

# Answers to test yourself questions

## Topic 10

### 10.1 Describing fields

1 a The net field at P is:  $g = \frac{Gm}{(d/5)^2} - \frac{16Gm}{(4d/5)^2} = \frac{25Gm}{d^2} - \frac{16 \times 25Gm}{16d^2} = 0$

b The net potential at P is:  $V = -\frac{Gm}{d/5} - \frac{16Gm}{4d/5} = -\frac{5Gm}{d} - \frac{16 \times 5Gm}{4d} = \frac{5Gm}{d} - \frac{20Gm}{d} = -\frac{15Gm}{d}$

2 a  $V = -\frac{GM}{5R_e}$   
 $= -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{5 \times 6.38 \times 10^6}$   
 $= -1.249 \times 10^7 \approx -1.25 \times 10^7 \text{ J kg}^{-1}$

b  $E_p = -\frac{GMm}{5R_e}$   
 $= -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 500}{5 \times 6.38 \times 10^6}$   
 $= -6.252 \times 10^9 \approx -6.25 \times 10^9 \text{ J}$

3 a  $E_p = -\frac{GM_{\text{earth}}M_{\text{moon}}}{r} = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}}{3.84 \times 10^8} = -7.62 \times 10^{28} \text{ J}$

b  $V = -\frac{GM_{\text{earth}}}{r} = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{3.84 \times 10^8} = -1.04 \times 10^6 \text{ J kg}^{-1}$

c  $v = \sqrt{\frac{GM_{\text{earth}}}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{3.84 \times 10^8}} = 1.02 \times 10^3 \text{ m s}^{-1}$

4 We must plot the function  $E_p = -\frac{GM_{\text{earth}}m}{r} - \frac{GM_{\text{moon}}m}{d-r}$  giving the graph in the answers. Here  $m$  is the mass

of the spacecraft and  $d$  the separation of the earth and the moon (center-to-center). Putting numbers in,

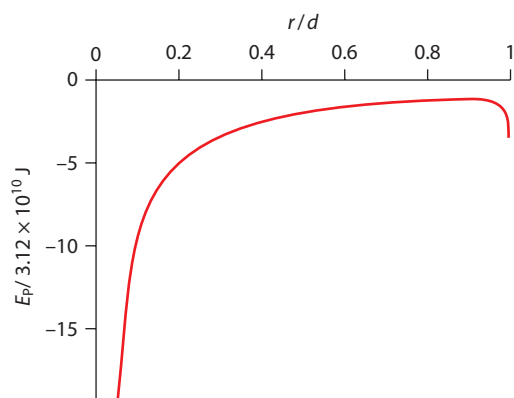
$$E_p = -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 3.0 \times 10^4}{r} - \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 3.0 \times 10^4}{3.84 \times 10^8 - r}$$

$$= \frac{1.2 \times 10^{19}}{r} - \frac{1.5 \times 10^{17}}{3.84 \times 10^8 - r}$$

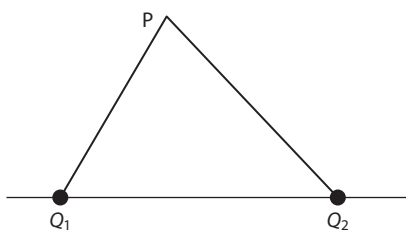
$$= \frac{1.2 \times 10^{19} / 3.84 \times 10^8}{r / 3.84 \times 10^8} - \frac{1.5 \times 10^{17} / 3.84 \times 10^8}{1 - r / 3.84 \times 10^8}$$

$$= \frac{3.1 \times 10^{10}}{x} - \frac{3.9 \times 10^8}{1 - x}$$

where  $x = \frac{r}{3.84 \times 10^8}$ . In this way the function can be plotted on a calculator to give the graph shown here.



- 5 a At  $r = 0.75$ ,  $g = \frac{GM_p}{(0.75d)^2} - \frac{GM_m}{(0.25d)^2} = 0$ . Hence  $\frac{M_p}{M_m} = \frac{(0.75d)^2}{(0.25d)^2} = 9$ .
- b The probe must have enough energy to get to the maximum of the graph. From then on the moon will pull it in. Then  $W = \frac{1}{2}mv^2 = m\Delta V \Rightarrow v = \sqrt{2\Delta V} = \sqrt{2(-0.20 \times 10^{12} - (-6.45 \times 10^{12}))} = 3.5 \times 10^6 \text{ m s}^{-1}$ .
- 6 The tangential component at A is in the direction of velocity and so the planet increases its speed. At B it is opposite to the velocity and so the speed decreases. The normal component does zero work since the angle between force and displacement is a right angle and  $\cos 90^\circ = 0$ .
- 7 The work done by an external agent in moving an object from  $r = a$  to  $r = b$  at a small constant speed.
- 8 a The pattern is not symmetrical and so the masses must be different. The spherical equipotential surfaces of the right mass are much less distorted and so this is the larger mass.
- b The gravitational field lines are normal to the equipotential surfaces.
- c From far away it looks like we have a single mass of magnitude equal to the sum of the two individual masses. The equipotential surfaces of a single point mass are spherical.
- 9 a  $V = \frac{kq}{d/2} + \frac{kq}{d/2} = \frac{4kq}{d}$
- b  $V = \frac{kq}{d/2} - \frac{kq}{d/2} = 0$
- 10 A diagram is:



The potential at P is  $V = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-6}}{0.4} - \frac{8.99 \times 10^9 \times 4.0 \times 10^{-6}}{0.6} = -1.5 \times 10^4 \text{ V}$ .

- 11 a  $V = 4 \frac{kq}{r}$  where  $r = 0.050\sqrt{2} \text{ m}$ . Hence  $V = 4 \times \frac{8.99 \times 10^9 \times 5.0 \times 10^{-6}}{0.050\sqrt{2}} = 2.5 \times 10^6 \text{ V}$ .
- b  $E = 0$
- c The potential at the centre has a maximum value. At a maximum value the derivative is zero.
- 12 a The work done is
- $$W = q\Delta V = q \left( \frac{kQ}{r_2} - \frac{kQ}{r_1} \right) = 1.0 \times 10^{-3} \times \left( \frac{8.99 \times 10^9 \times 10}{2.0} - \frac{8.99 \times 10^9 \times 10}{10} \right) = 3.6 \times 10^7 \text{ J}$$
- b No

- 13 The work done on the electron is

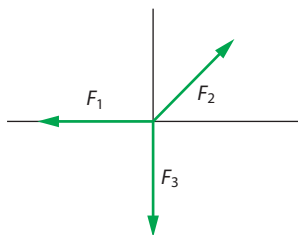
$$W = q\Delta V = q\left(\frac{kQ}{r} - 0\right) = (-1.6 \times 10^{-19}) \times \frac{8.99 \times 10^9 \times (-10)}{0.10} = +1.44 \times 10^{-7} \text{ J}.$$

- 14 The work done ( $W = q\Delta V$ ) is equal to the change in kinetic energy  $\left(\frac{1}{2}mv^2\right)$ . Hence

$$\frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 = 1.6 \times 10^{-19} \times (200 - 100)$$

$$\Rightarrow v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 100}{9.1 \times 10^{-31}}} = 5.9 \times 10^6 \text{ m s}^{-1}$$

- 15 a The forces are roughly as follows.



They have magnitudes:

$$F_1 = \frac{8.99 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{0.05^2} = 7.19 \text{ N}$$

$$F_2 = \frac{8.99 \times 10^9 \times 4 \times 10^{-6} \times 2 \times 10^{-6}}{0.05^2 + 0.05^2} = 14.4 \text{ N}$$

$$F_3 = \frac{8.99 \times 10^9 \times 3 \times 10^{-6} \times 2 \times 10^{-6}}{0.05^2} = 21.6 \text{ N}$$

We must find the components of  $F_2$ :

$F_{2x} = F_2 \cos 45^\circ = 10.2 \text{ N}$  and  $F_{2y} = F_2 \sin 45^\circ = 10.2 \text{ N}$ . So the net force has components:

$F_x = 10.2 - 7.2 = 3.0 \text{ N}$  and  $F_y = 10.2 - 21.6 = -11.4 \text{ N}$ . The net force is then  $F = \sqrt{(11.4)^2 + (3.0)^2} = 11.8 \text{ N}$ .

The direction of the net force is  $\arctan\left(\frac{-11.4}{3.0}\right) = -75^\circ$ .

- b The distance of the center of the square from each of the vertices is

$a = \sqrt{0.025^2 + 0.025^2} = 0.0354 \text{ cm}$ . So the potential at the center is

$$V = \frac{kQ_1}{a} + \frac{kQ_2}{a} + \frac{kQ_3}{a} + \frac{kQ_4}{a} = \frac{8.99 \times 10^9}{0.0354} \times (-1 \times 10^{-6} + 2 \times 10^{-6} - 3 \times 10^{-6} + 4 \times 10^{-6})$$

$$V = 5.1 \times 10^5 \text{ V}$$

- c The work done is  $W = q\Delta V = q(V - 0) = 1.0 \times 10^{-9} \times 5.1 \times 10^5 = 5.1 \times 10^{-4} \text{ J}$ .

- 16 a Charge will move until both spheres are at the same potential. Then  $\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$ . By conservation of charge,

$$q_1 + q_2 = Q \text{ where } Q \text{ is the charge on the one sphere originally. Thus } \frac{q_1}{10} = \frac{q_2}{15} \Rightarrow 3q_1 = 2q_2$$

$$\text{and } q_1 + q_2 = 2.0. \text{ Hence } q_1 = \frac{2}{5} \quad 2.0 = 0.80 \text{ C and } q_2 = \frac{3}{5} \quad 2.0 = 1.2 \text{ C.}$$

$$\text{b } \sigma_1 = \frac{0.80 \times 10^{-6}}{4\pi \times 0.10^2} = 6.4 \times 10^{-6} \text{ C m}^{-2} \text{ and } \sigma_2 = \frac{1.2 \times 10^{-6}}{4\pi \times 0.15^2} = 4.2 \times 10^{-6} \text{ C m}^{-2}.$$

$$\text{c } E_1 = \frac{kq_1}{r_1^2} = 4\pi k\sigma_1 = 4\pi \times 8.99 \times 10^9 \times 6.4 \times 10^{-6} = 7.2 \times 10^5 \text{ N C}^{-1} \text{ and}$$

$$E_2 = 4\pi k\sigma_2 = 4\pi \times 8.99 \times 10^9 \times 4.2 \times 10^{-6} = 4.8 \times 10^5 \text{ N C}^{-1}.$$

- d The electric field is largest for the sphere with the larger charge density. The wire has to be long so that the charge of one sphere will not affect the charge distribution on the other so that both are uniformly charged.

17 You must draw lines that are normal to the equipotentials.

18 a The potential a distance  $x$  from the bottom plate is given by

$$V = -250 + \frac{250 - (-250)}{0.15}x = (-250 + 3.33 \times 10^3 x) \text{ V and so at } x = 3.00 \text{ cm,}$$

$$V = (-250 + 3.33 \times 10^3 \times 0.0300) = -150 \text{ V. Therefore the electric potential energy of the charge is}$$

$$E_p = qV = (-2.00 \times 10^{-6}) \times (-150) = 0.300 \text{ mJ.}$$

b The potential at  $x = 12.0 \text{ cm}$  is  $V = (-250 + 3.33 \times 10^3 \times 0.120) = 150 \text{ V}$  and hence

$$E_p = qV = (-2.00 \times 10^{-6}) \times 150 = -0.300 \text{ mJ.}$$

c The work done must be  $W = q\Delta V = \Delta E_p = -0.300 - 0.300 = -0.600 \text{ J.}$

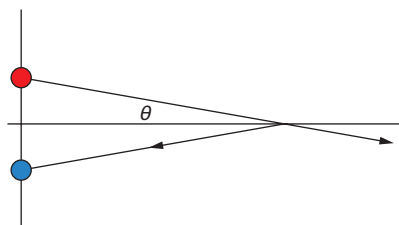
19 a The kinetic energy of the electron  $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.59 \times 10^6)^2 = 1.15 \times 10^{-18} \text{ J}$

gets converted to electric potential energy  $eV$  at the point where the electron stops. Hence the potential

$$\text{at P is } V = \frac{1.15 \times 10^{-18}}{-1.6 \times 10^{-19}} = -7.19 \text{ V.}$$

$$\text{b } V = \frac{kQ}{r} \Rightarrow Q = \frac{Vr}{k} = \frac{(-7.19) \times 2.0 \times 10^{-10}}{9 \times 10^9} = -1.6 \times 10^{-19} \text{ C.}$$

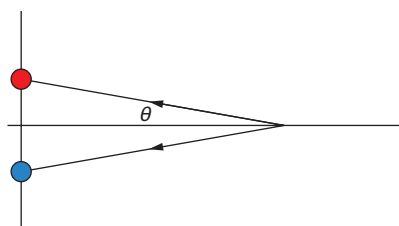
20 a The field due to each of the charges has the direction shown. It is clear that the net field will point in the negative  $y$ -direction.



The magnitude of the field due to one of the charges is  $E = \frac{kQ}{r^2} = \frac{kQ}{a^2 + d^2}$ . The  $y$ -component is

$$E_y = \frac{kQ}{a^2 + d^2} \sin \theta = \frac{kQ}{a^2 + d^2} \frac{a}{\sqrt{a^2 + d^2}} = \frac{kQa}{(a^2 + d^2)^{3/2}} \text{ and so the net field is } E_{\text{net}} = \frac{2kQa}{(a^2 + d^2)^{3/2}}.$$

b For two negative charges:



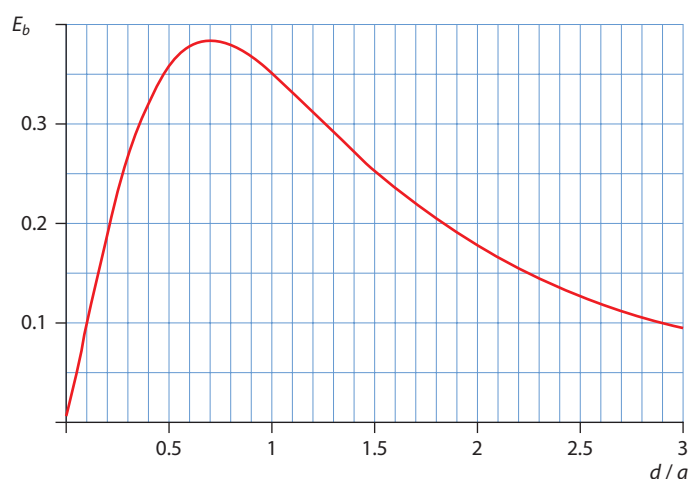
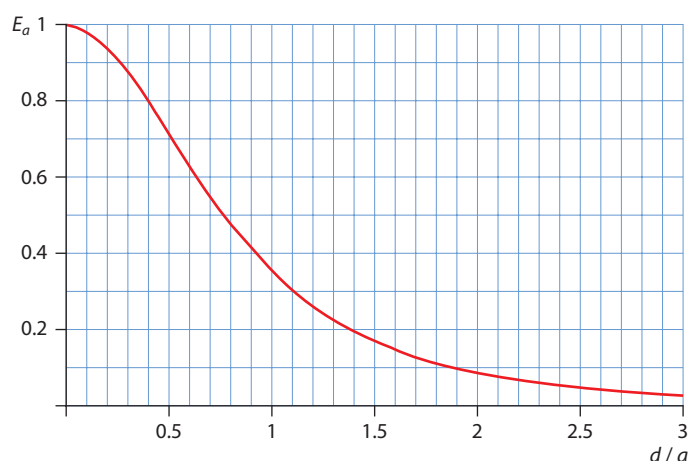
The net field is clearly directed to the left. It has magnitude

$$E_{\text{net}} = 2E_x = \frac{2kQ}{a^2 + d^2} \cos \theta = \frac{2kQ}{a^2 + d^2} \frac{d}{\sqrt{a^2 + d^2}} = \frac{2kQd}{(a^2 + d^2)^{3/2}}.$$

$$\text{c We have } E_a = \frac{2kQa}{(a^2 + d^2)^{3/2}} = \frac{2kQ}{a^2} \frac{1}{\left(1 + \frac{d^2}{a^2}\right)^{3/2}}$$

$$\text{and } E_b = \frac{2kQd}{(a^2 + d^2)^{3/2}} = \frac{2kQ}{a^3} \frac{d}{\left(1 + \frac{d^2}{a^2}\right)^{3/2}} = \frac{2kQ}{a^2} \frac{d/a}{\left(1 + \frac{d^2}{a^2}\right)^{3/2}}.$$

The plots are (the vertical axis is in units of  $\frac{2kQ}{a^2}$ ):



- 21** The initial potential energy of the three protons is zero. When at the vertices of the triangle of side  $a$  the potential energy is  $E_p = 3 \frac{k(e)(e)}{a} = \frac{3ke^2}{a}$  since there are three pairs of charges a distance  $a$  apart. This evaluates to

$$E_p = \frac{3 \times 8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{5.0 \times 10^{-16}} = 1.4 \times 10^{-12} \text{ J} \approx 8.6 \text{ MeV. This is the energy that must be supplied.}$$

## 10.2 Fields at work

**22 a**  $\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$

Substituting values:  $v = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.38 \times 10^6 + 500 \times 10^3}} = 7.6 \times 10^3 \text{ m s}^{-1}$

**b** From  $v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi \times (6.88 \times 10^6)}{7.6 \times 10^3} = 5688 \text{ s} = 94.8 \approx 95 \text{ min}$

**23** We know that  $\frac{GMm}{R^2} = m \frac{v^2}{R} \Rightarrow v^2 = \frac{GM}{R}$ . But  $v = \frac{2\pi r}{T}$  and so we deduce that  $T^2 = \frac{4\pi^2 R^3}{GM}$ .

- 24 a** From the previous problem, Therefore

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (24 \times 60 \times 60)^2}{4\pi^2}} = 4.2 \times 10^7 \text{ m. The distance from the surface is}$$

therefore  $r = 4.2 \times 10^7 - 6.38 \times 10^6 = 3.6 \times 10^4 \text{ km.}$

- b** No, it has to be above the equator.

- 25** The net force is the gravitational force and this must point towards the center of the earth. This happens only for orbit 2.

26 As shown in the text the reaction force from the spacecraft floor is zero giving the impression of weightlessness. More simply, both spacecraft and astronaut are in free fall with the same acceleration.

27 a Apply energy conservation to get: total energy at the point the fuel runs out is

$$E_T = \frac{1}{2}mv^2 - \frac{GMm}{2R} = \frac{1}{2}m \frac{GM}{2R} - \frac{GMm}{2R} = -\frac{GMm}{4R}. \text{ At the highest point the kinetic energy is zero and so}$$

$$-\frac{GMm}{4R} = -\frac{GMm}{r} \text{ leading to } r = 4R$$

b The total energy of the rocket at the point where the fuel runs out is negative so the rocket cannot escape, it will fall back down.

c Apply energy conservation again between the points where the fuel runs out and the crash point to get:

$$-\frac{GMm}{4R} = \frac{1}{2}mv^2 - \frac{GMm}{R} \text{ leading to}$$

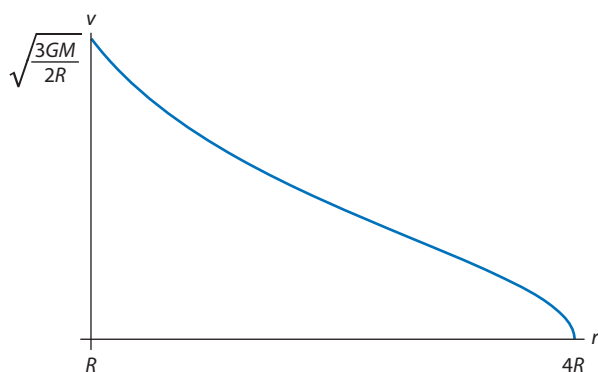
$$\frac{1}{2}v^2 = \frac{GM}{R} - \frac{GM}{4R} = \frac{3GM}{4R}$$

$$v = \sqrt{\frac{3GM}{2R}}$$

d From energy conservation, when the rocket is a distance  $r$  from the centre of the planet:

$$-\frac{GMm}{4R} = \frac{1}{2}mv^2 - \frac{GMm}{r}. \text{ This simplifies to } v = \sqrt{\frac{2GM}{r} - \frac{GM}{2R}} \text{ (where } R \leq r \leq 4R \text{). We need to plot this}$$

function. It is best to write the equivalent form:  $v = \sqrt{\frac{GM}{2R}} \sqrt{\frac{4R}{r} - 1}$ . The graph is then:



28 a We deduced many times that  $v^2 = \frac{GM}{r}$  and so  $E_T = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$ .

$$\text{b } E_T = -\frac{6.67 \times 10^{-11} \times 2.0 \times 10^{30} \times 6.0 \times 10^{24}}{2 \times 1.5 \times 10^{11}} = -2.7 \times 10^{33} \text{ J}$$

29 Using  $E_K = \frac{GMm}{2r}$ ,  $E_P = -\frac{GMm}{r}$  and  $E_T = -\frac{GMm}{2r}$  we deduce that

a B has the larger kinetic energy

b A has the larger potential energy

c A has the larger total energy

30 a The total energy is negative so the satellite cannot escape.

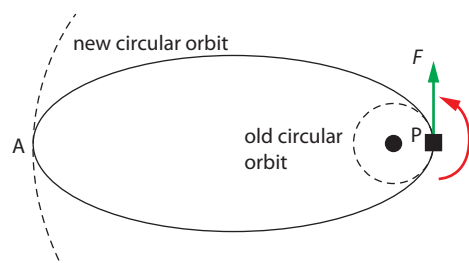
b From problem 30,  $E_T = -\frac{GMm}{2r}$ . Since we are told that  $E_T = -\frac{GMm}{5R}$  and energy is conserved,

$$-\frac{GMm}{2r} = -\frac{GMm}{5R} \Rightarrow r = \frac{5R}{2}.$$

31 The engines do positive work increasing the total energy of the satellite. Since  $E_T = -\frac{GMm}{2r}$  it follows that the orbit radius will increase.

**A bit more:** Since the kinetic energy is given by  $E_K = \frac{GMm}{2r}$  and the orbit radius has increased the speed in the new circular orbit will decrease.

The firing of the rockets when the satellite is in the lower orbit makes the satellite move on an elliptical orbit. After half a revolution the satellite will be at A and further from the earth than in the original position at P. As the satellite gets to A its kinetic energy is reduced and the potential energy increases. At A the speed is too low for the new circular orbit and the engines must again be fired to increase the speed to that appropriate to the new orbit. (If the engines are *not* fired at A then the satellite will remain in the elliptical orbit and will return to P.)



- 32 The potential energy is given by  $E_p = -\frac{GMm}{r}$ . This is least when the distance to the sun,  $r$ , is smallest (remember,  $E_p$  is negative). Therefore since the total energy is conserved, the kinetic energy and hence the speed are greatest at P.

- 33 The escape speed is  $v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$ . At the surface of the planet,  $g = \frac{GM}{R^2} \Rightarrow GM = gR^2$ . Substituting:
- $$v_{\text{esc}} = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}.$$

- 34 a We have done this before.

b  $T^2 = \frac{4\pi^2 r^3}{GM}$ . Now  $r \approx R$  and  $\rho = \frac{M}{\frac{4\pi R^3}{3}} = \frac{3M}{4\pi R^3}$ . Hence,  $\frac{M}{R^3} = \frac{4\pi\rho}{3}$ .

Substituting,  $T = \sqrt{\frac{4\pi^2}{G} \frac{3}{4\pi\rho}} = \sqrt{\frac{3\pi}{G\rho}}$ .

c  $\frac{T_{\text{planet}}}{T_{\text{earth}}} = \sqrt{\frac{\rho_{\text{earth}}}{\rho_{\text{planet}}}} \Rightarrow \frac{\rho_{\text{earth}}}{\rho_{\text{planet}}} = \left(\frac{169}{85}\right)^2 = 3.95 \approx 4$

- 35 a We must use the formula  $T^2 = \frac{4\pi^2 R^3}{GM}$  that we have derived many times already. Now

$$g = \frac{GM}{R^2} \Rightarrow GM = gR^2. \text{ Substituting, } T^2 = \frac{4\pi^2 R^3}{gR^2} = \frac{4\pi^2 R}{g}. \text{ Hence } T = 2\pi\sqrt{\frac{R}{g}}.$$

b  $T = 2\pi\sqrt{\frac{3.4 \times 10^6}{4.5}} = 5.5 \times 10^3 \text{ s} = 91 \text{ min}.$

- c From  $T^2 = \frac{4\pi^2 R^3}{GM}$  we deduce that  $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$  hence  $\frac{91^2}{140^2} = \frac{(3.4 \times 10^6)^3}{R_2^3}$  and so  $R_2 = 4.5 \times 10^6 \text{ m}$ . The height is therefore  $h = 4.5 \times 10^6 - 3.4 \times 10^6 = 1.1 \times 10^6 \text{ m}.$

36 a  $F = \frac{GM^2}{4R^2}$

b  $\frac{GM^2}{4R^2} = \frac{Mv^2}{R^2}$  and so  $v^2 = \frac{GM}{4R}$ . But  $v^2 = \left(\frac{2\pi R}{T}\right)^2$  and so  $\frac{GM}{4R} = \left(\frac{2\pi R}{T}\right)^2$ . Hence  $T^2 = \frac{16\pi^2 R^3}{GM}$

c  $T = \sqrt{\frac{16\pi^2 (1.0 \times 10^9)^3}{6.67 \times 10^{-11} \times 1.5 \times 2.0 \times 10^{30}}} = 2.8 \times 10^4 \text{ s} = 7.8 \text{ h}$

**d**  $E_T = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 - \frac{GM^2}{2R}$ . Since  $v^2 = \frac{GM}{4R}$  we have that

$$E_T = \frac{1}{2}M \frac{GM}{4R} \times 2 - \frac{GM^2}{2R} = \frac{GM^2}{4R} - \frac{GM^2}{2R} = -\frac{GM^2}{4R}.$$

**e** Since energy is being lost the total energy will decrease. This implies that the distance  $R$  will decrease. (From the period formula in (b) the period will decrease as well.)

**f i** The total energy is  $E_T = -\frac{GM^2}{4R}$  and the period is  $T^2 = \frac{16\pi^2 R^3}{GM}$ . Combining the two gives

$$E_T = -\frac{GM^2}{4\left(\frac{GMT^2}{16\pi^2}\right)^{3/2}} \text{ or } E_T = -cT^{-3/2} \text{ where } c \text{ is a constant. Working as we do with propagation of}$$

uncertainties (or using calculus) we have that  $\frac{\Delta E_T}{E_T} = \frac{3}{2} \frac{\Delta T}{T}$  or  $\frac{\Delta E_T}{E_T} = \frac{3}{2} \frac{\Delta T}{T}$ .

**ii**  $\frac{\frac{\Delta E_T}{E_T}}{\frac{\Delta T}{T}} = \frac{3}{2} \frac{\Delta T}{T} = \frac{3}{2} \times \frac{72 \times 10^{-6} \text{ s yr}^{-1}}{2.8 \times 10^4 \text{ s}} = 3.9 \times 10^{-9} \text{ yr}^{-1}$

**g** The lifetime is therefore  $\frac{1}{3.9 \times 10^{-9} \text{ yr}^{-1}} = 2.6 \times 10^8 \text{ yr}$ .

**37 a** Force towards the centre of the circle.

**b** We equate the electric force to the centripetal force to get:  $\frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = m \frac{v^2}{r}$ . Solving for the speed gives the answer.

**c** The total energy is kinetic plus electric potential energy:  $E = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0} \frac{q^2}{r}$ . Using the previous result for

speed:  $v^2 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{mr}$  gives  $E = \frac{1}{2}m \frac{1}{4\pi\epsilon_0} \frac{q^2}{mr} - \frac{1}{4\pi\epsilon_0} \frac{q^2}{r} = \frac{1}{8\pi\epsilon_0} \frac{q^2}{r} - \frac{1}{4\pi\epsilon_0} \frac{q^2}{r} = -\frac{1}{8\pi\epsilon_0} \frac{q^2}{r}$ .

**d** The change in energy is an increase of  $\Delta E = -\frac{1}{8\pi\epsilon_0} \frac{q^2}{2r} - \left(-\frac{1}{8\pi\epsilon_0} \frac{q^2}{r}\right) = +\frac{1}{16\pi\epsilon_0} \frac{q^2}{r}$ .

**38 a** As in the previous problem  $v^2 = k \frac{e^2}{mr}$ . Using also  $v = \frac{2\pi r}{T}$  we get  $\frac{4\pi^2 r^2}{T^2} = k \frac{e^2}{mr} \Rightarrow T^2 = \frac{4\pi^2 m}{ke^2} r^3$ .

**b**  $T = \sqrt{\frac{4\pi^2 \times 9.1 \times 10^{-31}}{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2} \times (0.5 \times 10^{-10})^3} = 1.397 \times 10^{-16} \approx 1.4 \times 10^{-16} \text{ s}$ .

**c** The change in energy is  $E = -\frac{ke^2}{2r}$ . In the first orbit this evaluates to

$$E_1 = \frac{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 0.5 \times 10^{-10}} \approx 2.30 \times 10^{-18} \text{ J. In the other orbit this becomes}$$

$$E_2 = \frac{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 2.0 \times 10^{-10}} \approx 5.75 \times 10^{-19} \text{ J. The change is } 1.7 \times 10^{-18} \text{ J.}$$