# Sequence *[66 marks]*

Consider a geometric sequence with a first term of 4 and a fourth term of −2.916.

1a. Find the common ratio of this sequence. *[3 marks]*

1b. Find the sum to infinity of this sequence. *[2 marks]*

The 3rd term of an arithmetic sequence is 1407 and the 10th term is 1183.

2. Calculate the number of positive terms in the sequence. *[3 marks]*

It is known that the number of fish in a given lake will decrease by 7% each year unless some new fish are added. At the end of each year, 250 new fish are added to the lake.

At the start of 2018, there are 2500 fish in the lake.

3a. Show that there will be approximately 2645 fish in the lake at the start of 2020. *[3 marks]*

3b. Find the approximate number of fish in the lake at the start of 2042. *[5 marks]*

An arithmetic sequence $u_1, u_2, u_3 \ldots$ has $u_1 = 1$ and common difference $d \neq 0$. Given that $u_2, u_3$ and $u_6$ are the first three terms of a geometric sequence

4a. find the value of $d$. *[4 marks]*

Given that $u_N = -15$

4b. *[3 marks]*

determine the value of $\sum_{r=1}^{N} u_r$.

The 1st, 4th and 8th terms of an arithmetic sequence, with common difference $d$, $d \neq 0$, are the first three terms of a geometric sequence, with common ratio $r$. Given that the 1st term of both sequences is 9 find

5a. the value of $d$; *[4 marks]*

5b. the value of $r$; *[1 mark]*

6. The fifth term of an arithmetic sequence is equal to 6 and the sum of the first 12 terms is 45. *[6 marks]*

Find the first term and the common difference.

Let $\{u_n\}, n \in \mathbb{Z}^+$, be an arithmetic sequence with first term equal to $a$ and common difference of $d$, where $d \neq 0$. Let another sequence $\{v_n\}, n \in \mathbb{Z}^+$, be defined by $v_n = 2^{u_n}$.

7a. (i) Show that $\frac{v_{n+1}}{v_n}$ is a constant. *[4 marks]*

(ii) Write down the first term of the sequence $\{v_n\}$.

(iii) Write down a formula for $v_n$ in terms of $a$, $d$ and $n$.

7b. Let $S_n$ be the sum of the first $n$ terms of the sequence $\{v_n\}$. *[8 marks]*

(i) Find $S_n$, in terms of $a$, $d$ and $n$.

(ii) Find the values of $d$ for which $\sum\limits_{i=1}^{\infty} v_i$ exists.

You are now told that $\sum\limits_{i=1}^{\infty} v_i$ does exist and is denoted by $S_\infty$.

(iii) Write down $S_\infty$ in terms of $a$ and $d$ .

(iv) Given that $S_\infty = 2^{a+1}$ find the value of $d$ .

7c. Let $\{w_n\},\ n \in \mathbb{Z}^+$, be a geometric sequence with first term equal to $p$ and common ratio $q$, where $p$ and $q$ are both greater than zero. Let another sequence $\{z_n\}$ be defined by $z_n = \ln w_n$. *[6 marks]*

Find $\sum\limits_{i=1}^{n} z_i$ giving your answer in the form $\ln k$ with $k$ in terms of $n$, $p$ and $q$.

The cubic equation $x^3 + px^2 + qx + c = 0$, has roots $\alpha, \beta, \gamma$. By expanding $(x - \alpha)(x - \beta)(x - \gamma)$ show that

8a. (i) $p = -(\alpha + \beta + \gamma)$; *[3 marks]*

(ii) $q = \alpha\beta + \beta\gamma + \gamma\alpha$;

(iii) $c = -\alpha\beta\gamma$.

8b. It is now given that $p = -6$ and $q = 18$ for parts (b) and (c) below. *[5 marks]*

(i) In the case that the three roots $\alpha,\ \beta,\ \gamma$ form an arithmetic sequence, show that one of the roots is $2$.

(ii) Hence determine the value of $c$.

8c. In another case the three roots $\alpha,\ \beta,\ \gamma$ form a geometric sequence. Determine the value of $c$. *[6 marks]*