Answers to test yourself questions

Option B

B1 Rotational dynamics

1 Use $\theta = \dfrac{(\omega+\omega_0)t}{2}$ to get $\theta = \dfrac{(15+3.5)\times 5.0}{2} = 46.25 \approx 46$ rad.

2 Use $\omega^2 = \omega_0^2 + 2\alpha\theta$ to get $\omega^2 = 5.0^2 + 2\times 2.5\times 54$ and so $\omega = 17.18 \approx 17\ \text{rad s}^{-1}$.

3 Use $\omega^2 = \omega_0^2 + 2\alpha\theta$ to get $12.4^2 = 3.2^2 + 2\times\alpha\times 20\times 2\pi$. Hence $\alpha = 0.571 \approx 0.57\ \text{rad s}^{-2}$.

4 Let the forces at each support be L (for left) and R (for right). These are vertically upwards. Then

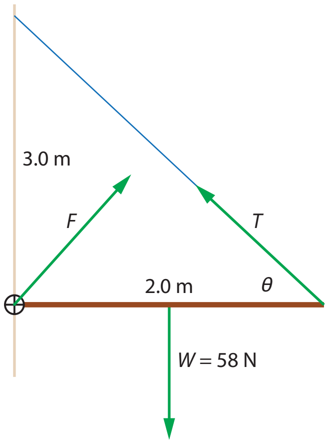
Translational equilibrium: $L + R = 450 + 120 = 570$ N

Rotational equilibrium: $120 \quad 2.0 + 450 \quad 2.5 = R \quad 5.0$ by taking torques about the support at the left. Then $R = 273 \approx 270$ N and hence $L = 297 \approx 300$ N.

5 Let the forces at each support be L (for left) and R (for right). These are vertically upwards. Then just before the rod tips over we have equilibrium and the left support force tends to become zero since the rod will no longer touch the support. Hence by taking torques about the right support we find:

$30 \quad 9.8 \quad 0.80 = 40 \quad 9.8x$ and $x = 0.60$ m.

6 **a** The forces are as shown in the diagram below:



Translational equilibrium demands that:

$T_x = F_x$

$T_y + F_y = 58$

Rotational equilibrium demands that (we take torques about an axis through the point of support at the wall):

$58\times 1.0 = T_y \times 2.0 \Rightarrow T_y = 29$ N.

Now $T_y = T\sin\theta$ and from the diagram, $\tan\theta = \dfrac{3.0}{2.0} = 1.5$. Hence $\theta = 56.31^\circ$. Thus

$T = \dfrac{T_y}{\sin\theta} = \dfrac{29}{\sin 56.31^\circ} = 34.854 \approx 35$ N.

b Since $T_y = 29$ N, $F_y = 58 - 29 = 29$ N also. Finally, $F_x = T_x = 34.854 \quad \cos 56.31^\circ = 19.33$ N. Hence the wall force is $F = \sqrt{F_x^2 + F_y^2} = \sqrt{19.33^2 + 29^2} = 34.854 \approx 35$ N. The angle it makes with the horizontal is $\tan^{-1}\dfrac{29}{19.33} = 56.31^\circ \approx 56^\circ$.

7 The torque provided by the force about the axis of rotation is $\Gamma = FR$. We know that $\Gamma = I\alpha$.

The moment of inertia of the cylinder is $I = \frac{1}{2}MR^2$ and so $\alpha = \frac{\Gamma}{I} = \frac{2F}{MR} = \frac{2\times 6.5}{5.0\times 0.20} = 13\ \text{rad s}^{-2}$.

Therefore $\omega = \omega_0 + \alpha t = 0 + 13\times 5.0 = 65\ \text{rad s}^{-1}$.

8 Let h be the height form which the bodies are released.

For the point particle will have: $Mgh = \frac{1}{2}Mv^2 \Rightarrow v = \sqrt{2gh}$.

For the others,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\frac{v^2}{R^2}$$

$$2gh = v^2\left(1 + \frac{I}{MR^2}\right)$$

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$$

So

Sphere: $$v = \sqrt{\frac{2gh}{1 + \frac{2}{5}\frac{MR^2}{MR^2}}} = \sqrt{\frac{2gh}{\frac{7}{5}}} = \sqrt{2gh}\quad\sqrt{\frac{5}{7}}$$

Cylinder: $$v = \sqrt{\frac{2gh}{1 + \frac{1}{2}\frac{MR^2}{MR^2}}} = \sqrt{\frac{2gh}{\frac{3}{2}}} = \sqrt{2gh}\quad\sqrt{\frac{2}{3}}$$

Ring: $$v = \sqrt{\frac{2gh}{1 + \frac{MR^2}{MR^2}}} = \sqrt{\frac{2gh}{2}} = \sqrt{2gh}\quad\sqrt{\frac{1}{2}}$$

Hence $v_{\text{ring}} < v_{\text{cylinder}} < v_{\text{sphere}} < v_{\text{point}}$

9 **a** One revolution corresponds to 2π radians and so the disc is making

$$\omega = \frac{45}{2\pi}\frac{\text{rev}}{\text{s}} = \frac{45}{2\pi}\frac{\text{rev}}{\frac{1}{60}\text{min}} = \frac{45\times 60}{2\pi} = 429.7 \approx 430\ \text{rpm}.$$

b The angular acceleration has to be $\alpha = \frac{45}{4.0} = 11.25\ \text{rad s}^{-2}$. From

$$\Gamma = I\alpha$$

$$FR = \frac{1}{2}MR^2\alpha$$

we deduce that $F = \frac{1}{2}MR\alpha = \frac{1}{2}\times 12\times 0.35\times 11.25 = 23.6 \approx 24\ \text{N}$.

c From $\omega^2 = \omega_0^2 + 2\alpha\theta$ we find $0 = 45^2 + 2\times(-11.25)\theta$ and so $\theta = 90$ rad. This corresponds to $\frac{90}{2\pi} = 14.3 \approx 14$ revolutions.

10 **a** $\Gamma = I\alpha \Rightarrow MgR = \frac{7}{5}MR^2\alpha$. Thus $\alpha = \frac{5g}{7R}$.

b It is not: the torque of the weight about the axis is decreasing because the perpendicular distance between the weight and the axis is decreasing.

c Conservation of energy: $MgR = \frac{1}{2}I\omega^2$. Hence $MgR = \frac{1}{2}\times\frac{7}{5}MR^2\omega^2$ and so $\omega = \sqrt{\frac{10g}{7R}}$.

11 **a** $\Gamma = I\alpha \Rightarrow Mg\frac{L}{2} = \frac{1}{3}ML^2\alpha$. Thus $\alpha = \frac{3g}{2L} = \frac{3}{2}\times\frac{9.81}{1.20} = 12.26 \approx 12.3\ \text{rad s}^{-2}$.

b It is not: the torque of the weight about the axis is decreasing because the perpendicular distance between the weight and the axis is decreasing.

c Conservation of energy: $Mg\frac{L}{2} = \frac{1}{2}I\omega^2$. Hence $MgL = \frac{1}{3}ML^2\omega^2$ and $\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3\times 9.81}{1.20}} = 4.95\ \text{rad s}^{-1}$.

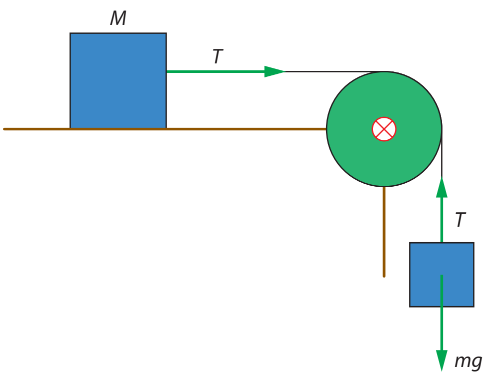
12 From Newton's second law: $F = Ma$ and $Fd = I\alpha = \frac{1}{2}MR^2\alpha$. Combing these two equations gives

$$Mad = \frac{1}{2}MR^2\alpha$$

$$M\alpha Rd = \frac{1}{2}MR^2\alpha$$

$$d = \frac{R}{2}$$

13 **a** The relevant forces are as shown.



Therefore:

$$mg - T = ma$$
$$T = Ma$$

Hence, adding side by side, $mg = Ma + ma$ and so $a = \frac{mg}{M+m}$.

b Using the hint we now have:

$$\begin{aligned} mg - T_1 &= ma \\ T_2 &= Ma \end{aligned} \quad \text{and } (T_1 - T_2)R = I\alpha.$$

Assuming no slipping at the pulley, $\alpha = \frac{a}{R}$ and so $(T_1 - T_2) = \frac{Ia}{R^2}$. Adding the first two equations gives

$mg - (T_1 - T_2) = (m+M)a$ or $mg - \frac{Ia}{R^2} = (m+M)a$ i.e. $mg - \frac{1}{2}Ma = (m+M)a$ so that, finally, $a = \frac{mg}{m+\frac{3}{2}M}$.

14 **a** The kinetic energy initially is $E_K = \frac{1}{2}I\omega^2 = \frac{1}{2}\times 280\times\left(\frac{320\times 2\pi}{60}\right)^2 = 1.57\times 10^5 \approx 1.6\times 10^5$ J. This is also the work needed to stop the disc.

b The power developed is $P = \frac{E}{t} = \frac{1.57\times 10^5}{12} = 1.31\times 10^4 \approx 1.3\times 10^4$ W.

15 X: $E_K = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{1}{12}ML^2\omega^2 = \frac{1}{24}\times 2.40\times 1.20^2\times 4.50^2 = 2.916 \approx 2.92\ \text{J}$;

$$L = I\omega = \frac{1}{12}ML^2\omega = \frac{1}{12}\times 2.40\times 1.20^2\times 4.50 = 1.296 \approx 1.30\ \text{J s}.$$

Y: $E_K = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{1}{3}ML^2\omega^2 = 1.17\ \text{J}$; $L = I\omega = \frac{1}{3}ML^2\omega = 5.18\ \text{J s}$.

16 There are no external torques so angular momentum is conserved: $I_1\omega_1 = I_2\omega_2$.

$$\frac{2}{5}MR^2\omega_1 = \frac{2}{5}\frac{M}{10}\left(\frac{R}{50}\right)^2\omega_2 \text{ leading to } \frac{\omega_2}{\omega_1} = 2.5\times 10^4.$$

17 The angular momentum before and after the ring begins to move must be the same. Before it moves, the angular momentum is zero. After it begins to move it is: $L = I\omega + mvR$.

Hence $I\omega + mvR = 0$, i.e. $\omega = -\frac{mvR}{I} = -\frac{0.18\times 0.50\times 0.80}{0.20} = -0.36\ \text{rad s}^{-1}$. (The minus sign indicates that the ring rotates in the opposite direction to that of the car.)

B2 Thermodynamics

18 Use $pV^{\frac{5}{3}} = c$ to get $8.1\times 10^5\times(2.5\times 10^{-3})^{\frac{5}{3}} = p\times(4.6\times 10^{-3})^{\frac{5}{3}}$ i.e.

$$p = 8.1\times 10^5\times\left(\frac{2.5\times 10^{-3}}{4.6\times 10^{-3}}\right)^{\frac{5}{3}}$$

$$p = 2.9\times 10^5\ \text{Pa}$$

19 We know that $pV^{\frac{5}{3}} = c$ and also $pV = nRT$. From the ideal gas law we find $p = \frac{nRT}{V}$ and substituting in the first equation we find $\frac{nRT}{V}V^{\frac{5}{3}} = c$ or $TV^{\frac{2}{3}} = c'$ (another constant). Hence,

$560\times(2.8\times 10^{-3})^{\frac{2}{3}} = T\times(4.8\times 10^{-3})^{\frac{2}{3}}$. Hence

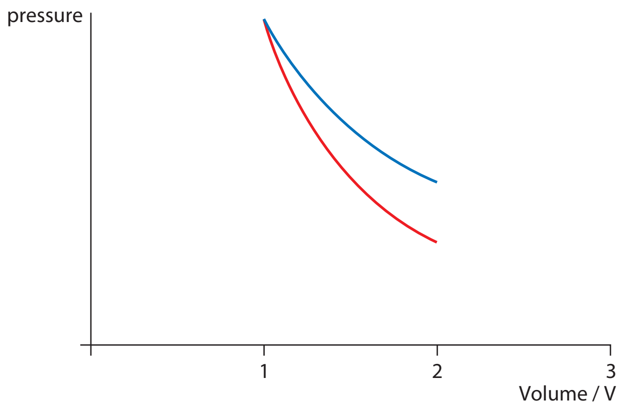
$$T = 560\times\left(\frac{2.8\times 10^{-3}}{4.8\times 10^{-3}}\right)^{\frac{2}{3}}$$

$$T = 391 \approx 390\ \text{K}$$

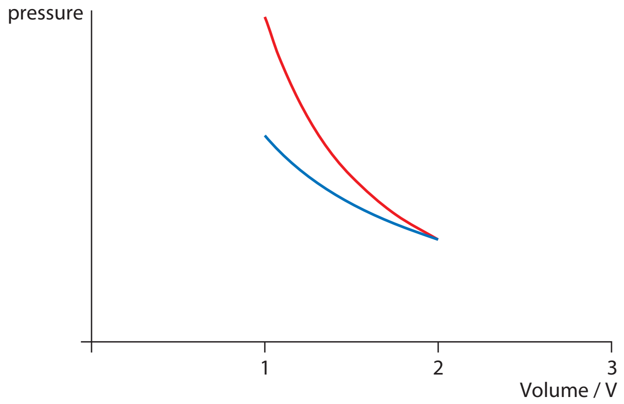
20 $W = p\Delta V = 4.0\times 10^5\times(4.3-3.6)\times 10^{-3} = 280\ \text{J}$.

21 **a** The process is isothermal and so $\Delta U = 0$. Hence $Q = 0 + W = 0 - 6500 = -6500\ \text{J}$.
b The adiabatic compression is steeper than the isotheermal and so there is more area under the graph and so more work.

22 Isothermal is the blue curve and the red is adiabatic. The area under the isothermal curve is larger and so the work done is larger.



23 Isothermal is the blue curve and the red is adiabatic. The area under theadiabatic curve is larger and so the work done is larger.



24

	W	ΔU	ΔT
X	positive	zero	zero
Y	zero	positive	positive
Z	positive	positive	positive

25 **a** From $Q = \Delta U + W$ and $Q = 0$ we find $\Delta U = -W$. The gas is compressed and so $W < 0$. Hence $\Delta U > 0$. In an ideal gas the internal energy is proportional to temperature (in kelvin) and so increasing U means increasing T.

b The gas is compressed by a pisotn that is rapidly moved in. The piston collides with molecules and gives kinetic energy to them increasing the average random kinetic energy of the molecules. Hence the temperature increases since temperature is proportional to the average random kinetic energy.

26 **a** Use $W = p\Delta V = 6.00 \times 10^6 \times (0.600 - 0.200) = 2.4$ MJ.

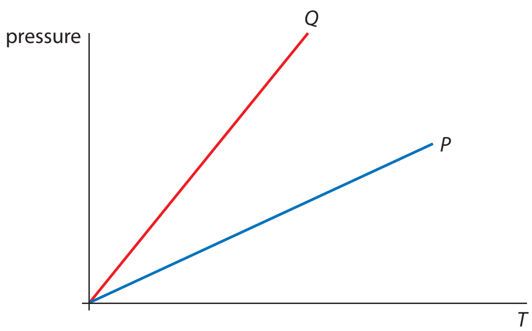
b From the gas law, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ and so $\frac{0.200}{300} = \frac{0.600}{T_2}$ giving $T_2 = 900$ K.

c The change in internal energy is $\Delta U = \frac{3}{2}Rn\Delta T = \frac{3}{2}\frac{pV}{T}\Delta T = \frac{3}{2} \times \frac{6.00 \times 10^6 \times 0.200}{300} = 3.6$ MJ.

Therefore $Q = \Delta U + W = 3.6 + 2.4 = 6.0$ MJ

27 From $Q = \Delta U + W$ we get $\Delta U = Q - W$. So we can have $Q > 0$ but as long as $Q - W < 0$, i.e. when the work done is greater than the heat supplied the temperature will actually drop.

28 **a** We must first determine if the gas is expanding or whether it is being compressed. Since $pV = nRT$ it follows that $p = \frac{nRT}{V}$. For *constant volume*, the graph of pressure versus temperature would be a straight line through the origin. This is not what we have so the volume is changing. The graph below shows two straight lines along each of which the volume is constant. Since $p = \frac{nRT}{V}$, the red line having a bigger slope corresponds to the smaller volume. Hence the volume from P to Q decreases and so $W < 0$.



b From $Q = \Delta U + W$ we see that $\Delta U < 0$ since the temperature is decreasing and since the gas is being compressed, we also have that $W < 0$. Hence $Q < 0$ and thermal energy is being removed from the gas.

29 For the constant volume case we have $Q = \Delta U + W = \Delta U + 0$. For the constant pressure case we have that $Q = \Delta U' + W$. Since the heats are equal $\Delta U = \Delta U' + W$ which shows that $\Delta U > \Delta U'$ since $W > 0$. The internal energy for an ideal gas is a function only of temperature and so the temperature increase is greater for the constant volume case.

30 Working as in the previous problem, $Q_V = \Delta U$ and $Q_P = \Delta U + W$. The two changes in internal energy are the same because the temperature differences are the same. Since $W > 0$, $Q_P > Q_V$.

31 a Steeper curve starting at 1.

b Larger area under the adiabatic so more work done.

c i $Q = \Delta U + W$ with $\Delta U = 0$ and $W = -25$ kJ so $Q = -25$ kJ. Hence the change in entropy for the gas is

$$\Delta S = \frac{Q}{T} = -\frac{25 \times 10^3}{300} = -83.3 \approx -83 \text{ J K}^{-1}$$

ii The surroundings receive heat 25 kJ and so their entropy increases by $+\frac{25 \times 10^3}{300} = +83.3 \approx +83 \text{ J K}^{-1}$.

d The entropy change for the universe is zero. This is possible only for idealised reversible processes and an isothermal compression is such a process.

32 a An adiabatic curve is a curve on a pressure–volume graph represents a process in which no heat enters or leaves a system.

b BC is an isobaric process and DA is isovolumetric.

c Along BC the temperature increases and so $\Delta U > 0$. The gas expands and so does work and hence $W > 0$. Therefore, from the first law, $Q = \Delta U + W > 0$ and so heat is given to the gas. Along DA the temperature drops and so $\Delta U < 0$; no work is done and so $Q < 0$. CD and AB are adiabatics and so $Q = 0$. So heat is supplied only along BC.

d i Heat is taken out along DA: the temperature at D is

$$T = \frac{pV}{nR} = \frac{4.0 \times 10^6 \times 8.6 \times 10^{-3}}{0.25 \times 8.31} = 1.66 \times 10^4 \approx 1.7 \times 10^4 \text{ K}$$ and that at A is

$$T = 1.4 \times \frac{1.66 \times 10^4}{4.0} = 5.80 \times 10^3 \approx 5.8 \times 10^3 \text{ K}.$$ Therefore

$$Q = \Delta U + W = \Delta U + 0 = \frac{3}{2} nR\Delta T = \frac{3}{2} \times 0.25 \times 8.31 \times (0.580 - 1.66) \times 10^4 = -3.37 \times 10^4 \approx -3.4 \times 10^4 \text{ J}.$$

ii The efficiency is $e = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}}$. Hence $0.36 = \frac{Q_{in} - 3.37 \times 10^4}{Q_{in}}$. This gives

$$Q_{in} = \frac{3.37 \times 10^4}{0.64} = 5.27 \times 10^4 \approx 5.3 \times 10^4 \text{ J}.$$

iii The area of the loop is the net work done which equals
$W = Q_{in} - Q_{out} = 5.27 \times 10^4 - 3.37 \times 10^4 = 1.90 \times 10^4 \approx 1.9 \times 10^4$ J.

33 Let a quantity of heat Q_1 be supplied to an ideal gas at constant volume: then $Q_1 = nc_V \Delta T$. Similarly, supply a quantity of heat Q_2 to another ideal gas of the same mass at constant pressure so that the change in temperature will be the same.: $Q_2 = nc_P \Delta T$. Q_1 and Q_2 will be different temperature because in the second case some of the heat will go into doing work as the gas expands at constant pressure. From the first law of thermodynamics we have that $Q_1 = nc_V \Delta T = \Delta U + 0$ and $Q_2 = nc_P \Delta T = \Delta U + p\Delta V$. Subtracting these two equations gives $nc_P \Delta T - nc_V \Delta T = p\Delta V$. From the ideal gas law we find $p\Delta V = nR\Delta T$ and so $nc_P \Delta T - nc_V \Delta T = nR\Delta T$ or $c_P - c_V = R$ as required.

34 **a** A and B have the same pressure and so $\dfrac{V_1}{T_1}=\dfrac{V_2}{T_2}$ i.e. $\dfrac{0.1}{800}=\dfrac{0.4}{T_2}$ hence $T_2 = 3200$ K. B and C have the same volume hence $\dfrac{P_1}{T_1}=\dfrac{P_2}{T_2}$ i.e. $\dfrac{4}{3200}=\dfrac{2}{T_2}$ hence $T_2 = 1600$ K. C and D have the same pressure and so $\dfrac{V_1}{T_1}=\dfrac{V_2}{T_2}$ i.e. $\dfrac{0.4}{1600}=\dfrac{0.1}{T_2}$ hence $T_2 = 400$ K.

b A to B: $\Delta U = \dfrac{3}{2}Rn\Delta T = \dfrac{3}{2}\dfrac{pV}{T}\Delta T = \dfrac{3}{2}\times\dfrac{4.0\times10^5\times0.10}{800}\times(3200-800) = 180$ kJ

B to C: $\Delta U = \dfrac{3}{2}Rn\Delta T = \dfrac{3}{2}\dfrac{pV}{T}\Delta T = \dfrac{3}{2}\times\dfrac{4.0\times10^5\times0.40}{3200}\times(1600-3200) = -120$ kJ

C to D: $\Delta U = \dfrac{3}{2}Rn\Delta T = \dfrac{3}{2}\dfrac{pV}{T}\Delta T = \dfrac{3}{2}\times\dfrac{2.0\times10^5\times0.40}{1600}\times(400-1600) = -90$ kJ

D to A: $\Delta U = \dfrac{3}{2}Rn\Delta T = \dfrac{3}{2}\dfrac{pV}{T}\Delta T = \dfrac{3}{2}\times\dfrac{2.0\times10^5\times0.10}{400}\times(800-400) = +30$ kJ

c From A to B: $Q = \Delta U + W$. $W = +p\Delta V = 4.0\times10^5\times0.3 = 120$ kJ. Hence $Q = 180 + 120 = 300$ kJ. From B to C: $W = 0$ so $Q = -120$ kJ. From C to D: $W = -p\Delta V = 2.0\times10^5\times0.3 = -60$ kJ. Hence $Q = -90 - 60 = -150$ kJ. Finally from D to A: $\Delta U = 30$ kJ and $W = 0$ so that $Q = +30$ kJ.

d The net work is the area of the loop which is 2.0 10^5 0.3 = 60 kJ. (As a check this must also be the net heat into the system i.e. $+300 - 120 - 150 + 30 = 60$ kJ.) The heat in is 300 kJ and so the efficiency is $\dfrac{60}{300} = 0.20$.

35 **a** The ice is receiving heat; the molecules are moving faster about their equilibrium positions. The disorder increases and so the entropy of the ice molecules increases.

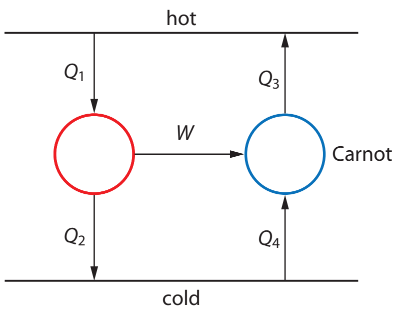
b During melting heat is provided to the ice molecules and so their entropy is again increasing.

c The water must have had a temperature greater than +15 °C. Therefore as the temperature drops to +15 °C the entropy of the water is decreasing. The overall change in entropy of the system must be positive.

36 The maximum possible efficiency for an engine working with these temperatures is, according to Carnot, $e = 1 - \dfrac{300}{500} = 0.40$. So the proposed engine is impossible.

37 **a** It is impossible for heat to flow from a cold to a warmer place without performing work.

b Imagine the work output of the engine to be the input work in a Carnot refrigerator.



Since the red engine has better efficiency than Carnot we must have that $\dfrac{W}{Q_1} > \dfrac{W}{Q_3}$ i.e. that $Q_3 > Q_1$. Notice that $W = Q_1 - Q_2 = Q_3 - Q_4$ so that $Q_4 - Q_2 = Q_3 - Q_1$ and each of these differences is **positive**. Now the combined effect of the two engines is to remove transfer heat $Q_4 - Q_2$ from the cold reservoir and deposit the (equal amount) $Q_3 - Q_1$ into the hot reservoir without performing work. This violates the Clausius form of the second law. Hence an engine with an efficiency greater than Carnot (for the same temperatures) is impossible.

B3 Fluids (HL)

38 Pressure only depends on depth and the pressure exerted on the surface so the pressures here are equal.

39 In the second case the fluid exerts an upward force on the floating wood and so the wood exerts and equal and opposite force on the fluid. Hence when put on a scale the second beaker will show a greater reading.

40 **a** The second beaker has a smaller mass of water and so a smaller weight by an amount equal to the weight of the displaced water. But there is an additional force pushing down on the liquid that is equal to the buoyant force. This force is equal to the weight of the displaced water and so the readings when the two beakers are put on a scale will be the same.

b They points are at the same depth so the pressure is the same.

41 Draw a horizontal line through X. Points on this line have the same pressure. Hence $P_X = P_{atm} + \rho gh = 1.0\times10^5 + 13600\times9.8\times0.55 = 1.7\times10^5$ Pa

42 Since the ice cube floats its weight is equal to the buoyant force. In turn the buoyant force is equal to the weight of the **displaced** water. Hence then the ice cube melts it will have a volume equal to the displaced volume of water and so the level will remain the same.

43 From Pascal's principle, in the first case the pressure is $p_0 + \rho gh$. In the second $p_0 + \frac{W}{A} + \rho gh$ and in the third $p_0 + \frac{2W}{A} + \rho gh$.

44 The buoyant force on the sphere equals the weight of the sphere and also $B = \rho g V_{imm}$. Hence

$W = B = 10^3\times9.8\times\frac{1}{2}\times\frac{4\pi R^3}{3} = 10^3\times9.8\times\frac{1}{2}\times\frac{4\pi\times0.15^3}{3} = 69.27$ N. The mass is therefore

$m = \frac{69.27}{9.8} = 7.07$ kg. The volume of the material in the sphere is $\frac{4\pi\times0.15^3}{3} - \frac{4\pi\times0.14^3}{3} = 2.64\times10^{-3}$ m^3.

Hence the density is $\frac{m}{V} = \frac{7.07}{2.64\times10^{-3}} = 2.678\times10^3 \approx 2.7\times10^3$ kg m^{-3}.

45 $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ and so $\frac{800}{\pi d^2/4} = \frac{1400\times9.8}{\pi\times9.0^2}$ giving $d = 4.3$ m.

46 Let v_1 be the speed of the water at the faucet of area A_1. After falling a distance of $h = 5.0$ cm the speed is $v_2 = \sqrt{v_1^2 + 2gh}$ and the cross sectional area is A_2. The equation of continuity gives $A_1v_1 = A_2v_2 = A_2\sqrt{v_1^2+2gh}$. Hence

$$A_1^2v_1^2 = A_2^2\left(v_1^2 + 2gh\right)$$

$$\left(A_1^2 - A_2\right)^2 v_1^2 = 2ghA_2^2$$

$$v_1 = \sqrt{\frac{2ghA_2^2}{A_1^2 - A_2^2}}$$

$$v_1 = \sqrt{\frac{2\times9.8\times0.050\times0.60^2}{1.4^2 - 0.60^2}} = 0.470 \text{ m s}^{-1}$$

The flow rate is $Q = A_1v_1 = 1.4\times10^{-4}\times0.470 = 6.6\times10^{-5}$ m^3 s^{-1}.

47 The equation of continuity gives $A_1v_1 = A_2v_2$ i.e. $\pi\times0.60^2\times1.1 = \pi\times0.10^2\times30\times v$ and so

$v = \frac{0.60^2}{30\times0.10^2}\times1.1 = 1.3$ m s^{-1}.

48 In one second a mass of water equal to $m = \rho\pi R^2 v = 10^3\times\pi\times0.012^2\times3.8 = 1.72$ kg. The energy of this mass of water as it leaves the pipe is $E = \frac{1}{2}mv^2 + mgh = \frac{1}{2}\times1.72\times3.8^2 + 1.72\times9.8\times4.0 = 79.8 \approx 80$ J. Since this is energy that is provided in 1 s the power is also 80 W.

49 We use Bernoulli's equation to find:

$$p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_2$$

$$220\times 10^3 + \frac{1}{2}\times 850 \times 2.0^2 + 0 = p_2 + \frac{1}{2}\times 850\times 4.0^2 + 850\times 9.8\times 8.0$$

$$p_2 = 148.3 \approx 150\ \text{kPa}$$

50 **a** Imagine a streamline joining the surface to the hole and apply Bernoulli's equation to find:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_{\text{atm}} + 0 + 1000\times 9.8\times 3.0 = P_{\text{atm}} + \frac{1}{2}\times 1000 \times v_2^2 + 0$$

$$v_2 = 7.67 \approx 7.7\ \text{m s}^{-1}$$

b The water from the upper hole has velocity $v = \sqrt{2g\quad 3.0}$ will hit the ground in a time given by

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\quad 6.0}{g}}$. It will then cover a horizontal distance of $vt = \sqrt{2g\quad 3.0}\sqrt{\frac{2\quad 6.0}{g}} = 2\sqrt{18} = 8.4853$ m.

The speed from the lower hole will be $v = \sqrt{2g\quad 6.0}$. The lower hole water will take a time of $t = \sqrt{\frac{2\quad 3.0}{g}}$

and cover a distance of $vt = \sqrt{2g\quad 6.0}\sqrt{\frac{2\quad 3.0}{g}} = 2\sqrt{18} = 8.4853$ m. The ratio is then 1.

51 The water will; exit with speed given by $v = \sqrt{2gd}$. The hole is a distance of $H-d$ from the ground and so will hit the ground in time $t = \sqrt{\frac{2(H-d)}{g}}$. The range is thus $\sqrt{2gd}\sqrt{\frac{2(H-d)}{g}} = 2\sqrt{d(H-d)}$. This is clearly a maximum (by graphing or differentiating) when $d = \frac{H}{2}$.

52 **a** $p_X = P_{\text{atm}} + \rho g h = 1.0\times 10^5 + 1000\times 9.8\times(220-95) = 1.325\times 10^6 \approx 1.3\times 10^6$ Pa;
$p_Y = P_{\text{atm}} + \rho g h = 1.0\times 10^5 + 1000\times 9.8\times 220 = 2.3\times 10^6$ Pa.

b **i** Assuming the surface does not move appreciably, Bernoulli's equation applied to a streamline joining the surface to Y gives

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_{\text{atm}} + 0 + 1000\times 9.8\times 220 = P_{\text{atm}} + \frac{1}{2}\times 1000\times v_2^2 + 0$$

$$v_2 = 65.66 \approx 66\ \text{m s}^{-1}$$

ii The speed at X can be found by applying the continuity equation from X to Y:
$\pi\times 0.40^2\times v = \pi\times 0.12^2\times 65.66$ hence $v = 5.909 \approx 5.9\ \text{m s}^{-1}$. Now applying Bernoulli's equation to a streamline joining the surface to X gives

$$P_{\text{atm}} + 0 + 1000\times 9.8\times(220-95) = P_X + \frac{1}{2}\times 1000\times 5.909^2$$

$$P_X = 1.308\times 10^6 \approx 1.3\times 10^6\ \text{Pa}$$

The pressure at Y is atmospheric.
(Comment: the speed at X is not large enough to make much of a difference in pressure between the static case and the case of water flowing.)

53 We use Bernoulli's equation $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$. Since the heights are approximately the same this becomes $p_1 + \frac{1}{2}\rho^2 = p_2 + \frac{1}{2}\rho v_2^2$ or $p_1 - p_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$. From then flow rate $A_1 v_1 = \pi(0.020)^2 v_1 = 1800\times 10^{-6}$ hence $v_1 = \frac{1800\times 10^{-6}}{\pi(0.020)^2} = 1.43\text{ m s}^{-1}$ and $v_2 = 1.43\times\frac{0.020^2}{0.004^2} = 35.8\text{ m s}^{-1}$. Hence $p_1 - p_2 = \frac{1}{2}\times 1.20(35.8^2 - 1.43^2) = 398\text{ Pa}$. Hence $\rho_{\text{Hg}} g h = 398\text{ Pa} \Rightarrow h = \frac{398}{13600\times 9.8} = 2.99 \approx 3.0\text{ mm}$.

54 $v = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2\times 12000}{0.35}} = 261.9 \approx 260\text{ m s}^{-1}$

55 The volume flow rate is $Q = 0.52 = \pi\times 0.40^2\times v$ and so the speed is $v = \frac{0.52}{\pi\times 0.40^2} = 1.03\text{ m s}^{-2}$. The Reynolds number is $R = \frac{v\rho r}{\eta} = \frac{1.03\times 850\times 0.40}{0.01} = 3.5\times 10^4$. This is larger than 1000 so the flow is turbulent.

56 We assume a wind speed of 10 m s^{-1} a distance between buildings of 10 m an air density of 1 kg m^{-3} and a viscosity of 10^{-5} Pa s. The Reynolds number is $R = \frac{v\rho r}{\eta} = \frac{10\times 1\times 5}{10^{-5}} = 5\times 10^6$, larger than 1000 so turbulent.

57 The terminal speed of the droplet is equal to $\frac{2\rho r^2 g}{9\eta} = 4.11\times 10^{-4}$ and so

$r = \sqrt{\frac{9\times 1.82\times 10^{-5}\times 4.11\times 10^{-4}}{2\times 870\times 9.8}} = 1.987\times 10^{-6}$ m. The mass of the droplet is then

$m = \rho V = \rho\frac{4\pi r^3}{3} = 870\times\frac{4\pi\times(1.987\times 10^{-6})^3}{3} = 2.8588\times 10^{-14}$ kg. When the droplet was balanced in the electric field, $mg = qE$ and so $q = \frac{mg}{E} = \frac{2.8588\times 10^{-14}\times 9.8}{1.25\times 10^5} = 2.241\times 10^{-18}$ C. Hence

$n = \frac{2.241\times 10^{-18}}{1.6\times 10^{-19}} = 14.01 \approx 14$.

B4 Forced vibrations and resonance (HL)

58 Oscillations are called free if no external force acts on the system other than the restoring force that tends to bring the system back to equilibrium. In forced oscillations there is an additional force acting on the system.

59 In critically damped oscillations the system, after being displaced, returns to its equilibrium position as fast as possible without performing oscillations. In overdamped oscillations the system will again return to its equilibrium position without oscillations but will do in a ling time.

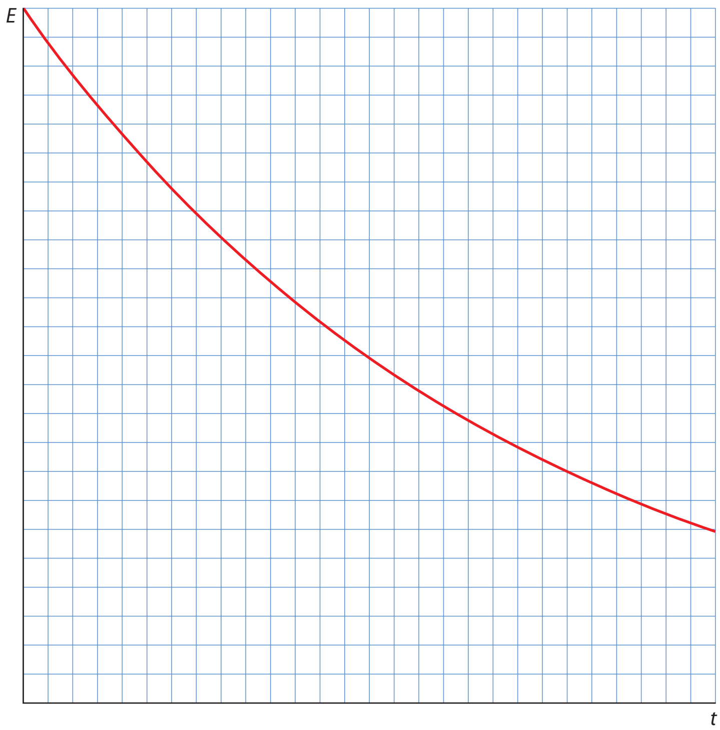
60 Damping denotes the loss of energy of an oscillating system which results in a gradual decrease of the amplitude of the oscillations.

61 When an oscillating system of natural frequency f_N is exposed to an external periodic force that varies with frequency f_D the system will to oscillate with the frequency f_D. The amplitude will be large when $f_D = f_N$ and when this happens we have resonance.

62 a The period is 4.0 s and so the frequency is 0.25 Hz.

b After one oscillation the amplitude drops from 30 cm to 27 cm and so $Q = 2\pi \dfrac{30^2}{30^2 - 27^2} \approx 33$.

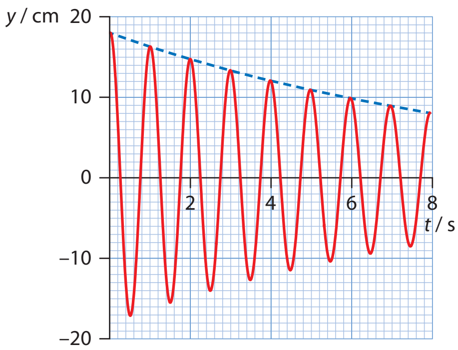
c The energy decreases exponentially with time:



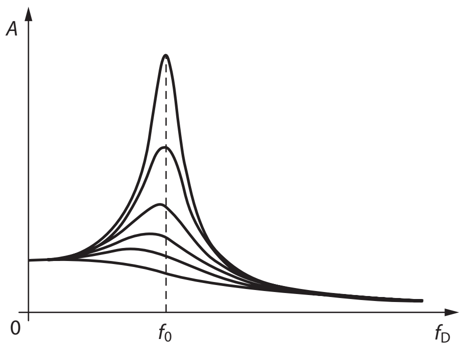
d Q will decrease; Q essentially measures how many oscillations the system will perform before coming to rest. With more damping the system will come to rest with fewer oscillations.

63 a The restoring force must be proportional to and opposite to displacement.

b See graph shown.

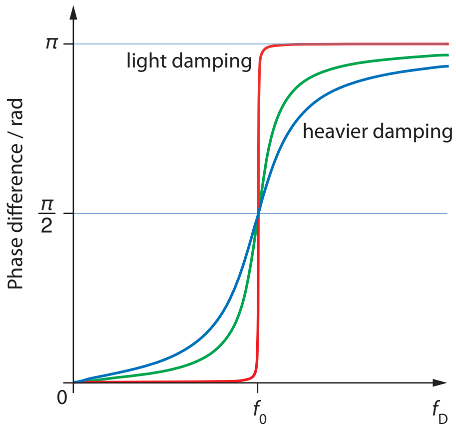


c **i** and **ii**. See graph shown



64 **a** 10 Hz

b See graph shown.



c At 15 Hz the frequency is greater than the natural frequency and so, for light damping, the phase difference between the displacement of the driver and the mass is π. This means that the pendulum mass will move to the left.