Answers to test yourself questions

Topic 7

7.1 Discrete energy and radioactivity

1 **a** Discrete energy means that the atom cannot have any continuous value of energy but rather one out of many separate i.e. discrete values.

b The existence of emission atomic spectra is the best evidence for the discreteness of energy in atoms: the emission lines have specific wavelengths implying specific energy differences between levels.

2 The bright lines are formed when an electron makes a transition from a high energy state H to a lower energy state L. The photon emitted will have a wavelength determined from $\frac{hc}{\lambda} = \Delta E_{LH} \Rightarrow \lambda = \frac{hc}{\Delta E_{LH}}$ where ΔE_{LH} is the difference in energy between state H and L. The dark lines are formed when a photon is absorbed by an electron in a low energy state L which then makes a transition to a high energy state H. For the absorption to be possible the photon energy must equal the difference ΔE_{LH}. Hence this photon will have the same wavelength as the emission line wavelength.

3 The energy difference is 2.55 eV. Hence,

$$\frac{hc}{\lambda} = 2.55 \text{ eV} = 2.55 \times 1.6 \times 10^{-19} = 4.08 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{4.08 \times 10^{-19}}$$

$$\lambda = 4.875 \times 10^{-7} \approx 4.9 \times 10^{-7} \text{ m}$$

4 The energy differences between levels get smaller as n increases. Therefore transitions down to $n = 2$ (the visible light transitions) have wavelengths that are close to each other.

5 **a** Ground state is the energy level with the least possible energy.

b The energy difference between the ground state and the first excited state is 10.2 eV. That between the ground state and the second excited state is 12.1 eV. The incoming photons do not have exactly these amounts of energy so the hydrogen atoms will not absorb any of these photons.

c With incoming electrons it is possible that some will give 10.2 eV of their energy to hydrogen atoms so that the atoms make a transition to the first excited state. The electrons that do give this energy will bounce off the atoms wityh a kinetic energy of about 0.2 eV.

6 $2e$

7 **a** Isotopes are nuclei of the same element (hence have the same proton (atomic) number) that differ in the number of neutrons, i.e. they have different nucleon (mass) number.

b They have different mass and different radius.

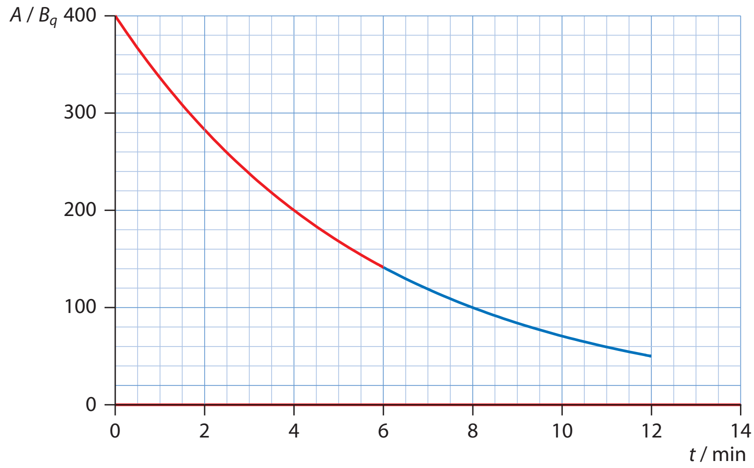
8 ${}^{210}_{83}\text{Bi} \rightarrow {}^{0}_{-1}e + \gamma + \bar{\nu} + {}^{210}_{84}\text{Po}$

9 ${}^{239}_{94}\text{Pu} \rightarrow {}^{4}_{2}\alpha + {}^{235}_{92}\text{U}$

10 18 min is 6 half-lives and so the sample will be reduced by $2^6 = 64$ times i.e. to 0.50 mg.

11 **a** Activity is the rate of decay.

b 4.0 min

c 

d We make the following table of numbers of X and Y nuclei:

Time/min	Number of X nuclei	Number fo Y nuclei	Ratio of Y to X
0	N	0	0
4	$N/2$	$N/2$	1
8	$N/4$	$3N/4$	3
12	$N/8$	$7N/8$	7

So the required time is 12 min.

12 The equation is $C = \dfrac{k}{(d+d_0)^2}$, i.e. $d + d_0 = \sqrt{\dfrac{k}{C}}$. So a graph of d versus $\dfrac{1}{\sqrt{C}}$ would give a straight line with slope $\sqrt{k}$ and intercept $-d_0$.

13 From $I = I_0 e^{-\mu x}$ we deduce that $\ln I = \ln I_0 - \mu x$, so a graph of $\ln I$ versus x gives a straight line with slope $-\mu$.

14 **a** strong nuclear force

b electric force

15 As the nucleus gets heavier more protons and neutrons must be added to the nucleus. The neutrons contribute to nuclear binding through the nuclear force but the protons contribute to repulsion through the electrical force in addition to binding through the nuclear force that they also participate in. However, the electrical force has infinite range and all the protons in the nucleus repel each other whereas only the very near neighbors attract through the nuclear force. To make up for this imbalance it is necessary to have more neutrons i.e. particles that contribute to only binding.

7.2 Nuclear reactions

16 Note that the problem has given an atomic mass for nickel and we need the nuclear mass. Hence we must subtract the electron masses. The mass defect is

$$\begin{aligned}\delta &= 28m_p + (62-28)m_n - (M_{Ni} - Zm_e)\\ &= 28 \times 1.007\,276 + 34 \times 1.008\,665 - (61.928\,348 - 28 \times 0.000549)\\ &= 0.585\,362 \text{ u}\end{aligned}$$

and so the binding energy is

$$\begin{aligned}E &= \delta c^2 = 0.585\,362 \times 931.5c^2 \quad \text{MeV c}^{-2}\\ &= 545.26 \quad \text{MeV}\end{aligned}$$

Hence the binding energy per nucleon is $\dfrac{E}{A} = \dfrac{545.26}{62} = 8.79 \quad \text{MeV}$. This is the highest binding energy per nucleon.

17 The mass defect is

$$\delta = 8m_p + 8m_n - (M_O - 8m_e)$$
$$= 8\times 1.007\,276 + 8\times 1.008\,665 - (15.994 - 8\times 0.000549)$$
$$= 0.137\,920 \text{ u}$$

and so the binding energy is

$$E = \delta c^2 = 0.137\,920 \times 931.5c^2 \text{ MeV c}^{-2}$$
$$= 128.47 \text{ MeV}$$

Hence the binding energy per nucleon is $\frac{E}{A} = \frac{128.47}{16} = 8.03\text{ MeV}$. Consider now the reaction ${}^{16}_{8}\text{O} \rightarrow {}^{1}_{1}\text{p} + {}^{15}_{7}\text{N}$. The mass difference is $15.994 - 8\times 0.000549 - (1.007\,276 + 15.000 - 7\times 0.000549) = -0.012\,727$ u. The negative sign implies that the reaction can take place only when energy is supplied to the oxygen nucleus. This energy is $E = 0.012\,727 \times 931.5c^2 \text{ MeV c}^{-2} = 11.9\text{ MeV}$

18 **a** Using, $E = hf = \frac{hc}{\lambda}$ we find $\lambda = \frac{hc}{E} = \frac{6.63\times 10^{-34} \times 3.0\times 10^{8}}{0.051\times 10^{6} \times 1.6\times 10^{-19}} = 2.44\times 10^{-11}$ m.

b This is in the gamma ray area of the spectrum.

19 **a** ${}^{236}_{92}\text{U} \rightarrow {}^{117}_{46}\text{Pd} + {}^{117}_{46}\text{Pd} + 2\,{}^{1}_{0}\text{n}$

b Two neutrons are produced as well as photons.

c The mass difference is $236.045\,5561 - (2\times 116.9178 + 2\times 1.008\,665) = 0.192\,626$ u. The energy is $E = 0.192\,626 \times 931.5c^2 \text{ MeV c}^{-2} = 179\text{ MeV}$. (Since equal numbers of electron masses have to be subtracted from the atomic masses on each side of the reaction equation, we are allowed to use atomic masses here.)

20 The mass difference is $235.043\,992 + 1.008\,665 - (97.912\,76 + 134.916\,5 + 3\times 1.008\,665) = 0.197\,402$ u. The energy released is $E = 0.197\,402 \times 931.5c^2 \text{ MeV c}^{-2} = 184\text{ MeV}$.

21 The mass difference is $2.014\,102 + 3.016\,049 - (1.008\,665 + 4.002\,603) = 0.018\,883$ u. This energy is $E = 0.018\,883 \times 931.5c^2 \text{ MeV c}^{-2} = 17.6 \approx 18\text{ MeV}$. (Since equal numbers of electron masses have to be subtracted from the atomic masses on each side of the reaction equation, we are allowed to use atomic masses here.)

22 The mass difference is $1.007\,276 + (7.016 - 3\times 0.000549) - 2\times(4.002\,603 - 2\times 0.000549) = 0.018\,619$ u. This corresponds to an energy $E = 0.018\,619 \times 931.5c^2 \text{ MeV c}^{-2} = 17.3\text{ MeV}$ not including the kinetic energy of the accelerated proton.

23 The formula for the mass defect given in the textbook is $\delta = Zm_p + (A - Z)m_n - M_{nucleus}$. Now, $M_{nucleus} = M_{atom} - Zm_e$. Hence,

$$\delta = Zm_p + (A - Z)m_n - (M_{atom} - Zm_e)$$
$$= Z(m_p + m_e) + (A - Z)m_n - M_{atom}$$
$$= ZM_H + (A - Z)m_n - M_{atom}$$

where $M_H = m_p + m_e$ is the mass of the hydrogen atom.

24 **a** $Q_1 = \left(M_D + M_T - M_{He} - m_n\right)c^2$. Now let us look at the binding energy of each nucleus involved in the reaction.

$$E_D = (m_p + m_n - M_D)c^2 \Rightarrow M_D c^2 = (m_p + m_n)c^2 - E_D$$
$$E_T = (m_p + 2m_n - M_T)c^2 \Rightarrow M_T c^2 = (m_p + 2m_n)c^2 - E_T$$
$$E_{He} = (2m_p + 2m_n - M_{He})c^2 \Rightarrow M_{He} c^2 = (2m_p + 2m_n)c^2 - E_{He}$$

Hence replacing the masses in the equation for Q_1,

$$Q_1 = \left(M_D + M_T - M_{He} - m_n\right)c^2$$
$$= \left((m_p + m_n - E_D) + (m_p + 2m_n - E_T) - (2m_p + 2m_n - E_{He}) - m_n\right)c^2$$
$$= E_{He} - (E_D + E_T)$$

b $Q_2 = \left(M_U + M_{Zr} - M_{Te} - 2m_n\right)c^2$. Working as in **a**

$$E_U = (92m_p + 143m_n - M_U)c^2 \Rightarrow M_U c^2 = (92m_p + 143m_n)c^2 - E_U$$
$$E_{Zr} = (40m_p + 58m_n - M_{Zr})c^2 \Rightarrow M_{Zr} c^2 = (40m_p + 58m_n)c^2 - E_{Zr}$$
$$E_{Te} = (52m_p + 83m_n - M_{Te})c^2 \Rightarrow M_{Te} c^2 = (52m_p + 83m_n)c^2 - E_{Te}$$
$$Q_2 = \left(M_U - M_{Zr} - M_{Te} - 2m_n\right)c^2$$
$$= \left((92m_p + 143m_n - E_U) - (40m_p + 58m_n - E_{Zr}) - (52m_p + 83m_n - E_{Te}) - 2m_n\right)c^2$$
$$= E_{Zr} + E_{Te} - E_U$$

c The results in **a** and **b** show that, in general, the energy released can be found from the difference of the total binding energy **after** the reaction minus that **before** the reaction. Thus, to have energy released the binding energy after the reaction must be greater than that before. The peak of the binding energy curve is at nickel. Elements to the right and left of nickel have lower binding energy per nucleon. The issue here is how to use the binding energy curve to show that energy will be released for fission (involving elements heavier than nickel) and fusion (involving elements lighter than nickel). Notice that we cannot prove mathematically that this is the case without knowing the mathematical equation of the binding energy curve. However, the fact that the curve rises for light elements up to nickel and then drops for elements heavier than nickel is indicative that energy is released in both fusion and fission reactions.

7.3 The structure of matter

25 **a** In order to avoid absorption of alpha particles as well as avoid multiple scatterings.
b In order to avoid collisions of alpha particles with air molecules which would have deflected the alphas.

26 **a** The neutron is $d\,d\,u$ and so the antineutron must be $\bar{d}\,\bar{d}\,\bar{u}$. The electric charge is $\left(+\frac{1}{3} + \frac{1}{2} - \frac{2}{3}\right)e = 0$.

b The proton is $u\,u\,d$ and so the antiproton is $\bar{u}\,\bar{u}\,\bar{d}$ with electric charge $\left(-\frac{2}{3} - \frac{2}{3} + \frac{1}{3}\right)e = -e$, as expected.

27 The antiparticle of the K^+ has quark structure $\bar{u}\ s$.

28 It is -1 since this is an antibaryon.

29 **a** Violates: $-1 \rightarrow 0 + 0$
b Conserves: $-1 + 1 \rightarrow 0 + 0$
c Conserves: $-1 + 1 \rightarrow 0 + 0 + 1 - 1$
d Violates: $+1 \rightarrow 0 + 0$

30 Consider a decay such as $\Lambda^0 \rightarrow p^+ + \pi^-$ where the lambda baryon is uds. Notice that there is a strange quark on the left hand side of the decay but none on the right hand side. If this were a strong interaction process (or electromagnetic) the lifetime would be very short (less than about 10^{-20} s). However, the decay of the lambda has a much larger lifetime (of order 10^{-10} s). To explain this it was hypothesised that this long lifetime decay (and many others like it) were due to the weak interaction. The weak interaction being weaker than the strong would naturally lead to a long lifetime decay. To prevent this decay from happening via the strong or electromagnetic interactions, a new quantum number called strangeness was introduced that was assumed to be conserved in strong and electromagnetic interactions but not in weak interactions.

31 **a** The charge of $d\,\bar{s}$ is zero $\left(-\frac{1}{3}+\frac{1}{3}\right)$; the strangeness is + 1.

b No, they have different stangeness

32 **a** The charge of $c\,\bar{d}$ is $e\left(+\frac{2}{3}+\frac{1}{3}=1\right)$

b The strangeness is zero since it does not have strange quarks in it.

33 **a** Conserves: $0+0 \rightarrow +1-1$

b Conserves: $0+0 \rightarrow +1-1$

c Violates: $+1 \rightarrow 0+0$

d Violates: $0+0 \rightarrow 0-1$

34 **a** Electron neutrino

b Muon neutrino

c Tau antineutrino

d Electron antineutrino

e Electron antineutrino and tau neutrino

35 **a** Electron lepton number

b electron and muon lepton number

c electric charge

d baryon number

e Energy

f baryon number

36 **a** Yes because they have electric charge

b No because they do not have electric charge

37 Yes because it is made out of charged quarks

38 Since $\eta_c = c\,\bar{c}$ the antiparticle of the η_c is $\bar{c}\;c$ i.e. is the same as the η_c itself. However the antiparticle of the meson $K^0 = d\,\bar{s}$ would be $s\,\bar{d}$ and is different.

39 **a** Confinement means that color cannot be observed. This implies that one cannot find isolated quarks or gluons.

b The gluons will be very short lived and will produce hadrons along their path. The energy of the gluons will create quark antiquark pairs out of the vacuum and these will combine to make hadrons.

40 **a** We may deduce that $2m_u + m_d = 938$ and $m_u + 2m_d = 940$ (units of mass are MeV c^{-2}). We solve this system of equations to obtain the individual quark masses: from the first equation, $m_d = 938 - 2m_u$ and substituting this into the second gives

$$m_u + 2(938 - 2m_u) = 940$$

$$3m_u = 2 \times 938 - 940$$

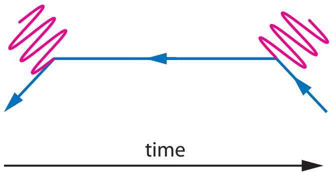
$$m_u = 312\,\text{MeV c}^{-2}$$

and so $m_d = 314\,\text{MeV c}^{-2}$.

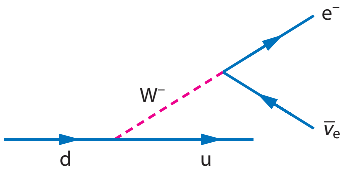
b It follows that we can predict a mass of $312 + 314 = 626\,\text{MeV c}^{-2}$ for the mass of the π^+ meson, which is clearly incorrect.

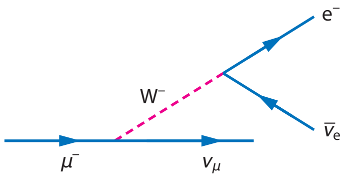
c The reason for the disagreement is that both in the calculation of the masses in **a** as well as in the calculation of the mass of the pion in **b** we have neglected to take into account the sizable binding energy of the quarks. (There are also other technical reasons having to do with exactly what one means by the "mass" of the quarks.)

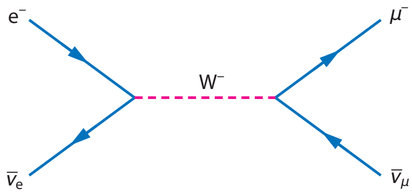
41 The Higgs particle is a crucial ingredient of the standard model of particles. Its interactions with other particles make those particles acquire mass.

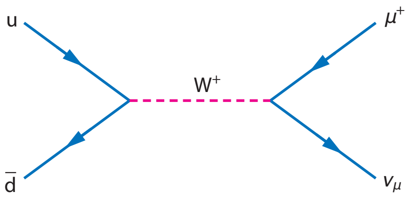
42 

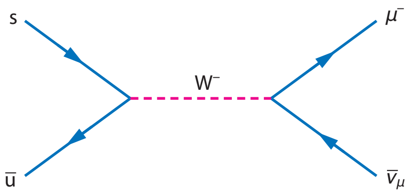
43 a d quark inside the neutron turns into a u quark, an electron and an electron antineutrino.

b 

44 a 

b 

c 

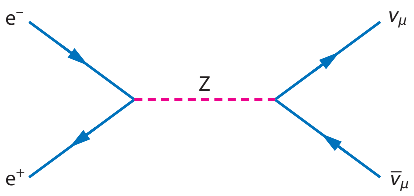
d 

45 $\mathrm{W}^- \rightarrow \mathrm{e}^- + \overline{\nu}_e$

$\mathrm{W}^- \rightarrow \mu^- + \overline{\nu}_\mu$

$\mathrm{W}^- \rightarrow \tau^- + \overline{\nu}_\tau$

46 **a**



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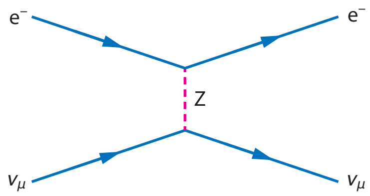

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