SL Paper 1

a. Find $\log_2 32$. [1]

b. Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px+qy$, find the value of p and of q. [4]

Find the value of each of the following, giving your answer as an integer.

a. $\log_6 36$ [2]

b. $\log_6 4+\log_6 9$ [2]

c. $\log_6 2-\log_6 12$ [3]

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a. Expand $(2+x)^4$ and simplify your result. [3]

b. Hence, find the term in x^2 in $(2+x)^4\left(1+\frac{1}{x^2}\right)$. [3]

An arithmetic sequence has the first term $\ln a$ and a common difference $\ln 3$.

The 13th term in the sequence is $8\ln 9$. Find the value of a.

Consider the following sequence of figures.

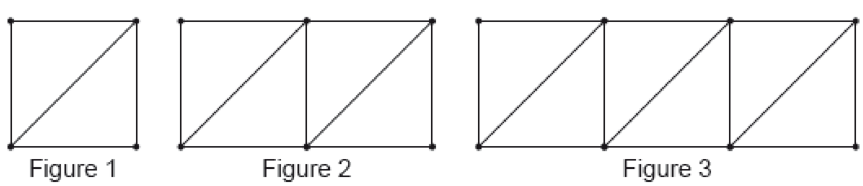

Figure 1 contains 5 line segments.

a. Given that Figure n contains 801 line segments, show that $n=200$. [3]

b. Find the total number of line segments in the first 200 figures. [3]

Let $f(x) = k\log_2 x$.

a. Given that $f^{-1}(1) = 8$, find the value of k . [3]

b. Find $f^{-1}\left(\frac{2}{3}\right)$. [4]

In an arithmetic sequence, the first term is 3 and the second term is 7.

a. Find the common difference. [2]

b. Find the tenth term. [2]

c. Find the sum of the first ten terms of the sequence. [2]

The first three terms of a infinite geometric sequence are $m - 1,\ 6,\ m + 4$, where $m \in \mathbb{Z}$.

a(i). Write down an expression for the common ratio, r. [2]

a(ii). Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0$. [2]

b(i). Find the two possible values of m. [3]

b(ii). Find the possible values of r. [3]

c(i). The sequence has a finite sum. [3]

State which value of r leads to this sum **and** justify your answer.

c(ii). The sequence has a finite sum. [3]

Calculate the sum of the sequence.

In an arithmetic sequence, the first term is 2 and the second term is 5.

a. Find the common difference. [2]

b. Find the eighth term. [2]

c. Find the sum of the first eight terms of the sequence. [2]

a. Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. [3]

b. Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$. [3]

The sums of the terms of a sequence follow the pattern

$S_1 = 1 + k,\ S_2 = 5 + 3k,\ S_3 = 12 + 7k,\ S_4 = 22 + 15k,\ \ldots,$ where $k \in \mathbb{Z}$.

a. Given that $u_1 = 1 + k$, find $u_2,\ u_3$ and u_4. [4]

b. Find a general expression for u_n. [4]

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a. Consider the arithmetic sequence $2,\ 5,\ 8,\ 11, \ldots$. [3]

Find u_{101} .

b. Consider the arithmetic sequence $2,\ 5,\ 8,\ 11, \ldots$. [3]

Find the value of n so that $u_n = 152$.

The first two terms of an infinite geometric sequence, in order, are

$$2\log_2 x,\ \log_2 x, \text{ where } x > 0.$$

The first three terms of an arithmetic sequence, in order, are

$$\log_2 x,\ \log_2\left(\frac{x}{2}\right),\ \log_2\left(\frac{x}{4}\right), \text{ where } x > 0.$$

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

a. Find r. [2]

b. Show that the sum of the infinite sequence is $4\log_2 x$. [2]

c. Find d, giving your answer as an integer. [4]

d. Show that $S_{12} = 12\log_2 x - 66$. [2]

e. Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$. [3]

The fifth term in the expansion of the binomial $(a+b)^n$ is given by $\binom{10}{4} p^6(2q)^4$.

a. Write down the value of n. [1]

b. Write down a and b, in terms of p and/or q. [2]

c. Write down an expression for the sixth term in the expansion. [3]

An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p,\ q > 0$.

a. Show that $d = \log_c(q)$. [2]

b. Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$. [6]

In an arithmetic sequence, the third term is 10 and the fifth term is 16.

a. Find the common difference. [2]

b. Find the first term. [2]

c. Find the sum of the first 20 terms of the sequence. [3]

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a. Consider the infinite geometric sequence $3,\ 3(0.9),\ 3(0.9)^2,\ 3(0.9)^3,\ \ldots$. [1]

Write down the 10th term of the sequence. Do not simplify your answer.

b. Consider the infinite geometric sequence $3,\ 3(0.9),\ 3(0.9)^2,\ 3(0.9)^3,\ \ldots$. [4]

Find the sum of the infinite sequence.

Let $f(x) = log_3\sqrt{x}$, for $x > 0$.

a. Show that $f^{-1}(x) = 3^{2x}$. [2]

b. Write down the range of f^{-1} . [1]

c. Let $g(x) = \log_3 x$, for $x > 0$. [4]

Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

The first three terms of an infinite geometric sequence are 32, 16 and 8.

a. Write down the value of r . [1]

b. Find u_6 . [2]

c. Find the sum to infinity of this sequence. [2]

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

a.i. Find an expression for r in terms of θ. [2]

a.ii. Find the possible values of r. [3]

b. Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. [4]

c. Find the values of θ which give the greatest value of the sum. [6]

Three consecutive terms of a geometric sequence are $x-3$, 6 and $x+2$.

Find the possible values of x.

The values in the fourth row of Pascal's triangle are shown in the following table.

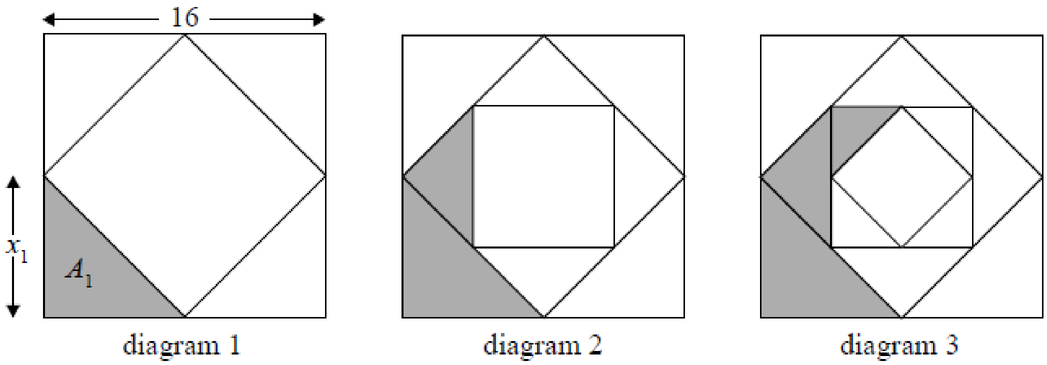
1	4	6	4	1

a. Write down the values in the fifth row of Pascal's triangle. [2]

b. Hence or otherwise, find the term in x^3 in the expansion of $(2x+3)^5$. [5]

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1).

The process is repeated twice, as shown in diagrams 2 and 3.



Let x_n denote the length of one of the equal sides of each new triangle.

Let A_n denote the area of each new triangle.

a. The following table gives the values of x_n and A_n, for $1 \leqslant n \leqslant 3$. **Copy** and complete the table. *(Do **not** write on this page.)* [4]

n	1	2	3
x_n	8		4
A_n	32	16	

b. The process described above is repeated. Find A_6. [4]

c. Consider an initial square of side length k cm. The process described above is repeated indefinitely. The total area of the shaded regions is k cm^2. Find the value of k. [7]

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a. Find the value of $\log_2 40 - \log_2 5$. [3]

b. Find the value of $8^{\log_2 5}$. [4]

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a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. [2]

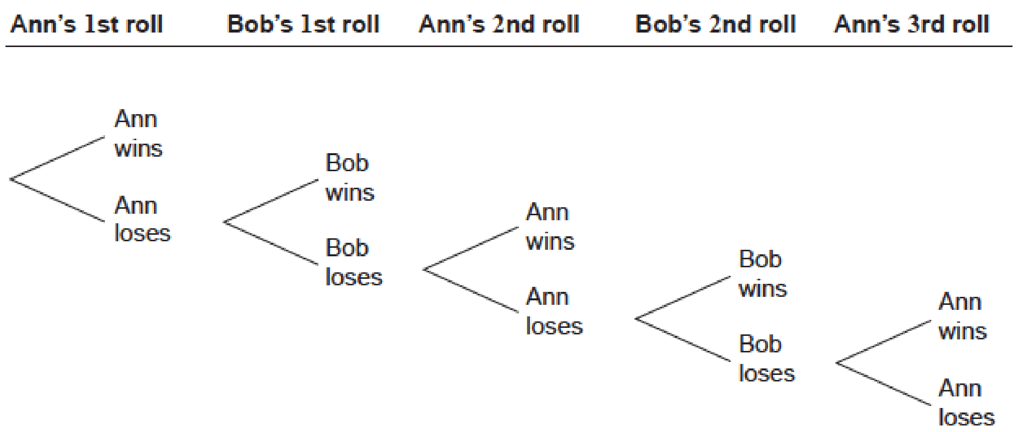
b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4]

The first three terms of a geometric sequence are $\ln x^{16}$, $\ln x^8$, $\ln x^4$, for $x > 0$.

a. Find the common ratio. [3]

b. Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$. [5]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.



a. Find the probability that Ann wins on her first roll. [2]

b. (i) The probability that Ann wins on her third roll is $\frac{5}{8} \times \frac{4}{8} \times p \times q \times \frac{3}{8}$. [6]

Write down the value of p and of q.

(ii) The probability that Ann wins on her tenth roll is $\frac{3}{8}r^k$ where $r \in \mathbb{Q},\ k \in \mathbb{Z}$.

Find the value of r and of k.

c. Find the probability that Ann wins the game. [7]

Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

Write down the value of

a(i).(i) $\log_3 27$; [1]

a(ii)(ii) $\log_8 \frac{1}{8}$; [1]

a(iii)(iii) $\log_{16} 4$. [1]

b. Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. [3]

Solve $\log_2 x + \log_2 (x-2) = 3$, for $x > 2$.

In the expansion of $(3x+1)^n$, the coefficient of the term in x^2 is $135n$, where $n \in \mathbb{Z}^+$. Find n.

Let $f(x) = 3\ln x$ and $g(x) = \ln 5x^3$.

a. Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. [4]

b. The graph of g is a transformation of the graph of f . Give a full geometric description of this transformation. [3]

In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.

a. Find d . [2]

b. Find u_{20} . [2]

c. Find S_{20} . [2]

Given that $\left(1 + \frac{2}{3}x \right)^n (3 + nx)^2 = 9 + 84x + \ldots$, find the value of n .

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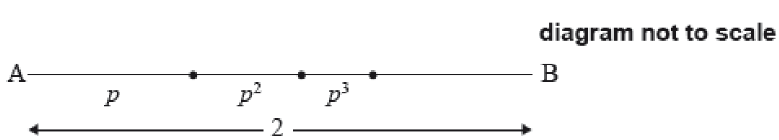
a. The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments. [5]

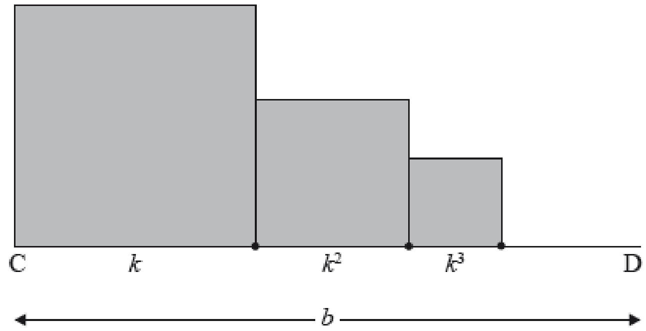


The length of the line segments are p cm, p^2 cm, p^3 cm, $\ldots$, where $0 < p < 1$.

Show that $p = \frac{2}{3}$.

b. The following diagram shows [CD], with length b cm, where $b > 1$. Squares with side lengths k cm, k^2 cm, k^3 cm, $\ldots$, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares. [9]

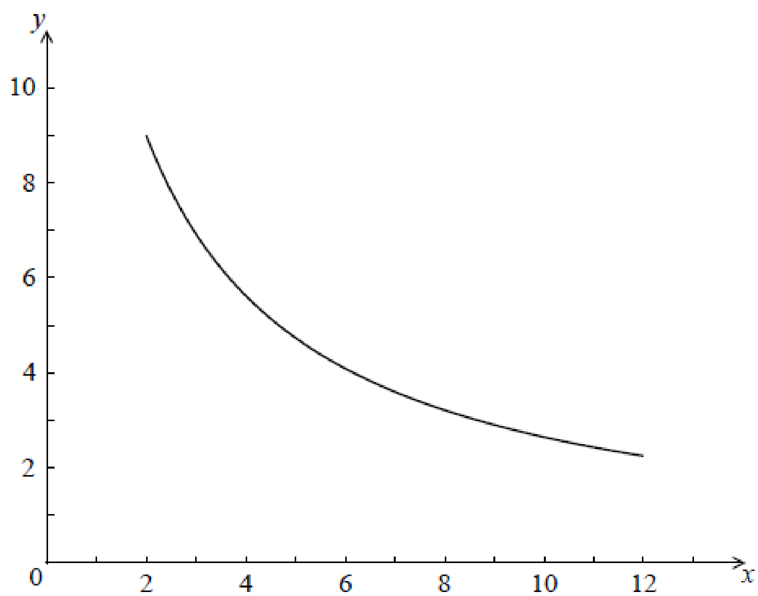
diagram not to scale



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.

Let $f(x) = \frac{1}{4}x^2 + 2$. The line L is the tangent to the curve of f at (4, 6) .

Let $g(x) = \frac{90}{3x+4}$, for $2 \leq x \leq 12$. The following diagram shows the graph of g .



a. Find the equation of L . [4]

b. Find the area of the region enclosed by the curve of g , the x-axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. [6]

c. The graph of g is reflected in the x-axis to give the graph of h . The area of the region enclosed by the lines L , $x = 2$, $x = 12$ and the x-axis is 120 120 cm^2 . [3]

Find the area enclosed by the lines L , $x = 2$, $x = 12$ and the graph of h .

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

a. $\ln\left(\frac{5}{3}\right)$. [2]

b. $\ln 45$. [4]

Let $\log_3 p = 6$ and $\log_3 q = 7$.

a. Find $\log_3 p^2$. [2]

b. Find $\log_3 \left(\frac{p}{q}\right)$. [2]

c. Find $\log_3(9p)$. [3]

Let $f(x) = \mathrm{e}^{x+3}$.

a. (i) Show that $f^{-1}(x) = \ln x - 3$. [3]

(ii) Write down the domain of f^{-1} .

b. Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$. [4]

In an arithmetic sequence, the first term is 8 and the second term is 5.

a. Find the common difference. [2]

b. Find the tenth term. [2]

c. Find the sum of the first ten terms. [2]

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

The y-coordinate of P is $\ln 27$.

a. Find the x-coordinate of P. [3]

b. Find $f(x)$, expressing your answer as a single logarithm. [8]

c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$. [[N/A

Find the value of a and of b, where $a,\ b \in \mathbb{N}$.