Answers to test yourself questions

Topic 3

3.1 Thermal concepts

1 **a** The thermal energy lost by one body must equal the thermal energy gained by the other because of energy conservation.

b The changes in temperature are not, however, necessarily equal because the masses and specific heat capacities may differ.

2 **a** From the definition, $Q = mc\Delta\theta \Rightarrow c = \dfrac{Q}{m\Delta\theta} = \dfrac{385}{0.150 \times 5.00} = 513\ \text{J kg}^{-1}\ \text{K}^{-1}$.

b It is the same.

3 The energy provided is $20 \times 3.0 \times 60 = 3600$ J. Hence
$0.090 \times 420 \times 4.0 + 0.310 \times c \times 4.0 = 3600 \Rightarrow c = 2.8 \times 10^3\ \text{J kg}^{-1}\ \text{K}^{-1}$.

4 The energy provided is $40 \times 4.0 \times 60 = 9600$ J. Hence
$25 \times 15.8 + 0.140 \times c \times 15.8 = 9600 \Rightarrow c = 4.2 \times 10^3\ \text{J kg}^{-1}\ \text{K}^{-1}$. The obvious assumptions are that the liquid and the calorimeter are heated uniformly and that none of the energy supplied gets lost to the surroundings.

5 The loss of potential energy is $mgh = 1360 \times 10 \times 86 = 1.17 \times 10^6$ J. Then,

$$C\Delta\theta = 1.17 \times 10^6 \Rightarrow \Delta\theta = \frac{1.17 \times 10^6}{16 \times 10^3} = 73\ \text{K}$$

6 **a** $C = m_1c_1 + m_2c_2 = 45.0 \times 450 + 23.0 \times 4200 = 1.17 \times 10^5\ \text{J K}^{-1}$.

b $\Delta Q = C\Delta\theta \Rightarrow \dfrac{\Delta Q}{\Delta t} = C\dfrac{\Delta\theta}{\Delta t}$. Hence $450 = 1.17 \times 10^5 \times \dfrac{\Delta\theta}{\Delta t} \Rightarrow \dfrac{\Delta\theta}{\Delta t} = 3.9 \times 10^{-3}\ \text{K s}^{-1}$. For a change of temperature of 20.0 K we then require a time of $\dfrac{20}{3.9 \times 10^{-3}} = 5.2 \times 10^3\ \text{s} = 87$ min.

7 The energy transferred from the water and the aluminum container is $Q = 0.300 \times 4200 \times 10 + 0.150 \times 900 \times 10 = 13950$ J. This is used to (a) raise the temperature of ice to the melting point of 0 °C, (b) melt the ice at 0 °C and (c) raise the temperature of the melted ice (which is now water) to the final temperature of 0 °C. Thus $13950 = m \times 2200 \times 10 + m \times 334 \times 10^3 + m \times 4200 \times 10$. Hence, $m = 0.035$ kg.

8 The mass of ice is $m = 20 \times 0.06 \times 900 = 1080$ kg. So we need $Q = 1080 \times 2200 \times 5 + 1080 \times 334 \times 10^3 = 3.7 \times 10^8$ J.

9 **a** Let the surface area (in square meters) of the pond be A. Then in time t the energy falling on the surface will be $Q = 600 \times A \times t$. The volume of ice is $V = A \times 0.01$ and so its mass is $m = (A \times 0.01) \times 900$. Then $600 \times A \times t = (A \times 0.01) \times 900 \times 334 \times 10^3$. We see that the unknown surface of area cancels out and is not required. Then, $t = \dfrac{0.01 \times 900 \times 334 \times 10^3}{600} = 5010\ \text{s} \approx 84$ min.

b This assumes that none of the incident radiation is reflected from the ice and that all the ice is uniformly heated.

10 **a** $Q_1 = 1.0 \times 2200 \times 10 = 2.2 \times 10^4$ J

b $Q_2 = 1.0 \times 334 \times 10^3 = 3.34 \times 10^5$ J

c $Q_3 = 1.0 \times 4200 \times 10 = 4.2 \times 10^4$ J

d In the melting stage.

11 The water will lose an amount of thermal energy $1.00 \times 4200 \times 10 = 42000\,\text{J}$. This energy is used to melt the ice and then raise the temperature of the melted ice to 10°C. Thus $m \times 334 \times 10^3 + m \times 4200 \times 10 = 42000 \Rightarrow m = 0.112\ \text{kg}$.

12 Since the specific latent heat of vaporisation of water is so much larger than the specific latent heat of fusion we expect that the final temperature will be greater than the initial 30 °C of the water. Then:
$0.150 \times 4200 \times (T - 30) + 0.100 \times 334 \times 10^3 + 0.100 \times 4200 \times T = 0.050 \times 2257 \times 10^3 + 0.050 \times$
$4200 \times (100 - T)$.
This long equation can be solved for T (preferably using the Solver of your calculator) to give $T = 95$ °C.

3.2 Modelling a gas

13 There are $\frac{28}{2} = 14$ moles of hydrogen and so $14 \times 6.02 \times 10^{23} \approx 8 \times 10^{24}$ molecules.

14 There are $\frac{6.0}{4.0} = 1.5$ moles.

15 $\frac{2.0 \times 10^{24}}{6.02 \times 10^{23}} \approx 3.3$ moles.

16 Krypton has $\frac{84}{21} = 4.0$ moles; 4.0 moles of carbon correspond to $\frac{12}{4.0} = 3.0$ g of carbon.

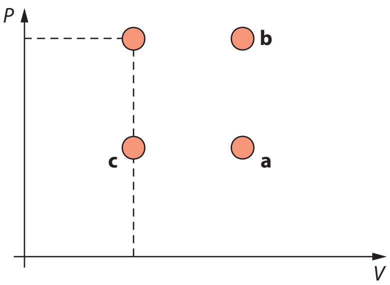
17 From $\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$ we deduce that $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ i.e. That $\frac{12.0 \times 10^5}{295} = \frac{P_2}{393}$ hence $P_2 = 16.0 \times 10^5$ Pa. (Notice the change to kelvin.)

18 From $\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$ we deduce that $P_1V_1 = P_2V_2$ i.e. That $8.2 \times 10^6 \times 2.3 \times 10^{-3} = 4.5 \times 10^6 \times V_2$ hence $V_2 = 4.2 \times 10^{-3}\ \text{m}^3$.

19 A quantity of 12.0 kg of helium corresponds to $\frac{12 \times 10^3}{4} = 3.0 \times 10^3$ mol. Then from the gas law, $pV = nRT$ we get $P = \frac{nRT}{V} = \frac{3.00 \times 10^3 \times 8.31 \times 293}{5.00 \times 10^{-3}} = 1.46 \times 10^9$ Pa.

20 From the gas law, $pV = nRT$ we get $n = \frac{PV}{RT} = \frac{4.00 \times 1.013 \times 10^5 \times 12.0 \times 10^{-3}}{8.31 \times 293} = 1.998$ mol. Since the mass of one mole of carbon dioxide (CO_2) is 44 g, we need $44 \times 1.9998 = 87.9$ g.

21 We use $\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$ to get $\frac{P}{n_1} = \frac{\frac{P}{2}}{n_2}$ and hence $n_2 = \frac{n_1}{2}$. In other words to reduce the pressure to half its original value, half the molecules must leave the container. The original number of molecules can be found using $pV = nRT$ to get $n = \frac{PV}{RT} = \frac{5.00 \times 10^5 \times 300 \times 10^{-6}}{8.31 \times 300} = 0.0602$ and hence $N = 0.0602 \times 6.02 \times 10^{23} = 3.62 \times 10^{22}$.
So we will have to lose $\frac{N}{2} = 1.81 \times 10^{22}$ molecules. This will take $\frac{1.81 \times 10^{22}}{3.00 \times 10^{19}} = 603\ \text{s} \approx 10$ min.

22 

23 Let there be n_1 moles of the gas in the left container and n_2 in the right. Then it must be true $\left(\text{using } n=\dfrac{PV}{RT}\right)$ that $n_1=\dfrac{12\times10^5\times6.0\times10^{-3}}{RT}$ and $n_1=\dfrac{6.0\times10^5\times3.0\times10^{-3}}{RT}$. When the gases mix we will have n_1+n_2 moles in a volume of 9.0 dm^3 and so $n_1+n_2=\dfrac{P\times9.0\times10^{-3}}{RT}$. Hence $\dfrac{12\times10^5\times6.0\times10^{-3}}{RT}+\dfrac{6.0\times10^5\times3.0\times10^{-3}}{RT}=\dfrac{P\times9.0\times10^{-3}}{RT}$.

This means that $P=\dfrac{12\times10^5\times6.0+6.0\times10^5\times3.0}{9.0}=10\times10^5\text{ Pa}=10\text{ atm}$.

24 **a** The cross sectional area of the piston is $A=\dfrac{V}{h}=\dfrac{0.050}{0.500}=0.10\text{ m}^2$. The pressure in the gas is constant and equal to $P=\dfrac{F}{A}=\dfrac{10.0\times10}{0.010}=1.0\times10^4\text{ Pa}$.

b From the gas law, $n=\dfrac{PV}{RT}=\dfrac{1.0\times10^4\times0.050}{8.31\times292}=0.206$. The number of molecules is then $N=0.206\times6.02\times10^{23}=1.24\times10^{23}$.

c From $\dfrac{V_1}{T_1}=\dfrac{V_2}{T_2}$ we get $\dfrac{0.050}{292}=\dfrac{V_2}{425}$ hence $V_2=7.3\times10^{-2}\text{ m}^3$.

25 The mass is just $28\times2=56$ g. The volume is found from $V=\dfrac{nRT}{P}=\dfrac{2.0\times8.31\times273}{1.0\times10^5}=0.045\text{ m}^3$.

26 The molar mass of helium is 4.00 g per mole. A mass of 70.0 kg of helium corresponds to $\dfrac{70.0\times10^3}{4.00}=1.75\times10^4$ mol. Thus $P=\dfrac{nRT}{V}=\dfrac{1.75\times10^4\times8.31\times290}{404}=1.04\times10^5\text{ Pa}$.

27 **a** $n=\dfrac{PV}{nRT}=\dfrac{150\times10^3\times5.0\times10^{-4}}{8.31\times300}=3.01\times10^{-2}\text{ mol}$.

b $N=nN_A=3.01\times10^{-2}\times6.02\times10^{23}=1.8\times10^{22}$

c $M=n\mu=3.01\times10^{-2}\times29=0.87\text{ g}$

28 **a** $V=\dfrac{nRT}{P}=\dfrac{1.0\times8.31\times273}{1.0\times10^5}=2.27\times10^{-2}\text{ m}^3$

b We have 1 mole and so 4.00 g of helium. The density is thus $\rho=\dfrac{M}{V}=\dfrac{4.00\times10^{-3}}{2.27\times10^{-2}}=0.176\text{ kg m}^{-3}$.

c The change for oxygen is just the molar mass and so $\rho=\dfrac{32}{4}\times0.176=1.41\text{ kg m}^{-3}$.

29 Under the given changes the volume will stay the same and so the density will be unchanged.

30 We use $\frac{1}{2}mc^2 = \frac{3}{2}kT$. The mass of a molecule is $\frac{4.0}{6.02\times10^{23}} = 6.64\times10^{-23}$ g $= 6.64\times10^{-27}$ kg. Hence $c = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\times1.38\times10^{-23}\times850}{6.64\times10^{-27}}} \approx 2300\ \mathrm{m\,s^{-1}}$.

31 From $\frac{1}{2}mc^2 = \frac{3}{2}kT$ we get $c = \sqrt{\frac{3kT}{m}}$. The mass of a molecule (in kg) is $\frac{M}{N_A}$

$\frac{4.0}{6.02\times10^{23}} = 6.64\times10^{-23}$ g $= 6.64\times10^{-27}$ kg. Hence $c = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kN_AT}{M}} = \sqrt{\frac{3RT}{M}}$ since $k = \frac{R}{N_A}$.

32 **a** $\frac{1}{2}mc^2 = \frac{3}{2}kT = \frac{3}{2}\times1.38\times10^{-23}\times300 = 6.2\times10^{-21}$ J

b $\frac{1}{2}m_1c_1^2 = \frac{1}{2}m_2c_2^2 \Rightarrow \frac{c_1}{c_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{\mu_2/N_A}{\mu_1/N_A}} = \sqrt{\frac{32}{4}} = \sqrt{8}$