3 Thermal physics

Learning objectives

- Describe solids, liquids and gases in terms of atoms and molecules.
- Use the concept of temperature and the relation of absolute temperature to the average kinetic energy of molecules.
- Understand and use the concept of internal energy.
- Solve problems in calorimetry using the specific heat capacities.
- Describe phase change and performing calculations using the concept of specific latent heat.

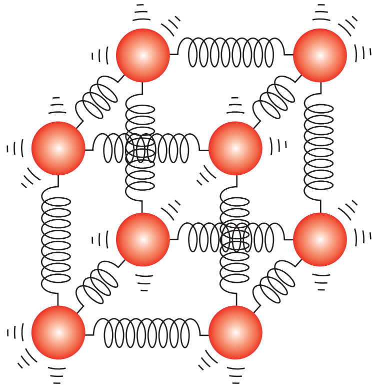


Figure 3.1 Particles in the solid phase oscillate about fixed positions but are not free to move inside the solid.

3.1 Thermal concepts

This section is devoted to the connections and the differences between the basic concepts of temperature, internal energy and heat. This section also deals with thermal equilibrium, phase changes and basic calorimetry problems.

The particle model of matter

As we look closer and closer into matter we discover smaller and smaller **structures**. We find that compounds are made out of molecules, molecules are made out of atoms and atoms contain nuclei and electrons. Nuclei, in turn, contain protons and neutrons. Today it is believed that electrons do not have any substructure but the nucleons (i.e. protons and neutrons) are known to be made out of quarks. It is not known if the quarks themselves are made out of smaller particles. In thermal physics we are mostly interested in molecules, atoms and electrons – we do not need to consider any smaller structures.

In a solid there are forces between the particles that can be **modelled** by springs joining neighbouring particles (Figure **3.1**). The springs then represent the **bonds** between the particles. In liquids the forces between the particles are weaker. The particles are able to move around the volume of the liquid and the liquid will take the shape of the container in which it is placed. However, the inter-particle forces between the particles in a liquid are sufficiently strong that the particles cannot move far from each other. In gases the inter-particle forces are very weak so as to be almost negligible. The only time significant forces exist between the particles is during collisions.

Temperature

We have an intuitive concept of **temperature** as the 'coldness' or 'hotness' of a body, but it wasn't until the 19th century that one of the greatest discoveries in physics related the concept of temperature to the random motion of molecules. This connection, which will be explored in greater detail in Subtopic **3.2**, is that temperature is proportional to the average random kinetic energy of the molecules.

This direct proportionality between temperature and the average random kinetic energy is only true for the absolute or kelvin temperature scale. In this scale zero is the lowest possible temperature, the **absolute zero** of temperature. There has to be an absolute zero in temperature since there is a lowest possible value of the average kinetic energy of molecules, namely zero kinetic energy. Since temperature is proportional to the average kinetic energy, the temperature must be zero when the kinetic energy is zero.

Many other temperature scales exist. In 1742, Anders Celsius (1701–1744) created the temperature scale that is still commonly used today and is known by his name. On the Celsius scale a value of zero degrees is assigned to the freezing point of water (Figure **3.2**) and a value of 100 degrees is assigned to the boiling point of water. The connection between the Celsius and Kelvin scales is:

T (in kelvin, K) = T (in degrees Celsius, °C) + 273

The magnitude of a kelvin is the same as that of a degree Celsius.

The lowest possible temperature on the absolute scale is zero kelvin, 0 K. On the Celsius scale the lowest possible temperature is, therefore, −273 °C. (Notice that we never say degrees kelvin, just kelvin.)

Temperature has varied a lot in the life of the Universe: at the time of the Big Bang, some 13.8 billion years ago, the temperature of the universe was about 10^{32} K. The Universe has been expanding ever since and so the temperature has been dropping. In the emptiness of space, far from stars and galaxies, its value today is only 2.7 K.

Exam tip

The magnitude of a kelvin is the same as that of a degree Celsius.

The need to agree on internationally accepted units, among them those for temperature, is a good example of international collaboration to establish international systems of measurement.

Worked example

3.1 The temperature of a body increases from 320 K to 340 K. State the temperature increase in degrees Celsius.

The temperature increase in kelvin is 340 − 320 = 20 K.

Since the magnitude of a kelvin is the same as that of a degree Celsius, the temperature increase is 20 °C.

(Another way to look at this is to convert both temperatures to kelvin. 320 K corresponds to 320 − 273 = 47 °C and 340 K corresponds to 340 − 273 = 67 °C, giving a change of 20 °C.)

Measuring temperature

Temperature can be measured with a thermometer, which is simply a device that has one property that changes in a predictable way as temperature changes. That property is volume in liquid-in-glass thermometers: the liquid column changes its volume and hence its length since the cross-sectional area stays the same when the temperature changes and so can be used to measure temperature if we first **calibrate** the thermometer. But properties other than volume can be used, for example, electrical resistance.

When a thermometer is used to measure the temperature of a body it has to come into contact with the body. A **thermal interaction** takes place and energy is transferred until the thermometer and the body are at the same temperature. When this happens we say that we have **thermal equilibrium**. The reading on the thermometer is then the temperature of the body. (For thermometers such as infrared thermometers thermal contact is not necessary – the thermometer absorbs radiation emitted by the body whose temperature is to be measured.) The average temperature on Earth is different at different locations. Figure **3.3** shows the temperature distribution in January.

Figure 3.2 A Celsius thermometer shows zero when immersed in melting ice.

Heat was once thought to be a fluid (called 'caloric') that moved from body to body. The more caloric a body contained the hotter it was, and as caloric left a body the body became colder. This idea was rejected when it was realised that you could warm your hands by rubbing them together. If caloric entered your hands it must have come from another body, making it colder. But this does not happen. In the 19th century heat was shown to just another form of energy.

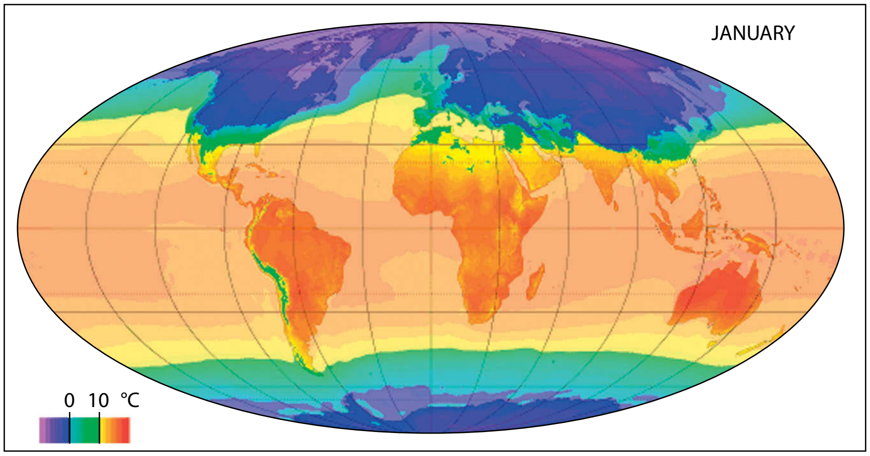


Figure 3.3 Temperature varies at different locations. This image shows the average surface temperature of the Earth in January for the period 1961–1990.

Heat

We have already mentioned that two bodies that are in thermal contact and have different temperatures will have a thermal interaction. So when a glass of cold water is placed in a warm room, **heat** flows from the room into the colder water until the temperature of the water becomes equal to that of its surroundings. We say that the colder body has been 'heated'.

> **Heat** is energy that is transferred from one body to another as a result of a difference in temperature.

Now, all substances consist of particles and, whether in the solid, liquid or gas phase, the particles are in constant motion. They therefore have kinetic energy. In a gas, the particles move randomly throughout the entire volume of the gas. In a solid the motion of the particles is on a very much smaller scale – the particles simply vibrate about their equilibrium positions. But this also requires kinetic energy.

In addition, there are forces between particles. For gases, these forces are very small – under reasonable conditions they are almost negligible (see ideal gases in Subtopic **3.2**). But forces between particles are substantial for solids. Increasing the average separation of two particles of a solid requires work to be done. This work goes into increasing the potential energy associated with inter-particle forces. Figure **3.4** shows the potential energy E_P of one pair of particles as a function of the distance r separating the two particles.

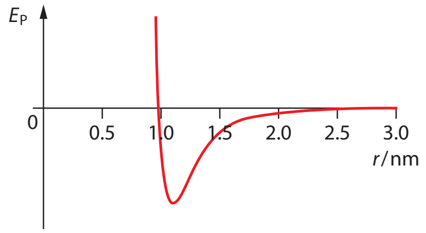


Figure 3.4 The average separation of the two particles is the separation at the minimum of the curve, i.e. at approximately 1.1 nm.

So, to describe the total energy in a substance we need to consider both the kinetic energy and the potential energy. We define the internal energy of a substance as follows:

> **Internal energy** is the total random kinetic energy of the particles of a substance, plus the total inter-particle potential energy of the particles.

Energy transferred from a hot to a cold body by **heating** increases the internal energy of the cold body (and decreases the internal energy of the hot body by the same amount). **Work** done on the particles of a substance increases the potential energy of the particles, and so increases the internal energy.

> **The internal energy of a system can change as a result of heat added or taken out and as a result of work performed.**

Internal energy, heat and work are thus three different concepts. What they have in common is that they are all measured in joules. Temperature is a measure of the random kinetic energy of a substance – not its internal energy.

We define the **specific heat capacity** c of a body to be the energy required to increase the temperature of a unit mass of the body by one kelvin. So, to increase the temperature of a body of mass m by ΔT degrees the heat Q required is:

$$Q = mc\Delta T$$

Exam tip

The term 'capacity' implies somehow that the body contains a certain amount of heat just as a water bottle contains water. This is incorrect. Heat is energy 'in transit' that moves from one body into another; it is not energy contained in any one body.

Worked examples

3.2 A quantity of heat equal to 9800 J is absorbed by a piece of iron of mass 1.8 kg and specific heat capacity 450 J kg^{-1} K^{-1}.

a Calculate the temperature increase of the iron.

b The heat of 9800 J was removed from 3.2 kg of water initially at 48 °C. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}. Calculate the final temperature of the water.

Substance	c / J kg^{-1} K^{-1}
aluminium	900
lead	128
iron	450
copper	385
silver	240
water	4200
ice	2200
ethanol	2430
marble	880

Table 3.1 Specific heat capacities for several substances.

a We need to use $Q = mc\Delta T$. This gives:

$$9800 = 1.8 \times 450 \times \Delta T$$

Solving for the change in temperature gives:

$$\Delta T = \frac{9800}{1.8 \times 450} = 12.1 \approx 12\text{ K}$$

(Notice that we do not need to know the initial temperature of the iron to answer this question.)

b We use $Q = mc\Delta T$ to get:

$$9800 = 3.2 \times 4200 \times \Delta T$$

Solving for the change in temperature gives:

$$\Delta T = \frac{9800}{3.2 \times 4200} = 0.729 \approx 0.73\,\text{K}$$

So the final temperature of the water is:

$$48 - 0.73 \approx 47\,°\text{C}$$

(Notice that the temperature changes of the iron and the water are very different. Notice also that it is unnecessary to convert between kelvin and °C since the temperature changes are the same in both scales.)

3.3 A piece of iron of mass 200 g and temperature 300 °C is dropped into 1.00 kg of water of temperature 20 °C. Predict the final equilibrium temperature of the water.

(Take c for iron as $450\,\text{J kg}^{-1}\,\text{K}^{-1}$ and for water as $4200\,\text{J kg}^{-1}\,\text{K}^{-1}$.)

Let T be the final unknown temperature. The iron will also be at this temperature, so:

amount of thermal energy lost by the iron $= m_{\text{iron}}c_{\text{iron}}(300 - T)$

and

amount of thermal energy gained by the water $= m_{\text{water}}c_{\text{water}}(T - 20)$

Conservation of energy demands that thermal energy lost = thermal energy gained, so:

$$m_{\text{iron}}c_{\text{iron}}(300 - T) = m_{\text{water}}c_{\text{water}}(T - 20)$$

$$0.200 \times 450 \times (300 - T) = 1.0 \times 4200 \times (T - 20)$$

$$\Rightarrow T = 25.9\,°\text{C} \approx 26\,°\text{C}$$

(Note how the large specific heat capacity of water results in a small increase in the temperature of the water compared with the huge drop in the temperature of the iron.)

Figure 3.5 Hot lava turns into a solid upon contact with water. The cold water takes heat away from the hot lava.

Change of phase

When heat is provided to a body or removed from it, the body may not necessarily change its temperature. The body may change **phase** instead. Changes of phase happen at constant temperature (Figure **3.5**) and include:

- **melting** – when a solid changes to a liquid (heat must be provided to the solid)
- **freezing** – when a liquid changes into a solid (heat must be taken out of the liquid)
- **vaporisation** (or **boiling**) – when a liquid changes into vapour (by giving heat to the liquid)
- **condensation** – when a vapour changes into a liquid (by taking heat out of the vapour).

Why does the heat absorbed or removed not result in a temperature change? Consider the process of melting. At the melting temperature, changing from solid to liquid means that the average distance between the molecules increases. But increasing the separation of the molecules requires work (because there are attractive forces between the molecules that need to be overcome). This is where heat supplied goes – it is used to 'break the bonds'. What the supplied heat does not do is to increase the kinetic energy of the molecules – hence the temperature stays the same.

We define the specific latent heat L to be the amount of energy required to change the phase of a unit mass at constant temperature. So the energy required to change the phase of a mass m is $Q = mL$. If the change is melting or freezing, we call it the **specific latent heat of fusion**, L_F. If the change is vaporisation or condensing then we call it **specific latent heat of vaporisation**, L_V.

Substance	Specific latent heat of fusion / kJ kg^{-1}	Melting temperature / °C	Specific latent heat of vaporisation / kJ kg^{-1}	Boiling temperature / °C
water	334	0	2260	100
ethanol	109	−114	840	78
aluminium	395	660	10550	2467
lead	23	327	850	1740
copper	205	1078	2600	5190
iron	275	1540	6300	2800

Table 3.2 Specific latent heats of fusion and vaporisation together with the melting and boiling temperatures.

Notice from Table **3.2** that the specific latent heat for vaporisation is greater than that for melting. This is because the increase in separation of the molecules is much larger when going from the liquid to the vapour phase than when going from the solid to the liquid phase. More work is needed to achieve the greater separation, and so more energy is required.

Worked examples

3.4 An ice cube of mass 25.0 g and temperature −10.0 °C is dropped into a glass of water of mass 300.0 g and temperature 20.0 °C. Calculate the final temperature.

(Specific heat capacity of ice = 2200 J kg^{-1} K^{-1}; specific latent heat of fusion of ice = 334 J kg^{-1} K^{-1}, specific heat capacity of water = 4200 J kg^{-1} K^{-1}.)

Let the final temperature be T. Ignoring any thermal energy lost by the glass itself, the water will cool down by losing thermal energy.

Using $Q = mc\Delta T$, the thermal energy lost by the water is:

$$0.3 \times 4200 \times (20 - T)$$

This thermal energy will be taken up by the ice to:

- increase its temperature from −10 °C to 0 °C: the thermal energy required is $25 \times 10^{-3} \times 2200 \times 10$ J
- melt the ice cube into water at 0 °C: the thermal energy required is $25 \times 10^{-3} \times 334 \times 10^{3}$ J
- increase the temperature of the former ice cube from 0 °C to the final temperature T: the thermal energy required is $25 \times 10^{-3} \times 4200 \times T$.

Thus:

$$0.3 \times 4200 \times (20 - T) = (25 \times 10^{-3} \times 2200 \times 10) + (25 \times 10^{-3} \times 334 \times 10^{3}) + (25 \times 10^{-3} \times 4200 \times T)$$

Exam tip

You can save yourself time and possible errors if you write this equation, as is, in the equation solver of your graphic display calculator (GDC) and ask the GDC to solve it for you.

Solving for T gives $T = 11.9$ °C.

3.5 A sample of 120 g of a solid initially at 20 °C is heated by a heater of constant power. The specific heat capacity of the solid is 2500 J kg^{-1} K^{-1}. The temperature of the sample varies with time as shown in Figure **3.6**.

Use the graph to determine:

a the power of the heater

b the melting temperature of the sample

c the specific latent heat of fusion of the sample

d the specific heat capacity of the sample in the liquid phase.

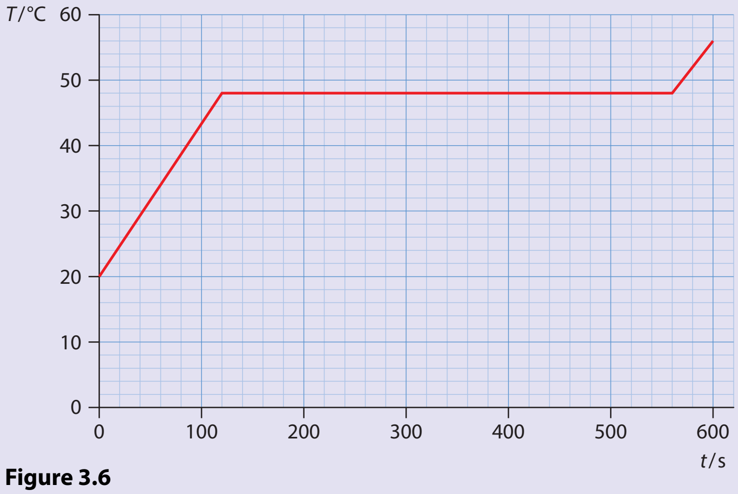


Figure 3.6

a It takes 120 s to raise the temperature of the solid sample from 20 °C to 48 °C.

Using $Q = mc\Delta T$, the heat required is:

$$0.120 \times 2500 \times (48 - 20) = 8400 \text{ J}$$

So the power is:

$$P = \frac{Q}{t} = \frac{8400}{120} = 70 \text{ W}$$

b The temperature is constant at melting, shown by the flat part of the graph, so the melting temperature is 48 °C.

c The sample is melting from 120 s to 560 s, i.e. for 440 s. The heat supplied during this time is therefore:

$$Q = Pt = 70 \times 440 = 30\,800\ \text{J}$$

So the specific latent heat of fusion is:

$$L_F = \frac{Q}{m} = \frac{30\,800}{0.120} = 2.6 \times 10^5\ \text{J kg}^{-1}$$

d The liquid increases its temperature from 48 °C to 56 °C in 40 s. In these 40 s the heat provided is:

$$Q = Pt = 70 \times 40 = 2800\ \text{J}$$

Using $Q = mc\Delta T$:

$$0.120 \times c \times (56 - 48) = 2800\ \text{J}$$

$$\Rightarrow c = 2.9 \times 10^3\ \text{J kg}^{-1}\ \text{K}^{-1}$$

The method of mixtures

The electrical method described in Worked example 3.5 is one method for measuring specific heat capacity and latent heat. Another method, the **method of mixtures**, measures the specific heat capacity of a solid as follows. A solid is put in a container of hot water and allowed time to reach a constant temperature. The temperature of the solid is thus that of the water and is recorded. The solid is then transferred into a calorimeter of known specific heat capacity and initial temperature, which contains a liquid such as water (Figure 3.7). The calorimeter is insulated. The final temperature of the water is recorded after thermal equilibrium has been reached.

For example, consider a mass of 0.400 kg of a solid at 80 °C that is put in a 100 g copper calorimeter containing 800 g of water at 20 °C. The final temperature of the water is measured to be 22 °C. From these values, we may deduce the specific heat capacity of the solid as follows.

Using $Q = mc\Delta T$, the amount of thermal energy (in joules) lost by the solid is:

$$0.400 \times c \times (80 - 22) = 23.2c$$

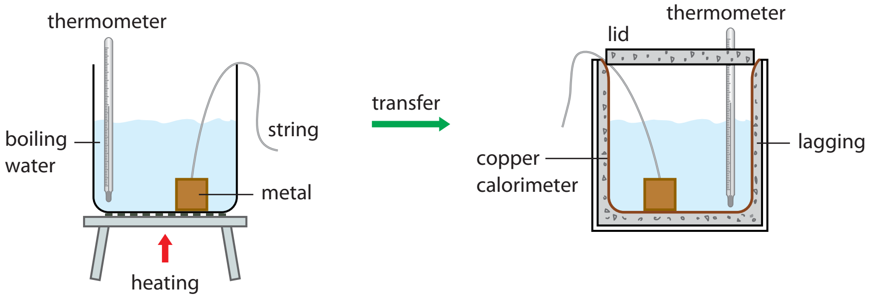


Figure 3.7 The hot metal is placed in the cold water in the calorimeter. The hot metal is removed from the container of boiling water and is quickly placed inside an insulated calorimeter containing cold water.

Exam tip
It is likely that the solid lost heat to the surrounding air while it was being transferred. This means that the actual temperature of the solid is less than we supposed. The actual specific heat capacity is therefore larger than the calculated value.

The amount of thermal energy gained by the calorimeter (see Table **3.1** for the value of c for copper) and the water is:

$$\underbrace{0.100 \times 385 \times (22 - 20)}_{\text{calorimeter}} + \underbrace{0.800 \times 4200 \times (22 - 20)}_{\text{water}} = 6797\,\text{J}$$

Equating the two we find that $c = 293\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$.

The same method can be applied to measure the specific latent heat of fusion of ice. To do this, place a quantity of ice at 0 °C (the ice must therefore come from a mixture with water at 0 °C) into a calorimeter containing water at a few degrees above room temperature. Blot the ice dry before putting it into the calorimeter. The mass of the ice can be determined by weighing the calorimeter at the end of the experiment.

For example, suppose that 25.0 g of ice at 0.00 °C is placed in an aluminium calorimeter of mass 250 g containing 300 g of water at 24.0 °C. The temperature of the water is measured at regular intervals of time until the temperature reaches a minimum value of 17.0 °C. The calorimeter and water lost heat, which the ice received.

Heat lost by calorimeter and water:

$$0.250 \times 900 \times (24 - 17) + 0.300 \times 4200 \times (24 - 17) = 10\,395\,\text{J}$$

Heat received by ice:

$$0.025 \times L_F + 0.025 \times 4200 \times 17 = 0.025 \times L_F + 1785$$

Equating the two gives:

$$1785 + 0.025 \times L_F = 10\,395 \Rightarrow L \approx 344\,\text{kJ}\,\text{kg}^{-1}$$

Nature of science

Models change

As already mentioned, heat was once thought to be a fluid (caloric). Conservation of energy was a natural consequence of this model of heat: a body lost a certain amount of fluid and another gained it. Energy was conserved. So the concept of heat as a fluid seemed natural. But there are phenomena that cannot be explained with this simple picture. For one thing, if heat is a fluid it must have mass. So when heat leaves a body, the body must lose mass. This is not observed, so the caloric theory must be wrong. The theory has many other failings and was abandoned in the 19th century. A major problem is that it does not take account the atomic theory of matter. The theory we use now is based on statistical mechanics, which uses probability theory to predict the average behaviour of very large numbers of particles.

Test yourself

1 A hot body is brought into contact with a colder body until their temperatures are the same. Assume that no other bodies are nearby.

  a Discuss whether the energy lost by one body is equal to the energy gained by the other.

  b Discuss whether the temperature drop of one body is equal to the temperature rise of the other.

2 **a** A body of mass 0.150 kg has its temperature increased by 5.00 °C when 385 J of energy is provided to it. Calculate the body's specific heat capacity.

  b Another body of mass 0.150 kg has its temperature increased by 5.00 K when 385 J of energy is provided to it. Calculate this body's specific heat capacity.

3 A calorimeter of mass 90 g and specific heat capacity $420\,\mathrm{J\,kg^{-1}\,K^{-1}}$ contains 310 g of a liquid at 15.0 °C. An electric heater rated at 20.0 W warms the liquid to 19.0 °C in 3.0 min. Assuming there are no energy losses to the surroundings, estimate the specific heat capacity of the liquid.

4 A calorimeter for which $mc = 25\,\mathrm{J\,K^{-1}}$ contains 140 g of a liquid. An immersion heater is used to provide energy at a rate of 40 W for a total time of 4.0 min. The temperature of the liquid increases by 15.8 °C. Calculate the specific heat capacity of the liquid. State an assumption made in reaching this result.

5 A car of mass 1360 kg descends from a hill of height 86 m at a constant speed. Assuming that all of the gravitational potential energy lost by the car goes into heating the brakes, estimate the rise in the temperature of the brakes. (It takes 16 kJ of energy to increase the temperature of the brake drums by 1 K; ignore any energy losses to the surroundings.)

6 A radiator made out of iron of specific heat capacity $450\,\mathrm{J\,kg^{-1}\,K^{-1}}$ has a mass of 45.0 kg and is filled with 23.0 kg of water of specific heat capacity $4200\,\mathrm{J\,kg^{-1}\,K^{-1}}$.

  a Determine the energy required to raise the temperature of the radiator–water system by 1 K.

  b If energy is provided to the radiator at the rate of 450 W, calculate how long it will take for the temperature to increase by 20.0 °C.

7 How much ice at −10 °C must be dropped into a cup containing 300 g of water at 20 °C in order for the temperature of the water to be reduced to 10 °C? The cup itself has a mass of 150 g and is made out of aluminium. Assume that no energy is lost to the surroundings.

8 The surface of a pond of area $20\,\mathrm{m^2}$ is covered by ice of uniform thickness 6 cm. The temperature of the ice is −5 °C. Calculate how much energy is required to melt this amount of ice into water at 0 °C. (Take the density of ice to be $900\,\mathrm{kg\,m^{-3}}$.)

9 Radiation from the Sun falls on the frozen surface of a pond at a rate of $600\,\mathrm{W\,m^{-2}}$. The ice temperature is 0 °C.

  a Calculate how long it will take to melt a 1.0 cm thick layer of ice. (Take the density of ice to be $900\,\mathrm{kg\,m^{-3}}$.)

  b Suggest why the actual mass of ices that melts is less than your answer to **a**.

10 **a** Calculate how much energy is required to warm 1.0 kg ice initially at −10 °C to ice at 0 °C.

  b Calculate how much energy is required to melt the ice at 0 °C.

  c Calculate how much energy is required to further increase the temperature of the water from 0 °C to 10 °C.

  d State in which stage (warming the ice, melting the ice, warming the water) the energy requirement is largest.

11 Ice at 0 °C is added to 1.0 kg of water at 20 °C, cooling it down to 10 °C. Determine how much ice was added.

12 A quantity of 100 g of ice at 0 °C and 50 g steam at 100 °C are added to a container that has 150 g water at 30 °C. Determine the final temperature in the container. Ignore the container itself in your calculations.

Learning objectives

- Use the concept of pressure.
- Solve problems using the equation of state of an ideal gas.
- Understand the assumptions behind the kinetic model of an ideal gas.
- Solve problems using moles, molar masses and the Avogadro constant.
- Describe differences between ideal and real gases.

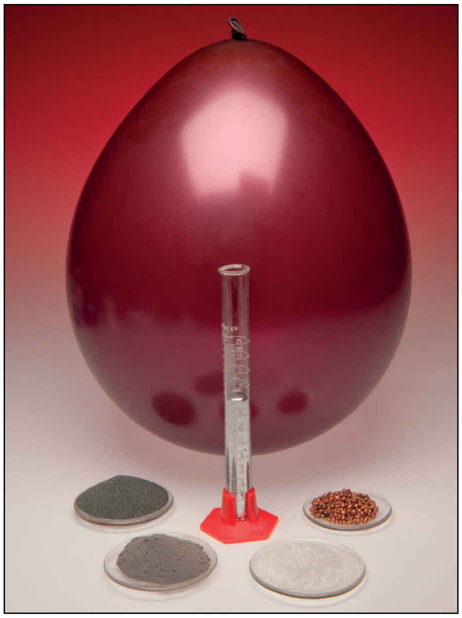


Figure 3.8 One mole of different substances.

3.2 Modelling a gas

This section introduces the equation of state of an ideal gas, which is the equation that relates the pressure, volume, absolute temperature and number of moles of an ideal gas. The connection between the average random kinetic energy of the molecules and the kelvin temperature is derived.

The Avogadro constant

By definition, one **mole** of any substance contains as many **particles** as there are atoms in 12 g of carbon-12. What we mean by 'particle' depends on the substance; it can be a single atom or a molecule. For example, in carbon the particles are single **atoms**, the particles in hydrogen gas (H_2) are diatomic **molecules**, in carbon dioxide gas (CO_2) they are triatomic molecules, and in methane gas (CH_4) they are molecules with five atoms.

Experiments show that the number of particles in a mole is $N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, a number known as the **Avogadro constant** and one of the basic constants of physics. So one mole of carbon, one mole of H_2, one mole of CO_2 and one mole of CH_4 all contain 6.02×10^{23} particles. This means 6.02×10^{23} atoms for carbon, $2 \times 6.02 \times 10^{23}$ atoms for H_2, $3 \times 6.02 \times 10^{23}$ atoms for CO_2 and $5 \times 6.02 \times 10^{23}$ atoms for CH_4. Figure **3.8** shows one mole of different substances.

If a substance contains N particles (atoms or molecules, as discussed above) then the number of moles n is:

$$n = \frac{N}{N_A}$$

The atomic mass scale defines one **atomic mass unit** (1 u) as $\frac{1}{12}$ of the mass of one atom of carbon-12, $^{12}_{6}C$. The mass of one atom of $^{12}_{6}C$ is therefore exactly 12 u. The notation $^{12}_{6}C$ means that the carbon atom has six protons and the number of protons and neutrons combined is 12 (i.e. six neutrons). The neutral atom also has six electrons. Neglecting the mass of the six electrons, the mass of the six protons and six neutrons is about 12 u. The proton and the neutron are approximately equal in mass and so approximately the mass of one proton and that of one neutron is 1 u. So an atom of helium ($^{4}_{2}He$) has a mass that is (approximately) 4 u and the mass of one atom of $^{56}_{26}Fe$ is (approximately) 56 u.

Now, remember that the mole is defined as the number of atoms in 12 g of $^{12}_{6}C$. We also defined the mass of one atom of $^{12}_{6}C$ to be 12 u. This means that:

$$\underbrace{N_A}_{\text{number of particles in 1 mol}} \times \underbrace{12\text{u}}_{\text{mass of 1 atom}} = \underbrace{12\,\text{g}}_{\text{mass in g of 1 mol}}$$

and so the u (in grams) is given by:

$$\text{u} = \frac{1\,\text{g}}{N_A} \quad (\approx 1.66 \times 10^{-24}\,\text{g} \approx 1.66 \times 10^{-27}\,\text{kg})$$

We know that A grams of the element ${}^{A}_{Z}X$ correspond to one mole of element X. So, for substances that are monatomic, one mole of a substance is also the quantity of the substance whose mass in grams is equal to the **atomic mass** (in u). Moving on to molecules, the **molar mass** is the sum of the atomic masses of the atoms making up the molecule. So CO_2 has molar mass $12 + 2 \times 16 = 44\,\text{g mol}^{-1}$. There are N_A molecules in 44 g of CO_2 because 44 g of CO_2 make one mole.

So, it is important to know that:

> One mole of a substance is a quantity of the substance that contains a number of particles equal to the Avogadro constant and whose mass in grams is equal to the molar mass of the substance.

The number of moles in a quantity of m grams of a substance with molar mass μ is then $n = \frac{m}{\mu}$.

Worked examples

3.6 Estimate the number of atoms of gold in 1.0 kg of gold (${}^{197}_{79}Au$).

The molar mass of gold is $197\,\text{g mol}^{-1}$. So 1000 g of gold (= 1 kg) contains $\frac{1000}{197} \approx 5.1$ mol of atoms.

Each mole contains 6.02×10^{23} atoms, so the number of atoms in 1 kg of gold is $6.02 \times 10^{23} \times 5.1 = 3 \times 10^{24}$.

3.7 Calculate how many grams of scandium, ${}^{45}_{21}Sc$, contain the same number of molecules as 8.0 g of argon, ${}^{40}_{18}Ar$.

The molar mass of argon is $40\,\text{g mol}^{-1}$, so a quantity of 8.0 g of argon corresponds to $\frac{8.0}{40} = 0.20$ mol.

Thus, we need 0.20 mol of scandium. This corresponds to $0.20 \times 45 = 9.0$ g.

3.8 Estimate the number of water molecules in an ordinary glass of water.

A glass contains about $200\,\text{cm}^3$ of water, which has a mass of 200 g.

Since the molar mass of water is $18\,\text{g mol}^{-1}$, the glass contains $\frac{200}{18} \approx 10$ mol or $6 \times 10^{23} \times 10 \approx 10^{25}$ molecules of water.

Pressure

Pressure is defined as the normal force applied per unit area. In Figure **3.9a** the force is normal to the area A, so the pressure is:

$$p = \frac{F}{A}$$

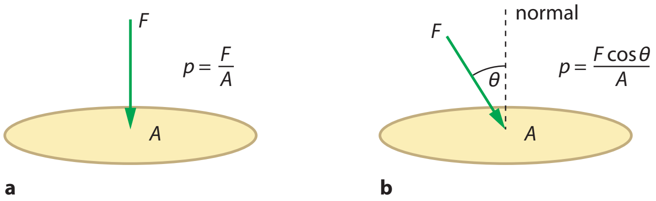


Figure 3.9 Pressure is the normal force per unit area.

The force in Figure **3.9b** acts at an angle θ, so the pressure on the area A is given by the expression:

$$p = \frac{F\cos\theta}{A}$$

The unit of pressure is newton per square metre, N m^{-2}, also known as the pascal, Pa. Another commonly used non-SI unit is the **atmosphere**, atm, which is equal to 1.013×10^5 Pa.

Worked example

3.9 Two hollow cubes of side 25 cm with one face missing are placed together at the missing face (Figure **3.10**). The air inside the solid formed is pumped out. Determine the force that is necessary to separate the cubes.

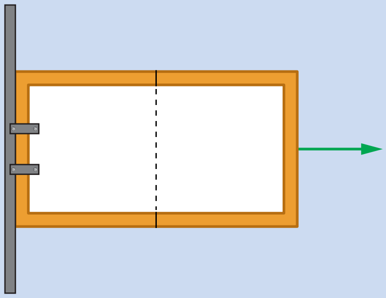


Figure 3.10

The pressure inside the solid is zero and outside it equals atmospheric pressure, 1.01×10^5 Pa.

Thus, the force is given by:

$$F = pA = 1.01 \times 10^5 \times (0.25)^2 = 6.3 \times 10^3 \text{ N}$$

Exam tip

You must be able to recall and describe a few of these assumptions in an exam

Ideal gases

An **ideal gas** is a theoretical model of a gas. It helps us to understand the behaviour of real, actual gases. We assume that an ideal gas obeys the following:

- The molecules are point particles, each with negligible volume.
- The molecules obey the laws of mechanics.
- There are no forces between the molecules except when the molecules collide.
- The duration of a collision is negligible compared to the time between collisions.
- The collisions of the molecules with each other and with the container walls are elastic.
- Molecules have a range of speeds and move randomly.

An ideal gas (unlike real gases) cannot be liquefied or solidified. You should be able to see how some of these assumptions may not be obeyed by a **real gas**. For example, there will always be forces between molecules of a real gas, not just when the molecules are in contact. In general, we expect that a real gas will behave like an ideal gas when the density is low (so that molecules are not close to each other and hence the forces between them are negligible). We do not expect ideal gas behaviour at high densities (molecules will be too close to each other and will exert forces on each other). Similarly, we do not expect ideal gas behaviour from a real gas at very low temperature, because the gas will then become a liquid or even a solid!

> A real gas may be approximated by an ideal gas when the density is low.

Exam tip

You must be able to describe the conditions under which a real gas may be approximated by that of an ideal gas. The main idea is that the density must be low. For a fixed quantity of gas, density will be low at low pressure and high temperature.

Figure **3.11** shows a molecule that collides with a container wall. The momentum normal to the wall before the collision is $mv\cos\theta$. After the collision momentum normal to the wall is $-mv\cos\theta$. So the change in momentum has magnitude $2mv\cos\theta$. The fact that the momentum of the molecule has changed means that a force acted on the molecule (from the wall). By Newton's third law, therefore, the molecule exerted on the wall an equal and opposite force. Taking into account the forces due to all the molecules colliding with the walls results in a force, and hence pressure, on the walls.

The **state of a gas** is determined when we know the values of the pressure, the volume, the temperature and the number of moles present. The parameters p, V, T and n are related to each other. The equation relating them is called the **equation of state**. Our objective is to discover the equation of state for a gas. To do this a number of simple experiments can be performed, as described in the following sections.

Exam tip

You must be able to give an explanation of pressure in terms of molecules colliding with their container walls.

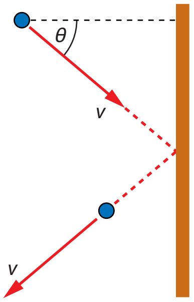


Figure 3.11 A molecule has its momentum changed when it collides with a wall. A force is exerted on the molecule and so the molecule exerts an equal and opposite force on the wall.

The pressure–volume law

The equipment shown in Figure **3.12** can be used to investigate the relationship between pressure and volume of a fixed quantity of gas that is kept at constant temperature.

The pump forces oil to move higher, decreasing the volume of the air trapped in the tube above the oil. A pressure gauge reads the pressure of the trapped air and so the relationship between pressure and volume may investigated. The changes in pressure and volume must take place slowly so that the temperature stays the same.

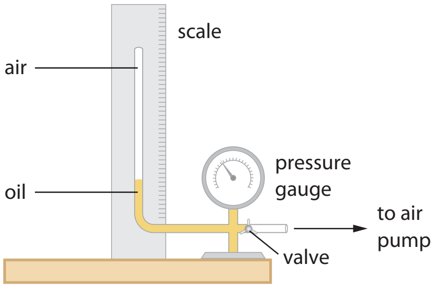


Figure 3.12 Apparatus for investigating the pressure–volume law. The pump forces oil to move up the tube, decreasing the volume of air.

The results of a typical experiment are shown in Figure 3.13. We have plotted pressure against the inverse of the volume and obtained a straight line.

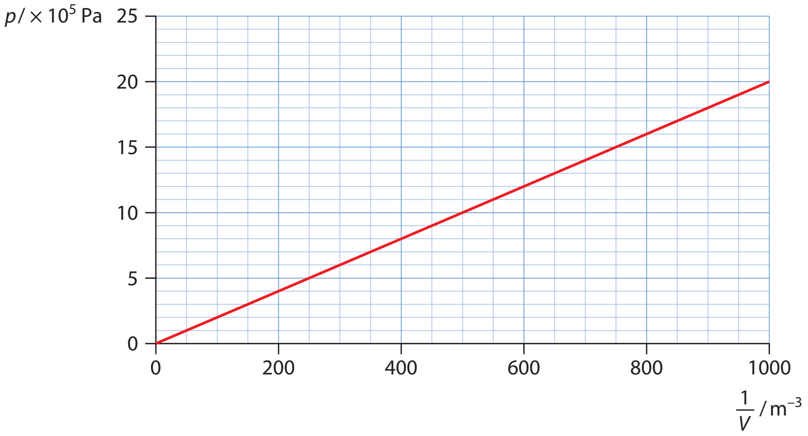


Figure 3.13 Graph of pressure against inverse volume at constant temperature.

Exam tip
In practice we use the relation pV = constant in the equivalent form $p_1V_1 = p_2V_2$ when the initial pressure and volume (p_1, V_1) change to a new pressure and volume (p_2, V_2) at constant temperature.

This implies that:

> At constant temperature and with a fixed quantity of gas, pressure is inversely proportional to volume, that is:
>
> $p \propto \frac{1}{V}$ or pV = constant

This relationship is known as the **Boyle's law**.

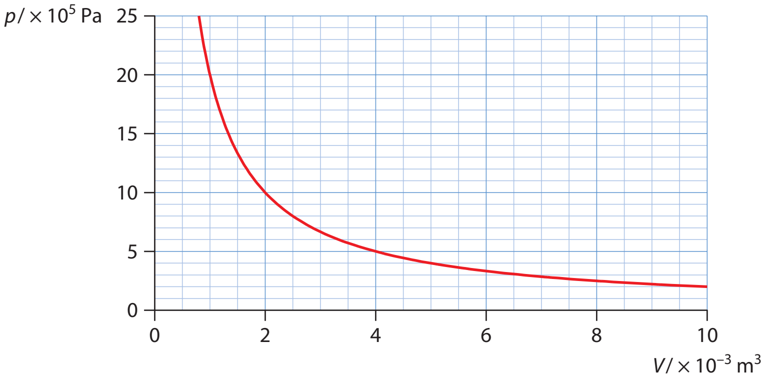


Figure 3.14 The relationship between pressure and volume at constant temperature for a fixed quantity of a gas. The product pV is the same for all points on the curve.

Exam tip
If you are asked to confirm the relationship pV = constant, take three points from a pressure–volume graph and show that their product is the same.

Figure 3.14 shows the same data now plotted as pressure against volume.

The curve in the pressure–volume diagram is a hyperbola and in physics it is known as an **isothermal** curve or **isotherm**: the temperature at all points on the curve is the same.

Worked example

3.10 The pressure of a fixed quantity of gas is 2.0 atm and its volume 0.90 dm^3. The pressure is increased to 6.0 atm at constant temperature. Determine the new volume.

Use $p_1V_1 = p_2V_2$. Substituting the known values we have:

Remember that 1 dm^3 = 1000 cm^3 = 1 litre.

$$2.0 \times 0.90 = 6.0 \times V$$

$$\Rightarrow \quad V = 0.30$$

The new volume is 0.30 dm^3.

(Notice that since this problem compares the pressure at two different volumes we do not have to change units to SI units.)

The volume–temperature law

The dependence of volume on temperature of a fixed quantity of gas kept at constant pressure can be investigated with the apparatus shown in Figure **3.15**. Air is trapped in a thin capillary tube that is immersed in heated water. The air is trapped by a thin thread of very concentrated sulfuric acid. The thread is exposed to the atmosphere and so the pressure of the trapped air is constant.

It is found that the volume increases uniformly with temperature. The striking fact is that when the straight line is extended backwards it always crosses the temperature axis at −273 °C, as in Figure **3.16**. This suggests that there exists a minimum possible temperature, namely −273 °C. (With a real gas the experiment cannot be conducted at very low temperatures since the gas would liquefy – hence the dotted line. With an ideal gas there would be no such restriction.)

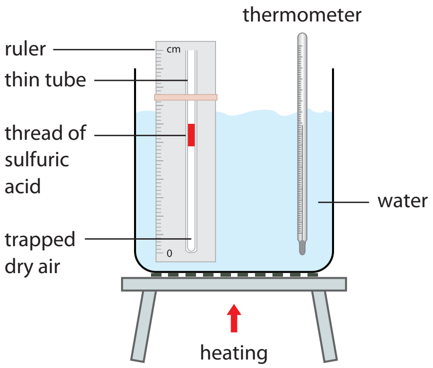


Figure 3.15 Apparatus for verifying the volume–temperature law.

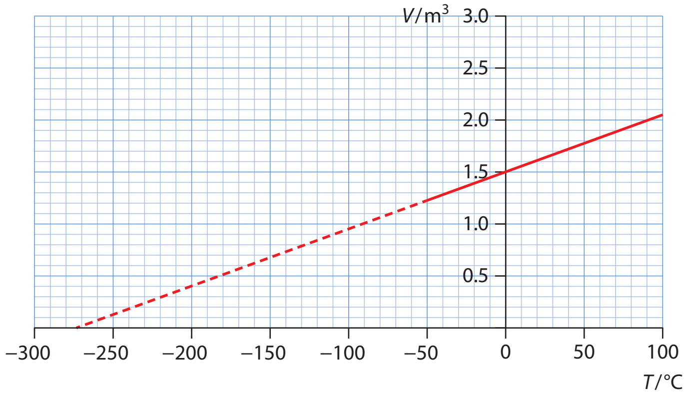


Figure 3.16 When the graph of volume versus temperature is extended backwards the line intersects the temperature axis at −273 °C.

If this same experiment is repeated with a different quantity of gas, or a gas at a different constant pressure, the result is the same. In each case, the straight-line graph of volume versus temperature crosses the temperature axis at −273 °C (Figure **3.17**). In Figure **3.18**, the same graphs are drawn using the Kelvin temperature scale.

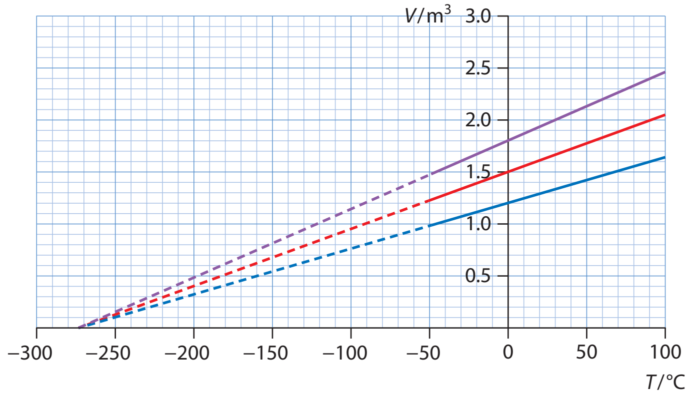


Figure 3.17 When the graph of volume versus temperature is extended backwards, all the lines intersect the temperature axis at the same point.

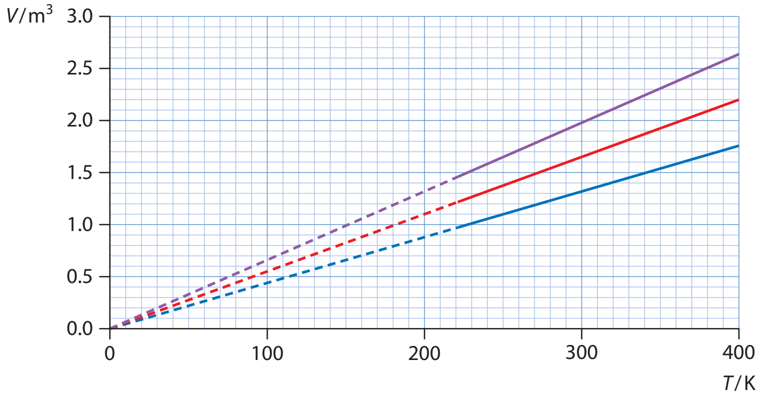


Figure 3.18 When temperature is expressed in kelvin, the lines start at zero temperature.

Exam tip

In practice we use the relation $\frac{V}{T}$ = constant in the equivalent form as $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ where the initial volume and temperature of the gas (V_1, T_1) change to a new volume and temperature (V_2, T_2) at constant pressure.

When the temperature is expressed in kelvin, this experiment implies that at constant pressure:

$$\frac{V}{T} = \text{constant}$$

This relationship is know as **Charles' law**.

Worked example

3.11 A gas expands at constant pressure from an original volume of 2.0 $\mathrm{dm^3}$ at 22 °C to a volume of 4.0 $\mathrm{dm^3}$. Calculate the new temperature.

Substituting in $\frac{V_2}{T_1} = \frac{V_2}{T_2}$ it follows that:

$$\frac{2.0}{295} = \frac{4.0}{T}$$

$\Rightarrow \quad T = 590\,\mathrm{K}$ or 317 °C

Note that we converted the original temperature into kelvin. (It is very easy to forget this conversion and get the incorrect answer of 44 °C.)

The pressure–temperature law

What remains now is to investigate the dependence of pressure on temperature of a fixed quantity of gas in a fixed volume. This can be done with the apparatus shown in Figure **3.19**. The gas container is surrounded by water whose temperature can be changed. A pressure gauge measures the pressure of the gas. We find that pressure increases uniformly with increasing temperature, as shown by the graph in Figure **3.20**.

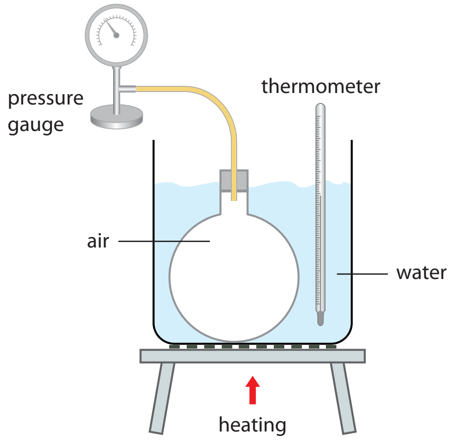


Figure 3.19 Investigating the pressure–temperature law.

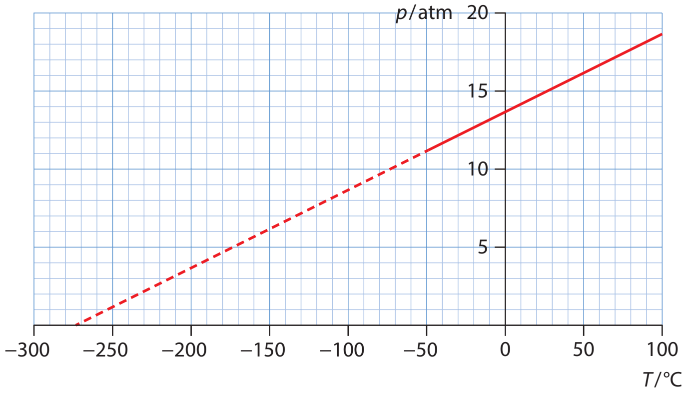


Figure 3.20 The graph of pressure versus temperature is a straight line that, when extended backwards, again intersects the temperature axis at −273 °C.

For quantities of gases containing different numbers of moles at different volumes the results are the same, as shown in Figure **3.21**. When the temperature is expressed in kelvin, the straight lines all pass through the origin (Figure **3.22**).

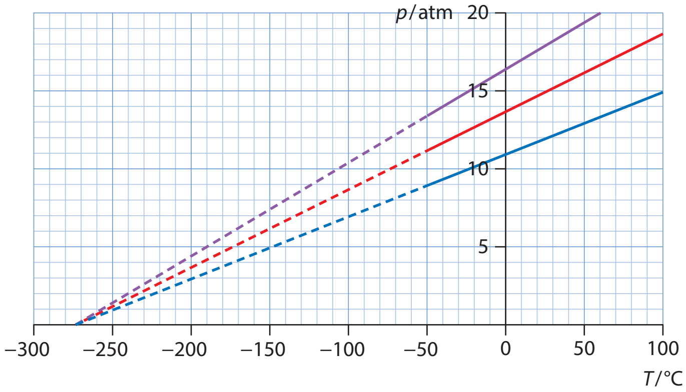


Figure 3.21 When extended backwards, the graphs of pressure versus temperature for three different quantities of gas all intersect the temperature axis at the same point.

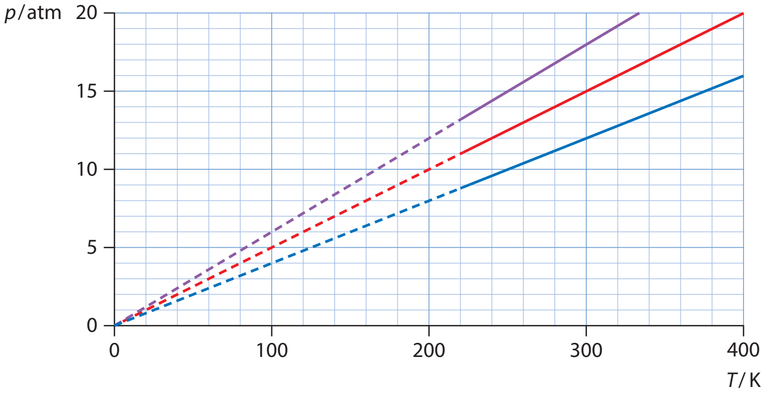


Figure 3.22 If temperature is expressed in kelvin, the lines start at zero temperature.

Exam tip

In practice we use the relation $\frac{p}{T}$ = constant in the equivalent form as $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ where the initial pressure and temperature of the gas (p_1, T_1) change to a new pressure and temperature (p_2, T_2) at constant volume. (Remember, T is in kelvin.)

> When the temperature is expressed in kelvin, this experiment implies that at constant volume:
>
> $$\frac{p}{T} = \text{constant}$$

This relationship is known as **Gay-Lussac's law** or **Amontons' law**.

Worked example

3.12 A gas in a container of fixed volume is heated from a temperature of 37 °C and pressure 3.0×10^5 Pa to a temperature of 87 °C. Calculate the new pressure.

Substituting in $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ we have:

$$\frac{3.0 \times 10^5}{310} = \frac{p}{360}$$

$$\Rightarrow \qquad p = 3.5 \times 10^5 \text{ Pa}$$

(Notice that we had to change the temperature into kelvin.)

The equation of state of an ideal gas

If we combine the results of the three preceding experiments, we find that:

$$\frac{pV}{T} = \text{constant}$$

What is the value of the constant? To determine that, we repeat all of the preceding experiments, this time using different quantities of the gas. We discover that the constant in the last equation is proportional to the number of moles n of the gas in question:

$$\frac{pV}{T} = n \times \text{constant}$$

We can now measure the pressure, temperature, volume and number of moles for a large number of different gases and calculate the value of $\frac{pV}{nT}$. We find that this constant has the same value for all gases – it is a universal constant. We call this the gas constant R. It has the numerical value:

$$R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$$

Thus, finally, the equation of state is:

$$pV = RnT$$

(Remember that temperature must always be in kelvin.)

Exam tip

In practice we use this in the form $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ when a gas changes from values (p_1, V_1, T_1) to (p_2, V_2, T_2). Cancel out any quantities that stay the same.

Worked examples

3.13 Estimate how many molecules there are in a gas of temperature 320 K, volume 0.025 $\mathrm{m^3}$ and pressure 4.8×10^5 Pa.

First we find the number of moles:

$$n = \frac{pV}{RT}$$

$$n = \frac{4.8 \times 10^5 \times 0.025}{8.31 \times 320} = 4.51\,\mathrm{mol}$$

Each mole contains the Avogadro number of molecules, so the number of molecules is:

$$4.51 \times 6.02 \times 10^{23} \approx 2.7 \times 10^{24}$$

3.14 A container of hydrogen of volume 0.10 $\mathrm{m^3}$ and temperature 25 °C contains 3.2×10^{23} molecules. Calculate the pressure in the container.

The number of moles present is:

$$n = \frac{3.2 \times 10^{23}}{6.02 \times 10^{23}} = 0.53$$

So: $p = \dfrac{RnT}{V} = \dfrac{8.31 \times 0.53 \times 298}{0.10} = 1.3 \times 10^4\,\mathrm{Pa}$

3.15 A fixed quantity of gas of volume $3.0 \times 10^{-3}\,\mathrm{m^3}$, pressure 3.0×10^5 Pa and temperature 300 K expands to a volume of $4.0 \times 10^{-3}\,\mathrm{m^3}$ and a pressure of 6.0×10^5 Pa. Calculate the new temperature of the gas.

Use $\dfrac{p_1 V_1}{n_1 T_1} = \dfrac{p_2 V_2}{n_2 T_2}$ to get:

$$\frac{3.0 \times 10^5 \times 3.0 \times 10^{-3}}{300} = \frac{6.0 \times 10^5 \times 4.0 \times 10^{-3}}{T}$$

Solving for T gives: $T = 800\,\mathrm{K}$

3.16 Figure **3.23** shows two isothermal curves for equal quantities of two ideal gases. State and explain which gas is at the higher temperature.

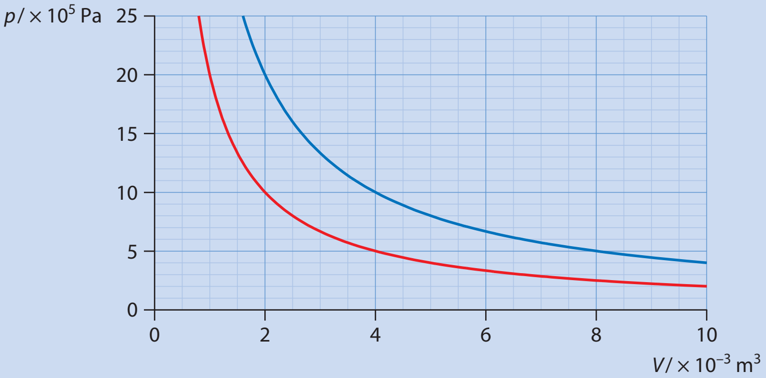


Figure 3.23 Two isothermal curves for equal quantities of two gases.

Draw a vertical line that intersects the two isotherms at two points. At these points both gases have the same volume, and as the quantities of gas are equal n is the same. So for these points $\frac{p}{T}$ is constant. The point on the blue curve has higher pressure, so it must have the higher temperature.

The Boltzmann equation

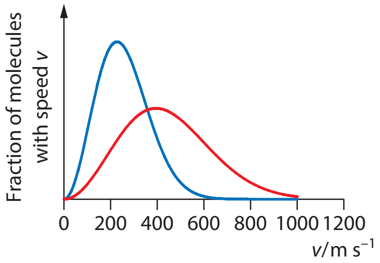


Figure 3.24 The distribution of speeds at two different temperatures.

The molecules of a gas move about randomly with a range of speeds. The graph in Figure **3.24** shows the distribution of speeds for oxygen molecules kept at two different temperatures: the blue curve is at 100 K and the red curve at 300 K. The vertical axis shows the fraction of molecules having a given speed v. You will not be examined on this graph but knowing a few of its features helps a lot in understanding how gases behave.

We see that there is a speed that corresponds to the peak of the curve. For the blue curve this is about 225 m s^{-1} and for the red curve at 400 m s^{-1}. The speed at the peak represents the most probable speed that would be found if you picked a molecule at random. Two other speeds are important:

- the average speed of the molecules, $\overline{v} = \dfrac{v_1 + v_2 + v_3 + \cdots + v_N}{N}$
- the r.m.s. speed or root mean square speed c, which is the square root of the average of the squares of the speeds of the molecules, i.e.

$$c = \sqrt{\frac{v_1{}^2 + v_2{}^2 + v_3{}^2 + \cdots + v_N{}^2}{N}}$$

Why do we bother to work with an r.m.s. speed? Consider the *average kinetic energy* for the N molecules, which is given by:

$$\overline{E}_K = \frac{\frac{1}{2}mv_1{}^2 + \frac{1}{2}mv_2{}^2 + \frac{1}{2}v_3{}^2 + \cdots + mv_N{}^2}{N}$$

$$= \frac{1}{2}m\left(\frac{v_1{}^2 + v_2{}^2 + v_3{}^2 + \cdots + v_N{}^2}{N}\right)$$

$$= \frac{1}{2}mc^2$$

So we see that the average kinetic energy involves the r.m.s. speed. These three speeds (most probable, average and r.m.s. speed) are all different but numerically close to each other. So, even though it is not technically correct, we may assume that all three speeds mean the same thing and we will use the symbol c for all of them.

Now, it can be proven that the pressure of a gas is $p=\frac{1}{3}\rho c^2$, where the quantity c stands for the r.m.s. speed and ρ is the density of the gas. (You will not need to know this equation for the exam.) We get a very interesting result if we combine this equation with the equation of state for an ideal gas, i.e. the equation $pV=nRT$. There are many steps in the derivation in the box below. N stands for the number of molecules and m for the mass of one molecule.

Since $\frac{1}{2}mc^2$ is equal to E, the average random kinetic energy of the molecules, we can write:

$pV=nRT$	
$(\frac{1}{3}\rho c^2)\,V=nRT$	replacing the pressure with $p=\frac{1}{3}\rho c^2$
$\frac{1}{3}\frac{M}{Vc^2V}=nRT$	replacing the density by mass ÷ volume
$\frac{1}{3}Mc^2=nRT$	cancelling the volume
$\frac{1}{3}Nmc^2=\frac{N}{N_A}RT$	writing $M=Nm$ and $n=\frac{N}{N_A}$
$\frac{1}{2}Nmc^2=\frac{3}{2}\frac{R}{N_A}T$	multiplying both sides by $\frac{3}{2}$

$$k_B=\frac{R}{N_A}=\frac{8.31}{6.02\times10^{23}}$$
$$=1.38\times10^{-23}\,\mathrm{J\,K^{-1}}$$

The product of all this algebra is the very important result that relates the average random kinetic energy to the absolute temperature.

$$\bar{E}_K=\frac{3}{2}\frac{R}{N_A}T$$

The ratio $\frac{R}{N_A}$ is called the Boltzmann constant, k_B. So the final result is that the average random kinetic energy of the particles is directly proportional to the kelvin temperature:

$$\bar{E}_K=\frac{3}{2}k_BT$$

Using this equation we can find an expression for the internal energy of an ideal gas. Remember that the internal energy of an ideal gas consists only of the random kinetic energy of its molecules and no potential energy. Suppose that the gas has N molecules. Then, since the average kinetic energy is $\frac{3}{2}k_BT$, the **total** random kinetic energy, i.e. the internal energy U, is:

$$U = \frac{3}{2}Nk_BT$$

But recall that $k_B = \frac{R}{N_A}$, so that another expression is:

$$U = \frac{3}{2}nRT$$

Yet another expression comes from using the equation of state, $pV = nRT$, which gives:

$$U = \frac{3}{2}pV$$

Exam tip
You must be able to obtain an expression for the internal energy of an ideal gas even though this formula is not in the IB data booklet.

Worked examples

3.17 The kelvin temperature of a gas is doubled. By what factor does the average speed increase?

From $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$ we find that when T is doubled then c^2 will double, so c itself will increase by a factor of $\sqrt{2}$.

3.18 Calculate the ratio of the average speed of oxygen (O_2) to carbon dioxide (CO_2) molecules when both gases are at the same temperature.

Since the temperature is the same for both gases, using $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$ we find that:

$$\frac{1}{2}m_Oc_O{}^2 = \frac{1}{2}m_{CO_2}c_{CO_2}{}^2 \quad \text{and so} \quad \frac{c_O{}^2}{c_{CO_2}{}^2} = \frac{m_{CO_2}}{m_O}$$

So we need to find the ratio of the masses of the molecules. One mole of oxygen has a mass of 32 g so one molecule has a mass (in grams) of $\frac{32}{N_A}$. Similarly, the mass in grams of a carbon dioxide molecule is $\frac{44}{N_A}$. So:

$$\frac{c_O{}^2}{c_{CO_2}{}^2} = \frac{44/N_A}{32/N_A} = \frac{44}{32} = 1.375 \Rightarrow \frac{c_O}{c_{CO_2}} = \sqrt{1.375} = 1.17 \approx 1.2$$

3.19 Calculate the average speed of helium (${}^4_2He$) molecules at a temperature of −15 °C.

We use $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$. First we need to find the mass m of a helium atom. One mole of helium has a mass of 4.0 g so the mass of one molecule is given by:

$$m = \frac{4.0}{N_A} = \frac{4.0}{6.02 \times 10^{23}} = 6.64 \times 10^{-24}\,\text{g} = 6.64 \times 10^{-27}\,\text{kg}$$

Now remember to convert the temperature into kelvin: $273 - 15 = 258\,\text{K}$. So we have:

$$\frac{1}{2} \times 6.64 \times 10^{-27} \times c^2 = \frac{3}{2} \times 1.38 \times 10^{-23} \times 258$$

This gives $c^2 = 1.61 \times 10^6$ and so $c = 1.3 \times 10^3\,\text{m}\,\text{s}^{-1}$.

Nature of science

Models must be correct but also simple

Boyle thought that a gas consists of particles joined by springs. Newton thought that a gas consists of particles that exert repulsive forces on each other. Bernoulli thought that a gas is a collection of a very large number of particles that exert forces on each other only when they collide. All three could explain why a gas exerts a pressure on its container but it is Bernoulli's picture that is the simplest. We assume that the ordinary laws of mechanics apply to the individual particles making up the gas. Even though the laws apply to each individual particle we cannot observe or analyse each particle individually since there are so many of them. By concentrating on average behaviours of the whole gas and using probability and statistics, physicists developed a new field of physics known as statistical mechanics. This has had enormous success in advancing our understanding of gases and other systems, including where the approximation to an ideal gas breaks down.

Test yourself

13 Calculate the number of molecules in 28 g of hydrogen gas (molar mass 2 g mol^{-1}).

14 Calculate the number of moles in 6.0 g of helium gas (molar mass 4 g mol^{-1}).

15 Determine the number of moles in a sample of a gas that contains 2.0×10^{24} molecules.

16 Determine the mass in grams of carbon (molar mass 12 g mol^{-1}) that contains as many molecules as 21 g of krypton (molar mass 84 g mol^{-1}).

17 A sealed bottle contains air at 22.0 °C and a pressure of 12.0×10^5 Pa. The temperature is raised to 120.0 °C. Calculate the new pressure.

18 A gas has pressure 8.2×10^6 Pa and volume 2.3×10^{-3} m^3. The pressure is reduced to 4.5×10^6 Pa at constant temperature. Calculate the new volume of the gas.

19 A mass of 12.0 kg of helium is required to fill a bottle of volume 5.00×10^{-3} m^3 at a temperature of 20.0 °C. Determine the pressure in helium.

20 Determine the mass of carbon dioxide required to fill a tank of volume 12.0×10^{-3} m^3 at a temperature of 20.0 °C and a pressure of 4.00 atm.

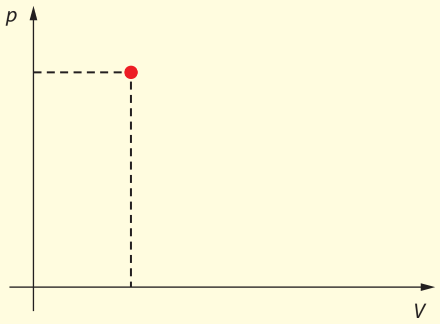
21 A flask of volume 300.0×10^{-6} m^3 contains air at a pressure of 5.00×10^5 Pa and a temperature of 27.0 °C. The flask loses molecules at a rate of 3.00×10^{19} per second. Estimate how long it takes for the pressure in the flask to fall to half its original value. (Assume that the temperature of the air remains constant during this time.)

22 The point marked in the diagram represents the state of a fixed quantity of ideal gas in a container with a movable piston. The temperature of the gas in the state shown is 600 K. Copy the diagram. Indicate on the diagram the point representing the new state of the gas after the following separate changes.

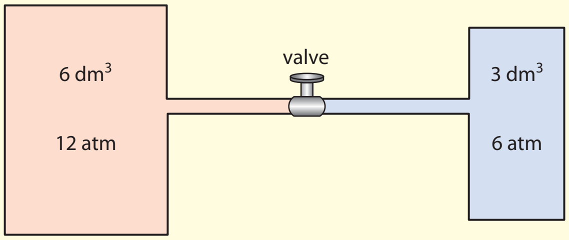
a The volume doubles at constant temperature.

b The volume doubles at constant pressure.

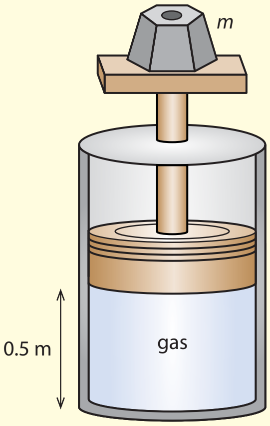
c The pressure halves at constant volume.



23 Two ideal gases are kept at the same temperature in two containers separated by a valve, as shown in the diagram. Estimate the pressure when the valve is opened. (The temperature stays the same.)



24 The diagram shows a cylinder in a vacuum, which has a movable, frictionless piston at the top. An ideal gas is kept in the cylinder. The piston is at a distance of 0.500 m from the bottom of the cylinder and the volume of the cylinder is 0.050 m^3. The weight on top of the cylinder has a mass of 10.0 kg. The temperature of the gas is 19.0 °C.



a Calculate the pressure of the gas.

b Determine how many molecules there are in the gas.

c The temperature is increased to 152 °C. Calculate the new volume of the gas.

25 The molar mass of a gas is 28 g mol^{-1}. A container holds 2.00 mol of this gas at 0.00 °C and a pressure of 1.00×10^5 Pa. Determine the mass and volume of the gas.

26 A balloon has a volume of 404 m^3 and is filled with helium of mass 70.0 kg. The temperature inside the balloon is 17.0 °C. Determine the pressure inside the balloon.

27 A flask has a volume of 5.0×10^{-4} m^3 and contains air at a temperature of 300 K and a pressure of 150 kPa.

a Calculate the number of moles of air in the flask.

b Determine the number of molecules in the flask.

c Estimate the mass of air in the flask. You may take the molar mass of air to be 29 g mol^{-1}.

28 The molar mass of helium is 4.00 g mol^{-1}.

a Calculate the volume of 1.0 mol of helium at standard temperature and pressure (stp) i.e. at $T = 273$ K, $p = 1.0 \times 10^5$ Pa.

b Determine the density of helium at stp.

c Estimate the density of oxygen gas at stp (the molar mass of, oxygen gas is 32 g mol^{-1}).

29 The density of an ideal gas is 1.35 kg m^{-3}. The temperature in kelvin and the pressure are both doubled. Calculate the new density of the gas.

30 Calculate the average speed (r.m.s.) of helium atoms at a temperature of 850 K. The molar mass of helium is 4.0 g mol^{-1}.

31 Show that the average (r.m.s.) speed of molecules of a gas of molar mass M (in kg mol^{-1}) kept at a temperature T is given by $c = \sqrt{\frac{3RT}{M}}$.

32 **a** Calculate the average random kinetic energy of a gas kept at a temperature of 300 K.

b Determine the ratio of the average speeds (r.m.s. speeds) of two ideal gases of molar mass 4.0 g mol^{-1} and 32 g mol^{-1}, which are kept at the same temperature.

Exam-style questions

1 Body X whose temperature is 0 °C is brought into thermal contact with body Y of equal mass and temperature 100 C. The only exchanges of heat that take place are between X and Y. The specific heat capacity of X is greater than that of Y. Which statement about the final equilibrium temperature T of the two bodies is correct?

A $T = 50\,°\mathrm{C}$
B $0 < T < 50\,°\mathrm{C}$
C $100\,°\mathrm{C} > T > 50\,°\mathrm{C}$
D Answer depends on value of mass

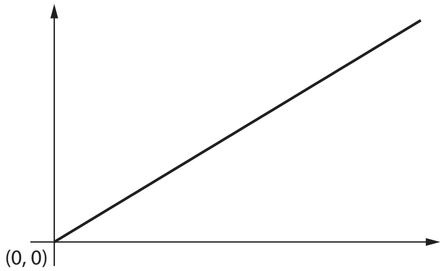
2 Energy is provided to a liquid at its boiling point at a rate of P joules per second. The rate at which mass is boiling away is μ kg per second. The specific latent heat of vaporisation of the liquid is

A μP **B** $\dfrac{P}{\mu}$ **C** $\dfrac{\mu}{P}$ **D** $\dfrac{1}{\mu P}$

3 The following are all assumptions of the kinetic theory of gases, except which one?

A The duration of a collision is very small compared to the time in between collisions.
B The collisions are elastic.
C The average kinetic energy of molecules is proportional to temperature.
D The volume of molecules is negligible compared to the volume of the gas.

4 In the context of a fixed mass of an ideal gas, the graph could represent the variation of:



A pressure with volume at constant temperature
B volume with Celsius temperature at constant pressure
C pressure with Celsius temperature at constant volume
D pressure with inverse volume at constant temperature

5 The temperature of an ideal gas of pressure 200 kPa is increased from 27 °C to 54 °C at constant volume. Which is the best estimate for the new pressure of the gas?

A 400 kPa **B** 220 kPa **C** 180 kPa **D** 100 kPa

6 A container of an ideal gas that is isolated from its surroundings is divided into two parts. One part has double the volume of the other. The pressure in each part is p and the temperature is the same. The partition is removed. What is the pressure in the container now?

A p **B** $2p$ **C** $\frac{3p}{2}$ **D** $4p$

7 Different quantities of two ideal gases X and Y are kept at the same temperature. Which of the following is a correct comparison of the average kinetic energy and internal energy of the two gases?

	Average kinetic energy	**Internal energy**
A	same	same
B	same	different
C	different	same
D	different	different

8 The temperature of an ideal gas is doubled. The average speed of the molecules increases by a factor of

A $\sqrt{2}$ **B** 2 **C** $2\sqrt{2}$ **D** 4

9 Two ideal gases X and Y are kept at the same temperature. Gas X has molar mass m_X and gas Y has molar mass μ_Y. The ratio of average speeds of the molecules of gas X to that of gas Y is

A $\frac{\mu_X}{\mu_Y}$ **B** $\frac{\mu_Y}{\mu_X}$ **C** $\sqrt{\frac{\mu_X}{\mu_Y}}$ **D** $\sqrt{\frac{\mu_Y}{\mu_X}}$

10 The pressure of a fixed quantity of ideal gas is doubled. The average speed of the molecules is also doubled. The original density of the gas is ρ. Which is the new density of the gas?

A $\frac{\rho}{2}$ **B** ρ **C** 2ρ **D** 4ρ

11 **a** Calculate the volume of 1 mol of helium gas (molar mass 4 g mol−1) at temperature 273 K and pressure 1.0 × 105 Pa. **[2]**

b **i** Find out how much volume corresponds to each molecule of helium. **[2]**

ii The diameter of an atom of helium is about 31 pm. Discuss whether or not the ideal gas is a good approximation to the helium gas in **a**. **[2]**

c Consider now 1 mol of lead (molar mass 207 g mol^{-1}, density 11.3 × 10^3 kg m^{-3}). How much volume corresponds to each atom of lead? **[3]**

d Find the ratio of these volumes (helium to lead) and hence determine the order of magnitude of the ratio: separation of helium atoms to separation of lead atoms. **[2]**

12 **a** Define what is meant by **specific heat capacity** of a substance. **[1]**

b Consider two metals that have different specific heat capacities. The energies required to increase the temperature of 1 mol of aluminium and 1 mol of copper by the same amount are about the same. Yet the specific heat capacities of the two metals are very different. Suggest a reason for this. **[2]**

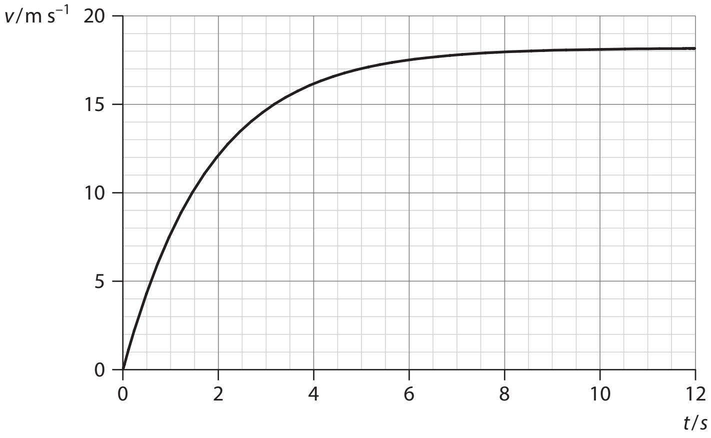
A hair dryer consists of a coil that warms air and a fan that blows the warm air out. The coil generates thermal energy at a rate of 600 W. Take the density of air to be 1.25 kg m^{-3} and its specific heat capacity to be 990 $\text{J kg}^{-1}\text{K}^{-1}$. The dryer takes air from a room at 20 °C and delivers it at a temperature of 40 °C.

c What mass of air flows through the dryer per second? **[2]**

d What volume of air flows per second? **[1]**

e The warm air makes water in the hair evaporate. If the mass of water in the air is 180 g, calculate how long it will take to dry the hair. (The heat required to evaporate 1 g of water at 40 °C is 2200 J.) **[2]**

13 The graph shows the variation with time of the speed of an object of mass 8.0 kg that has been dropped (from rest) from a certain height.



The body hits the ground 12 seconds later. The specific heat capacity of the object is 320 $\text{J kg}^{-1}\text{K}^{-1}$.

a **i** Explain how we may deduce that there must be air resistance forces acting on the object. **[2]**

ii Estimate the height from which the object was dropped. **[2]**

iii Calculate the speed the object would have had if there were no air resistance forces. **[2]**

b Estimate the change in temperature of the body from the instant it was dropped to just before impact. List any assumptions you make. **[4]**

14 A piece of tungsten of mass 50 g is placed over a flame for some time. The metal is then quickly transferred to a well-insulated aluminium calorimeter of mass 120 g containing 300 g of water at 22 °C. After some time the temperature of the water reaches a maximum value of 31 °C.

a State what is meant by the internal energy of a piece of tungsten. **[1]**

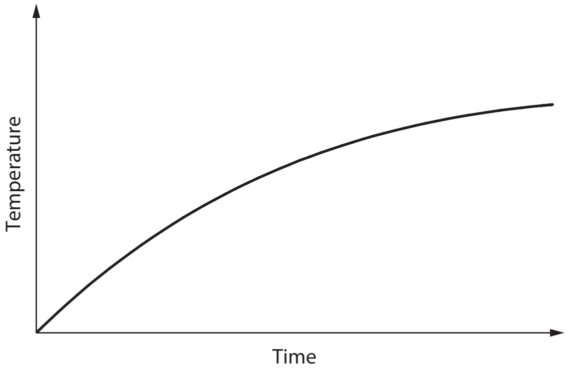
b Calculate the temperature of the flame. You may use these specific heat capacities: water $4.2\times10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$, tungsten $1.3\times10^{2}\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and aluminum $9.0\times10^{2}\,\mathrm{J\,kg^{-1}\,K^{-1}}$. **[3]**

c State and explain whether the actual flame temperature is higher or lower than your answer to **b**. **[2]**

15 **a** Describe what is meant by the internal energy of a substance. **[1]**

b A student claims that the kelvin temperature of a body is a measure of its internal energy. Explain why this statement is not correct by reference to a solid melting. **[2]**

c In an experiment, a heater of power 35 W is used to warm 0.240 kg of a liquid in an uninsulated container. The graph shows the variation with time of the temperature of the liquid.



The liquid never reaches its boiling point.

Suggest why the temperature of the liquid approaches a constant value. **[2]**

d After the liquid reaches a constant temperature the heater is switched off. The temperature of the liquid decreases at a rate of $3.1\,\mathrm{K\,min^{-1}}$.

Use this information to estimate the specific heat capacity of the liquid. **[3]**

16 The volume of air in a car tyre is about $1.50\times10^{-2}\,\mathrm{m^3}$ at a temperature of 0.0 °C and pressure 250 kPa.

a Calculate the number of molecules in the tyre. **[2]**

b Explain why, after the car is driven for a while, the pressure of the air in the tyre will increase. **[3]**

c Calculate the new pressure of the tyre when the temperature increases to 35 °C and the volume expands to $1.60\times10^{-2}\,\mathrm{m^3}$.

d The car is parked for the night and the volume, pressure and temperature of the air in the tyre return to their initial values. A small leak in the tyre reduces the pressure from 250 kPa to 230 kPa in the course of 8 h. Estimate (stating any assumptions you make):

i the average rate of loss of molecules (in molecules per second) **[2]**

ii the total mass of air lost (take the molar mass of air to be $29\,\mathrm{g\,mol^{-1}}$). **[3]**