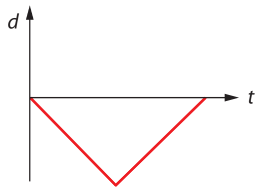
Answers to test yourself questions

Topic 2

2.1 Uniform motion

1 Distance traveled in first 1.5 h is $s = vt = 70 \times 1.5 = 105$ km. Remaining distance is 15 km and must be covered in 1.0 hr so average speed must be $v = \frac{15}{1.0}\frac{\text{km}}{\text{hr}} = 15\ \text{km hr}^{-1}$.

2 The velocity is initially constant and negative so displacement graph will be a straight line with negative slope. The velocity is then constant and positive so displacement graph is a straight line with positive slope.



3 The relative speed of the cyclists is $v = 35$ km hr^{-1}. They will then meet in a time of $v = \frac{70}{35} = 2.0\,\text{hr}$.

a The common displacement is $s = 15 \times 2.0 = 30$ km.
b The fly will travel a distance of $s = 30 \times 2.0 = 60$ km.

4 **a** The distance traveled is 80 m. The average speed is then $v = \frac{80}{20}\ \text{m s}^{-1} = 4.0\ \text{m s}^{-1}$.
b The displacement is zero and so the average velocity is zero.

5 Use $v = u + at$. So $8.0 = 2.0 + a \times 2.0 \Rightarrow a = \frac{8.0 - 2.0}{2.0} = 3.0\ \text{m s}^{-2}$.

6 From $v = u + at$, $28 = 0 + a \times 9.0 \Rightarrow a = \frac{28}{9.0} = 3.1\ \text{m s}^{-2}$, hence from $s = ut + \frac{1}{2}at^2$ we have that $s = \frac{1}{2}3.1 \times 9.0^2 = 126\ \text{m} \approx 130\ \text{m}$.

7 From $v^2 = u^2 + 2as$ we find $0^2 = 12^2 + 2a \times 45 \Rightarrow a = -\frac{144}{90} = -1.6\ \text{m s}^{-2}$.

8 From $s = ut + \frac{1}{2}at^2$ we get $16 = -6.0t + \frac{1}{2} \times 2.0 \times t^2$. Solving the quadratic equation gives $t = 8.0$ s.

9 Use $s = ut + \frac{1}{2}at^2$. So $450 = \frac{1}{2}a \times 15.0^2 \Rightarrow a = \frac{900}{225} = 4.00\ \text{m s}^{-2}$. Then from $v = u + at$, $v = 4.00 \times 15.0 = 60.0$ m s^{-1}.

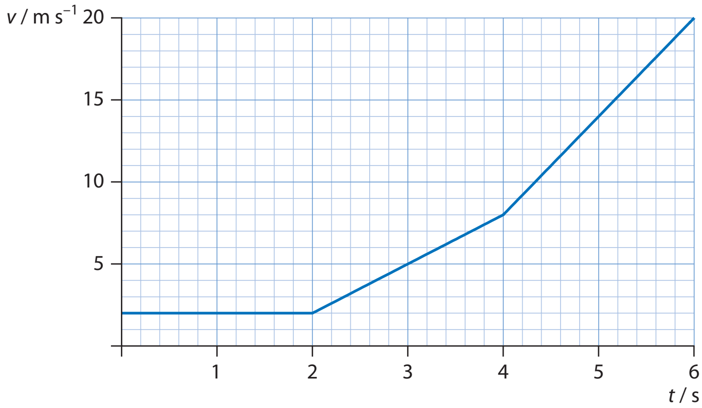
10 **a** The distance traveled before the brakes are applied is $s = 40.0 \times 0.50 = -20$ m. Once the brakes are applied the distance is given from $v^2 = u^2 + 2as$, i.e. $0 = 40^2 + 2 \times (-4.0) \times s \Rightarrow s = 200$ m. The total distance is thus 220 m.
b 200 m as done in **a**.
c $s = 220 - 200 = 20$ m.
d It would be less since the speed is less.

11 **a** $s_1 = -\frac{1}{2} \times 10t^2 = -5t^2$ and $s_2 = -\frac{1}{2} \times 10(t-1)^2$. Two seconds after the second ball was dropped means that $t = 3.0$ s. Then, $s_1 = -45$ m and $s_2 = -20$ m. The separation is thus 25 m.

b $s_1 - s_2 = -5t^2 + \frac{1}{2} \times 10(t-1)^2 = -10t + 5$ so in magnitude this increases.

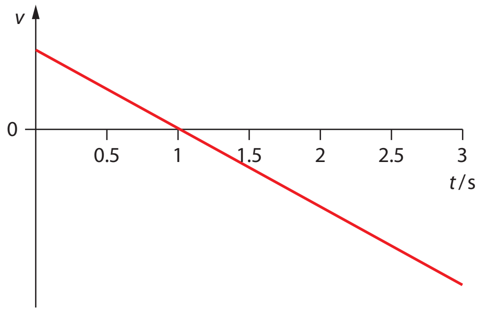
12 **a** The velocity at 2 s is $v_2 = 2 + 0 \times 2 = 2.00$ m s^{-1}. The velocity at 4 s is $v_4 = 2 + 3 \times 2 = 8.00$ m s^{-1}. The velocity at 6 s is $v_6 = 8 + 6 \times 2 = 20.0$ m s^{-1}. Alternatively, the area under the graph is 18.0 m s^{-1} and this gives the *change* in velocity. Since the initial velocity is 2.00 m s^{-1}, the final velocity is 20.0 m s^{-1}.

b

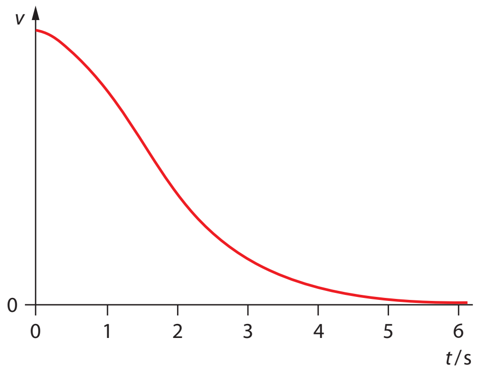


13 The acceleration is the slope of the velocity – time graph. Drawing a tangent to the curve at 2 s we find a slope of approximately $a = 2.0$ m s^{-2}.

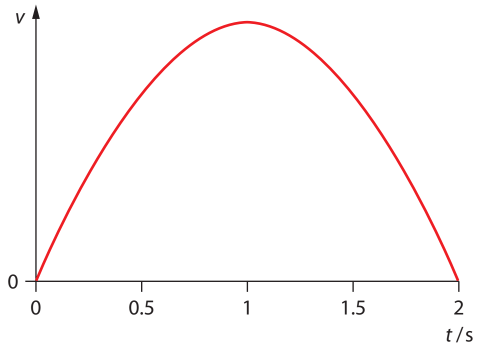
14 Velocity is the slope of the displacement – time graph. So we observe that the velocity is initially positive and begins to decrease. It becomes zero at 1 s and then becomes negative. The displacement graph is in fact a parabola and so the velocity is in fact a linear function. Of course we are not told that, so any shape showing the general features described above would be acceptable here.



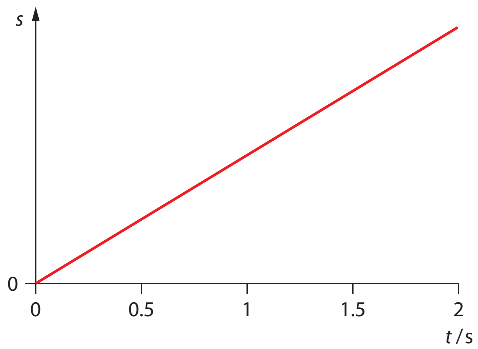
15 Velocity is the slope of the displacement – time graph. So we observe that the velocity is initially very large and continues to decrease all the time approaching a small value. The slope and hence the velocity are always positive. This explains the graph in the answers in the textbook.



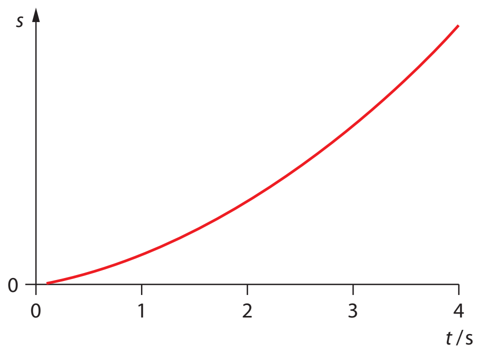
16 Velocity is the slope of the displacement – time graph. So we observe that the velocity is initially very small, becomes greatest at 1 s and starts decreasing thereafter becoming very small again. The slope and hence the velocity are always positive. This explains the graph in the answers in the textbook.



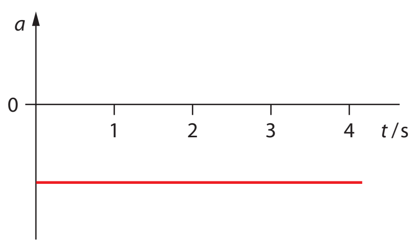
17 The acceleration is zero here so $x = vt$, i.e. The graph of displacement is a linear function with a positive slope (that may or may not go through the origin depending on the initial displacement).



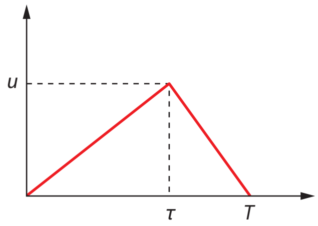
18 Here we have a constant positive acceleration and so $x = ut + \frac{1}{2}at^2$ which is the graph of a (concave down) parabola.



19 The acceleration is the slope of the velocity – time graph. The slope is large and positive initially and decreases to become zero at 1 s. It then becomes negative increasing in magnitude (i.e. becoming more negative).



20 You must push the car as hard as you can but then you must also pull back on it to stop it before it crashes on the garage. The velocity – time graph must be something like:



We know that: $u = 2\tau$. During pullback, we have that the velocity is given by $v = u - 3(t - \tau) = 2\tau - 3(t - \tau)$ $= 5\tau - 3t$. The velocity becomes zero at time T and so $0 = 5\tau - 3T$, i.e. $\tau = \frac{3T}{5}$. The area under the curve (triangle) is 15 m and is given by $\frac{1}{2}Tu = \frac{1}{2}T(2\tau) = \frac{1}{2}T\frac{6T}{5} = \frac{3T^2}{5}$.

Hence $\frac{3T^2}{5} = 15 \Rightarrow T^2 = 25 \Rightarrow T = 5$ s.

21 **a** The velocity is the slope of the displacement – time graph. Therefore the velocity is negative from A to B.

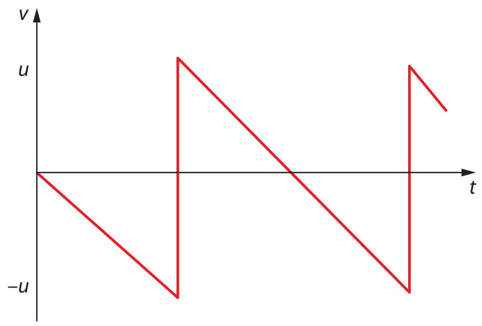
b Between B and C the slope and so the velocity is zero.

c From A to B the velocity is becoming less negative and so it is increasing. So the acceleration is positive.

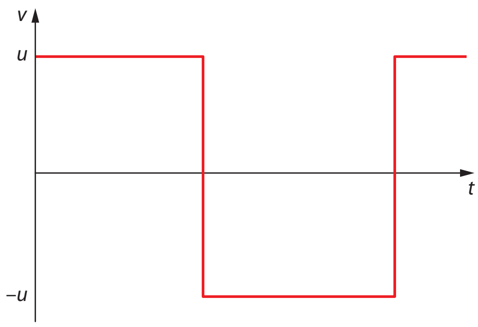
d From C to D the slope is increasing meaning the velocity is increasing. Hence the acceleration is positive.

22 See graphs below.

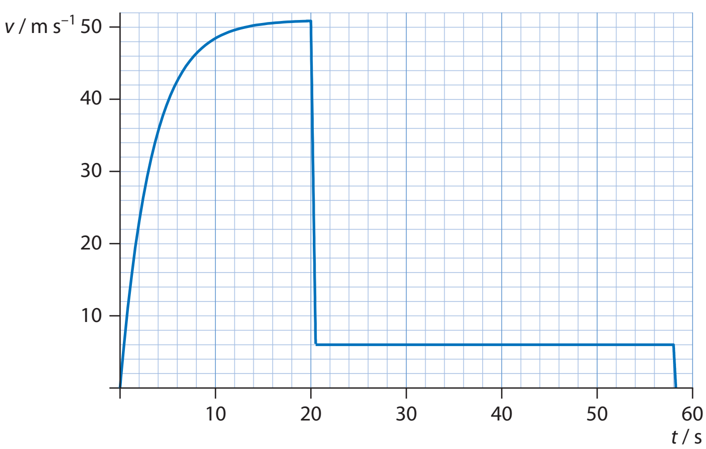
a



b



c



23 **a** Use $v^2 = u^2 + 2as$ to get $0 = 8^2 + 2 \times (-10) \times s \Rightarrow s = 3.2$ m from the cliff.

b Use $s = ut + \frac{1}{2}at^2$ to get $-35 = 8 \times t + \frac{1}{2}(-10) \times t^{22}$ and solve for time to get $t = 3.56$ s.

c $v = u + at = 8 - 10 \times 3.56 = -27.6$ m s^{-1}.

d $3.2 + 3.2 + 35 = 41.4$ m.

e average speed is $\frac{41.4}{3.56} = 11.6$ m s^{-1} and average velocity is $\frac{-35}{3.56} = -9.83$ m s^{-1}.

24 **a** 60 m^2

b 40 m s^{-1}

25 The time to fall to the floor is given by $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 1.3}{10}} = 0.51$ s. The horizontal distance traveled is therefore $x = v_x t = 2.0 \times 0.51 = 1.02 \approx 1.0$ m.

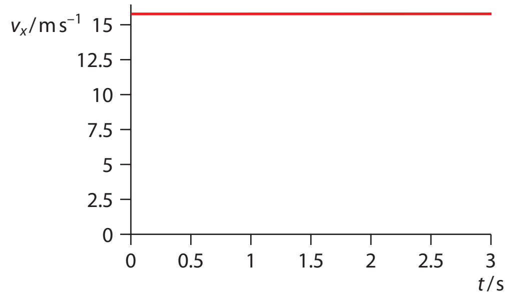
26 **a** The times to hit the ground are found from $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 4.0}{10}} = 0.894$ s and $\sqrt{\frac{2 \quad 8.0}{10}} = 1.265$ s.

The objects are thus separated by $4.0 \times (1.265 - 0.894) = 1.48 \approx 1.5$ m when they land.

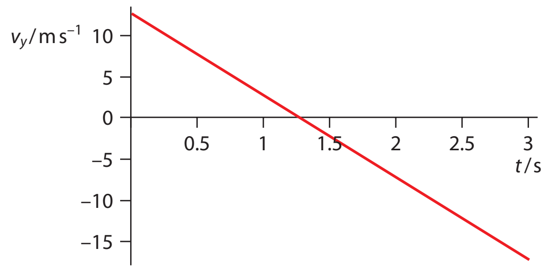
b The horizontal distance traveled by the object falling from 8.0 m is (see previous problem) $x = v_x t = 4.0 \quad 1.265 = 5.06$ m. Thus the speed of the other object must be $v_x = \frac{5.06}{0.894} = 5.66 \approx 5.7$ m s^{-1}.

27 The components of velocity are:

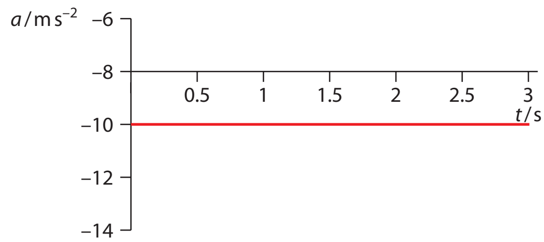
a $v_x = v\cos 40° = 20\cos 40° = 15.3$ m s^{-1} and so the graph is a horizontal straight line.



b $v_y = v\cos 40° - gt = (12.9 - 10t)$ m s^{-1} so that graph is a straight line with negative slope as shown in graph.



c The acceleration is constant so graph is a horizontal straight line.



28 $v_y = v\sin 40° - gt$. At the highest point this component is zero and so $t = \frac{v\sin 40°}{g} = \frac{24 \quad \sin 40°}{10} = 1.54$ s. Then from $y = vt\sin 40° - \frac{1}{2}gt^2$ we find $y = 24 \times 1.54 \times \sin 40° - \frac{1}{2} \times 10 \times 1.54^2 \approx 12$ m.

29 Graphs are shown in the answers to the textbook. They correspond to:

a The horizontal displacement is given by $x = 20t \quad \cos 50° = 12.6t$ whose graph is a straight line.

b The vertical displacement is $y = 20t \times \sin 50° - \frac{1}{2} \times 10t^2 = 15.3t - 5t^2$ whose graph is a concave down parabola.

30 In time t the monkey will fall a vertical distance $y = \frac{1}{2}gt^2$ but so will the bullet and hence the bullet will hit the monkey.

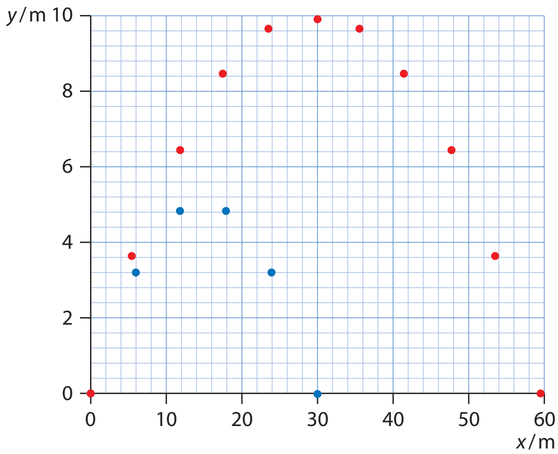
31 a i The ball covers a horizontal distance of 60 m in 2.0 s and so the horizontal velocity component is $u_x = \frac{60}{2.0} = 30 \text{ m s}^{-1}$. The ball climbs to a height of 10 m in 1.0 s and so from $y = \frac{u_y + v_y}{2}t$ we have $10 = \frac{u_y + 0}{2} \times 1.0 \Rightarrow u_y = 20 \text{ m s}^{-1}$.

ii The angle of launch is $\theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}\left(\frac{20}{30}\right) = 34°$.

iii The vertical component of velocity becomes zero at 1.0 s and so $v_y = u_y - gt \Rightarrow 0 = 20 - g \times 1.0 \Rightarrow g = 20 \text{ m s}^{-2}$.

b The velocity is horizontal to the right and the acceleration is vertically down.

c With $g = 40 \text{ m s}^{-2}$, the ball will stay in the air for half the time and so will have half the range. The maximum height is reached in 0.50 s and is $y = u_y t - \frac{1}{2}gt^2 = 20 \times 0.50 - \frac{1}{2} \times 40 \times 0.50^2 = 5.0$ m, i.e. half as great as before.



32 a The initial velocity components are: $u_x = 20.0\cos 48° = 13.38 \text{ m s}^{-1}$ and $u_y = 20.0\sin 48° = 14.86 \text{ m s}^{-1}$.

The ball hits the sea when the vertical displacement is $y = -60.0$ m. Thus $y = u_y t - \frac{1}{2}gt^2 \Rightarrow$ $-60.0 = 14.86t - 4.90t^2$. Solving for the positive root we find $t = 5.33$ s. Hence $v_x = u_x = 13.38 \text{ m s}^{-1}$ and $v_y = u_y - gt = 14.86 - 9.81 \times 5.33 = 37.43 \text{ m s}^{-1}$. The speed at impact is thus $v = \sqrt{13.38^2 + (-37.43)^2} = 39.7 \approx 40 \text{ m s}^{-1}$ at $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(-\frac{37.74}{13.38}\right) = -70°$ to the horizontal.

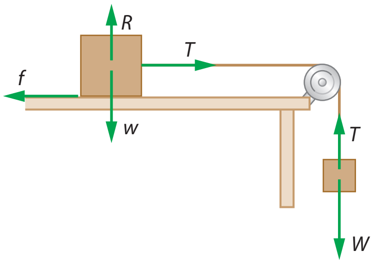
b Some of the kinetic energy of the ball will be converted into thermal energy and so the speed at impact will be less. The horizontal component of velocity will decrease in the course of the motion and will tend to go to zero but the vertical component will never become zero (after reaching the maximum height). This means that the angle of impact will be steeper.

33 **a** Terminal speed is the eventual constant speed reached by a projectile as a result of an air resistance force that increases with speed.

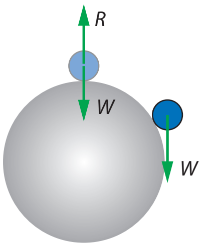
b Initially the net force on the particle is just the weight. As the speed increases so does the resistance force. Eventually the resistance force will equal the weight and from then on the particle will move with zero acceleration, i.e. with a constant terminal speed.

2.2 Forces

34

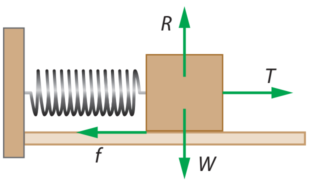


35 **a** and **b**



36 The tension is the same in both cases since the wall exerts a force of 50 N to the left on the string just as in the second diagram.

37



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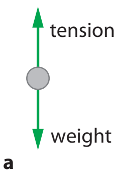
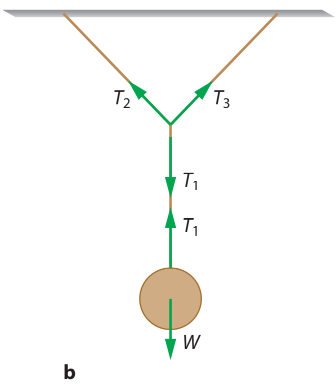




39 A 30 N to the right
B 6 N to the right
C 8 N to the left
D 15 N to the right
E 10 N down
F 20 N up

40 The horizontal components clearly cancel put leaving a net force of $2 \quad 20 \quad \sin 45° = 28$ N in the up direction.

41 Because there would be no vertical force to cancel the weight of the block.

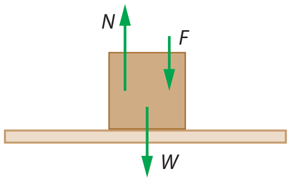
42 **a** Since the string is being pulled slowly we have equilibrium until one of the strings breaks. If the lower string is being pulled with force F then the tension in the lower string will be F and the tension in the upper string will be T where $T = mg + F$. The tension in the upper string is thus greater and will reach breaking point first.

b If the lower string is pulled down very abruptly, the inertia of the block will keep it momentarily motionless and so the tension in the lower string will reach a high value before the upper one does. Hence it will break.

43 The largest frictional force that can develop between the mass and the table is $f_{max} = \quad {}_{s}N =$ $0.60 \quad 2.00 \quad 9.8 = 11.8$ N. This is also the tension holding the hanging mass up. Hence $mg = 11.8 \Rightarrow m = 1.2$ kg.

44 Equilibrium demands that $W = F + N \Rightarrow N = W - F = 220 - 140 = 80$ N.

45



Equilibrium demands that $W + F = N \Rightarrow N = 150 + 50 = 200$ N. This is the force that the table exerts on the block. By Newton's third law this is also the force exerted on the table by the block.

46 The component of the weight down the plane is $Mg\sin\theta$ and for equilibrium this is also the tension in the string. To have equilibrium for the hanging mass its weight must equal the tension and so $Mg\sin\theta = mg$. Hence $\theta = \sin^{-1}\dfrac{m}{M}$.

47 **a** One possibility is to have the mass of the body decrease as in the case of a rocket where the fuel is being burned and ejected from the rocket.

b That happens when the mass increases as for example cart that is being filled with water or sand while being pulled with a constant force.

48 The maximum force can only be 575 N and so the maximum acceleration is $a = \dfrac{600}{1400} = 0.43\ \text{m s}^{-2}$.

49 The forces on the man are his weight, mg and the reaction force R from the floor.

a **i** The acceleration is zero and so $R - mg = 0$, i.e. $R = mg$.

ii The acceleration is zero and so $R - mg = 0$, i.e. $R = mg$.

iii The net force is in the downward direction and equals $mg - R$. Hence, $mg - R = ma$ and so $R = mg - ma$.

iv From iii we have that $R = mg - mg = 0$.

b The man will be hit by the ceiling of the elevator that is coming down faster than the man.

50 As the elevator goes up the force that must be supplied by the arm on the book upwards must increase. Because you are not aware that you have to do that the book "feels" heavier and so moves down a bit. As the elevator comes to a stop the force necessary to keep it up decreases and so the book "fells" less heavy and so moves up a bit. The same thing happens when you start going down. As the elevator comes to a stop on the way down, the force needed to keep it up is again greater than the weight so the book falls.

51 **a** The forces on the man are: the tension from the rope, T, on his hands upward (this is the same as the force with which he pulls down on the rope); his weight 700 N downward; the reaction, R, from the elevator floor upward.

b On the elevator they are: the weight downward; the reaction, R, from the man downward; the tension, T, in the rope upward.

c The forces on the man *and* the elevator **together** are $2T$ upwards (one T on the elevator at the top and one T on the man from the rope). The combined weight is 1000 N. Thus

$2T - 1000 = 100a = 100 \times 0.50 \Rightarrow T = 525$ N. The net force on the man is

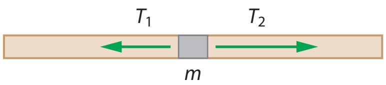
$R + T - 700 = 70a = 35 \Rightarrow R = 700 - 525 + 35 \Rightarrow R = 210$ N.

d We know that

$$\begin{aligned} 2T - 1000 &= 100a \\ R + T - 700 &= 70a \end{aligned} \quad \text{and so} \quad \begin{aligned} 2T - 1000 &= 100a \\ 2R + 2T - 1400 &= 140a \end{aligned}$$

Subtracting we get $2R - 400 = 40a$ and so $a = \dfrac{2R - 400}{40} = \dfrac{600 - 400}{40} = 5.0 \text{ m s}^{-2}$.

52 Suppose the tensions at some point were different.



The net force on the bit of string of mass m is $T_2 - T_1 = ma$. But the string is massless, $m = 0$ and so $T_2 - T_1 = 0$ meaning that the tensions are the same.

53 **a** Treat the two masses as one body. The net force is 60.0 N and so the acceleration is $a = \dfrac{60.0}{40.0} = 1.50 \text{ m s}^{-2}$. The net force on the back block is the tension in the string and so $T = ma = 10.0 \quad 1.50 = 15.0$ N.

b The tension would now be $T = Ma = 30.0 \quad 1.50 = 45.0$ N.

54 For three (planar) forces to be in equilibrium, any one force must have a magnitude that is in between the sum and the difference of the other two forces. This is the case here. Now, the resultant of the 4.0 N and the 6.0 N forces must have a magnitude of 9.0 N. This when the 9.0 N force is suddenly removed, the net force on the body is 9.0 N. The acceleration is therefore $a = \dfrac{9.0}{3.0} = 3.0 \text{ m s}^{-2}$.

2.3 Work, energy and power

55 The work done is $W = Fd\cos\theta = 24 \times 5.0 \times \cos 0° = 120$ J.

56 The work done is $W = Fd\cos\theta = 2.4 \times 3.2 \times \cos 180° = -7.7$ J.

57 The work done is $W = Fd\cos\theta = 25 \times 15 \times \cos 20° = 352 \approx 3.5 \times 10^2$ J.

58 The change in kinetic energy is $\Delta E_K = \dfrac{1}{2} \times 2.0(0 - 5.4^2) = -29.16$ J. This equal the work done by the resistive force i.e. $f \times 4.0 \times \cos 180° = -29.16 \Rightarrow f = 7.3$ N.

59 The work done has gone to increase the elastic potential energy of the spring i.e.

$W = \dfrac{1}{2} \times 200 \times (0.050^2 - 0.030^2) = 0.16$ J.

60 **a** **i** The minimum energy is required to just get the ball at A. Then,

$\dfrac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 4.0} = 8.9 \text{ m s}^{-1}$.

ii At position B, $\dfrac{1}{2} \times 1.0 \times 8.9^2 = 1.0 \times 9.8 \times 2.0 + \dfrac{1}{2} \times 1.0 \times v^2 \Rightarrow v = 6.3 \text{ m s}^{-1}$.

b At A: $\dfrac{1}{2} \times 1.0 \times 12.0^2 = \dfrac{1}{2} \times 1.0 \times v^2 + 1.0 \times 9.8 \times 4.0 \Rightarrow v = 8.1 \text{ m s}^{-1}$.

At B: $\dfrac{1}{2} \times 1.0 \times 12.0^2 = \dfrac{1}{2} \times 1.0 \times v^2 + 1.0 \times 9.8 \times 2.0 \Rightarrow v = 10 \text{ m s}^{-1}$.

61 The total energy at A is $E_A = 8.0 \quad 9.8 \quad 12 + \frac{1}{2} \quad 8.0 \quad 6.0^2 = 1085$ J. At B it is $E_B = \frac{1}{2} \quad 8.0 \quad 12^2 = 576$ J.

The total energy decreased by $1085 - 576 = 509$ J and this represents the work done by the resistive forces. The distance traveled down the plane is 24 m and so $f \times 24 = 509 \Rightarrow f = 21$ N.

62 **a** The work done is the area under the curve. This is a trapezoid and so $W = \frac{15+7.0}{2} \quad 8.0 = 88$ J.

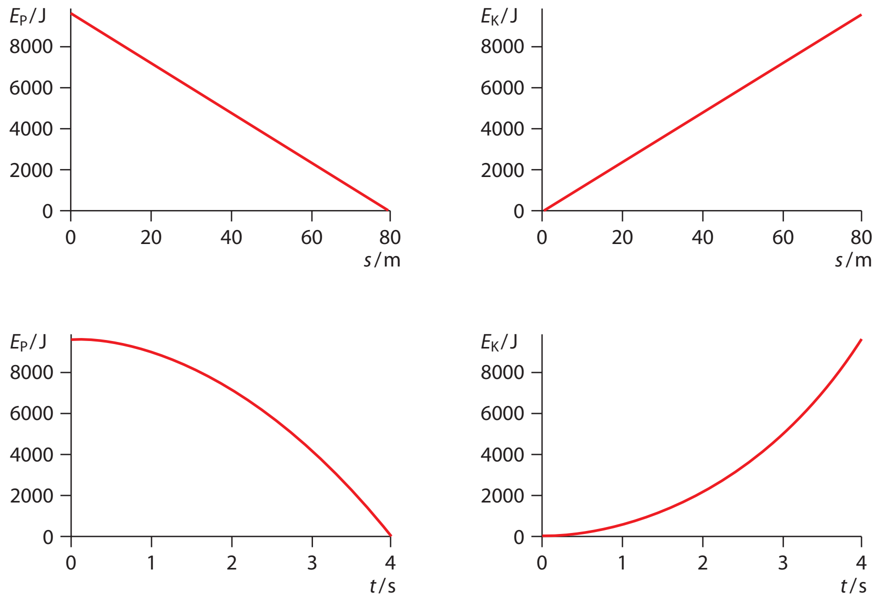
b The work done is the change in kinetic energy and so $\frac{1}{2}mv^2 = W$, giving $v = \sqrt{\frac{2 \times 88}{2.0}} \approx 9.4 \text{ m s}^{-1}$.

63 **a** **i** The potential energy is given by $E_P = mgH - mgs = 12 \times 10 \times 80 - 12 \times 10s = 9600 - 120s$.

ii The kinetic energy is $E_K = E_{Total} - E_P = 9600 - (9600 - 120s) = 120s$.

b **i** Since the distance fallen s is given by $s = \frac{1}{2}gt^2$ the answers in **a** become $E_P = 9600 - 600t^2$

ii $E_P = 600t^2$. These four equations are in the graphs here.



c In the presence of a *constant* resistance force, the graph of potential energy against distance will not be affected. The graph of kinetic energy against distance will be a straight line with a smaller slope since the final kinetic energy will be less. The graph of potential energy against time will have the same shape but will reach zero in a longer time. Similarly, the kinetic energy – time graph will reach a smaller maximum value in a longer time.

64 From $P = Fv$, $F \times \frac{100 \times 10^3}{3600} = 90 \times 10^3 \Rightarrow F = 3240 = 3.24 \times 10^3 \text{ N} \approx 3 \times 10^3$ N.

65 **a** From $P = Fv$, and $F = Mg = 1.2 \quad 10^4$ N we find $v = \frac{2.5 \times 10^3}{1.2 \times 10^4} = 0.21 \text{ m s}^{-1}$.

b Most likely some of the power produced by the motor gets dissipated in the motor itself due to frictional forces and get converted into thermal energy and is not used to raise the block.

66 **a** The work done is $mgh = 50 \quad 9.8 \quad 15 = 7350$ J. The power is thus $\frac{7350}{125} = 59$ W.

b $e = \frac{59}{80} = 0.74$

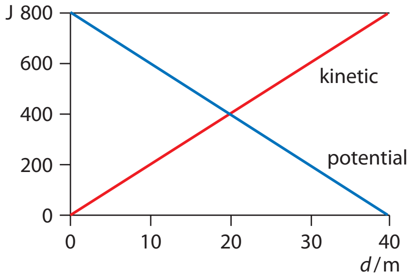
c The work required is double and the time is therefore also double, 250 s.

67 From $P = Fv$, $F \times \dfrac{240 \times 10^3}{3600} = 250 \times 10^3 \Rightarrow F = 3750 \approx 3.8 \times 10^3$ N.

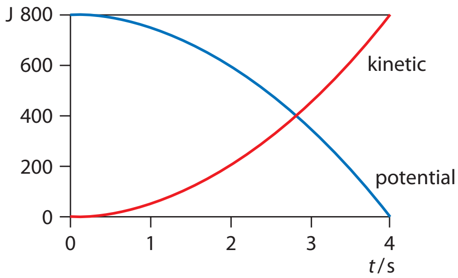
68 Electrical energy from the motor is converted to potential energy and thermal energy if the elevator is just pulled up. Normally a counterweight is being lowered as the elevator is being raised which means that the net change in gravitational potential energy is zero (assuming that the counterweight is equal in weight to the elevator). In this case all the electrical energy goes into thermal energy.

69 **a** The acceleration of the mass is $g \sin 30° = 5.0$ m s^{-2} and so the speed is $v = 5.0 \quad t$. Hence the kinetic energy (in joule) as a function of time is $E_K = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \quad 4.0 \quad (5.0 \quad t)^2 = 50t^2$. The distance s traveled down the plane is given by $s = \dfrac{1}{2}at^2 = \dfrac{1}{2} \quad 5.0t^2 = 2.5t^2$ and so the vertical distance h from the ground is given by $h = 20 - s \sin 30° = 20 - 1.25t^2$. Hence the potential energy is $E_P = mgh = 4.0 \times 10 \times (20 - 1.25t^2) = 800 - 50t^2$. These are the functions to be graphed with the results as shown in the answers in the textbook, page 532.

b



c



70 **a** The net force is zero since the velocity is constant and so $T = mg \sin\theta$.

b $W_T = Fd = mgd \sin\theta$

c $W_W = -mgh = -mgd \sin\theta$

d $W_N = 0$ since the angle is a right angle.

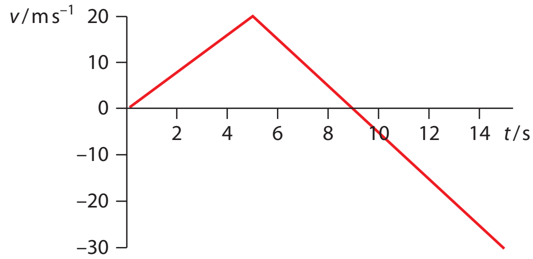
e Zero since the kinetic energy is constant (or zero because $W_W + W_W + W_N = mgd \sin\theta - mgd \sin\theta + 0 = 0$).

71 **a** From $s = \dfrac{1}{2}at^2$ we find $s = \dfrac{1}{2} \quad 4.0 \quad 5.0^2 = 50$ m.

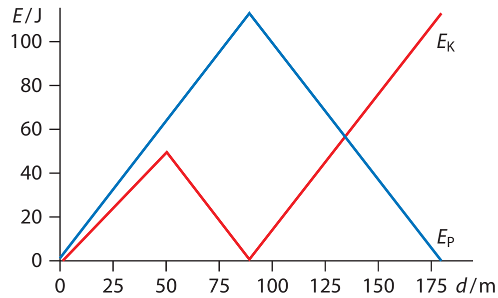
b At 5.0 s the speed acquired is $v = at = 4.0 \times 5.0 = 20$ m s^{-1}. From then on the acceleration becomes a deceleration of $a = g \sin\theta = 10 \times 0.5 = 5.0$ m s^{-2}. Then from $v^2 = u^2 - 2as$ we find $0 = 20^2 - 2 \times 5.0 \times s$ giving $s = \dfrac{20^2}{2 \quad 5.0} = 40$ m. The total distance up the plane is thus 90 m.

c The car will travel the distance of 90 m from rest with an acceleration of 5.0 m s^{-2} and so from $s = \dfrac{1}{2}at^2$ we get $90 = \dfrac{1}{2} \quad 5.0 \quad t^2$ giving $t^2 = \dfrac{180}{5.0} = 36 \Rightarrow t = 6.0$ s. (The car took 5.0 s to get to the 50 m up the hill. The remaining 40 m were covered in $0 = 20 - 5.0 \times t \Rightarrow t = 4.0$ s. The time from the start to get back down again is thus 15 s.)

d For the first 5 s the velocity is given by $v = 4.0t$. For the rest of the motion the velocity is $v = 20 - 5.0t$.



e Potential energy: For a distance d traveled up the plane the vertical distance is $h = \frac{d}{2}$ and so the potential energy is $E_P = mgh = 0.250 \quad 10 \quad \frac{d}{2} = 1.25d$. At 90 m, (the highest the car gets on the plane) we have $E_P = 1.25 \times 90 = 112.5 \approx 112$ J. For the last 90 m (the way down) the graph is decreasing symmetrically. These facts give the graph ion the answers in the textbook. Kinetic energy: For the first 50 m traveled we have that: the speed is given by $v^2 = u^2 + 2as = 0 + 2 \quad 4.0 \quad s = 8s$ and so the kinetic energy is $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \quad 0.250 \quad 8s = s$. (The kinetic energy attained at 50 m is thus 50 J.) In the next 40 m the speed is given by $v^2 = u^2 + 2as = 20^2 - 2 \times 5.0 \times s = 400 - 10s$ and so the kinetic energy decreases according to $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.250(400 - 10s) = 50 - 1.25s$. At 40 m the kinetic energy becomes zero. From then on it increases according to $E_K = \frac{1}{2} \quad 0.250 \quad 2 \quad 5.0 \quad s = 1.25s$. Putting all these together gives the graph below.



f The mechanical energy (kinetic plus potential) will be conserved when there are no external forces acting on the car (other than gravity) i.e. after the first 5.0 s.

g The motor was exerting a force F up the plane given by

$F - mg\sin\theta = ma \Rightarrow F = ma + mg\sin\theta = 0.250 \times 4.0 + 0.250 \times 10 \times \frac{1}{2} = 2.25$ N. The average speed up the plane was $\bar{v} = \frac{0 + 20}{2} = 10$ m s^{-1} and so the average power is $\bar{P} = F\bar{v} = 2.25 \quad 10 = 22.5$ W.

The maximum power was $P = Fv = 2.25 \quad 20 = 45$ W.

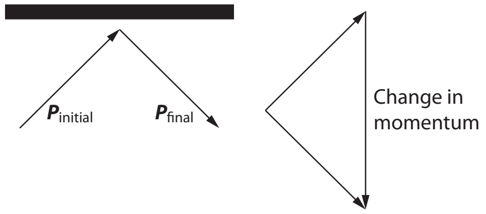
2.4 Momentum and impulse

72 $F_{ave} = \frac{\Delta p}{\Delta t} = \frac{12.0}{2.00} = 6.00$ N

73 **a** Impulse $= \Delta p = p_{final} - p_{initial} = 0.150 \times (-3.00) - 0.150 \times 3.00 = -0.900$ N s. (b) $|F_{ave}| = \left|\frac{\Delta p}{\Delta t}\right| = \frac{0.900}{0.125} = 7.20$ N.
This is the force exerted on the ball by the wall and so by Newton's third law this is also the force the ball exerted on the wall.

74 The total momentum before the collision is $m \times v + 2m \times \left(-\frac{v}{2}\right) = 0$. This is also the momentum after. If u is the speed after the collision then $3m \times u = 0 \Rightarrow u = 0$.

75 a The change in momentum is given by the following vector diagram. The angle between the vectors is a right angle.



The magnitude of the initial and of the final momentum is $p = 0.250 \quad 4.00 = 1.00$ N s. The direction of the change of momentum is given in the diagram. Its magnitude is $\sqrt{1.00^2 + 1.00^2} = \sqrt{2.00} = 1.41$ N s.

b It depends on what we take the system to be. If the system is just the ball then its momentum is not conserved since there is an external force acting on the ball (the force from the wall). If, on the other hand, we take the system to be the ball and the wall then the total momentum is conserved since there is now no external force acting on the system. The wall will acquire a momentum equal and opposite to the momentum change of the ball.

76 The initial total momentum is $4.0 \times 24 - 12.0 \times 2.0 = +72$ N s. The final total momentum is $-4.0 \times 3.0 + 12 \times v$.

Hence $-12 + 12v = 72 \Rightarrow v = +7.0 \text{ m s}^{-1}$. (The ball moves to the right.)

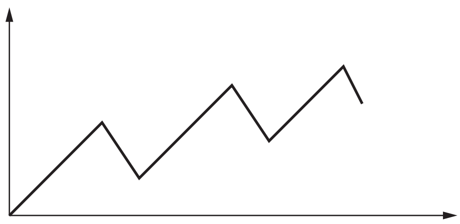
77 a The impulse is the area under the graph and so equals (we use the area of a trapezoid) $\frac{17+7}{2} \quad 8.0 = 96$ N s.

b Since the impulse is the change in momentum: $96 = mv - 0 \Rightarrow v = \frac{96}{3.00} = 32 \text{ m s}^{-1}$.

c Now, $96 = 0 - mv \Rightarrow v = -\frac{96}{3.00} = -32 \text{ m s}^{-1}$.

78 a The impulse supplied to the system is (area under curve) $3 \times 0.5 \times 100 - 25 \times 4 = 50$ N s. This is the change in momentum i.e. $25 \times \Delta v = 50 \text{ N s} \Rightarrow \Delta v = 2.0 \text{ m s}^{-1}$.

b (You can easily work out the numbers and slopes on the axes.)



79 a The ball is dropped from a height of h_1 so its speed right before impact will be given by (applying conservation of energy) $mgh_1 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{2gh_1}$. The ball will leave the floor on its way up with a speed found in the same way: $\frac{1}{2}mv_2^2 = mgh_2 \Rightarrow v_2 = \sqrt{2gh_2}$. The change in momentum is therefore $mv_2 - (-mv_1) = m(\sqrt{2gh_2} + \sqrt{2gh_1})$ and hence the net average force is $\frac{\Delta p}{\Delta t} = m\frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\tau}$.

b $F = 0.250 \times \frac{\sqrt{2 \times 9.81 \times 6.0} + \sqrt{2 \times 9.81 \times 8.0}}{0.125} = 46.8 \approx 47$ N. This is the average net force on the ball. The forces on the ball are the reaction from the floor R and its weight so $R - mg = F \Rightarrow R = mg + F = 46.8 + 0.250 \times 9.81 = 49.2 \approx 49$ N. By Newton's third law this is also the force on the floor exerted by the ball.

80 **a** The question assumes the ball hits normally. From the previous problem the net force is
$$\frac{\Delta p}{\Delta t} = m\frac{v_2 - (-v_1)}{\tau} = m\frac{v_2 + v_1}{\tau}.$$

b This equals $R - mg$ where R is the average force exerted on the ball by the floor. Hence $R = m\frac{v_2 + v_1}{\tau} + mg$.

81 **a** From 0.5 s to 1.5 s i.e. for 1 s.

b A rough approximation would be to treat the area a triangle (of area $\frac{1}{2}$ 1.0 120 = 60 N s) but this too rough and would not be acceptable in a exam. There are roughly 120 rectangles in the area and each has area 0.1 4.0 = 0.4 N s so that the total area is $0.4 \times 120 = 48 \approx 50$ N s.

c From $F_{average}\Delta t = \Delta p$ we thus find $F_{average} = 50$ N.

82 The initial momentum is zero and will remain zero. Therefore the speed with which the 4.0 kg moves off is $2.0 \times 3.0 = 4.0 \times v \Rightarrow v = 1.5 \text{ m s}^{-1}$

The total kinetic energy of the two bodies is $\frac{1}{2} \times 2.0 \times 3.0^2 + \frac{1}{2} \times 4.0 \times 1.5^2 = 13.5 \approx 14$ J.

83 The initial momentum is zero and so must remain zero for the rocket-fuel system. $0 = (5000 - m)v - m \times (1500 - v)$.

In the first 1 second, $v = 15 \text{ m s}^{-1}$ and so $(5000 - m) \times 15 = m \times 1485 \Rightarrow m = \frac{75000}{1500} = 50.0$ kg in 1 second.