Learning objectives

- Use the concept of pressure.
- Solve problems using the equation of state of an ideal gas.
- Understand the assumptions behind the kinetic model of an ideal gas.
- Solve problems using moles, molar masses and the Avogadro constant.
- Describe differences between ideal and real gases.

3.2 Modelling a gas

This section introduces the equation of state of an ideal gas, which is the equation that relates the pressure, volume, absolute temperature and number of moles of an ideal gas. The connection between the average random kinetic energy of the molecules and the kelvin temperature is derived.

The Avogadro constant

By definition, one **mole** of any substance contains as many **particles** as there are atoms in 12 g of carbon-12. What we mean by 'particle' depends on the substance; it can be a single atom or a molecule. For example, in carbon the particles are single **atoms**, the particles in hydrogen gas (H_2) are diatomic **molecules**, in carbon dioxide gas (CO_2) they are triatomic molecules, and in methane gas (CH_4) they are molecules with five atoms.

Experiments show that the number of particles in a mole is $N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$, a number known as the **Avogadro constant** and one of the basic constants of physics. So one mole of carbon, one mole of H_2, one mole of CO_2 and one mole of CH_4 all contain 6.02×10^{23} particles. This means 6.02×10^{23} atoms for carbon, $2 \times 6.02 \times 10^{23}$ atoms for H_2, $3 \times 6.02 \times 10^{23}$ atoms for CO_2 and $5 \times 6.02 \times 10^{23}$ atoms for CH_4. Figure **3.8** shows one mole of different substances.

Figure 3.8 One mole of different substances.

If a substance contains N particles (atoms or molecules, as discussed above) then the number of moles n is:

$$n = \frac{N}{N_A}$$

The atomic mass scale defines one **atomic mass unit** (1 u) as $\frac{1}{12}$ of the mass of one atom of carbon-12, ${}^{12}_{6}\text{C}$. The mass of one atom of ${}^{12}_{6}\text{C}$ is therefore exactly 12 u. The notation ${}^{12}_{6}\text{C}$ means that the carbon atom has six protons and the number of protons and neutrons combined is 12 (i.e. six neutrons). The neutral atom also has six electrons. Neglecting the mass of the six electrons, the mass of the six protons and six neutrons is about 12 u. The proton and the neutron are approximately equal in mass and so approximately the mass of one proton and that of one neutron is 1 u. So an atom of helium (${}^{4}_{2}\text{He}$) has a mass that is (approximately) 4 u and the mass of one atom of ${}^{56}_{26}\text{Fe}$ is (approximately) 56 u.

Now, remember that the mole is defined as the number of atoms in 12 g of ${}^{12}_{6}\text{C}$. We also defined the mass of one atom of ${}^{12}_{6}\text{C}$ to be 12 u. This means that:

$$\underset{\text{number of particles in 1 mol}}{N_A} \times \underset{\text{mass of 1 atom}}{12\text{u}} = \underset{\text{mass in g of 1 mol}}{12\,\text{g}}$$

and so the u (in grams) is given by:

$$\text{u} = \frac{1\,\text{g}}{N_A} \quad (\approx 1.66 \times 10^{-24}\,\text{g} \approx 1.66 \times 10^{-27}\,\text{kg})$$

We know that A grams of the element ${}^{A}_{Z}\mathrm{X}$ correspond to one mole of element X. So, for substances that are monatomic, one mole of a substance is also the quantity of the substance whose mass in grams is equal to the **atomic mass** (in u). Moving on to molecules, the **molar mass** is the sum of the atomic masses of the atoms making up the molecule. So CO_2 has molar mass $12+2\times16=44\,\mathrm{g\,mol^{-1}}$. There are N_A molecules in 44 g of CO_2 because 44 g of CO_2 make one mole.

So, it is important to know that:

> **One mole of a substance is a quantity of the substance that contains a number of particles equal to the Avogadro constant and whose mass in grams is equal to the molar mass of the substance.**

The number of moles in a quantity of m grams of a substance with molar mass μ is then $n=\frac{m}{\mu}$.

Worked examples

3.6 Estimate the number of atoms of gold in 1.0 kg of gold (${}^{197}_{79}\mathrm{Au}$).

The molar mass of gold is $197\,\mathrm{g\,mol^{-1}}$. So 1000 g of gold (= 1 kg) contains $\frac{1000}{197}\approx5.1$ mol of atoms.

Each mole contains 6.02×10^{23} atoms, so the number of atoms in 1 kg of gold is $6.02\times10^{23}\times5.1=3\times10^{24}$.

3.7 Calculate how many grams of scandium, ${}^{45}_{21}\mathrm{Sc}$, contain the same number of molecules as 8.0 g of argon, ${}^{40}_{18}\mathrm{Ar}$.

The molar mass of argon is $40\,\mathrm{g\,mol^{-1}}$, so a quantity of 8.0 g of argon corresponds to $\frac{8.0}{40}=0.20$ mol.

Thus, we need 0.20 mol of scandium. This corresponds to $0.20\times45=9.0$ g.

3.8 Estimate the number of water molecules in an ordinary glass of water.

A glass contains about $200\,\mathrm{cm^3}$ of water, which has a mass of 200 g.

Since the molar mass of water is $18\,\mathrm{g\,mol^{-1}}$, the glass contains $\frac{200}{18}\approx10$ mol or $6\times10^{23}\times10\approx10^{25}$ molecules of water.

Pressure

Pressure is defined as the normal force applied per unit area. In Figure **3.9a** the force is normal to the area A, so the pressure is:

$$p=\frac{F}{A}$$

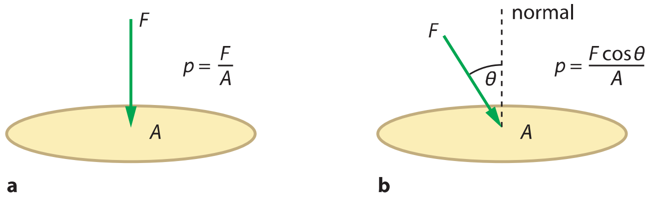


Figure 3.9 Pressure is the normal force per unit area.

The force in Figure **3.9b** acts at an angle θ, so the pressure on the area A is given by the expression:

$$p = \frac{F\cos\theta}{A}$$

The unit of pressure is newton per square metre, $\mathrm{N\,m^{-2}}$, also known as the pascal, Pa. Another commonly used non-SI unit is the **atmosphere**, atm, which is equal to 1.013×10^5 Pa.

Worked example

3.9 Two hollow cubes of side 25 cm with one face missing are placed together at the missing face (Figure **3.10**). The air inside the solid formed is pumped out. Determine the force that is necessary to separate the cubes.

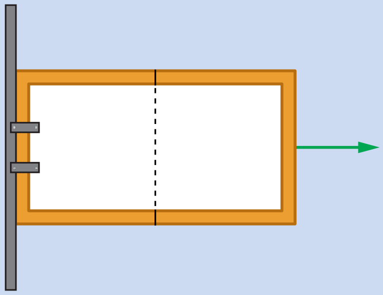


Figure 3.10

The pressure inside the solid is zero and outside it equals atmospheric pressure, 1.01×10^5 Pa.

Thus, the force is given by:

$$F = pA = 1.01 \times 10^5 \times (0.25)^2 = 6.3 \times 10^3\,\mathrm{N}$$

Exam tip
You must be able to recall and describe a few of these assumptions in an exam

Ideal gases

An **ideal gas** is a theoretical model of a gas. It helps us to understand the behaviour of real, actual gases. We assume that an ideal gas obeys the following:

- The molecules are point particles, each with negligible volume.
- The molecules obey the laws of mechanics.
- There are no forces between the molecules except when the molecules collide.
- The duration of a collision is negligible compared to the time between collisions.
- The collisions of the molecules with each other and with the container walls are elastic.
- Molecules have a range of speeds and move randomly.

An ideal gas (unlike real gases) cannot be liquefied or solidified. You should be able to see how some of these assumptions may not be obeyed by a **real gas**. For example, there will always be forces between molecules of a real gas, not just when the molecules are in contact. In general, we expect that a real gas will behave like an ideal gas when the density is low (so that molecules are not close to each other and hence the forces between them are negligible). We do not expect ideal gas behaviour at high densities (molecules will be too close to each other and will exert forces on each other). Similarly, we do not expect ideal gas behaviour from a real gas at very low temperature, because the gas will then become a liquid or even a solid!

> A real gas may be approximated by an ideal gas when the density is low.

Exam tip
You must be able to describe the conditions under which a real gas may be approximated by that of an ideal gas. The main idea is that the density must be low. For a fixed quantity of gas, density will be low at low pressure and high temperature.

Figure **3.11** shows a molecule that collides with a container wall. The momentum normal to the wall before the collision is $mv\cos\theta$. After the collision momentum normal to the wall is $-mv\cos\theta$. So the change in momentum has magnitude $2mv\cos\theta$. The fact that the momentum of the molecule has changed means that a force acted on the molecule (from the wall). By Newton's third law, therefore, the molecule exerted on the wall an equal and opposite force. Taking into account the forces due to all the molecules colliding with the walls results in a force, and hence pressure, on the walls.

Exam tip
You must be able to give an explanation of pressure in terms of molecules colliding with their container walls.

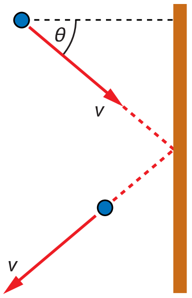


Figure 3.11 A molecule has its momentum changed when it collides with a wall. A force is exerted on the molecule and so the molecule exerts an equal and opposite force on the wall.

The **state of a gas** is determined when we know the values of the pressure, the volume, the temperature and the number of moles present. The parameters p, V, T and n are related to each other. The equation relating them is called the **equation of state**. Our objective is to discover the equation of state for a gas. To do this a number of simple experiments can be performed, as described in the following sections.

The pressure–volume law

The equipment shown in Figure **3.12** can be used to investigate the relationship between pressure and volume of a fixed quantity of gas that is kept at constant temperature.

The pump forces oil to move higher, decreasing the volume of the air trapped in the tube above the oil. A pressure gauge reads the pressure of the trapped air and so the relationship between pressure and volume may investigated. The changes in pressure and volume must take place slowly so that the temperature stays the same.

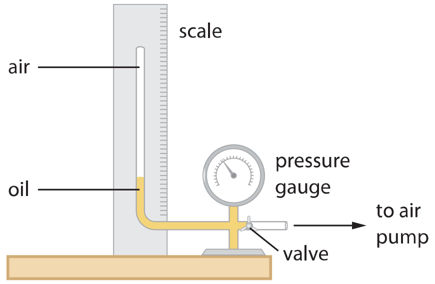


Figure 3.12 Apparatus for investigating the pressure–volume law. The pump forces oil to move up the tube, decreasing the volume of air.

The results of a typical experiment are shown in Figure **3.13**. We have plotted pressure against the inverse of the volume and obtained a straight line.

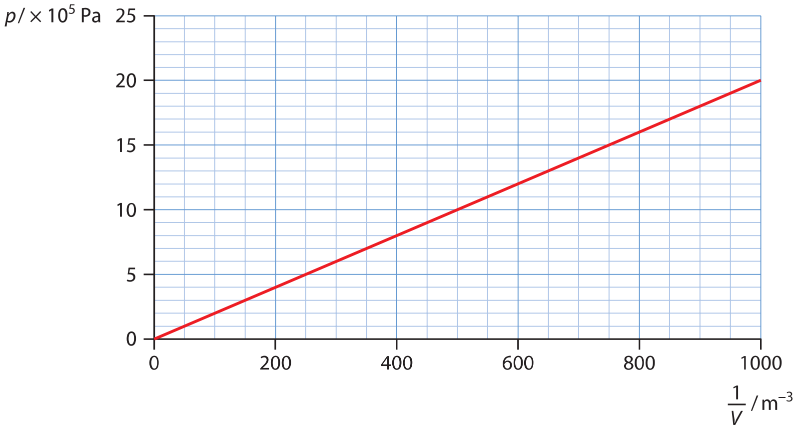


Figure 3.13 Graph of pressure against inverse volume at constant temperature.

Exam tip

In practice we use the relation pV = constant in the equivalent form $p_1V_1 = p_2V_2$ when the initial pressure and volume (p_1, V_1) change to a new pressure and volume (p_2, V_2) at constant temperature.

This implies that:

> At constant temperature and with a fixed quantity of gas, pressure is inversely proportional to volume, that is:
>
> $p \propto \frac{1}{V}$ or pV = constant

This relationship is known as the **Boyle's law**.

Figure 3.14 The relationship between pressure and volume at constant temperature for a fixed quantity of a gas. The product pV is the same for all points on the curve.

Exam tip

If you are asked to confirm the relationship pV = constant, take three points from a pressure–volume graph and show that their product is the same.

Figure **3.14** shows the same data now plotted as pressure against volume.

The curve in the pressure–volume diagram is a hyperbola and in physics it is known as an **isothermal** curve or **isotherm**: the temperature at all points on the curve is the same.

Worked example

3.10 The pressure of a fixed quantity of gas is 2.0 atm and its volume 0.90 dm^3. The pressure is increased to 6.0 atm at constant temperature. Determine the new volume.

Remember that 1 $dm^3 = 1000\ cm^3 = 1$ litre.

Use $p_1V_1 = p_2V_2$. Substituting the known values we have:

$$2.0 \times 0.90 = 6.0 \times V$$

$$\Rightarrow \quad V = 0.30$$

The new volume is 0.30 dm^3.

(Notice that since this problem compares the pressure at two different volumes we do not have to change units to SI units.)

The volume–temperature law

The dependence of volume on temperature of a fixed quantity of gas kept at constant pressure can be investigated with the apparatus shown in Figure **3.15**. Air is trapped in a thin capillary tube that is immersed in heated water. The air is trapped by a thin thread of very concentrated sulfuric acid. The thread is exposed to the atmosphere and so the pressure of the trapped air is constant.

It is found that the volume increases uniformly with temperature. The striking fact is that when the straight line is extended backwards it always crosses the temperature axis at −273 °C, as in Figure **3.16**. This suggests that there exists a minimum possible temperature, namely −273 °C. (With a real gas the experiment cannot be conducted at very low temperatures since the gas would liquefy – hence the dotted line. With an ideal gas there would be no such restriction.)

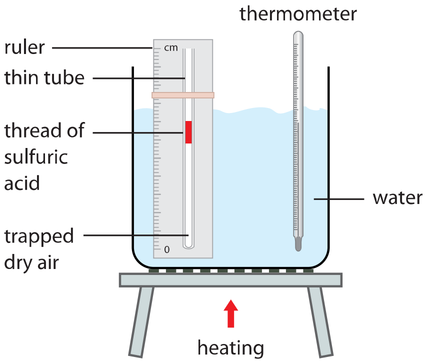


Figure 3.15 Apparatus for verifying the volume–temperature law.

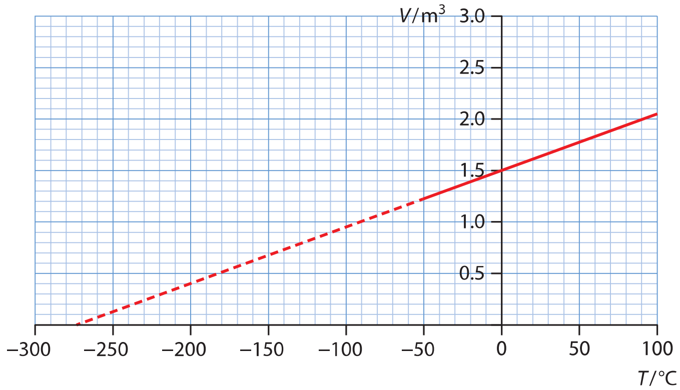


Figure 3.16 When the graph of volume versus temperature is extended backwards the line intersects the temperature axis at −273 °C.

If this same experiment is repeated with a different quantity of gas, or a gas at a different constant pressure, the result is the same. In each case, the straight-line graph of volume versus temperature crosses the temperature axis at −273 °C (Figure **3.17**). In Figure **3.18**, the same graphs are drawn using the Kelvin temperature scale.

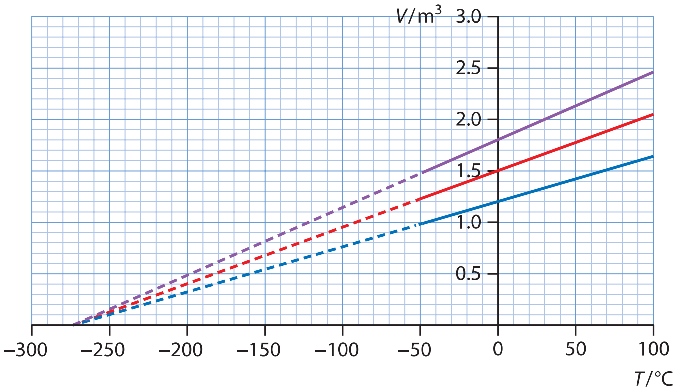


Figure 3.17 When the graph of volume versus temperature is extended backwards, all the lines intersect the temperature axis at the same point.

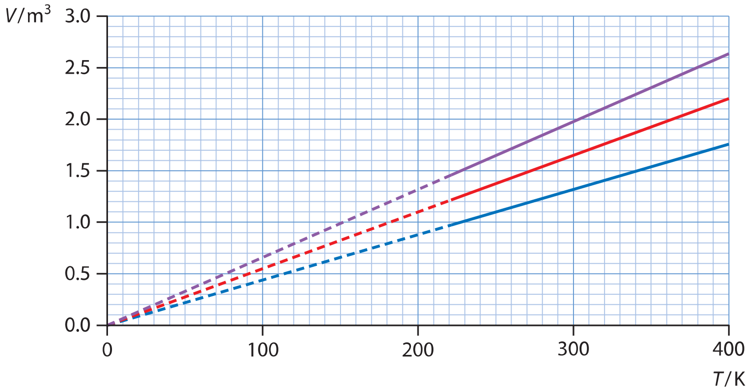


Figure 3.18 When temperature is expressed in kelvin, the lines start at zero temperature.

Exam tip

In practice we use the relation $\frac{V}{T}$ = constant in the equivalent form as $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ where the initial volume and temperature of the gas (V_1, T_1) change to a new volume and temperature (V_2, T_2) at constant pressure.

When the temperature is expressed in kelvin, this experiment implies that at constant pressure:

$$\frac{V}{T} = \text{constant}$$

This relationship is know as **Charles' law**.

Worked example

3.11 A gas expands at constant pressure from an original volume of 2.0 dm^3 at 22 °C to a volume of 4.0 dm^3. Calculate the new temperature.

Substituting in $\frac{V_2}{T_1} = \frac{V_2}{T_2}$ it follows that:

$$\frac{2.0}{295} = \frac{4.0}{T}$$

$\Rightarrow \quad T = 590\,\text{K}$ or $317\,^\circ\text{C}$

Note that we converted the original temperature into kelvin. (It is very easy to forget this conversion and get the incorrect answer of 44 °C.)

The pressure–temperature law

What remains now is to investigate the dependence of pressure on temperature of a fixed quantity of gas in a fixed volume. This can be done with the apparatus shown in Figure **3.19**. The gas container is surrounded by water whose temperature can be changed. A pressure gauge measures the pressure of the gas. We find that pressure increases uniformly with increasing temperature, as shown by the graph in Figure **3.20**.

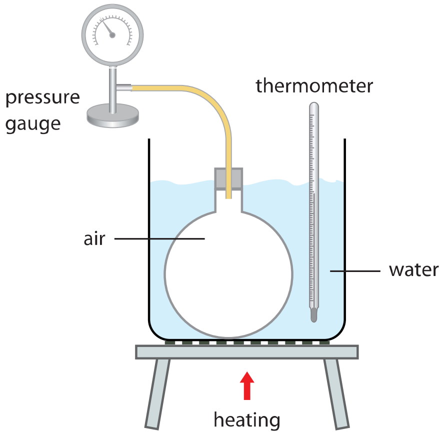


Figure 3.19 Investigating the pressure–temperature law.

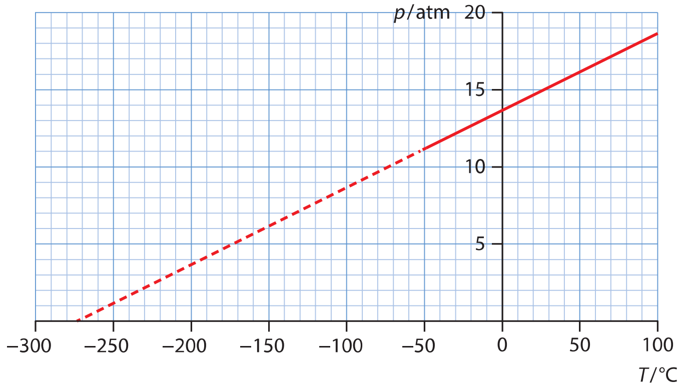


Figure 3.20 The graph of pressure versus temperature is a straight line that, when extended backwards, again intersects the temperature axis at −273 °C.

For quantities of gases containing different numbers of moles at different volumes the results are the same, as shown in Figure **3.21**. When the temperature is expressed in kelvin, the straight lines all pass through the origin (Figure **3.22**).

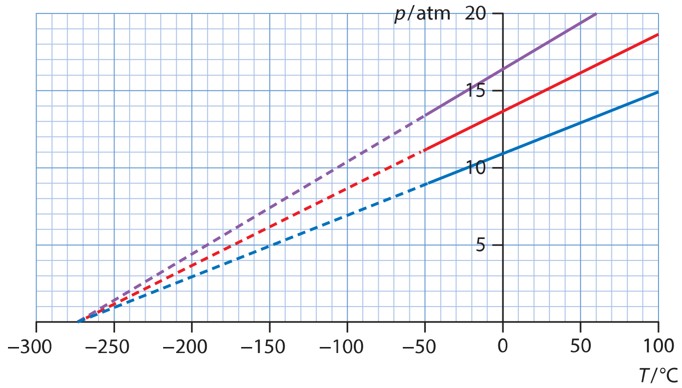


Figure 3.21 When extended backwards, the graphs of pressure versus temperature for three different quantities of gas all intersect the temperature axis at the same point.

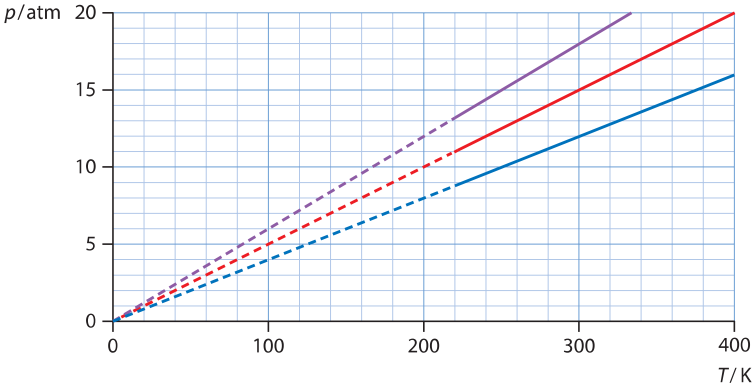


Figure 3.22 If temperature is expressed in kelvin, the lines start at zero temperature.

Exam tip

In practice we use the relation $\frac{p}{T}$ = constant in the equivalent form as $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ where the initial pressure and temperature of the gas (p_1, T_1) change to a new pressure and temperature (p_2, T_2) at constant volume. (Remember, T is in kelvin.)

> **When the temperature is expressed in kelvin, this experiment implies that at constant volume:**
>
> $$\frac{p}{T} = \text{constant}$$

This relationship is known as **Gay-Lussac's law** or **Amontons' law**.

Worked example

3.12 A gas in a container of fixed volume is heated from a temperature of 37 °C and pressure 3.0×10^5 Pa to a temperature of 87 °C. Calculate the new pressure.

Substituting in $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ we have:

$$\frac{3.0 \times 10^5}{310} = \frac{p}{360}$$

$$\Rightarrow \qquad p = 3.5 \times 10^5\,\text{Pa}$$

(Notice that we had to change the temperature into kelvin.)

The equation of state of an ideal gas

If we combine the results of the three preceding experiments, we find that:

$$\frac{pV}{T} = \text{constant}$$

Exam tip

In practice we use this in the form $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ when a gas changes from values (p_1, V_1, T_1) to (p_2, V_2, T_2). Cancel out any quantities that stay the same.

What is the value of the constant? To determine that, we repeat all of the preceding experiments, this time using different quantities of the gas. We discover that the constant in the last equation is proportional to the number of moles n of the gas in question:

$$\frac{pV}{T} = n \times \text{constant}$$

We can now measure the pressure, temperature, volume and number of moles for a large number of different gases and calculate the value of $\frac{pV}{nT}$. We find that this constant has the same value for all gases – it is a universal constant. We call this the gas constant R. It has the numerical value:

$$R = 8.31\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1}$$

Thus, finally, the equation of state is:

$$\boldsymbol{pV = RnT}$$

(Remember that temperature must always be in kelvin.)

Worked examples

3.13 Estimate how many molecules there are in a gas of temperature 320 K, volume 0.025 m^3 and pressure 4.8×10^5 Pa.

First we find the number of moles:

$$n = \frac{pV}{RT}$$

$$n = \frac{4.8 \times 10^5 \times 0.025}{8.31 \times 320} = 4.51\ \mathrm{mol}$$

Each mole contains the Avogadro number of molecules, so the number of molecules is:

$$4.51 \times 6.02 \times 10^{23} \approx 2.7 \times 10^{24}$$

3.14 A container of hydrogen of volume 0.10 m^3 and temperature 25 °C contains 3.2×10^{23} molecules. Calculate the pressure in the container.

The number of moles present is:

$$n = \frac{3.2 \times 10^{23}}{6.02 \times 10^{23}} = 0.53$$

So: $p = \dfrac{RnT}{V} = \dfrac{8.31 \times 0.53 \times 298}{0.10} = 1.3 \times 10^4\ \mathrm{Pa}$

3.15 A fixed quantity of gas of volume $3.0 \times 10^{-3}\ \mathrm{m}^3$, pressure 3.0×10^5 Pa and temperature 300 K expands to a volume of $4.0 \times 10^{-3}\ \mathrm{m}^3$ and a pressure of 6.0×10^5 Pa. Calculate the new temperature of the gas.

Use $\dfrac{p_1 V_1}{n_1 T_1} = \dfrac{p_2 V_2}{n_2 T_2}$ to get:

$$\frac{3.0 \times 10^5 \times 3.0 \times 10^{-3}}{300} = \frac{6.0 \times 10^5 \times 4.0 \times 10^{-3}}{T}$$

Solving for T gives: $T = 800\ \mathrm{K}$

3.16 Figure **3.23** shows two isothermal curves for equal quantities of two ideal gases. State and explain which gas is at the higher temperature.

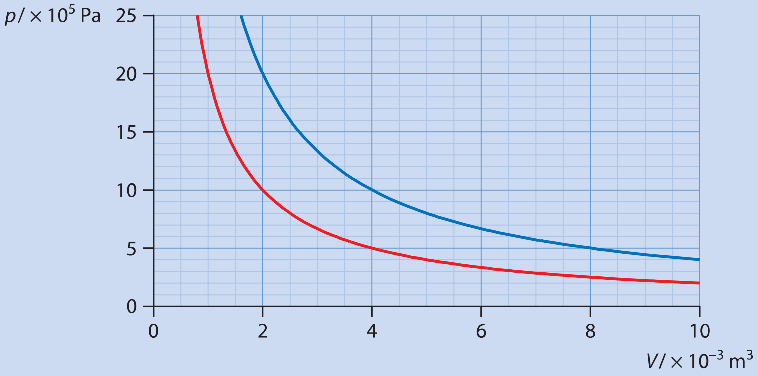


Figure 3.23 Two isothermal curves for equal quantities of two gases.

Draw a vertical line that intersects the two isotherms at two points. At these points both gases have the same volume, and as the quantities of gas are equal n is the same. So for these points $\frac{p}{T}$ is constant. The point on the blue curve has higher pressure, so it must have the higher temperature.

The Boltzmann equation

The molecules of a gas move about randomly with a range of speeds. The graph in Figure **3.24** shows the distribution of speeds for oxygen molecules kept at two different temperatures: the blue curve is at 100 K and the red curve at 300 K. The vertical axis shows the fraction of molecules having a given speed v. You will not be examined on this graph but knowing a few of its features helps a lot in understanding how gases behave.

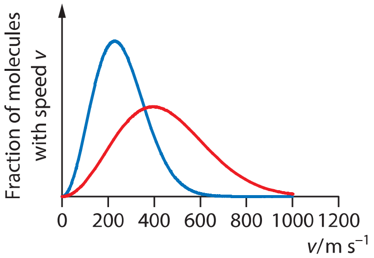


Figure 3.24 The distribution of speeds at two different temperatures.

We see that there is a speed that corresponds to the peak of the curve. For the blue curve this is about 225 m s^{-1} and for the red curve at 400 m s^{-1}. The speed at the peak represents the most probable speed that would be found if you picked a molecule at random. Two other speeds are important:

- the average speed of the molecules, $\bar{v} = \dfrac{v_1 + v_2 + v_3 + \cdots + v_N}{N}$
- the r.m.s. speed or root mean square speed c, which is the square root of the average of the squares of the speeds of the molecules, i.e.

$$c = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \cdots + v_N^2}{N}}$$

Why do we bother to work with an r.m.s. speed? Consider the *average kinetic energy* for the N molecules, which is given by:

$$\bar{E}_K = \frac{\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}v_3^2 + \cdots + mv_N^2}{N}$$

$$= \frac{1}{2}m\left(\frac{v_1^2 + v_2^2 + v_3^2 + \cdots + v_N^2}{N}\right)$$

$$= \frac{1}{2}mc^2$$

So we see that the average kinetic energy involves the r.m.s. speed. These three speeds (most probable, average and r.m.s. speed) are all different but numerically close to each other. So, even though it is not technically correct, we may assume that all three speeds mean the same thing and we will use the symbol c for all of them.

Now, it can be proven that the pressure of a gas is $p = \frac{1}{3}\rho c^2$, where the quantity c stands for the r.m.s. speed and ρ is the density of the gas. (You will not need to know this equation for the exam.) We get a very interesting result if we combine this equation with the equation of state for an ideal gas, i.e. the equation $pV = nRT$. There are many steps in the derivation in the box below. N stands for the number of molecules and m for the mass of one molecule.

Since $\frac{1}{2}mc^2$ is equal to E, the average random kinetic energy of the molecules, we can write:

$pV = nRT$	
$(\frac{1}{3}\rho c^2)V = nRT$	replacing the pressure with $p = \frac{1}{3}\rho c^2$
$\frac{1}{3}\frac{M}{Vc^{2V}} = nRT$	replacing the density by mass ÷ volume
$\frac{1}{3}Mc^2 = nRT$	cancelling the volume
$\frac{1}{3}Nmc^2 = \frac{N}{N_A}RT$	writing $M = Nm$ and $n = \frac{N}{N_A}$
$\frac{1}{2}Nmc^2 = \frac{3}{2}\frac{R}{N_A}T$	multiplying both sides by $\frac{3}{2}$

The product of all this algebra is the very important result that relates the average random kinetic energy to the absolute temperature.

$$k_B = \frac{R}{N_A} = \frac{8.31}{6.02 \times 10^{23}}$$
$$= 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$$

$$\bar{E}_K = \frac{3}{2}\frac{R}{N_A}T$$

The ratio $\frac{R}{N_A}$ is called the Boltzmann constant, k_B. So the final result is that the average random kinetic energy of the particles is directly proportional to the kelvin temperature:

$$\bar{E}_K = \frac{3}{2}k_B T$$

Using this equation we can find an expression for the internal energy of an ideal gas. Remember that the internal energy of an ideal gas consists only of the random kinetic energy of its molecules and no potential energy. Suppose that the gas has N molecules. Then, since the average kinetic energy is $\frac{3}{2}k_\mathrm{B}T$, the **total** random kinetic energy, i.e. the internal energy U, is:

$$U = \frac{3}{2}Nk_\mathrm{B}T$$

But recall that $k_\mathrm{B} = \frac{R}{N_\mathrm{A}}$, so that another expression is:

$$U = \frac{3}{2}nRT$$

Yet another expression comes from using the equation of state, $pV = nRT$, which gives:

$$U = \frac{3}{2}pV$$

Exam tip
You must be able to obtain an expression for the internal energy of an ideal gas even though this formula is not in the IB data booklet.

Worked examples

3.17 The kelvin temperature of a gas is doubled. By what factor does the average speed increase?

From $\frac{1}{2}mc^2 = \frac{3}{2}k_\mathrm{B}T$ we find that when T is doubled then c^2 will double, so c itself will increase by a factor of $\sqrt{2}$.

3.18 Calculate the ratio of the average speed of oxygen (O_2) to carbon dioxide (CO_2) molecules when both gases are at the same temperature.

Since the temperature is the same for both gases, using $\frac{1}{2}mc^2 = \frac{3}{2}k_\mathrm{B}T$ we find that:

$$\frac{1}{2}m_\mathrm{O}c_\mathrm{O}{}^2 = \frac{1}{2}m_{\mathrm{CO_2}}c_{\mathrm{CO_2}}{}^2 \quad \text{and so} \quad \frac{c_\mathrm{O}{}^2}{c_{\mathrm{CO_2}}{}^2} = \frac{m_{\mathrm{CO_2}}}{m_\mathrm{O}}$$

So we need to find the ratio of the masses of the molecules. One mole of oxygen has a mass of 32 g so one molecule has a mass (in grams) of $\frac{32}{N_\mathrm{A}}$. Similarly, the mass in grams of a carbon dioxide molecule is $\frac{44}{N_\mathrm{A}}$. So:

$$\frac{c_\mathrm{O}{}^2}{c_{\mathrm{CO_2}}{}^2} = \frac{44/N_\mathrm{A}}{32/N_\mathrm{A}} = \frac{44}{32} = 1.375 \Rightarrow \frac{c_\mathrm{O}}{c_{\mathrm{CO_2}}} = \sqrt{1.375} = 1.17 \approx 1.2$$

3.19 Calculate the average speed of helium (${}^4_2\mathrm{He}$) molecules at a temperature of −15 °C.

We use $\frac{1}{2}mc^2 = \frac{3}{2}k_\mathrm{B}T$. First we need to find the mass m of a helium atom. One mole of helium has a mass of 4.0 g so the mass of one molecule is given by:

$$m = \frac{4.0}{N_\mathrm{A}} = \frac{4.0}{6.02 \times 10^{23}} = 6.64 \times 10^{-24}\,\mathrm{g} = 6.64 \times 10^{-27}\,\mathrm{kg}$$

Now remember to convert the temperature into kelvin: 273 − 15 = 258 K. So we have:

$$\frac{1}{2} \times 6.64 \times 10^{-27} \times c^2 = \frac{3}{2} \times 1.38 \times 10^{-23} \times 258$$

This gives $c^2 = 1.61 \times 10^6$ and so $c = 1.3 \times 10^3\,\mathrm{m\,s^{-1}}$.

Nature of science

Models must be correct but also simple

Boyle thought that a gas consists of particles joined by springs. Newton thought that a gas consists of particles that exert repulsive forces on each other. Bernoulli thought that a gas is a collection of a very large number of particles that exert forces on each other only when they collide. All three could explain why a gas exerts a pressure on its container but it is Bernoulli's picture that is the simplest. We assume that the ordinary laws of mechanics apply to the individual particles making up the gas. Even though the laws apply to each individual particle we cannot observe or analyse each particle individually since there are so many of them. By concentrating on average behaviours of the whole gas and using probability and statistics, physicists developed a new field of physics known as statistical mechanics. This has had enormous success in advancing our understanding of gases and other systems, including where the approximation to an ideal gas breaks down.

Test yourself

13 Calculate the number of molecules in 28 g of hydrogen gas (molar mass $2\,\mathrm{g\,mol^{-1}}$).

14 Calculate the number of moles in 6.0 g of helium gas (molar mass $4\,\mathrm{g\,mol^{-1}}$).

15 Determine the number of moles in a sample of a gas that contains 2.0×10^{24} molecules.

16 Determine the mass in grams of carbon (molar mass $12\,\mathrm{g\,mol^{-1}}$) that contains as many molecules as 21 g of krypton (molar mass $84\,\mathrm{g\,mol^{-1}}$).

17 A sealed bottle contains air at 22.0 °C and a pressure of 12.0×10^5 Pa. The temperature is raised to 120.0 °C. Calculate the new pressure.

18 A gas has pressure 8.2×10^6 Pa and volume $2.3 \times 10^{-3}\,\mathrm{m^3}$. The pressure is reduced to 4.5×10^6 Pa at constant temperature. Calculate the new volume of the gas.

19 A mass of 12.0 kg of helium is required to fill a bottle of volume $5.00 \times 10^{-3}\,\mathrm{m^3}$ at a temperature of 20.0 °C. Determine the pressure in helium.

20 Determine the mass of carbon dioxide required to fill a tank of volume $12.0 \times 10^{-3}\,\mathrm{m^3}$ at a temperature of 20.0 °C and a pressure of 4.00 atm.

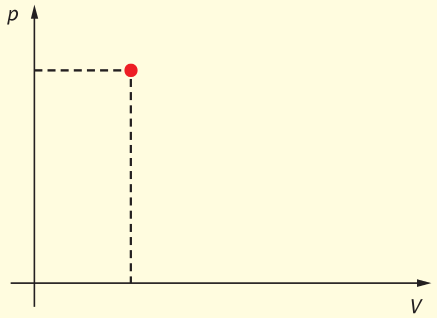
21 A flask of volume $300.0 \times 10^{-6}\,\mathrm{m^3}$ contains air at a pressure of 5.00×10^5 Pa and a temperature of 27.0 °C. The flask loses molecules at a rate of 3.00×10^{19} per second. Estimate how long it takes for the pressure in the flask to fall to half its original value. (Assume that the temperature of the air remains constant during this time.)

22 The point marked in the diagram represents the state of a fixed quantity of ideal gas in a container with a movable piston. The temperature of the gas in the state shown is 600 K. Copy the diagram. Indicate on the diagram the point representing the new state of the gas after the following separate changes.

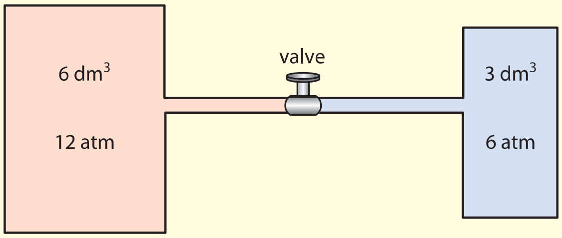
a The volume doubles at constant temperature.

b The volume doubles at constant pressure.

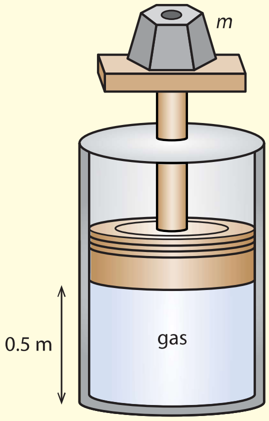
c The pressure halves at constant volume.



23 Two ideal gases are kept at the same temperature in two containers separated by a valve, as shown in the diagram. Estimate the pressure when the valve is opened. (The temperature stays the same.)



24 The diagram shows a cylinder in a vacuum, which has a movable, frictionless piston at the top. An ideal gas is kept in the cylinder. The piston is at a distance of 0.500 m from the bottom of the cylinder and the volume of the cylinder is 0.050 m^3. The weight on top of the cylinder has a mass of 10.0 kg. The temperature of the gas is 19.0 °C.



a Calculate the pressure of the gas.

b Determine how many molecules there are in the gas.

c The temperature is increased to 152 °C. Calculate the new volume of the gas.

25 The molar mass of a gas is 28 g mol^{-1}. A container holds 2.00 mol of this gas at 0.00 °C and a pressure of 1.00×10^5 Pa. Determine the mass and volume of the gas.

26 A balloon has a volume of 404 m^3 and is filled with helium of mass 70.0 kg. The temperature inside the balloon is 17.0 °C. Determine the pressure inside the balloon.

27 A flask has a volume of 5.0×10^{-4} m^3 and contains air at a temperature of 300 K and a pressure of 150 kPa.

a Calculate the number of moles of air in the flask.

b Determine the number of molecules in the flask.

c Estimate the mass of air in the flask. You may take the molar mass of air to be 29 g mol^{-1}.

28 The molar mass of helium is 4.00 g mol^{-1}.

a Calculate the volume of 1.0 mol of helium at standard temperature and pressure (stp) i.e. at $T = 273$ K, $p = 1.0 \times 10^5$ Pa.

b Determine the density of helium at stp.

c Estimate the density of oxygen gas at stp (the molar mass of, oxygen gas is 32 g mol^{-1}).

29 The density of an ideal gas is 1.35 kg m^{-3}. The temperature in kelvin and the pressure are both doubled. Calculate the new density of the gas.

30 Calculate the average speed (r.m.s.) of helium atoms at a temperature of 850 K. The molar mass of helium is 4.0 g mol^{-1}.

31 Show that the average (r.m.s.) speed of molecules of a gas of molar mass M (in kg mol^{-1}) kept at a temperature T is given by $c = \sqrt{\frac{3RT}{M}}$.

32 **a** Calculate the average random kinetic energy of a gas kept at a temperature of 300 K.

b Determine the ratio of the average speeds (r.m.s. speeds) of two ideal gases of molar mass 4.0 g mol^{-1} and 32 g mol^{-1}, which are kept at the same temperature.