$P(A|B) = \frac{0.3}{0.6}$ *(M1)*

$= 0.5$ *(A1)(ft)*   *(C2)*

Note: Award *(M1)* for correct substitution in conditional probability formula. Follow through from their answer to part (a), provided probability is not greater than one.

[2 marks]

16c.   *[2 marks]*

Markscheme

$P(A \cap B) = P(A) \times P(B)$ or $0.3 = 0.5 \times 0.6$ *(R1)*

OR

$P(A|B) = P(A)$ *(R1)*

they are independent. (Yes)   *(A1)*(ft)   *(C2)*

Note: Follow through from their answers to parts (a) or (b).

Do not award *(R0)(A1)*.

[2 marks]

17a.   *[1 mark]*

Markscheme

$a = 0$
$\left(\frac{0}{4}\right)$ *(A1)*

[1 mark]

17b.   *[2 marks]*

Markscheme

$b = \frac{3}{4}(0.75,\ 75\%)$ *(A2)(G2)*

[2 marks]

17c.
Markscheme

$\frac{4}{5} \times \frac{3}{4}$ *(M1)(A1)*

$\frac{12}{20} \left(\frac{3}{5}, 0.6, 60\%\right)$ *(A1)(ft)(G2)*

Note: Award *(M1)* for multiplying two probabilities, *(A1)* for using their probabilities, *(A1)* for answer.

[3 marks]

Examiners report

This question was well handled by most of the candidates except for (c)(ii) in which they had to find a conditional probability. Some candidates did not copy the second tree diagram in the answer sheets and instead wrote their answers in the exam booklet thus losing the 3 marks allocated to part (b).

17d.
Markscheme

(A1)(A1)(A1)
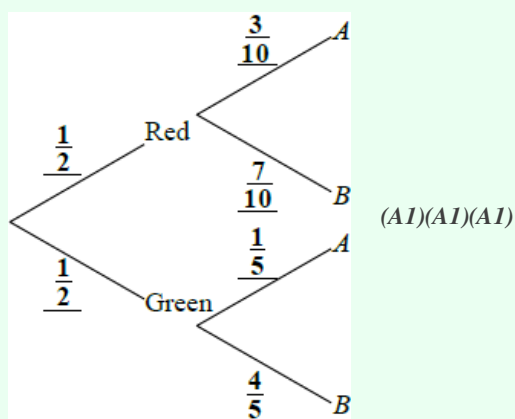
Note: Award *(A1)* for each pair.

[3 marks]

Examiners report

This question was well handled by most of the candidates except for (c)(ii) in which they had to find a conditional probability. Some candidates did not copy the second tree diagram in the answer sheets and instead wrote their answers in the exam booklet thus losing the 3 marks allocated to part (b).

17e.
Markscheme

$\frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{1}{5}$ *(M1)(M1)*

$= \frac{5}{20} \left(\frac{1}{4}, 0.25, 25\%\right)$ *(A1)(ft)(G2)*

Note: Award *(M1)* for two products seen with numbers from the problem, *(M1)* for adding two products. Follow through from their tree diagram.

[3 marks]

17f.   *[3 marks]*

Markscheme

$\dfrac{\frac{1}{2} \times \frac{3}{10}}{\frac{1}{4}}$ *(M1)(A1)*

$= \dfrac{3}{5}$ (0.6, 60 %)   *(A1)(ft)(G2)*

Note: Award *(M1)* for substituted conditional probability formula, *(A1)* for correct substitution.

Follow through from their part (b) and part (c) (i).

[3 marks]

Examiners report

18.   *[6 marks]*

Markscheme
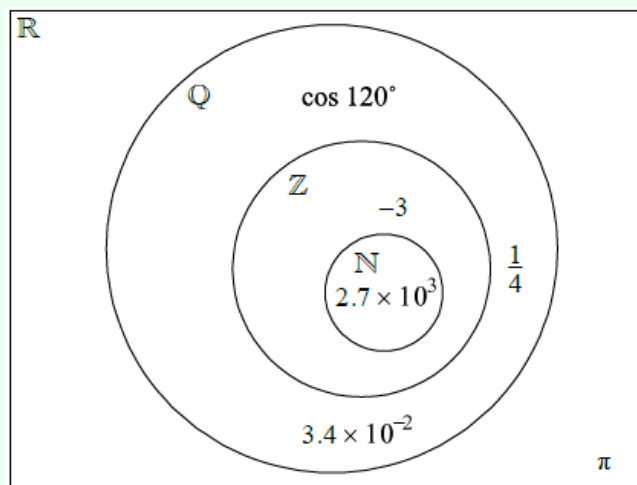
(A1)(A1)(A1)(A1)(A1)(A1)   *(C6)*

Note: Award *(A1)* for each number placed once in the correct section. Accept equivalent forms for numbers.

[6 marks]

Examiners report

About half of the students answered this question correctly. The placement of cos120 and π appeared to cause the most problems.

19a. [2 marks]

Markscheme

If a quadrilateral is not a square (then) the four sides of the quadrilateral are not equal.   *(A1)(A1)*   *(C2)*

Note: Award *(A1)* for "if…(then)", *(A1)* for the correct phrases in the correct order.

[2 marks]

Examiners report

There was confusion among some students about which was the inverse and converse of the given statement.

19b. [2 marks]

Markscheme

If the four sides of the quadrilateral are equal (then) the quadrilateral is a square.   *(A1)(A1)*(ft)   *(C2)*

Note: Award *(A1)* for "if…(then)", *(A1)*(ft) for the correct phrases in the correct order.

Note: Follow through in (b) if the inverse and converse in (a) and (b) are correct and reversed.

[2 marks]

Examiners report

There was confusion among some students about which was the inverse and converse of the given statement.

19c. [2 marks]

Markscheme

The converse is not always true, for example a rhombus (diamond) is a quadrilateral with four equal sides, but it is not a square. *(A1)(R1)*   *(C2)*

Note: Do not award *(A1)(R0)*.

[2 marks]

Examiners report

There was confusion among some students about which was the inverse and converse of the given statement. Part (c) was poorly done with very few students able to provide an example that shows that the converse is not always true.

20a. [2 marks]

Markscheme

Total $= 2 + 3 + 5 + 7 + 11 + 5 + 6 + 9 + 2 + 1$ *(M1)*

(M1) is for a sum of frequencies.

$= 51$ *(A1)(G2)*

[2 marks]

20b.   *[5 marks]*

Markscheme

Unit penalty (UP) is applicable where indicated in the left hand column.

(i) modal interval is 60 – 70

Award (A0) for 65 **(A1)**

(ii) median is length of fish no. 26, **(M1)(A1)**

also 60 – 70 **(G2)**

Can award (A1)(ft) or (G2)(ft) for 65 if (A0) was awarded for 65 in part (i).

(iii) mean is

$$\frac{2 \times 25 + 3 \times 35 + 5 \times 45 + 7 \times 55 + \ldots}{51}$$ **(M1)**

(UP) = 69.5 cm (3sf) **(A1)(ft)(G1)**

Note: (M1) is for a sum of (frequencies multiplied by midpoint values) divided by candidate's answer from part (a). Accept mid-points 25.5, 35.5 etc or 24.5, 34.5 etc, leading to answers 70.0 or 69.0 (3sf) respectively. Answers of 69.0, 69.5 or 70.0 (3sf) with no working can be awarded (G1).

[5 marks]

Examiners report

a) b), c) There was much confusion about how to present the intervals. Often the mid-point only was seen. (eg. 65 instead of 60-70). Understanding of mode, median and mean was usually good but too many candidates wasted time calculating standard deviation by hand and got it wrong. In c(ii) 'greater than three' caused no problems but 'above the mean' was often ignored.

20c.   *[3 marks]*

Markscheme

Unit penalty (UP) is applicable where indicated in the left hand column.

(UP) (i) standard deviation is 21.8 cm **(G1)**

For any other answer without working, award (G0). If working is present then (G0)(AP) is possible.

(ii)
$69.5 + 3 \times 21.8 = 134.9 > 120$ **(M1)**

no fish **(A1)(ft)(G1)**

For 'no fish' without working, award (G1) regardless of answer to (c)(i). Follow through from (c)(i) only if method is shown.

[3 marks]

Examiners report

a) b), c) There was much confusion about how to present the intervals. Often the mid-point only was seen. (eg. 65 instead of 60-70). Understanding of mode, median and mean was usually good but too many candidates wasted time calculating standard deviation by hand and got it wrong. In c(ii) 'greater than three' caused no problems but 'above the mean' was often ignored.

20d. *[2 marks]*

Markscheme

5 fish are less than 40 cm in length,   *(M1)*

Award (M1) for any of

$\frac{5}{51}$,

$\frac{46}{51}$, *0.098 or 9.8%, 0.902, 90.2% or 5.1 seen.*

hence no fine.   *(A1)(ft)*

Note: There is no G mark here and (M0)(A1) is never allowed. The follow-through is from answer in part (a).

[2 marks]

Examiners report

d) This was often well done, even if earlier parts were poorly done.

20e. *[2 marks]*

Markscheme

(i) and (iii) are correct.   *(A1)(A1)*

[2 marks]

Examiners report

e) Rather mixed performance here. It was hard to identify any consistency in the errors made.

Too much time was spent on this question. It was only worth two marks and candidates ought to have realised that it relied on a general pictorial understanding of the concepts, possibly supplemented by a little elementary arithmetic only, to compare (iii) and (vi). With good understanding, many of the options could be ruled out in a few seconds.

21a. *[3 marks]*

Markscheme

(i) P (a dog is grey and has the yellow bowl)

$= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} (= 0.111)$ *(M1)(A1)(G2)*

The (M1) is for multiplying two values along any branch of the tree.

(ii) P (dog does not get yellow bowl)
$= \frac{2}{3}$ (= 0.667 (3sf) or 0.6)   *(A1)*

[3 marks]

21b.   *[9 marks]*

Markscheme

(i) The tree diagram should show the values
$\frac{1}{2}, \frac{1}{2}$ for the brown branch and
$\frac{1}{4}, \frac{3}{4}$ in the correct positions for the grey branch.   *(A1)(A1)*(ft)

Follow through if the branches are interchanged.

(ii) P (the dog is grey or is playing with a stick, but not both)

$= \frac{1}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{1}{2}$ *(M1)*

$= \frac{7}{12}$ ($= 0.583$)   *(A1)*(ft)*(G1)*

The (M1) is for showing two correct products (whether added or not). Follow through from b(i). Award (M1) for \(\frac{1}{3} + \frac{1}{4}\) (must be a sum).

(iii) P (dog is grey given that it is playing with stick)

$\frac{P(G \cap S)}{P(S)} = \frac{\frac{1}{3} \times \frac{1}{4}}{\left(\frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{1}{3} \times \frac{1}{4}\right)}$ or
$\frac{1}{12} / \frac{5}{12}$ *(M1)(A1)*(ft)

(M1) for substituted conditional probability formula, (A1) for correct substitutions.

$= \frac{1}{5}$ ($= 0.2$)   *(A1)*(ft)*(G2)*

(iv) P (grey and fed from yellow bowl and not playing with stick)
$= \frac{1}{3} \times \frac{1}{3} \times \frac{3}{4} = \frac{1}{12}$ (=
$\frac{3}{36} = 0.0833$ 3sf).   *(M1)(A1)*(ft)*(G1)*

(M1) is for product of 3 reasonable probability values.

[9 marks]

Markscheme

(A1)(A1)(A1)(ft)(A1)(ft)
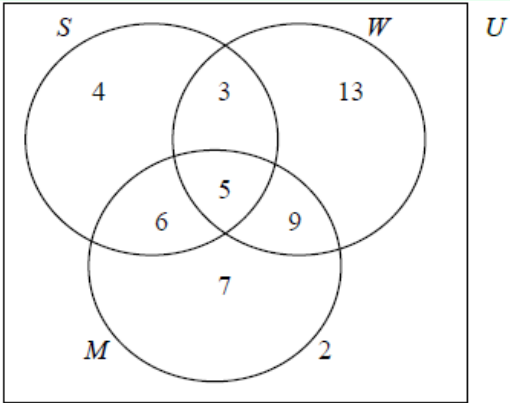
*Award **(A1)** for 2 (must be in a box), **(A1)** for 7, **(A1)(ft)** for 6 and 4, **(A1)(ft)** for 9 and 13. Observe the assignment of **(ft)** marks strictly here. Example A common error is likely to be 11 instead of 6 **(A0)**. In this case follow through to 4 and 18 **(A1)(ft)** for the final pair. Here the 4 follows from the total of 27 for n(M).*

[4 marks]

Examiners report

(ii) Most candidates had some idea how to fill in the numbers on the diagram. Full marks were common here and most candidates got some of the marks.

Markscheme

(i)
$n(M \cap W) = 14$ **(A1)(ft)**

(ii)
$n(M' \cup S) = 22 + 11$ **OR**
$15 + 18$ **(A1)(ft)**

$= 33$ **(A1)(ft)**

*Award **(A2)** if answer 33 is seen. Award **(A1)** for any of 22, 11, 15 or 18 seen but 33 absent.*

[3 marks]

Examiners report

Part b(i) was handled better than b(ii), with the complement causing problems. Extensive follow-through was used here from (a).

21e. *[2 marks]*

Markscheme

P (both mice short-tailed)
$= \frac{18}{49} \times \frac{17}{48} = \frac{306}{352}$ (= 0.130).   *(M1)(A1)(ft)(G1)*

(Allow alternatives such as 153/1176 or 51/392.) Award (M1) for any of
$\frac{18}{49}$ *and*
$\frac{17}{48}$ *or*
$\frac{18}{49} \times \frac{17}{49}$ *or*
$\frac{18}{49} + \frac{17}{48}$ *seen.*

[2 marks]

Examiners report

Part (c) was rarely completed, perhaps due to time constraints, but also due to lack of understanding.

22a. *[1 mark]*

Markscheme

Both are 'p or q', the first is 'but not both'   *(A1)*

Note: Award mark for clear understanding if wording is poor.   (C1)

[1 mark]

Examiners report

a) The majority of candidates were able to explain the difference between inclusive and exclusive correctly but many used "and" and "or" to distinguish between the two.

22b. *[4 marks]*

Markscheme

$\neg q$	$p \veebar q$	$\neg p \veebar \neg q$	$p \veebar q \Rightarrow \neg p \veebar \neg q$
	F		T
T			
		F	

(A1)(A1)(ft)(A1)(A1)

Note: Follow through is for final column.   (C4)

[4 marks]

Examiners report

b) Less than half were able to find the truth value of the two disjunctions in the table correctly. Most candidates did gain some marks but a number of them left at least one cell blank even though it was a 50% chance of getting the correct value.

22c. *[1 mark]*

Markscheme

Tautology.   *(A1)(ft)   (C1)*

[1 mark]

23a.   *[4 marks]*

Markscheme

(i)

$B = 2,3,5,7$ *(A1)*

Brackets not required

(ii)

$C \cap B = 2,3,5$ *(A1)(ft)*

Follow through only from incorrect B

(iii)

$C' = 0,1,7,8,9$ *(A1)(ft)*

$B \cup C' = 0,1,2,3,5,7,8,9$ *(A1)(ft)*

Note: Award (A1) for correct
C' seen. The first (A1)(ft) in (iii) can be awarded only if C was listed incorrectly and a mark was lost as a result in (a)(ii). If C was not listed and
C' is wrong, the first mark is lost. The second mark can (ft) within part (iii) as well as from (i).   (C4)

[4 marks]

23b.   *[2 marks]*

Markscheme

"If x is not a positive integer between 1 and 7, then x is not a prime number less than 10."   *(A1)(A1)*

*Award (A1) for **both** (not) statements, (A1) for correct order.   (C2)*

[2 marks]

24a. [3 marks]

Markscheme

If Alex does not play the flute then he is **either** a scientist **or** from Uruguay.   *(A1)(A1)(A1)*   *(C3)*

Note: Award *(A1)* if… then, correct *(A1)* antecedent, *(A1)* correct consequent.

Examiners report

[N/A]

24b. [2 marks]

Markscheme

p	q	r	$\neg r$	$q \vee p$	$\neg r \Rightarrow (q \vee p)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	F

(A1)(A1)   *(C2)*

Examiners report

[N/A]

24c. [1 mark]

Markscheme

Not all entries in the final column are T.   *(R1)*   *(C1)*

Examiners report

[N/A]

25a. [2 marks]

Markscheme

If ABCD is a square, then ABCD has four equal sides.   *(A1)(A1)*   *(C2)*

Note: Award *(A1)* for if… then, *(A1)* for propositions in the correct order.

Examiners report

[N/A]

25b.

Markscheme

If ABCD is not a square, then ABCD does not have four equal sides.   *(A1)(A1)*   *(C2)*

Note: Award *(A1)* for if… then, *(A1)* for propositions in the correct order.

Examiners report

[N/A]

25c.

Markscheme

Not a valid argument. ABCD may have 4 equal sides but will not **necessarily** be a square. (It may be a rhombus)   *(A1)(R1)*   *(C2)*

Note: Award *(R1)* for correct reasoning, award *(A1)* for a consistent conclusion with their answer in part (b).

It is therefore possible that *(R1)(A0)* may be awarded, but *(R0)(A1)* can never be awarded.
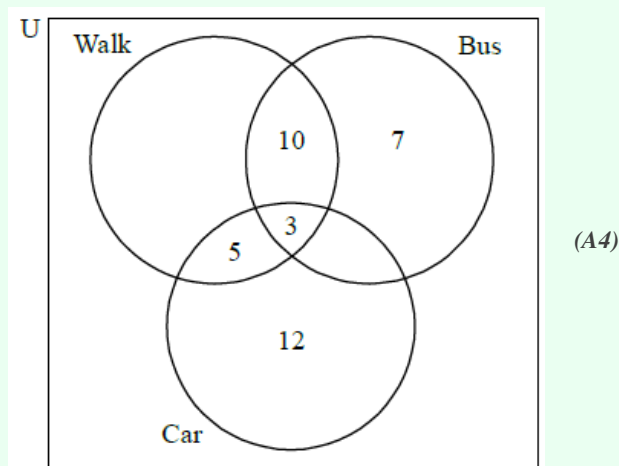
Note: Simple examples of determining the validity of an argument without the use of a truth table may be tested.

Examiners report

[N/A]

26a.

Markscheme

(A4)

Note: Award *(A1)* for rectangle and three labelled intersecting circles, *(A1)* for
3, *(A1)* for
5 and
10, *(A1)* for
7 and
12.

Examiners report

[N/A]

26b. [4 marks]

Markscheme

(i)

$28 - (10 + 3 + 7) = 8$ *(M1)(A1)(ft)(G2)*

Note: Follow through from their Venn diagram.

(ii)

$5 + 3 + 8 + 12 = 28$ *(M1)(A1)(ft)(G2)*

Note: Follow through from part (b)(i) and their Venn diagram.

Examiners report

[N/A]

26c. [2 marks]

Markscheme

$P(\text{walk}|\text{bus}) = \frac{13}{28}$
$(0.464, 46.4\%)$ (
$0.464285\ldots$)   *(A1)(A1)(ft)(G2)*

Note: Award *(A1)(ft)* for the numerator, *(A1)* for denominator.

Examiners report

[N/A]

26d. [7 marks]

Markscheme

(i)

$\frac{23}{50} \times \frac{22}{49}$ *(A1)(M1)(M1)*

Note: Award *(A1)* for
23 seen, *(M1)* for non replacement, *(M1)* for multiplying their fractions.

$= \frac{506}{2450}$
$(0.207, 20.7\%)$ (
$0.206530\ldots$)   *(A1)(G3)*

(ii)

$\frac{23}{50} \times \frac{27}{49} + \frac{27}{50} \times \frac{23}{49}$ *(A1)(ft)(M1)*

Notes: Award *(A1)(ft)* for two products, *(M1)* for adding two products. Do not penalise in (ii) for consistent use of with replacement.

$= \frac{1242}{2450}$
$(0.507, 50.7\%)$ (
$0.509638\ldots$)   *(A1)(ft)(G2)*

Examiners report

[N/A]