sequences & series - 1

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4 questions – progressing from 'accessible' to 'discriminating'

1. How many terms are in the following arithmetic sequence? [*calculator allowed*]

$$-9, -2, 5, \ldots, 103$$

2. The sum of the first three terms of an arithmetic sequence is 6 and the fourth term is 16. Find the first term, u_1, and the common difference, d, of the sequence. [***no*** *calculator*]

3. a, 1, b are three consecutive terms of an arithmetic series, and b, a, $\frac{8}{3}$ are the first three terms of an infinite geometric series that has a sum of S. Find a, b and S.

[*calculator allowed*]

4. Show that the series $\log_2 x + \log_4 x + \log_{16} x + \cdots$ is geometric and find the sum of the series for infinite terms. [***no*** *calculator*]

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Answers

1. 17 terms

2. $u_1 = -5,\ d = 7$

3. $a = -4,\ b = 6,\ S = \frac{18}{5}$

4. $\frac{\log_4 x}{\log_2 x} = \frac{\frac{\log_2 x}{\log_2 4}}{\frac{\log_2 x}{\log_2 2}} = \frac{\frac{\log_2 x}{2}}{\frac{\log_2 x}{1}} = \frac{1}{2}$ and $\frac{\log_{16} x}{\log_4 x} = \frac{\frac{\log_2 x}{\log_2 16}}{\frac{\log_2 x}{\log_2 4}} = \frac{\frac{\log_2 x}{4}}{\frac{\log_2 x}{2}} = \frac{1}{2} \Rightarrow$ series is geometric

$S_\infty = 2\log_2 x \quad \left[\text{or} \quad \log_2\left(x^2\right)\right]$