Answers to test yourself questions

Topic 6

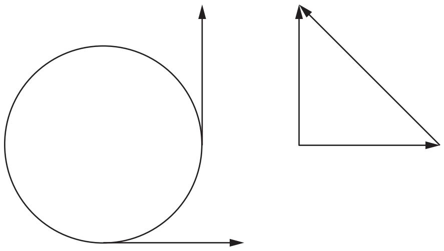
6.1 Circular motion

1 **a** The angular speed is just $\omega = \frac{2\pi}{T} = \frac{2\pi}{1.24} = 5.07$ rad s^{-1}. The linear speed is $v = \omega R = 5.07 \times 3.50 = 17.7$ m s^{-1}.

b The frequency is $f = \frac{1}{T} = \frac{1}{1.24} = 0.806$ s^{-1}.

2 $a = 4\pi^2 rf^2 = 4\pi^2 \times 2.45 \times (3.5)^2 = 1.2 \times 10^3$ m s^{-2}.

3 **a** The average acceleration is defined as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$. The velocity vectors at A and B and the change in the velocity $\Delta\vec{v}$ are shown below.



The magnitude of the velocity vector is 4.0 m s^{-1} and it takes a time of $\frac{2\pi \times 2.0}{4.0} = 3.14$ s to complete a full revolution. Hence a time of $\frac{3.14}{4} = 0.785$ s to complete a quarter of revolution from A to B. The magnitude of $\Delta\vec{v}$ is $\sqrt{4.0^2 + 4.0^2} = 5.66$ m s^{-1} and so the magnitude of the average acceleration is $\frac{5.66}{0.785} = 7.2$ m s^{-2}. This is directed towards north-west and if this vector is made to start at the midpoint of the arc AB it is then directed towards the center of the circle.

b The centripetal acceleration has magnitude $\frac{v^2}{r} = \frac{16.0}{2.0} = 8.0$ m s^{-2} directed towards the center of the circle.

4 The centripetal acceleration is $a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 rf^2$. Hence

$f = \sqrt{\frac{a}{4\pi^2 r}} = \sqrt{\frac{50}{4\pi^2 \times 10}} = 0.356$ s$^{-1} \approx 21$ min^{-1}.

5 **a** The centripetal acceleration is $\frac{v^2}{r} = \frac{4.00}{0.400} = 10.0$ m s^{-2}. The tension is the force that provides the centripetal acceleration and so $T = ma = 1.00 \quad 10.0 = 10.0$ N.

b From $T = ma = 20.0$ N we have $a = \frac{v^2}{r} = 20.0$ m s^{-2} and so $v = \sqrt{20 \times 0.40} = 2.83$ m s^{-1}.

c $20.0 = 1.00 \times \frac{4.00^2}{r} \Rightarrow r = \frac{16.0}{20.0} = 0.800$ m

6 With $a = 9.8$ m s^{-2} we have that $a = \frac{4\pi^2 r}{T^2} \Rightarrow T = \sqrt{\frac{4\pi^2 \times 6.4 \times 10^6}{9.8}} = 5.08 \times 10^3$ s ≈ 85 min.

7 **a** $a = \dfrac{v^2}{r} = \dfrac{\left(\dfrac{2\pi r}{T}\right)^2}{r} = \dfrac{4\pi^2 r}{T^2} = \dfrac{4\pi^2 \times 50.0 \times 10^3}{(25.0 \times 10^{-3})^2} = 3.2 \times 10^9 \text{ m s}^{-2}$

b The forces on the probe are (i) its weight, mg, and (ii) the normal reaction force N from the surface. Assuming the probe to stay on the surface the net force would be

$$mg - N = \frac{mv^2}{r} \Rightarrow N = mg - \frac{mv^2}{r} = m\left(g - \frac{v^2}{r}\right) = m(8.0 \times 10^{10} - 3.2 \times 10^9) > 0.$$

This is positive so the probe can stay on the surface.

8 **a** $v = \dfrac{2\pi R}{T} = \dfrac{2\pi \times 1.5 \times 10^{11}}{365 \times 24 \times 60 \times 60} = 2.99 \times 10^4 \approx 30 \text{ km s}^{-1}$

b $a = \dfrac{v^2}{r} = \dfrac{(2.99 \times 10^4)^2}{1.5 \times 10^{11}} = 5.95 \times 10^{-3} \approx 6.0 \times 10^{-3} \text{ m s}^{-2}$

c $F = ma = \dfrac{mv^2}{r} = 6.0 \times 10^{24} \times 5.95 \times 10^{-3} \approx 3.6 \times 10^{22} \text{ N}$

9 The components of L are:

$L_x = L\sin\theta, \quad L_y = L\cos\theta$

We have that

$L\sin\theta = m\dfrac{v^2}{R}$

$L\cos\theta = mg$

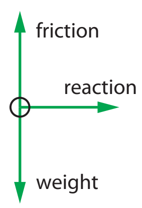
Dividing side by side:

$$\frac{L\sin\theta}{L\cos\theta} = \frac{m\dfrac{v^2}{R}}{mg}$$

$$\tan\theta = \frac{v^2}{gR}$$

This gives $\Rightarrow R = \dfrac{v^2}{g\tan\theta} = \dfrac{180^2}{9.8 \times \tan 35°} = 4.7 \text{ km}$

10 **a**



b Let the normal reaction force from the wall be N. Then

$N = m\dfrac{v^2}{r}$

$mg = f_s$

For the minimum rotation speed the frictional force must be a maximum i.e. $f_s = {}_s N$. I.e.

$N = m\dfrac{v^2}{r}$

$mg = {}_s N$

Combining gives $mg = {}_{s}m\frac{v^2}{r}$ i.e. $v = \sqrt{\frac{gr}{\mu_s}} = \sqrt{\frac{9.8 \times 5.0}{0.60}} = 9.04\ \text{m s}^{-1}$. From $v = 2\pi rf$ we find

$f = \frac{v}{2\pi r} = \frac{9.04}{2\pi \times 5.0} = 0.288\ \text{rev s}^{-1} \approx 17\ \text{rev min}^{-1}$.

11 **a** Let v be the speed on the flat part of the road before the loop is entered. At the top the net force on the cart is its weight and the normal reaction force from the road, both directed vertically downwards. Then, $N + mg = \frac{mu^2}{R} \Rightarrow N = \frac{mu^2}{R} - mg$ where u is the speed at the top. For the cart not to fall off the road, we must have $N > 0$ i.e. $u^2 > gR$. From conservation of energy, $\frac{1}{2}mv^2 = mg(2R) + \frac{1}{2}mu^2$ and so $u^2 = v^2 - 4gR$.

Hence $v^2 - 4gR > gR$, i.e. $v > \sqrt{5gR} = 29.7 \approx 30\ \text{m s}^{-1}$.

b For just about equal to $\sqrt{5gR}$ we get $u = \sqrt{gR} = 13.3 \approx 13\ \text{m s}^{-1}$.

12 The tension in the string must equal the weight of the hanging mass i.e. $T = Mg$. The tension serves as the centripetal force on the smaller mass and so $T = m\frac{v^2}{r}$. Hence $m\frac{v^2}{r} = Mg \Rightarrow v = \sqrt{\frac{Mgr}{m}}$.

13 Let the tension in the upper string be T_U and T_L in the lower string. Both strings make an angle θ with the horizontal. We have that:

$$T_U \sin\theta = mg + T_L \sin\theta$$

$$T_U \cos\theta + T_L \cos\theta = m\frac{v^2}{r}$$

We may rewrite these as:

$$T_U \sin\theta - T_L \sin\theta = mg$$

$$T_U \cos\theta + T_L \cos\theta = m\frac{v^2}{r}$$

From trigonometry, $\sin\theta = \frac{0.50}{1.0} = 0.50 \Rightarrow \theta = 30°$. Further, $r = \sqrt{1.0^2 - 0.50^2} = 0.866\ \text{m}$. Therefore the equations simplify to

$$\begin{aligned} 0.50 \times (T_U - T_L) &= 2.45 \\ 0.866 \times (T_U + T_L) &= 18.48 \end{aligned} \quad \text{or} \quad \begin{aligned} T_U - T_L &= 4.90 \\ T_U + T_L &= 21.33 \end{aligned}$$

Finally, $T_U = 13.1\ \text{N}$, $T_L = 8.22\ \text{N}$.

14 **a** By conservation of energy, $mgh = \frac{1}{2}mv^2$ and so $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 120} = 48.9 \approx 49\ \text{m s}^{-1}$ (with this speed, this amusement park should not have a licence to operate!).

b The forces on a passenger are the weight and the reaction force R both in the vertically down direction. Thus $R + mg = m\frac{v^2}{r} \Rightarrow R = m\frac{v^2}{r} - mg$. The speed at the top is found from energy conservation as

$mgH = \frac{1}{2}mv^2 + mg(2r) \Rightarrow v^2 = 9.81 \times 240 - 2 \times 9.81 \times 60 = 1177$. Hence

$R = 60 \times \frac{1177}{30} - 60 \times 9.81 = 1765 \approx 1800\ \text{N}$.

c Using $v^2 = u^2 - 2as$ we get $0 = 49^2 - 2a \times 40$ and so $a = \frac{50^2}{2 \times 40} = 30\ \text{m s}^{-2}$ (some passengers will be fainting now, assuming they are still alive!).

6.2 The law of gravitation

15 a $F = G\dfrac{Mm}{R^2} = 6.67\times10^{-11}\times\dfrac{5.98\times10^{24}\times7.35\times10^{22}}{(3.84\times10^{8})^2} = 1.99\times10^{20}\ \text{N}$

b $F = G\dfrac{Mm}{R^2} = 6.67\times10^{-11}\times\dfrac{1.99\times10^{30}\times1.90\times10^{27}}{(7.78\times10^{11})^2} = 4.17\times10^{23}\ \text{N}$

c $F = G\dfrac{Mm}{R^2} = 6.67\times10^{-11}\times\dfrac{1.67\times10^{-27}\times9.11\times10^{-31}}{(1.0\times10^{-10})^2} = 1.0\times10^{-47}\ \text{N}$

16 a Zero since it is being pulled equally from all directions.

b Zero, by Newton's third law.

c $F = G\dfrac{m^2}{4R^2}$, (d) $F = G\dfrac{m^2}{4R^2} + G\dfrac{Mm}{4R^2} = G\dfrac{m(m+M)}{4R^2}$

17 $\dfrac{g_A}{g_B} = \dfrac{\left(\dfrac{GM}{(9R)^2}\right)}{\left(\dfrac{GM}{R^2}\right)} = \dfrac{1}{81}$

18 $\dfrac{g_A}{g_B} = \dfrac{\left(\dfrac{G2M}{(2R)^2}\right)}{\left(\dfrac{GM}{R^2}\right)} = \dfrac{1}{2}$

19 Since star A is 27 times as massive and the density is the same the volume of A must be 27 times as large. Its radius must therefore be 3 times as large. Hence $\dfrac{g_A}{g_B} = \dfrac{\left(\dfrac{G27M}{(3R)^2}\right)}{\left(\dfrac{GM}{R^2}\right)} = 3$.

20 $\dfrac{g_{new}}{g_{old}} = \dfrac{\left(\dfrac{GM/2}{(R/2)^2}\right)}{\left(\dfrac{GM}{R^2}\right)} = 2$

21 Let this point be a distance x from the center of the Earth and let d be the center to center distance between the earth and the moon. Then

$$\frac{G81M}{x^2} = \frac{GM}{(d-x)^2}$$

$$81(d-x)^2 = x^2$$

$$9(d-x) = x$$

$$\frac{x}{d} = \frac{9}{10} = 0.9$$

22 a At point P the gravitational field strength is obviously zero.

b The gravitational field strength at Q from each of the masses is

$g = \dfrac{GM}{R^2} = 6.67\times10^{-11}\times\dfrac{3.0\times10^{22}}{(\sqrt{2}\times10^{9})^2} = 1.0\times10^{6}\ \text{N kg}^{-1}$. The net field, taking components, is directed from Q to P and has magnitude $2g\cos45° = 2\times1\times10^{6}\cos45° = 1.4\times10^{6}\ \text{N kg}^{-1}$.

23 We know that $\frac{GMm}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r}$. But $v = \frac{2\pi r}{T}$ and so we deduce that $T^2 = \frac{4\pi^2 r^3}{GM}$. Therefore

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67\times10^{-11}\times6.0\times10^{24}\times(24\times60\times60)^2}{4\pi^2}} = 4.2\times10^7\ \text{m}.$$

24 **a** From $v^2 = \frac{GM}{r}$ we calculate $v = \sqrt{\frac{6.67\times10^{-11}\times6.0\times10^{24}}{(6.4+0.560)\times10^6}} = 7.5828754\times10^3 \approx 7.6\times10^3\ \text{m s}^{-1}$.

b The shuttle speed is $v = \sqrt{\frac{6.67\times10^{-11}\times6.0\times10^{24}}{6.9595\times10^6}} = 7.5831478\times10^3\ \text{m s}^{-1}$. The relative speed of the shuttle and Hubble is 0.2724 m s^{-1} and so the distance of 10 km will be covered in $\frac{10^4}{0.2724} = 36711\ \text{s} \approx 10\,\text{hrs}$.

25 **a** $\frac{Gm_1m_2}{r^n} = m_2\frac{v^2}{r} \Rightarrow v^2 = \frac{Gm_1}{r^{n-1}}$. But $v = \frac{2\pi r}{T}$ and so $\left(\frac{2\pi r}{T}\right)^2 = \frac{Gm_1}{r^{n-1}}$ giving

$$\frac{4\pi^2 r^2}{T^2} = \frac{Gm_1}{r^{n-1}}$$

$$T^2 = \frac{4\pi^2 r^{n+1}}{Gm_1}$$

b For this to be consistent with Kepler's third law we need $n + 1 = 3 \Rightarrow n = 2$