Topic 6—Calculus

40 hours

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their applications.

	Content	Further guidance	Links
6.1	Informal ideas of limit and convergence. Limit notation. Definition of derivative from first principles as $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$. Derivative interpreted as gradient function and as rate of change. Tangents and normals, and their equations. **Not required:** analytic methods of calculating limits.	*Example*: 0.3, 0.33, 0.333, ... converges to $\frac{1}{3}$. Technology should be used to explore ideas of limits, numerically and graphically. *Example*: $\lim_{x\to\infty}\left(\frac{2x+3}{x-1}\right)$ Links to 1.1, infinite geometric series; 2.5–2.7, rational and exponential functions, and asymptotes. Use of this definition for derivatives of simple polynomial functions only. Technology could be used to illustrate other derivatives. Link to 1.3, binomial theorem. Use of both forms of notation, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $f'(x)$, for the first derivative. Identifying intervals on which functions are increasing or decreasing. Use of both analytic approaches and technology. Technology can be used to explore graphs and their derivatives.	**Appl:** Economics 1.5 (marginal cost, marginal revenue, marginal profit). **Appl:** Chemistry 11.3.4 (interpreting the gradient of a curve). **Aim 8:** The debate over whether Newton or Leibnitz discovered certain calculus concepts. **TOK:** What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life? **TOK:** Opportunities for discussing hypothesis formation and testing, and then the formal proof can be tackled by comparing certain cases, through an investigative approach.

	Content	Further guidance	Links
6.2	Derivative of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$. Differentiation of a sum and a real multiple of these functions.		
	The chain rule for composite functions.	Link to 2.1, composition of functions.	
	The product and quotient rules.	Technology may be used to investigate the chain rule.	
	The second derivative.	Use of both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$.	
	Extension to higher derivatives.	$\frac{d^ny}{dx^n}$ and $f^{(n)}(x)$.	

	Content	Further guidance	Links
6.3	Local maximum and minimum points. Testing for maximum or minimum.	Using change of sign of the first derivative and using sign of the second derivative. Use of the terms "concave-up" for $f''(x) > 0$, and "concave-down" for $f''(x) < 0$.	**Appl:** profit, area, volume.
	Points of inflexion with zero and non-zero gradients.	At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.	
	Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''. Optimization.	Both "global" (for large $\lvert x \rvert$) and "local" behaviour. Technology can display the graph of a derivative without explicitly finding an expression for the derivative. Use of the first or second derivative test to justify maximum and/or minimum values.	
	Applications. **Not required:** points of inflexion where $f''(x)$ is not defined: for example, $y = x^{1/3}$ at $(0,0)$.	Examples include profit, area, volume. Link to 2.2, graphing functions.	

	Content	Further guidance	Links
6.4	Indefinite integration as anti-differentiation.		
	Indefinite integral of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x.	$\int \frac{1}{x} \mathrm{d}x = \ln x + C$, $x > 0$.	
	The composites of any of these with the linear function $ax + b$.	*Example*: $f'(x) = \cos(2x+3) \Rightarrow f(x) = \frac{1}{2}\sin(2x+3) + C$.	
	Integration by inspection, or substitution of the form $\int f(g(x))g'(x)\mathrm{d}x$.	*Examples*: $\int 2x\left(x^2+1\right)^4 \mathrm{d}x$, $\int x \sin x^2 \mathrm{d}x$, $\int \frac{\sin x}{\cos x} \mathrm{d}x$.	
6.5	Anti-differentiation with a boundary condition to determine the constant term.	*Example*: if $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + x$ and $y = 10$ when $x = 0$, then $y = x^3 + \frac{1}{2}x^2 + 10$.	**Int:** Successful calculation of the volume of the pyramidal frustum by ancient Egyptians (Egyptian Moscow papyrus). Use of infinitesimals by Greek geometers.
	Definite integrals, both analytically and using technology.	$\int_a^b g'(x)\mathrm{d}x = g(b) - g(a)$. The value of some definite integrals can only be found using technology.	Accurate calculation of the volume of a cylinder by Chinese mathematician Liu Hui
	Areas under curves (between the curve and the x-axis). Areas between curves. Volumes of revolution about the x-axis.	Students are expected to first write a correct expression before calculating the area. Technology may be used to enhance understanding of area and volume.	**Int:** Ibn Al Haytham: first mathematician to calculate the integral of a function, in order to find the volume of a paraboloid.
6.6	Kinematic problems involving displacement s, velocity v and acceleration a.	$v = \frac{\mathrm{d}s}{\mathrm{d}t}$; $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$.	**Appl:** Physics 2.1 (kinematics).
	Total distance travelled.	Total distance travelled $= \int_{t_1}^{t_2} \lvert v \rvert \, \mathrm{d}t$.	