* *Terry's Messy Unchecked Solutions * *

Physics 8B – Summer 2016 – Midterm 2

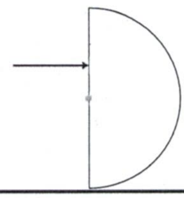
Correct Answers without supporting work or justification will receive NO CREDIT, except Mult. Choice

1. (10 pts)

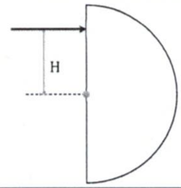
Using a laser and a semi-circular piece of plastic (radius, R and index of refraction, n_{plas}) like in lab, you shine a beam of light as shown. You can consider $n_{air} = 1$. The dot shown is the center of the semi-circle. ***REDRAW the sketches in exam book. ***

a) (2 pts) Sketch the path the light takes as it passes through the plastic and emerges on the other side

b) (2 pts) Sketch the path the light takes as it passes through the plastic and emerges on the other side



c) (6 pts) If you raise the beam from b) up, you eventually reach a height, H, where the beam no longer emerges on the other side of the plastic. Calculate this height.



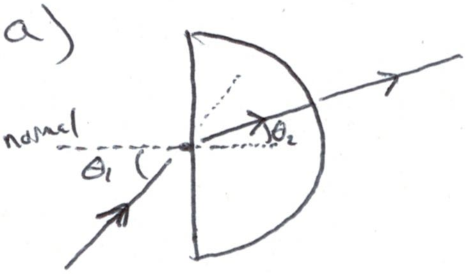
Snell's Law: $n_{in} \sin\theta_{in} = n_{out} \sin\theta_{out}$

Comments * higher n $\Longleftrightarrow$ "closer to the normal."

* Lower n $\Longleftrightarrow$ "farther away from normal."

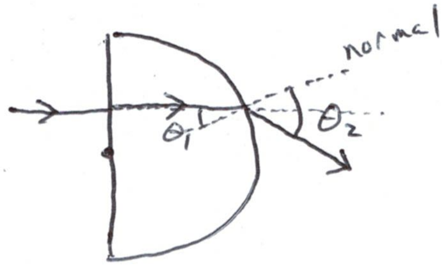
* Also $\theta_{in} = 0 \Longleftrightarrow \theta_{out} = 0$

a)



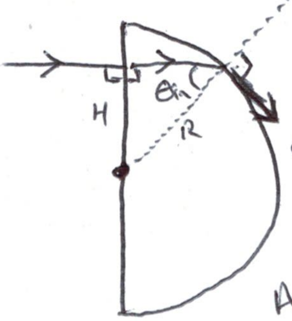
- air $n=1$ to plastic $n>1$ — θ_2 smaller (ie closer to normal)
- plastic to air — $\theta_{in} = 0 \Rightarrow \theta_{out} = 0$ (Since radial dir. in plastic hits interface with $\theta_{in} = 0$)

b)



- air to plastic — $\theta_{in} = 0 \Rightarrow \theta_{out} = 0$
- plastic $n>1$ to air $n=1$ — θ_2 larger (ie farther from normal)

c)



normal is radial

cutoff is $\theta_{out} = 90^\circ$

After that TIR

$$n_{in} \sin\theta_{in} = n_{out} \sin\theta_{out}$$

$$n_{plas} \sin\theta_{in} = (1) \sin 90^\circ$$

$$n_{plas} \frac{H}{R} = 1$$

$$\boxed{H = \frac{R}{n_{plas}}}$$

2. (30 pts)

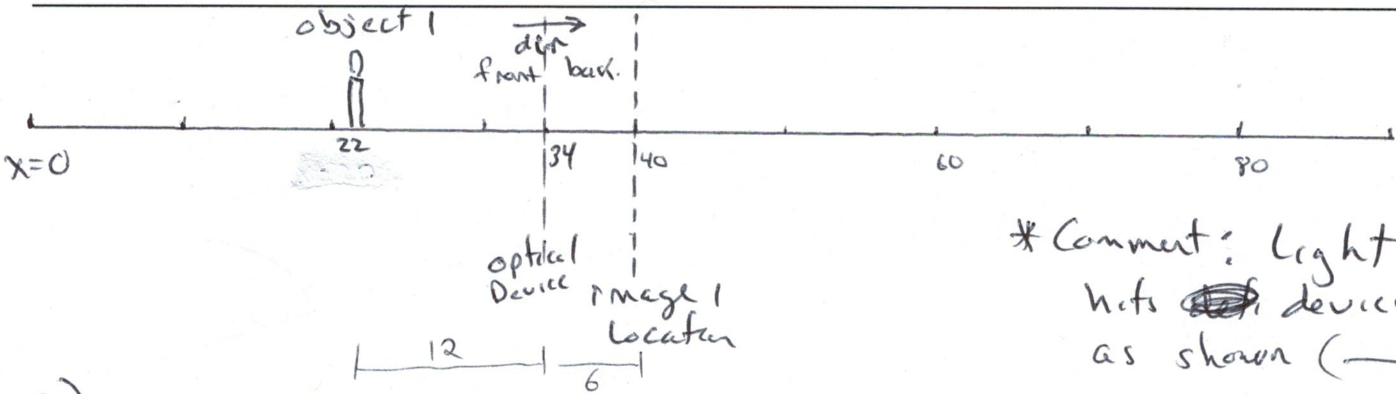
A 2 cm tall candle is placed at x = 22 cm. An unknown optical device (i.e. mirror or lens) is placed at x = 34 cm. You are told that this device forms an image (with unspecified characteristics) at x = 40 cm.

a) **(10 pts) IF the device is a mirror :**
   i) Is the image real or virtual? Explain.
   ii) Find the focal length of the device. State whether it is converging, diverging or plane.
   iii) Draw a qualitatively accurate ray-tracing diagram for this arrangement. ***Put arrows on your rays so we know which way they go.***

b) **(10 pts) IF the device is a lens :**
   i) Is the image real or virtual? Explain.
   ii) Find the focal length of the device. State whether it is converging or diverging.
   iii) Draw a qualitatively accurate ray-tracing diagram for this arrangement. ***Put arrows on your rays so we know which way they go.***

For the above arrangement, you add a lens placed at x = 50 cm. The combined system of the unknown device (x=34cm) and added lens (x=50cm) forms a sharp clear image of the original candle (x=22cm) on a screen at x = 90 cm.

c) (10 pts)
   i) Compared to the original candle, is the final image on screen the same orientation, inverted, or it depends on whether the unknown device is a mirror/lens. Explain.
   ii) Calculate the height (absolute value) of the final image on the screen?



* Comment: Light hits device as shown ($\longrightarrow$)

a) i) For a mirror the image location of x = 40 is behind mirror, ie $\boxed{\text{Image1 is virtual}}$

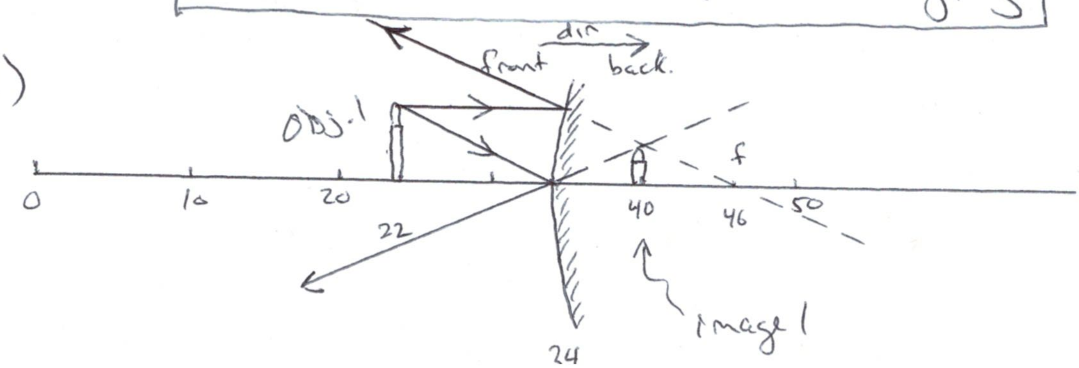
so $s_1' < 0$

ii) $$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$$

$$\frac{1}{+12} + \frac{1}{-6} = \frac{1}{f_1}$$

$$\boxed{f_1 = -12 \text{ cm} \quad \text{ie diverging}}$$

iii)



(Comments: virtual, same orient., smaller)

Prob. 2 continued

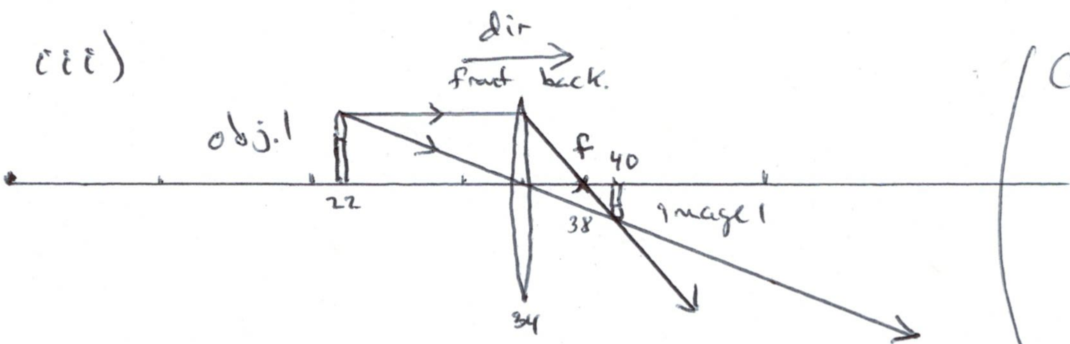
b) i) For a lens, the image location $x = 40\,cm$ is on the outgoing ray side (behind lens)
so $\boxed{\text{image 1 is real}}$
$s_1' > 0$

ii) $$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$$

$$\frac{1}{+12} + \frac{1}{+6} = \frac{1}{f_1}$$

$$\boxed{f_1 = +4\,cm \quad \text{ie converging}}$$

iii)



(Comments
real
inverted
smaller)

c) * Since light goes through added lens and forms image on screen at $x = 90\,cm$, unknown device must be lens. For mirror light would never reach added lens nor screen.

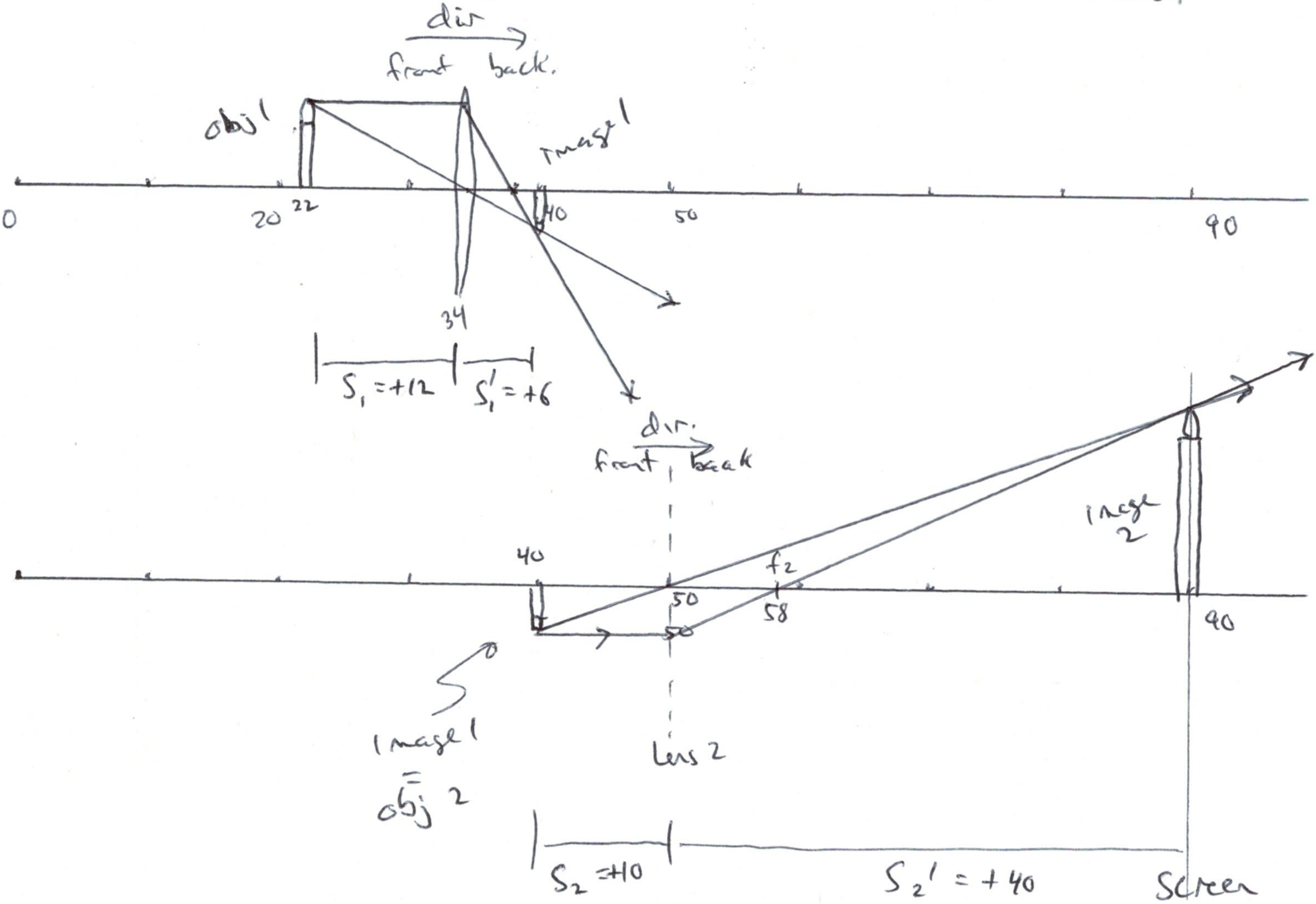
$$M_{tot} = M_1 M_2 = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \left(-\frac{+6}{+12}\right)\left(-\frac{+40}{+10}\right) = +2$$

So final image is $\boxed{\text{same orientation as orig. obj.}}$

+ final image is $\boxed{\text{twice as big ie } h_f = 4\,cm}$

Prob. 2 continued (Just Comments)

Here is full diagrams to help see distances.



Not asked for but

$$\frac{1}{S_2} + \frac{1}{S_2'} = \frac{1}{f_2}$$

$$\frac{1}{10} + \frac{1}{40} = \frac{1}{f_2}$$

$f_2 = +8$ cm ie converging

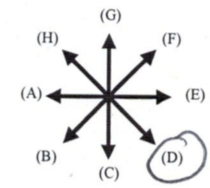
3. (20 pts)

A satellite radio transmitter with power, P, uses an LC circuit (with inductor, L, and capacitor, C) to generate its signal. The satellite is at the origin. A space shuttle is located at (x,y) = (d,d).

a) (13 pts) At the location of the shuttle, find the frequency, wavelength, E_{max}, and B_{max} of the radio signal.

b) (2 pts) **MULT. CHOICE – NO PARTIAL CREDIT**

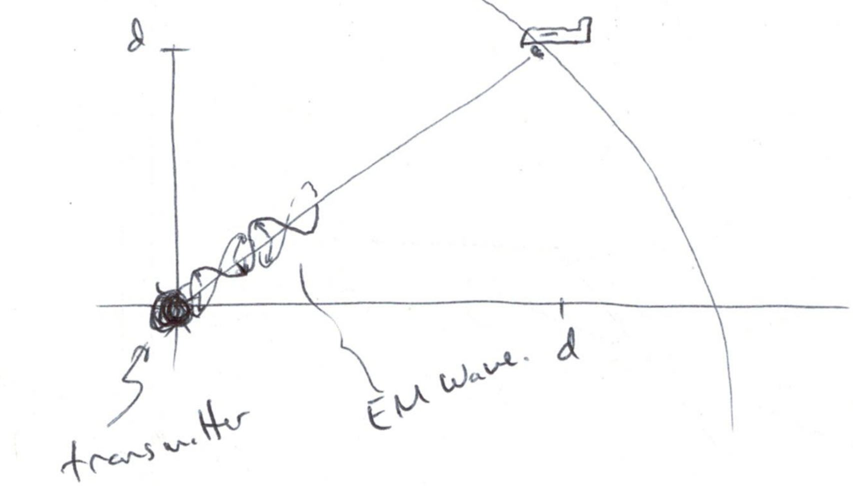
At some instant in time, the magnetic field of the signal at the location of the shuttle is directed INTO the page (-z dir), what is the direction of the electric field at that same instant at the shuttle location?



Choices for b)

*** For this part, just answer based on an oscillating LC circuit, don't worry about the transmitter ***

c) (5 pts) When the capacitor has 2/3 of its max charge, what fraction of the energy is stored in the inductor?



shuttle is distance

$$r = \sqrt{d^2 + d^2} = \sqrt{2}\, d$$

$$(r^2 = 2d^2)$$

a) $\omega = \frac{1}{\sqrt{LC}}$

$2\pi f = \frac{1}{\sqrt{LC}}$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

$f\lambda = v = c$ ← for EM wave — small "c" speed of light.

$\lambda = \frac{c}{f}$

$$\boxed{\lambda = 2\pi c\sqrt{LC}}$$

$$\frac{P}{A} = I = \frac{1}{2c\mu_0} E_{max}^2$$

$$E_{max}^2 = 2c\mu_0 \frac{P}{(4\pi r^2)} = 2c\mu_0 \frac{P}{(4\pi(2d^2))}$$

$$\boxed{E_{max} = \sqrt{\frac{c\mu_0 P}{4\pi d^2}}}$$

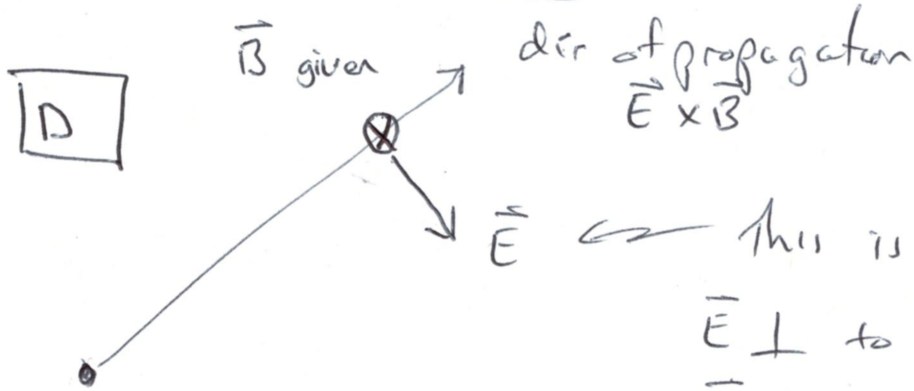
$$\frac{E_{max}}{B_{max}} = c$$

$$B_{max} = \frac{E_{max}}{c}$$

$$\boxed{B_{max} = \sqrt{\frac{\mu_0 P}{c\, 4\pi d^2}}}$$

Prob. 3 continued

b)



This is $\vec{E}$ we need

$\vec{E} \perp$ to $\vec{B}$

$\vec{E} \perp$ to $\vec{E} \times \vec{B}$

and $\vec{E} \times \vec{B}$ gives dir. of propagation

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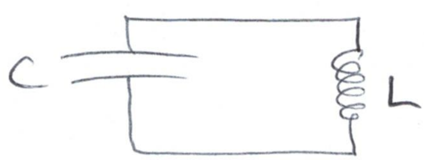
c)



Compare case of $Q = Q_{max}$ to $Q = \frac{2}{3} Q_{max}$.

$$E_0 = E_f$$

$$U_{C_0} + U_{L_0} = U_{C_f} + U_{L_f}$$

$$\frac{1}{2} \frac{Q_{max}^2}{C} + 0 = \frac{1}{2} \frac{\left(\frac{2}{3} Q_{max}\right)^2}{C} + U_{Lf}$$

$$\frac{1}{2} \frac{Q_{max}^2}{C} \left(1 - \frac{4}{9}\right) = U_{Lf}$$

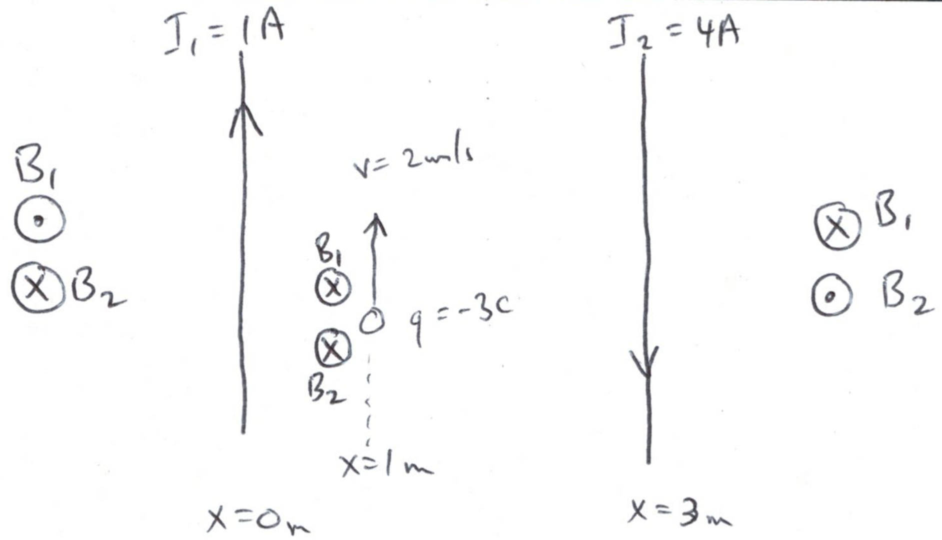
$$\frac{1}{2} \frac{Q_{max}^2}{C} \frac{5}{9} = U_{Lf}$$

$$\boxed{\frac{5}{9} E_{tot} = U_{Lf}}$$

4. (10 pts)
A long straight vertical wire at x = 0 m has a current $I_1 = 1$ A in the +y dir. A second vertical wire at x = 3 m has a current $I_2 = 4$ A in the –y dir. A small charge (q = -3 C) located at x = 1 m is shot with speed v = 2 m/s in the +y direction.

a) (8 pts) What is the force (magnitude and direction) on the small ball? Explain your direction.
b) (2 pts) **MULT. CHOICE – NO PARTIAL CREDIT**

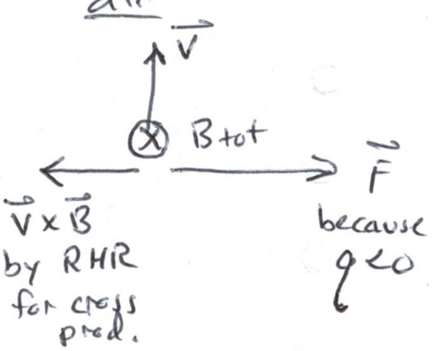
In which region is there a location that has zero magnetic field?
- i) x < 0 m
- ii) 0 m < x < 3 m
- iii) x > 3 m
- iv) Both x < 0 m and x > 3 m.



$* \; |B_2| > |B_1|$ because I_2 is closer and larger.
So $B_{tot} \neq 0$

a) $\vec{F} = q\vec{v} \times \vec{B}$

dir



magnitude

$$|F| = |qvB \sin 90°|$$

$$= |qv(B_1 + B_2)|$$

$$= \left| qv\left(\frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2}\right) \right|$$

$$= \left| (-3)(2)\left[\left(\frac{\mu_0 \cdot 1}{2\pi \; 1}\right) + \left(\frac{\mu_0 \; 4}{2\pi \; 2}\right)\right] \right|$$

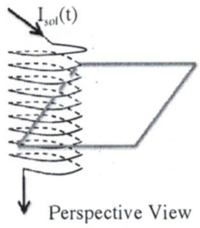
$$\boxed{\vec{F} = \frac{9\mu_0}{\pi} \text{ to the right}}$$

b) $\boxed{i}$

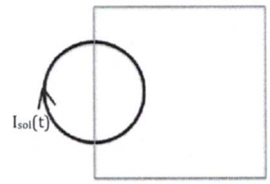
(Need ~~B~~ $\vec{B}_1$ and $\vec{B}_2$ opposite dir + equal magnitude
Can't be x>3 because B_2 is larger there

5. (30 pts)

A square loop of side, x, and resistance, R, has the left side passing through the center of a very long solenoid with length, L, radius, r, and N_{sol} turns. (See perspective view and top view). The solenoid has a current that varies in time as $I_{sol}(t) = 4 - 2t$ (Amps) in the direction shown.



Perspective View



Top View

***** ANSWER all questions about directions based on TOP VIEW*****

a) (4 pts) Mult. Choice.
   i) What is the direction of the magnetic field in the solenoid at $t = 0$ s?
      Up the page, Down the page, Left, Right, Into page, Out of Page, Zero.
   ii) What is the direction of the magnetic field in the solenoid at $t = 2$ s?
      Up the page, Down the page, Left, Right, Into page, Out of Page, Zero.

b) (5 pts) What is the direction of the induced current in the loop at t = 1s? Explain.

c) (3 pts) What is the direction of the force on the square loop at t = 1s? Explain.

d) (18 pts) What is the magnitude of the force on the square loop at t = 1s? (You may assume the loop is massive enough that the amount it moves is negligible.)

a) i) $t = 0$ s $B = \mu_0 i_{sol} n_{sol} = \mu_0 i_{sol} \frac{N_{sol}}{L}$

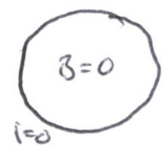
$i_{sol}(0) = 4A$

$\boxed{\text{Into page}}$ by RHR for B due to I.

ii) $t = 2$ s

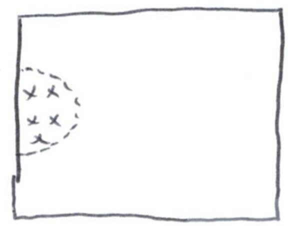
$i_{sol}(2) = 0$

$\boxed{\text{Zero}}$



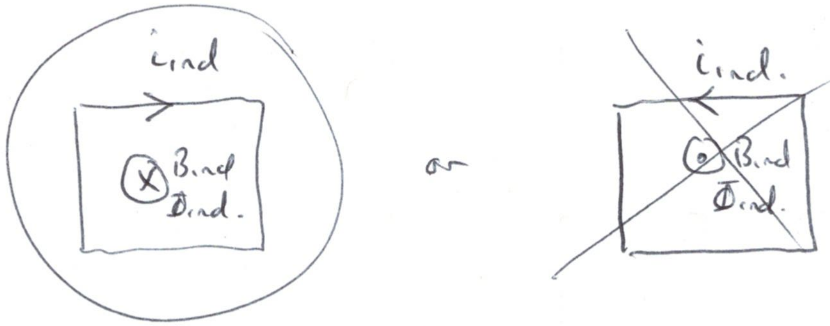
Problem 5 Continued

b) The flux through the loop is only in the semi-circle region shown.



	$t=0$	$t=2$
B	×× ××	0
Φ	×× ××	0

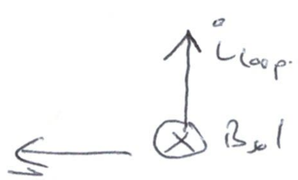
$\frac{d\Phi}{dt}$ ⊙ ← So need B_{ind} and Φ_{ind} in loop ⊗ to oppose change in flux $\frac{d\Phi}{dt}$.

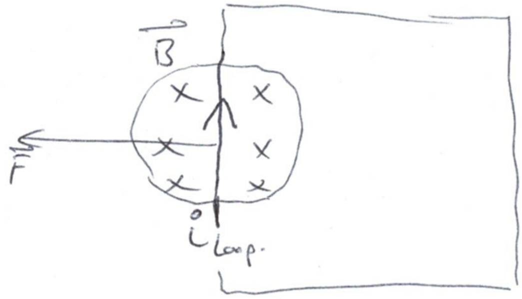


$$\boxed{i_{ind} = \text{clockwise}}$$

c) The loop has an induced current. That sits (partly) in the B_{sol}.

$$\underset{\substack{\text{on}\\\text{Loop}\\\text{due}\\\text{to}\\\text{Solenoid}}}{\vec{F}} = \underbrace{i\, d\vec{\ell}}_{\text{Loop.}} \times \underbrace{\vec{B}}_{\text{solenoid.}}$$





$\underline{\underline{\vec{F} \text{ is Left}}}$ by RHR for cross products.

Prob. 5 continued

d) $\vec{F}_{\substack{\text{on loop}\\ \text{due to sol.}}} = i_{loop}\, d\vec{l} \times \vec{B}_{solenoid}$

induced current in loop.

only the portion in $\vec{B}$ field experiences force.

$\vec{B}$ of solenoid. (the environment the loop is sitting in)

$|i_{ind}| = \left|\frac{\mathcal{E}_{ind}}{R}\right|$

$dl = 2r$

at $t = 1s$

$B_{sol} = \mu_0 i_{sol} n_{sol}$

$= \mu_0 i_{sol} \frac{N_{sol}}{L}$

$= \mu_0 (2) \frac{N_{sol}}{L}$

So need to do Faraday's law to find this.

step 1

$\Phi_{Loop} = \Phi_D + \Phi_{\square}$

$= B_D A_D + \cancel{B_{\square}}^{0} A_{\square}$

$= B_{sol} \cdot A_D + 0$

$= \mu_0 i_{sol} \frac{N_{sol}}{L} \left(\frac{1}{2}\pi r^2\right)$

step 2

$|\mathcal{E}_{ind\,loop}| = \left|-N_{loop}\frac{d\Phi_{loop}}{dt}\right|$

$= \left|-(1)\frac{\mu_0 N_{sol} \pi r^2}{2L}\frac{di_{sol}}{dt}\right|$

$i_{sol} = 4 - 2t$

$\frac{di_{sol}}{dt} = -2$

$= \left|-\frac{\mu_0 N_{sol} \pi r^2}{\cancel{2}L}(\cancel{-2})\right|$

$= \frac{\mu_0 N_{sol} \pi r^2}{L}$

So $|i_{ind}| = \left|\frac{\mathcal{E}_{ind}}{R}\right| = \frac{\mu_0 N_{sol} \pi r^2}{LR}$

Prob. 5 continued

Thus $|F_{Loop.}| = |i \, d\vec{\ell} \times B|$

$$= \frac{\mu_0 N_{sol} \pi r^2}{L R} (2r) \left(\mu_0 \, 2 \, \frac{N_{sol}}{L} \right)$$

$$\boxed{|F| = \frac{4 \mu_0^2 N_{sol}^2 \pi r^3}{L^2 R}}$$

to the left as seen in c)

Comment

Another way to think of dir. of Force is

as time passes $i_{sol} \downarrow$

$B_{sol} \downarrow$

Φ_{Loop} decreases (less Φ into page)

So Force is such that loop would try to increase area/flux to counter the decreasing flux. ie <u>move left</u>