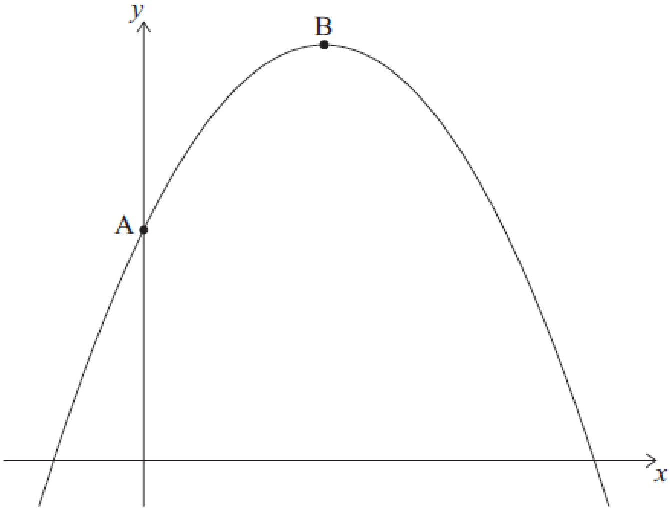
Topic 6 Part 1 *[251 marks]*

The graph of the quadratic function
$f(x) = c + bx - x^2$ intersects the y-axis at point A(0, 5) and has its vertex at point B(2, 9).



1a. Write down the value of c. *[1 mark]*

1b. Find the value of b. *[2 marks]*

1c. Find the x-intercepts of the graph of f. *[2 marks]*

1d. Write down $f(x)$ in the form $f(x) = -(x - p)(x + q)$. *[1 mark]*

In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

$$D(t) = 1.2 \times (0.87)^t,\ t \geqslant 0$$

where
D is the amount of the drug in the bloodstream in mg per litre
$(\text{mg}\,\text{l}^{-1})$ and
t is the time in hours.

2a. Write down the amount of the drug in the bloodstream at $t = 0$. *[1 mark]*

2b. Calculate the amount of the drug in the bloodstream after 3 hours. *[2 marks]*

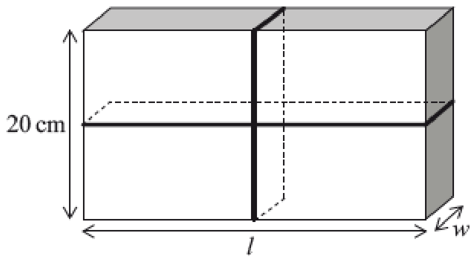
2c. Use your graphic display calculator to determine the time it takes for the amount of the drug in the bloodstream to decrease to $0.333\ \text{mg}\text{l}^{-1}$. *[3 marks]*

A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length
l cm, width
w cm and height of
20 cm.
The total volume of the parcel is
$3000\ \text{cm}^3$.

3a. Express the volume of the parcel in terms of l and w. *[1 mark]*

3b. Show that $l = \frac{150}{w}$. *[2 marks]*

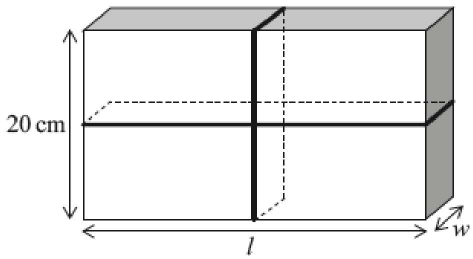
3c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



Show that the length of string, S cm, required to tie up the parcel can be written as

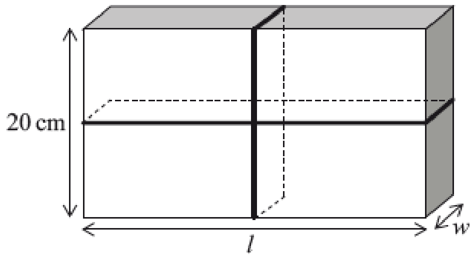
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

3d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



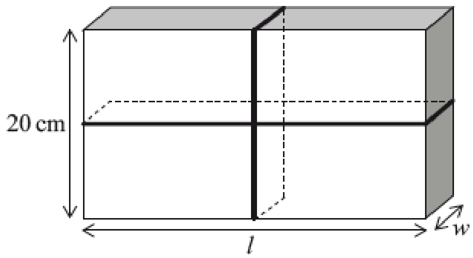
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

3e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[3 marks]*



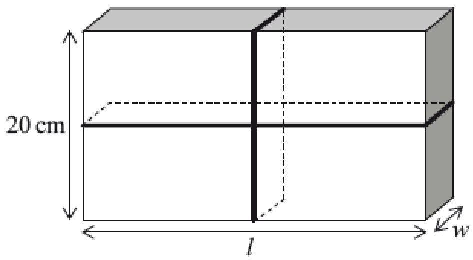
Find
$\frac{dS}{dw}$.

3f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*



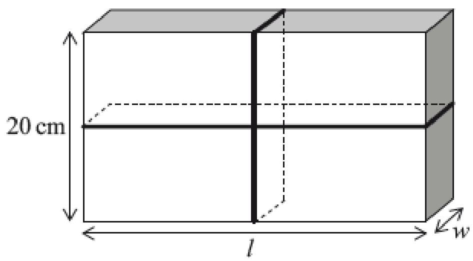
Find the value of
w for which
S is a minimum.

3g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[1 mark]*



Write down the value,
l, of the parcel for which the length of string is a minimum.

3h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. *[2 marks]*

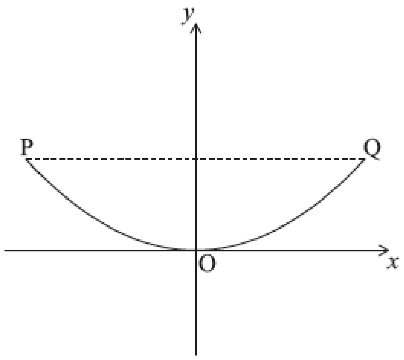


Find the minimum length of string required to tie up the parcel.

The front view of the edge of a water tank is drawn on a set of axes shown below.

The edge is modelled by

$y = ax^2 + c$.



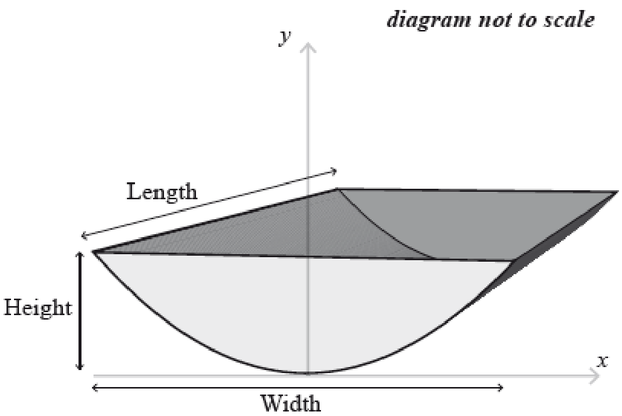
Point P has coordinates $(-3, 1.8)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(3, 1.8)$.

4a. Write down the value of c. *[1 mark]*

4b. Find the value of a. *[2 marks]*

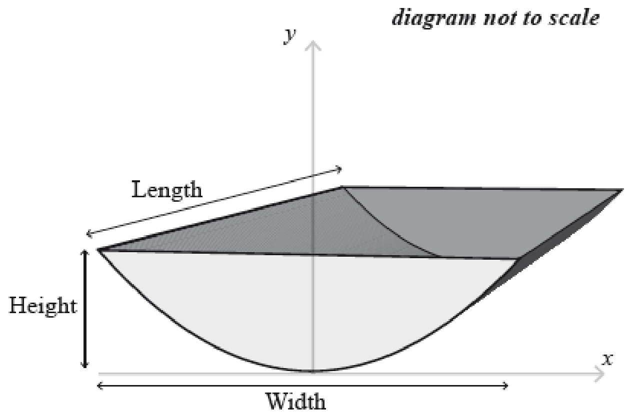
4c. Hence write down the equation of the quadratic function which models the edge of the water tank. *[1 mark]*

4d. The water tank is shown below. It is partially filled with water. *[2 marks]*



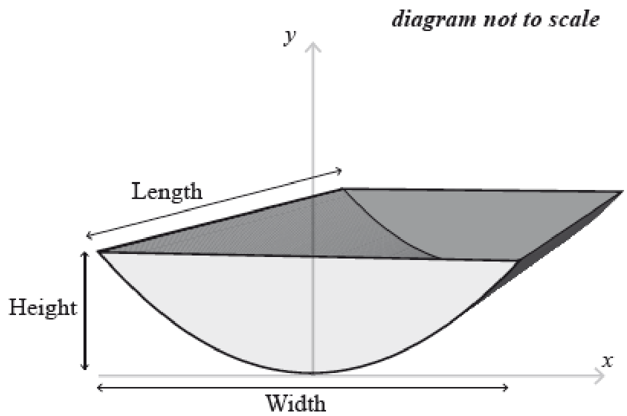
Calculate the value of y when $x = 2.4$ m.

4e. The water tank is shown below. It is partially filled with water. *[2 marks]*



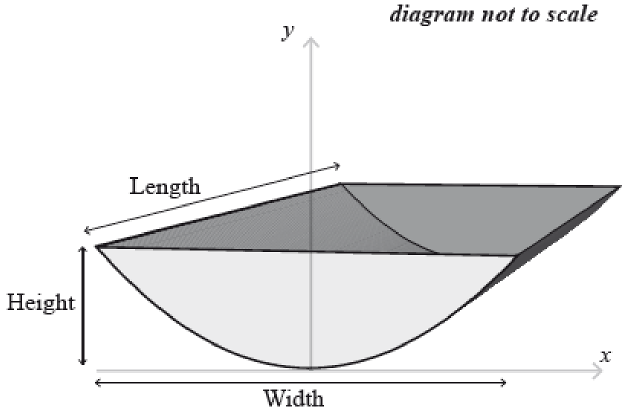
State what the value of
x and the value of
y represent for this water tank.

4f. The water tank is shown below. It is partially filled with water. *[2 marks]*



Find the value of
x when the height of water in the tank is
0.9 m.

4g. The water tank is shown below. It is partially filled with water. *[2 marks]*



When the water tank is filled to a height of $0.9\,\text{m}$, the front cross-sectional area of the water is $2.55\,\text{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\,\text{m}^3$.

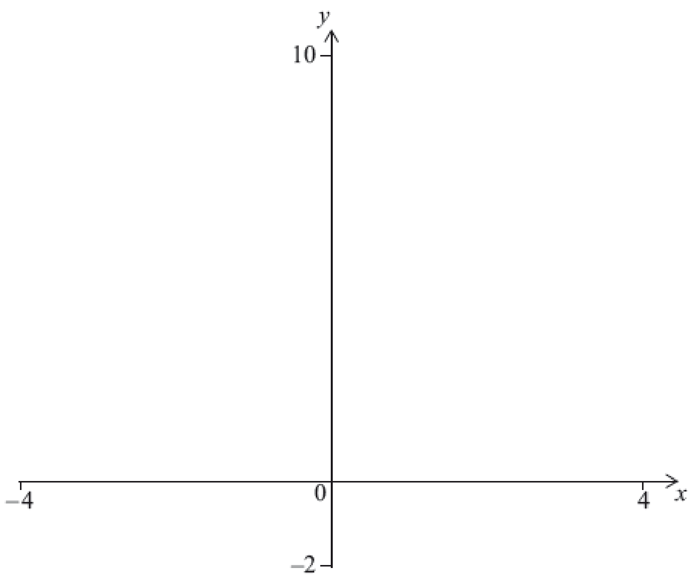
(ii) Calculate the percentage of water in the tank.

Consider the two functions, f and g, where

$$f(x) = \frac{5}{x^2+1}$$

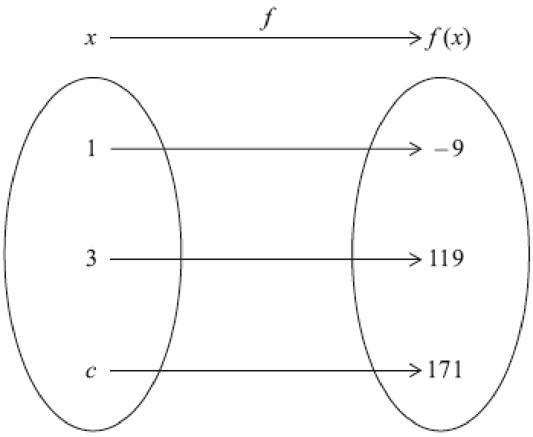
$$g(x) = (x-2)^2$$

5a. Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below. Indicate clearly the points where each graph intersects the *y*-axis. *[4 marks]*



5b. Use your graphic display calculator to solve $f(x) = g(x)$. *[2 marks]*

A quadratic function
$f: x \mapsto ax^2 + b$, where
a and
$b \in \mathbb{R}$ and
$x \geqslant 0$, is represented by the mapping diagram.



6a. Using the mapping diagram, write down two equations in terms of a and b. *[2 marks]*

6b. Solve the equations to find the value of *[2 marks]*

(i) a;

(ii) b.

6c. Find the value of c. *[2 marks]*

A computer virus spreads according to the exponential model

$$N = 200 \times (1.9)^{0.85t},\ t \geqslant 0$$

where
N is the number of computers infected, and
t is the time, in hours, after the initial infection.

7a. Calculate the number of computers infected after 6 hours. *[2 marks]*

7b. Calculate the time for the number of infected computers to be greater than 1 000 000. *[4 marks]*
Give your answer correct to the nearest hour.

Consider the function
$f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20$.

8a. Find $f(-2)$. *[2 marks]*

8b. Find $f'(x)$. *[3 marks]*

8c. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. *[5 marks]*

Using your answer to part (b), show that there is a second local minimum at $x = 3$.

8d. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. *[4 marks]*

Sketch the graph of the function $f(x)$ for $-5 \leqslant x \leqslant 5$ and $-40 \leqslant y \leqslant 50$. Indicate on your sketch the coordinates of the y-intercept.

8e. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. *[2 marks]*

Write down the coordinates of the local maximum.

8f. Let T be the tangent to the graph of the function $f(x)$ at the point $(2,-12)$. *[2 marks]*

Find the gradient of T.

8g. The line L passes through the point $(2, -12)$ and is perpendicular to T. *[5 marks]*

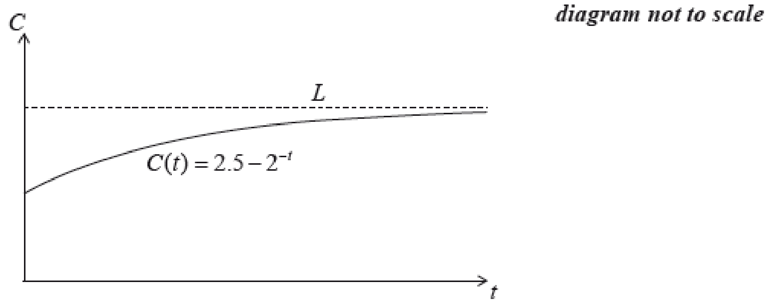
L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$.

Find

(i) the gradient of L;

(ii) the value of b and the value of c.

The amount of electrical charge, C, stored in a mobile phone battery is modelled by $C(t) = 2.5 - 2^{-t}$, where t, in hours, is the time for which the battery is being charged.



9a. Write down the amount of electrical charge in the battery at $t = 0$. [1 mark]

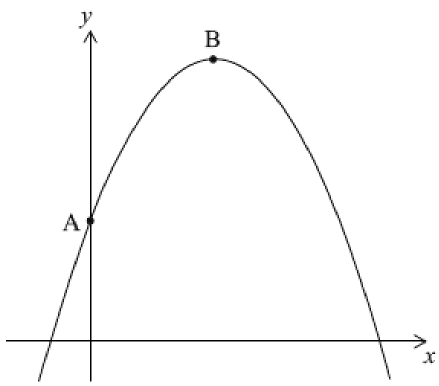
9b. The line L is the horizontal asymptote to the graph. [2 marks]

Write down the equation of L.

9c. To download a game to the mobile phone, an electrical charge of 2.4 units is needed. [3 marks]

Find the time taken to reach this charge. Give your answer correct to the nearest minute.

The graph of the quadratic function $f(x) = ax^2 + bx + c$ intersects the y-axis at point A (0, 5) and has its vertex at point B (4, 13).



10a. Write down the value of c. [1 mark]

10b. By using the coordinates of the vertex, B, or otherwise, write down **two** equations in a and b. [3 marks]

10c. Find the value of a and of b. [2 marks]

Consider the sequence
$u_1, u_2, u_3, \ldots, u_n, \ldots$ where

$$u_1 = 600,\ u_2 = 617,\ u_3 = 634,\ u_4 = 651.$$

The sequence continues in the same manner.

11a. Find the value of u_{20}. *[3 marks]*

11b. Find the sum of the first 10 terms of the sequence. *[3 marks]*

11c. Now consider the sequence *[3 marks]*
$v_1, v_2, v_3, \ldots, v_n, \ldots$ where

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$$

This sequence continues in the same manner.
Find the exact value of
v_{10}.

11d. Now consider the sequence *[3 marks]*
$v_1, v_2, v_3, \ldots, v_n, \ldots$ where

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$$

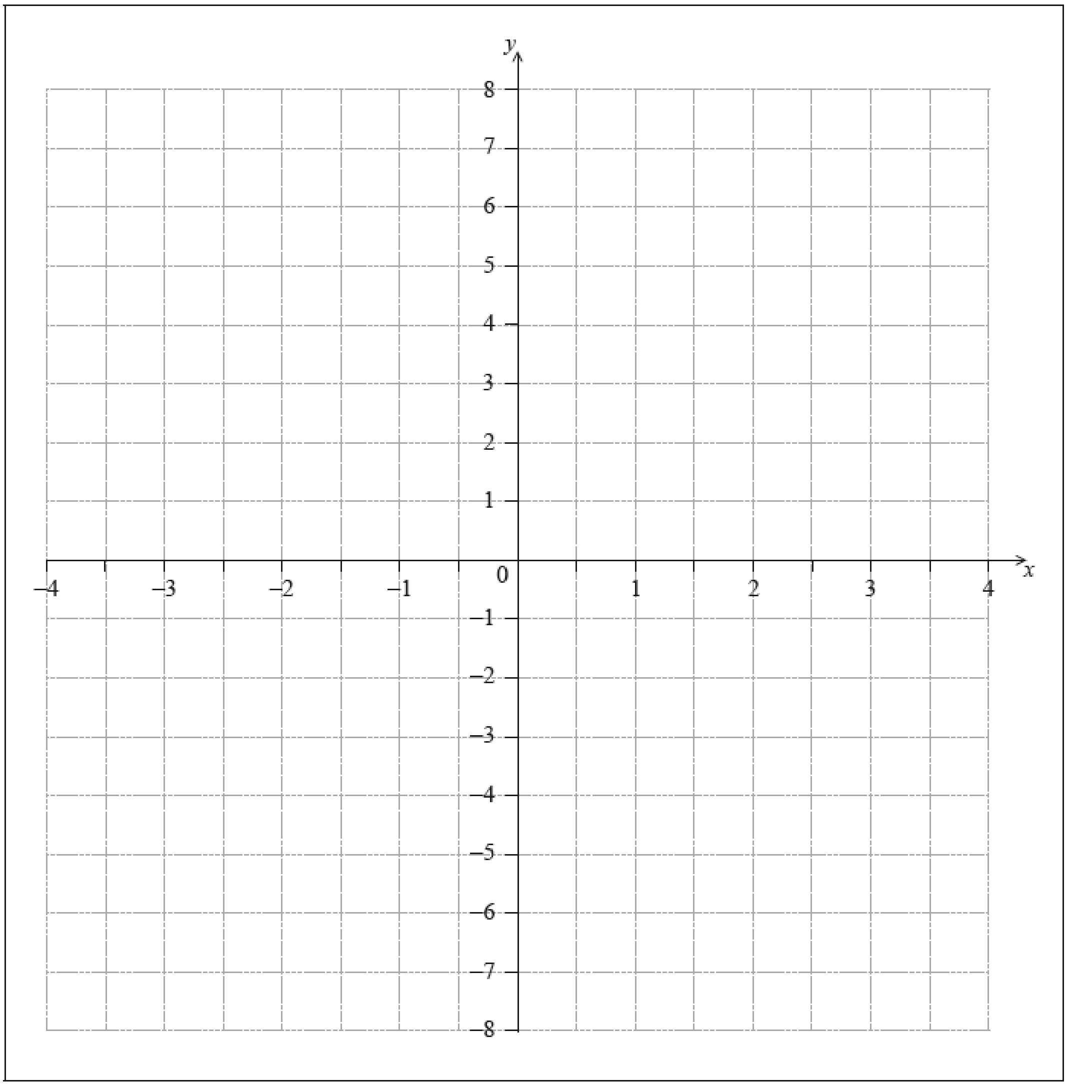
This sequence continues in the same manner.
Find the sum of the first 8 terms of this sequence.

11e. k is the smallest value of *[3 marks]*
n for which
v_n is greater than
u_n.
Calculate the value of
k.

The axis of symmetry of the graph of a quadratic function has the equation $x = -\frac{1}{2}$

.

12a. Draw the axis of symmetry on the following axes. *[1 mark]*



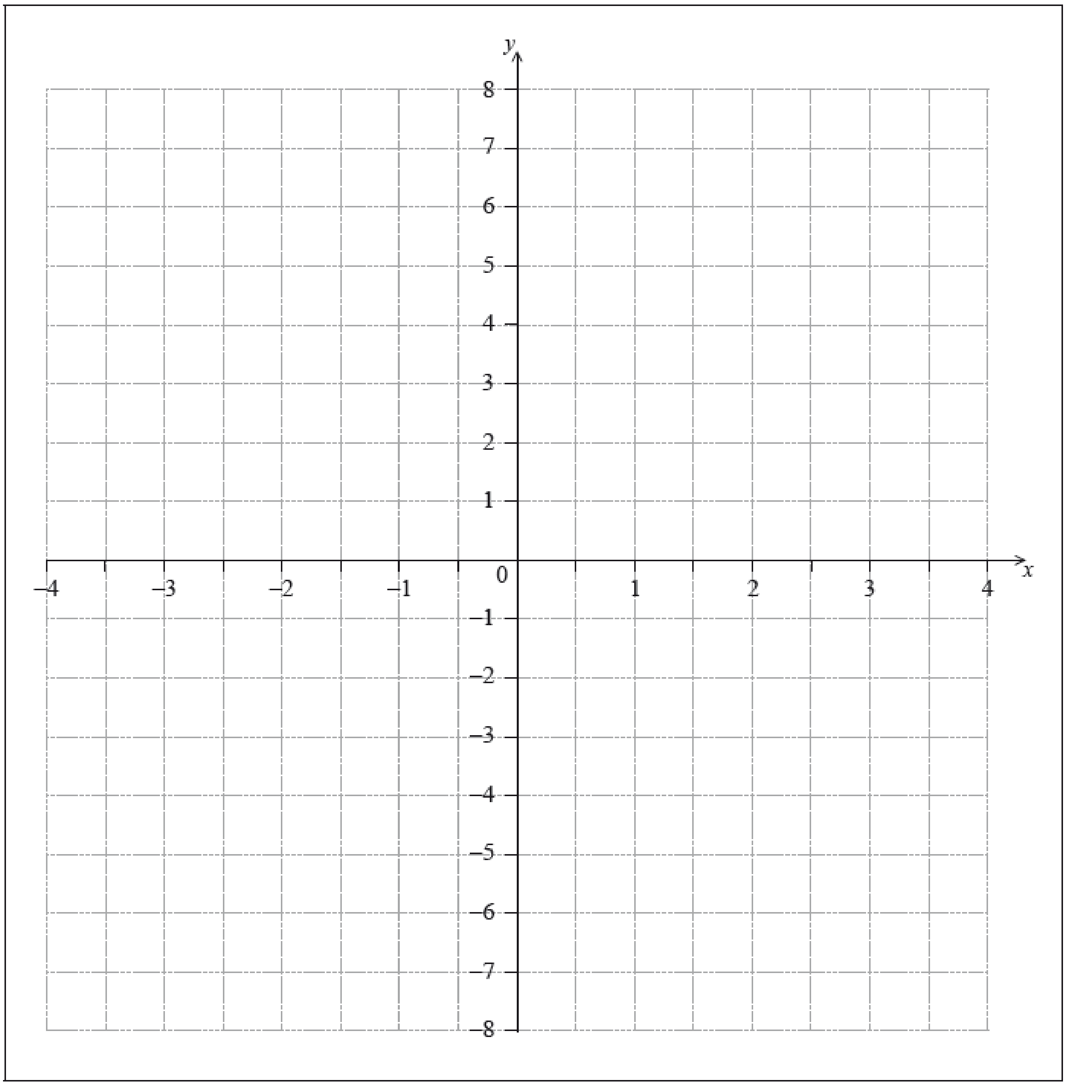
The graph of the quadratic function intersects the *x*-axis at the point N(2, 0) . There is a second point, M, at which the graph of the quadratic function intersects the *x*-axis.

. . . .

. . . .

. . . .

12b. Draw the axis of symmetry on the following axes. *[1 mark]*



12c. The graph of the quadratic function intersects the x-axis at the point $\mathrm{N}(2, 0)$. There is a second point, M, at which the graph of the quadratic function intersects the x-axis. *[1 mark]*

Clearly mark and label point M on the axes.

. . . .

. . . .

. . . .

12d. (i) Find the value of b and the value of c. *[4 marks]*

(ii) Draw the graph of the function on the axes.

. . . .

. . . .

. . . .

A biologist is studying the relationship between the number of chirps of the Snowy Tree cricket and the air temperature. He records the chirp rate, x, of a cricket, and the corresponding air temperature, T, in degrees Celsius.

The following table gives the recorded values.

Cricket's chirp rate, x, (chirps per minute)	20	40	60	80	100	120
Temperature, T (°C)	8.0	12.8	15.0	18.2	20.0	21.1

13a. Draw the scatter diagram for the above data. Use a scale of 2 cm for 20 chirps on the horizontal axis and 2 cm for 4°C on the vertical axis. *[4 marks]*

. . . .

. . . .

. . . .

13b. Use your graphic display calculator to write down the Pearson’s product–moment correlation coefficient, r, between x and T. *[2 marks]*

. . . .

. . . .

. . . .

13c. Interpret the relationship between x and T using your value of r. *[2 marks]*

. . . .

. . . .

. . . .

13d. Use your graphic display calculator to write down the equation of the regression line T on x. Give the equation in the form $T = ax + b$. *[2 marks]*

. . . .

. . . .

. . . .

13e. Calculate the air temperature when the cricket's chirp rate is 70. *[2 marks]*

13f. Given that $\bar{x} = 70$, draw the regression line T on x on your scatter diagram. *[2 marks]*

13g. A forest ranger uses her own formula for estimating the air temperature. She counts the number of chirps in 15 seconds, z, multiplies this number by 0.45 and then she adds 10. *[1 mark]*

Write down the formula that the forest ranger uses for estimating the temperature, T.

Give the equation in the form $T = mz + n$.

13h. A cricket makes 20 chirps in **15** seconds. *[6 marks]*

For this chirp rate

(i) calculate an estimate for the temperature, T, **using the forest ranger's formula**;

(ii) determine the actual temperature recorded by the biologist, **using the table above**;

(iii) calculate the percentage error in the forest ranger's estimate for the temperature, compared to the actual temperature recorded by the biologist.

. . . .

. . . .

. . . .

A function is defined by
$f(x) = \frac{5}{x^2} + 3x + c,\ x \neq 0,\ c \in \mathbb{Z}$.

14a. Write down an expression for $f'(x)$. *[4 marks]*

14b. Consider the graph of *f*. The graph of *f* passes through the point P(1, 4). *[2 marks]*

Find the value of *c*.

14c. There is a local minimum at the point Q. *[4 marks]*

Find the coordinates of Q.

14d. There is a local minimum at the point Q. *[3 marks]*

Find the set of values of *x* for which the function is decreasing.

14e. Let *T* be the tangent to the graph of *f* at P. *[2 marks]*

Show that the gradient of *T* is –7.

14f. Let *T* be the tangent to the graph of *f* at P. *[2 marks]*

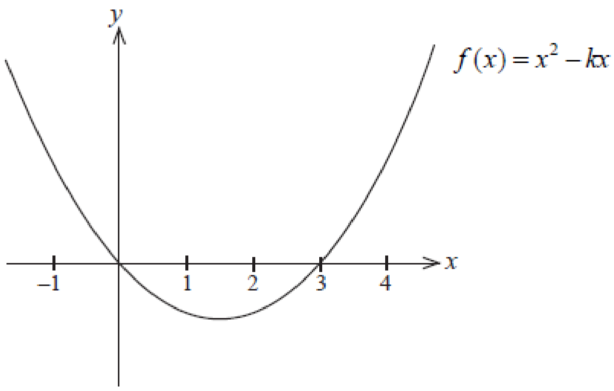
Find the equation of *T*.

14g. *T* intersects the graph again at R. Use your graphic display calculator to find the coordinates of R. *[2 marks]*

15a. Factorise the expression $x^2 - kx$. *[1 mark]*

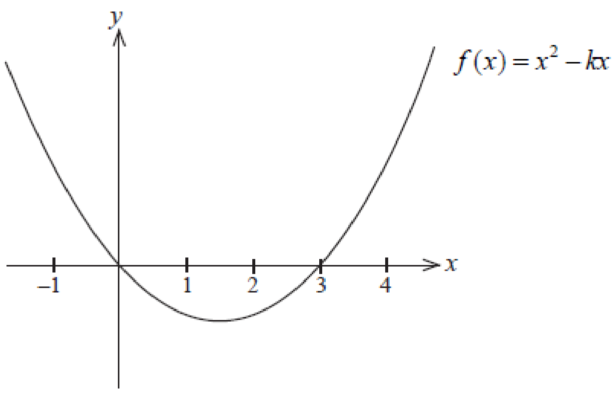
15b. Hence solve the equation $x^2 - kx = 0$. *[1 mark]*

15c. The diagram below shows the graph of the function $f(x) = x^2 - kx$ for a particular value of k. *[1 mark]*



Write down the value of k for this function.

15d. The diagram below shows the graph of the function $f(x) = x^2 - kx$ for a particular value of k. *[3 marks]*



Find the minimum value of the function $y = f(x)$.

Throughout this question *all* the numerical answers must be given correct to the nearest whole number.

16a. Park School started in January 2000 with 100 students. Every full year, there is an increase of 6% in the number of students. *[4 marks]*

Find the number of students attending Park School in

(i) January 2001;

(ii) January 2003.

16b. Park School started in January 2000 with 100 students. Every full year, there is an increase of 6% in the number of students. *[2 marks]*

Show that the number of students attending Park School in January 2007 is 150.

16c. Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year. *[2 marks]*

Find the number of students attending Grove School in January 2003.

16d. Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year. *[4 marks]*

Find the year in which the number of students attending Grove School will be first 60% **more than** in January 2000.

16e. Each January, one of these two schools, the one that has more students, is given extra money to spend on sports equipment. *[5 marks]*

(i) Decide which school gets the money in 2007. Justify your answer.

(ii) Find the first year in which Park School will be given this extra money.

The temperature in $^\circ$C of a pot of water removed from the cooker is given by $T(m) = 20 + 70 \times 2.72^{-0.4m}$, where m is the number of minutes after the pot is removed from the cooker.

17a. Show that the temperature of the water when it is removed from the cooker is 90°C. *[2 marks]*

17b. The following table shows values for m and $T(m)$. *[9 marks]*

m	1	2	4	6	8	10
$T(m)$	66.9	51.4	34.1	26.3	22.8	s

(i) Write down the value of s.

(ii) Draw the graph of $T(m)$ for $0 \leqslant m \leqslant 10$. Use a scale of 1 cm to represent 1 minute on the horizontal axis and a scale of 1 cm to represent 10°C on the vertical axis.

(iii) **Use your graph** to find how long it takes for the temperature to reach 56°C. Show your method clearly.

(iv) Write down the temperature approached by the water after a long time. Justify your answer.

17c. Consider the function $S(m) = 20m - 40$ for $2 \leqslant m \leqslant 6$. *[2 marks]*

The function $S(m)$ represents the temperature of soup in a pot placed on the cooker two minutes after the water has been removed. The soup is then heated.

Draw the graph of $S(m)$ on the same set of axes used for part (b).

17d. Consider the function $S(m) = 20m - 40$ for $2 \leqslant m \leqslant 6$. *[1 mark]*

The function $S(m)$ represents the temperature of soup in a pot placed on the cooker two minutes after the water has been removed. The soup is then heated.

Comment on the meaning of the constant 20 in the formula for $S(m)$ in relation to the temperature of the soup.

17e. Consider the function $S(m) = 20m - 40$ for $2 \leqslant m \leqslant 6$. *[4 marks]*

The function $S(m)$ represents the temperature of soup in a pot placed on the cooker two minutes after the water has been removed. The soup is then heated.

(i) **Use your graph** to solve the equation $S(m) = T(m)$. Show your method clearly.

(ii) Hence describe by using inequalities the set of values of m for which $S(m) > T(m)$.

Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$.

18a. (i) Write down the value of y when x is 2. *[3 marks]*

(ii) Write down the coordinates of the point where the curve intercepts the y-axis.

18b. Sketch the curve for $-4 \leqslant x \leqslant 3$ and $-10 \leqslant y \leqslant 10$. Indicate clearly the information found in (a). *[4 marks]*

18c. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[3 marks]*

18d. Let L_1 be the tangent to the curve at $x = 2$. *[8 marks]*

Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

18e. It is known that $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ for $x < -2$ and $x > b$ where b is positive. [5 marks]

(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$.

19a. Consider the numbers 2, $\sqrt{3}$, $-\frac{2}{3}$ and the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$. [3 marks]

Complete the table below by placing a tick in the appropriate box if the number is an element of the set, and a cross if it is not.

		$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
(i)	2				
(ii)	$\sqrt{3}$				
(iii)	$-\frac{2}{3}$				

19b. A function f is given by $f(x) = 2x^2 - 3x, x \in \{-2, 2, 3\}$. [1 mark]

Write down the range of function f.

In an experiment it is found that a culture of bacteria triples in number every four hours. There are 200 bacteria at the start of the experiment.

Hours	0	4	8	12	16
No. of bacteria	200	600	a	5400	16200

20a. Find the value of a. [1 mark]

20b. Calculate how many bacteria there will be after one day. [2 marks]

20c. Find how long it will take for there to be two million bacteria. [3 marks]

Consider the functions
$f(x) = \frac{2x+3}{x+4}$ and
$g(x) = x + 0.5$.

21a. Sketch the graph of the function $f(x)$, for $-10 \leqslant x \leqslant 10$. Indicating clearly the axis intercepts and any asymptotes. *[6 marks]*

21b. Write down the equation of the vertical asymptote. *[2 marks]*

21c. On the same diagram as part (a) sketch the graph of $g(x) = x + 0.5$. *[2 marks]*

21d. Using your graphical display calculator write down the coordinates of **one** of the points of intersection on the graphs of f and g, **giving your answer correct to five decimal places**. *[3 marks]*

21e. Write down the gradient of the line $g(x) = x + 0.5$. *[1 mark]*

21f. The line L passes through the point with coordinates $(-2,\ -3)$ and is perpendicular to the line $g(x)$. Find the equation of L. *[3 marks]*

22a. Factorise the expression $x^2 - 3x - 10$. *[2 marks]*

22b. A function is defined as $f(x) = 1 + x^3$ for $x \in \mathbb{Z}, -3 \leqslant x \leqslant 3$. *[4 marks]*

(i) List the elements of the domain of $f(x)$.

(ii) Write down the range of $f(x)$.