Answers to test yourself questions

Topic 1

1.1 Measurement in physics

1 Taking the diameter of a proton to be order 10^{-15} m we find $\dfrac{10^{-15}}{3\times 10^{8}} = 0.3\times 10^{-23} = 3\times 10^{-24} \approx 10^{-24}$ s.

2 The mass of the Earth is about $6 \quad 10^{24}$ kg and the mass of a hydrogen atom about 2×10^{-27} kg so we need $\dfrac{6\times 10^{24}}{2\times 10^{-27}} = 3\times 10^{51} \approx 10^{51}$.

3 $\dfrac{10^{17}}{10^{-43}} = 10^{60}$

4 A heartbeat lasts or 1 s so $\dfrac{75\times 365\times 24\times 3600}{1} \approx 8\times 4\times 2\times 4\times 10^{7} \approx 2.6\times 10^{9} \approx 10^{9}$.

5 $\dfrac{10^{41}}{10^{30}} = 10^{11}$

6 $\dfrac{10^{21}}{1.5\times 10^{11}} \approx 10^{10}$

7 There are 300 g of water in the glass and hence $\dfrac{300}{18} \approx \dfrac{300}{20} = 15$ moles of water. Hence the number of molecules is $15\times 6\times 10^{23} = 90\times 10^{23} \approx 10^{25}$.

8 There are $6 \quad 10^{4}$ g of water in the body and hence $\dfrac{6\times 10^{4}}{18} \approx 0.3\times 10^{4} = 3\times 10^{3}$ moles of water. Hence the number of molecules is $3\times 10^{3}\times 6\times 10^{23} = 18\times 10^{26} \approx 10^{27}$.

9 The mass is about 1.7×10^{-27} kg and the radius about 10^{-15} m so the density is $\dfrac{1.7\times 10^{-27}}{\frac{4\pi}{3}\times (10^{-15})^{3}} \approx \dfrac{1.7\times 10^{-27}}{4\times 10^{-45}} = 0.5\times 10^{18} = 5\times 10^{17}$ kg m^{-3}.

10 $\dfrac{10^{21}}{3\times 10^{8}} \approx 0.3\times 10^{13} = 3\times 10^{12}$ s $\approx 10^{5}$ yr

11 **a** $E = 2.5\times 1.6\times 10^{-19} = 4.0\times 10^{-19}$ J

b $E = \dfrac{8.6\times 10^{-18}}{1.6\times 10^{-19}} = 54$ eV

12 $V = (2.8\times 10^{-2})^{3} = 2.2\times 10^{-5}$ m^{3}

13 $a = (588\times 10^{-9})^{1/3} = 8.38\times 10^{-3}$ m

14 **a** 200 g
b 1 kg
c 400 g

15 The mass is about 10^{30} kg and the radius is $6.4 \quad 10^6$ m so the density is of about

$$\frac{10^{30}}{\frac{4\pi}{3}(6.4\times10^6)^3} \approx 9\times10^8 \approx 10^9 \text{ kg m}^{-3}.$$

16 In SI units the acceleration is $\dfrac{100\times\dfrac{10^3}{3600}}{4} = \dfrac{\dfrac{10^5}{4\times10^3}}{4} = \dfrac{10^2}{16} \approx 6.25 \text{ m s}^{-2} \approx 0.7g.$

17 Assuming a mass of 70 kg made out of water we have $7 \quad 10^4$ g of water in the body and hence $\dfrac{7\times10^4}{18} \approx 0.5\times10^4 = 5\times10^3$ moles of water. Hence the number of molecules is $5\times10^3\times6\times10^{23} = 30\times10^{26} \approx 3\times10^{27}$. Each molecule contains 2 electrons from hydrogen and 8 from oxygen for a total of $10\times3\times10^{27} \approx 10^{28}$ electrons.

18 The ratio is $\dfrac{F_e}{F_g} = \dfrac{ke^2}{Gm^2} = \dfrac{9\times10^9\times(1.6\times10^{-19})^2}{6.7\times10^{-11}\times(9.1\times10^{-31})^2} \approx \dfrac{9\times10^9\times3\times10^{-38}}{7\times10^{-11}\times81\times10^{-62}} \approx \dfrac{3\times10^{44}}{63} \approx \dfrac{10^{44}}{20} \approx 5\times10^{42}.$

19 $f = cm^x k^y$. The units of m is kg i.e. M and those of k are $\dfrac{\text{N}}{\text{m}} = \dfrac{\text{kg m s}^{-2}}{\text{m}} = \text{kg s}^{-2} = \text{M T}^{-2}$. Hence

$$\text{T} = \text{M}^x(\text{M T}^{-2})^y = \text{M}^{x+y}\ \text{T}^{2y}.$$

From this we deduce that

$$x + y = 0$$

$$2y = 1 \Rightarrow y = \frac{1}{2} \Rightarrow x = -\frac{1}{2}$$

Thus, $f = c\sqrt{\dfrac{k}{m}}$.

20 $P = \dfrac{1.2 \quad 9.81 \quad 5.55}{2.450} = 2.6667 \quad 10^1$ W. The answer must be given to 2 s.f. and so

$$P = \frac{1.2 \quad 9.81 \quad 5.55}{2.450} = 2.7 \quad 10^1 \text{ W}.$$

21 $E_K = \dfrac{1}{2} \quad 5.00 \quad 12.5^2 = 3.9063 \quad 10^2$ J. The answer must be given to 3 s.f. and so $E_K = 3.91 \quad 10^2$ J.

22 **a** $\dfrac{243}{43} \approx \dfrac{250}{50} = 5$

b $2.80\times1.90 \approx 3\times2 = 6$

c $\dfrac{312\times480}{160} \approx \dfrac{300\times500}{150} = 1000$

d $\dfrac{8.99\times10^9\times7\times10^{-16}\times7\times10^{-6}}{(8\times10^2)^2} \approx \dfrac{10^{10}\times50\times10^{-22}}{60\times10^4} \approx 10^{-16}$

e $\dfrac{6.6\times10^{-11}\times6\times10^{24}}{(6.4\times10^6)^2} \approx \dfrac{50\times10^{13}}{40\times10^{12}} \approx 10$

1.2 Uncertainties and errors

23 $sum = (180 \pm 8)\ \text{N} = (1.8 \pm 0.8) \quad 10^2\ \text{N}$

$dif = (60 \pm 8)\ \text{N} = (6.0 \pm 0.8) \quad 10^1\ \text{N}$

24 **a** $Q_0 = \frac{a}{b} = \frac{20}{10} = 2;\ \frac{\Delta Q}{Q_0} = \frac{\Delta a}{a} + \frac{\Delta b}{b} = \frac{1}{20} + \frac{1}{10} = 0.15 \Rightarrow \Delta Q = 2.0 \times 0.15 = 0.30$. Hence $Q = 2.0 \pm 0.3$.

b $Q_0 = 2 \quad 20 + 3 \quad 15 = 85;\ \Delta Q = 2 \times 2 + 3 \times 3 = 13$. Hence $Q = 85 \pm 13 \approx (8.5 \pm 0.1) \times 10^1$

c $Q_0 = 50 - 2 \times 24 = 2;\ \Delta Q = 1 + 2 \times 1 = 3$. Hence $Q = 2 \pm 3$

d $Q_0 = 1.00 \quad 10^2;\ \frac{\Delta Q}{Q_0} = 2 \times \frac{\Delta a}{a} = 2 \times \frac{0.3}{10.0} = 6.00 \times 10^{-2} \Rightarrow \Delta Q = 100 \times 6.00 \times 10^{-2} = 0.06 \times 10^{-2}$.

Hence $Q = 1.00 \quad 10^2 \pm 0.06 \quad 10^2 = (1.00 \pm 0.06) \quad 10^2$

e $Q_0 = \frac{100^2}{20^2} = 25;\ \frac{\Delta Q}{Q_0} = 2 \times \frac{\Delta a}{a} + 2 \times \frac{\Delta b}{b} = 2 \times \frac{5}{100} + 2 \times \frac{2}{20} = 3.0 \times 10^{-1} \Rightarrow \Delta Q = 25 \times 3.0 \times 10^{-1} = 7.5 \approx 8$

Hence $Q = 25 \pm 8$

25 $F_0 = \frac{2.8 \quad 14^2}{8.0} = 68.6\ \text{N}$

$\frac{\Delta F}{F_0} = \frac{\Delta m}{m} + 2 \times \frac{\Delta v}{v} + \frac{\Delta r}{r} = \frac{0.1}{2.8} + 2 \times \frac{2}{14} + \frac{0.2}{8.0} = 0.3464 \Rightarrow \Delta F = 68.6 \times 0.3464 = 23.7 \approx 20\ \text{N}.$

Hence $F = (68.6 \pm 20)\ \text{N} \approx (7 \pm 2) \times 10^1\ \text{N}$

26 **a** $A_0 = \pi R^2 = 18.096\ \text{cm}^2.\ \frac{\Delta A}{A_0} = 2 \times \frac{\Delta R}{R} = 2 \times \frac{0.1}{2.4} = 0.0833 \Rightarrow \Delta A = 18.096 \times 0.0833 = 1.51 \approx 2\ \text{cm}^2.$

Hence $A = (18.096 \pm 2)\ \text{cm}^2 \approx (18 \pm 2)\ \text{cm}^2.$

b $S_0 = 2\pi R = 15.08\ \text{cm}.\ \frac{\Delta S}{S_0} = \frac{\Delta R}{R} = \frac{0.1}{2.4} = 0.04167 \Rightarrow \Delta S = 15.08 \times 0.04167 = 0.628\ \text{cm}^2.$

Hence $S_0 = (15.08 \pm 0.628)\ \text{cm} \approx (15 \pm 1)\ \text{cm}.$

27 $A_0 = ab = 37.4\ \text{cm}^2.\ \frac{\Delta A}{A_0} = \frac{\Delta a}{a_0} + \frac{\Delta b}{b_0} = \frac{0.2}{4.4} + \frac{0.3}{8.5} = 0.080749 \Rightarrow \Delta A = 37.4 \times 0.080749 = 3.02 \approx 3\ \text{cm}^2.$

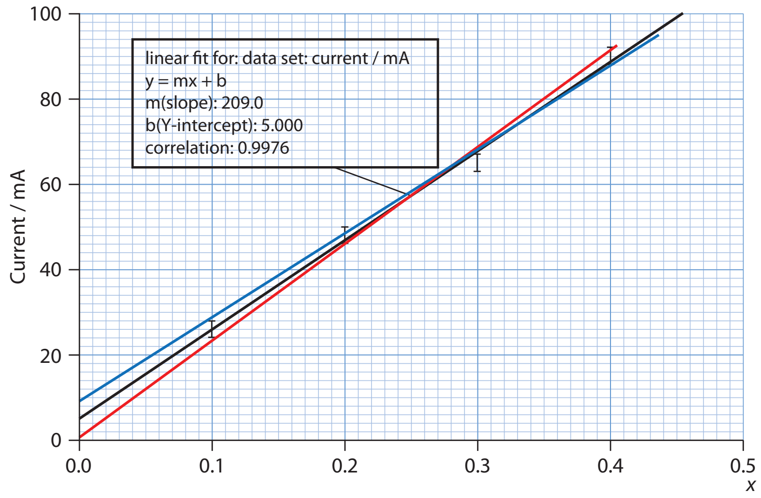
Hence $A = (37.4 \pm 3)\ \text{cm}^2 \approx (37 \pm 3)\ \text{cm}^2.$

$P_0 = 2(a + b) = 25.8\ \text{cm}$. $\Delta P = 2 \times \Delta a + 2 \times \Delta b = 2 \times 0.2 + 2 \times 0.3 = 1.0\ \text{cm}$. Hence $P = (25.8 \pm 1)\ \text{cm} \approx (26 \pm 1)\ \text{cm}$.

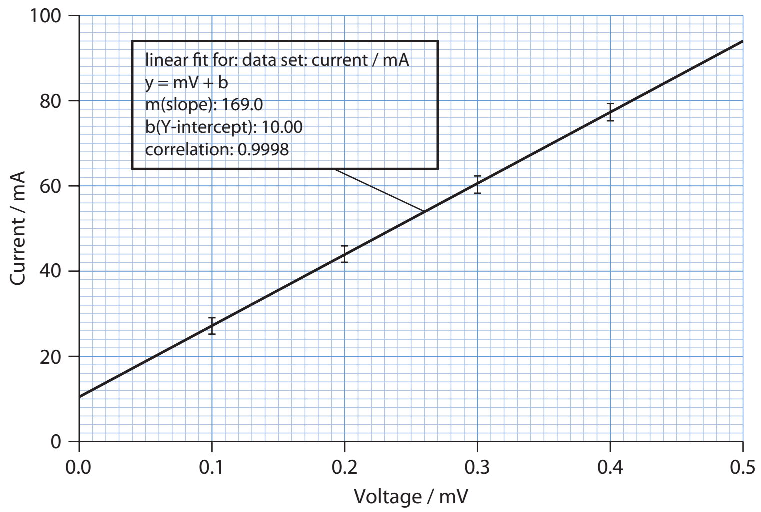
28 $\frac{\Delta T}{T_0} = \frac{1}{2}\frac{\Delta L}{L_0}$ (assuming g is accurately known). Hence $\frac{\Delta T}{T_0} = \frac{1}{2} \times 2\% = 1\%$.

29 $\frac{\Delta V}{V_0} = 2 \times \frac{\Delta R}{R_0} + \frac{\Delta h}{h_0} = 2 \times 4\% + 4\% = 12\%$

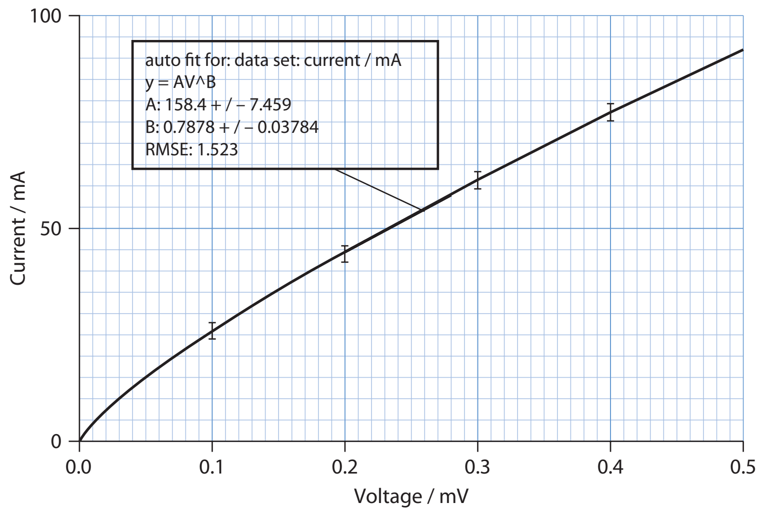
30 The line of best-fit does not go through the origin. There is a vertical intercept of about 4 mA. Lines of maximum and minimum slope give intercepts of about 0 and 9 mA implying an error in the intercept of about 4 mA. The intercept is thus (4 ± 4) mA. This just barely includes the origin so the conclusion has to be that they can be proportional.



31 The vertical intercept is about 10 mA. No straight line can be made to pass through the origin and the error bars unless a systematic error of about 10 mA in the current is invoked.



However, a line of best fit that is a curve can also be fitted through the data and that does go through the origin. (However, it may be objected that this particular functional form is chosen – at low voltages we might expect a straight line (Ohm's law). So a different functional form may have to be tried.)



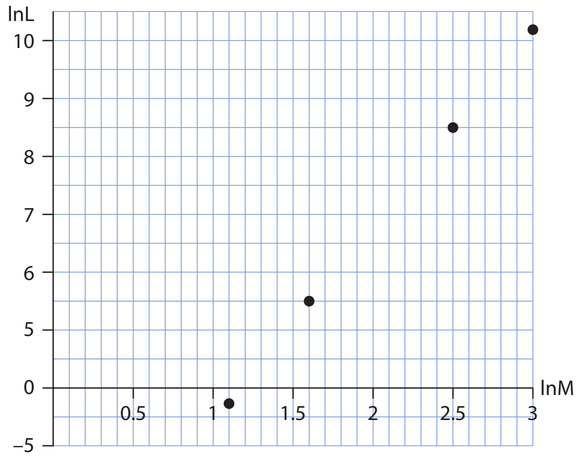
32 Let P the common perimeter. Then the radius of the circle satisfies $2\pi R = P \Rightarrow R = \frac{P}{2\pi}$ and the side of the square $4a = P \Rightarrow a = \frac{P}{4}$. The circle area is then $A_c = \pi\left(\frac{P}{2\pi}\right)^2 = \frac{P^2}{4\pi}$. The square area is $A_s = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$ and is smaller.

33 **a** The initial voltage V_0 is such that $\ln V_0 = 4 \Rightarrow V_0 = e^4 = 55$ V.

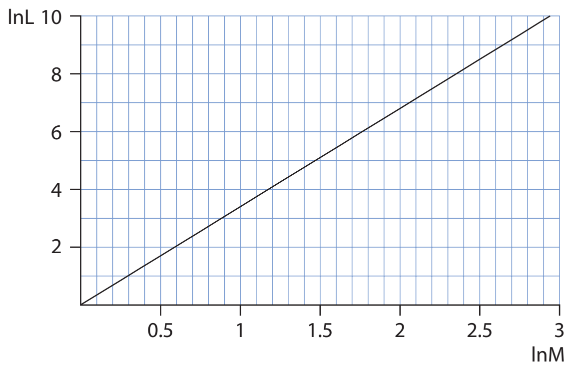
b When $V = \frac{V_0}{2} \approx 27$ V, $\ln V = \ln 27 \approx 3.29$. From the graph when $\ln V \approx 3.29$ we find $t \approx 7$ s.

c Since $V = V_0 e^{-t/RC}$, taking logs, $\ln V = \ln V_0 - \frac{t}{RC}$ so a graph of $\ln V$ versus time gives a straight line with slope equal to $-\frac{1}{RC}$. The slope of the given graph is approximately $\frac{4-2}{0-20} = -0.10$. Hence $-\frac{1}{RC} = -0.10 \Rightarrow R = \frac{1}{0.10 \times C} = \frac{1}{0.10 \times 5 \times 10^{-6}} = 2 \times 10^6\ \Omega$.

34 We expect $L = kM^{\alpha}$ and so $\ln L = \ln k + \alpha \ln M$. A graph of $\ln L$ versus $\ln M$ is shown below. The slope is α.



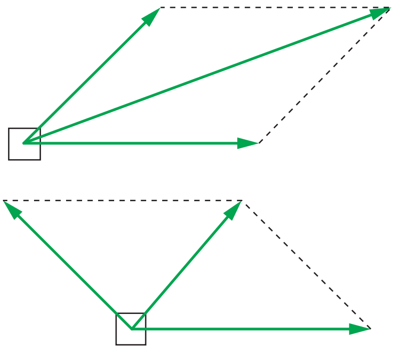
Drawing a best-fit line gives:



Measuring the slope gives $\alpha = 3.4$.

1.3 Vectors and scalars

35

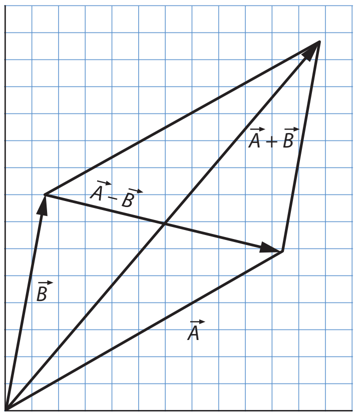


36 **a** $\vec{A}+\vec{B}$:

length 9 cm

$\Rightarrow$ F $\simeq$ 18 N

$\theta \simeq 49°$



b $\vec{A}-\vec{B}$:

scale:
1 cm $\leftrightarrow$ 2 N

length 4.5 cm

$\Rightarrow$ F $\simeq$ 9 N

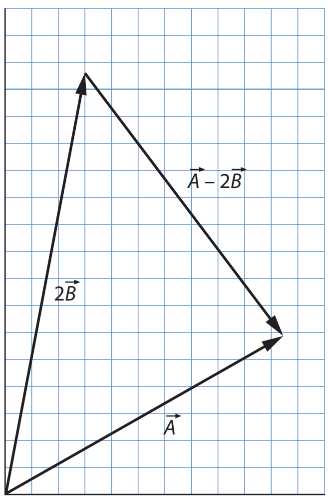
$\Theta \simeq 14°$ below horizontal.

c $\vec{A}-2\vec{B}$:

length 6.1 cm

$\Rightarrow$ F $\simeq$ 12.2 N

$\Theta \simeq 50°$ below horizontal.



37 The components are:

$A_x = 12 \cos 30° = 10.39$ $\quad B_x = 8.00 \cos 80° = 1.389$

$A_y = 12 \sin 30° = 6.00$ $\quad A_y = 8.00 \sin 80° = 7.878$

Hence

a $(A+B)_x = 10.39 + 1.389 = 11.799$

$(A+B)_y = 6.00 + 7.878 = 13.878$

The vector $\vec{A}+\vec{B}$ has magnitude $\sqrt{11.799^2 + 13.878^2} = 18.2$ and is directed at an angle

$\theta = \arctan\frac{13.878}{11.799} = 49.6°$ to the horizontal.

b $(A-B)_x = 10.39 - 1.389 = 9.001$

$(A-B)_y = 6.00 - 7.878 = -1.878$

The vector $\vec{A}-\vec{B}$ has magnitude $\sqrt{9.001^2 + 1.878^2} = 9.19$ and is directed at an angle

$\theta = \arctan-\frac{1.878}{9.001} = -11.8°$ (below) the horizontal.

c $(A-2B)_x = 10.39 - 2 \times 1.389 = 7.612$

$(A-2B)_y = 6.00 - 2 \times 7.878 = -9.756$

The vector $\vec{A}-2\vec{B}$ has magnitude $\sqrt{7.612^2 + 9.756^2} = 12.4$ and is directed at an angle

$\theta = \arctan-\frac{9.756}{7.612} = -52.0°$ (below) the horizontal.

38 **a** $\sqrt{4.0^2 + 4.0^2} = 5.66$ cm in a direction $\theta = 180° + \arctan\frac{4.0}{4.0} = 225°$.

b $\sqrt{124^2 + 158^2} = 201$ km in a direction $\theta = \arctan-\frac{158}{124} = -52°$.

c $\sqrt{0^2 + 5.0^2} = 5.0$ m at $\theta = 270°$ or $\theta = -90°$.

d $\sqrt{8.0^2 + 0^2} = 8.0$ N at $\theta = 0°$.

39 **a** $\sqrt{2.00^2 + 3.00^2} = 3.61$ at $\theta = \arctan\frac{3.00}{2.00} = 56.3°$

b $\sqrt{2.00^2 + 5.00^2} = 5.39$ at $\theta = 180° - \arctan\frac{5.00}{2.00} = 112°$

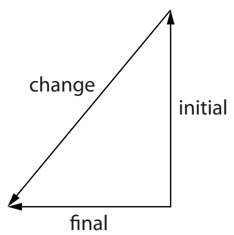
c $\sqrt{0^2 + 8.00^2} = 8.00$ at $\theta = 90°$

d $\sqrt{4.00^2 + 2.00^2} = 4.47$ at $\theta = \arctan-\frac{2.00}{4.00} = -26.6°$

e $\sqrt{6.00^2 + 1.00^2} = 6.08$ at $\theta = \arctan\frac{1.00}{6.00} = 9.46°$

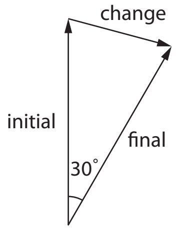
40 The displacement has components $\Delta r_x = 4 - 2 = 2$ and $\Delta r_y = 8 - 2 = 6$.

41 A diagram is:



The magnitude of the change n the velocity vector is $\sqrt{10^2+10^2}=14.1\ \text{m s}^{-1}$. The vector makes an angle of 45° with the horizontal as shown in the diagram.

42 A diagram is:



The other two angles of the triangle are each $\frac{1}{2}(180°-30°)=75°$. Using the sine rule we find

$$\frac{\Delta p}{\sin 30°}=\frac{p}{\sin 75°}\Rightarrow \Delta p = p\times\frac{\sin 30°}{\sin 75°}=0.518p\approx 0.52p.$$

43 The components of the velocity vector at the various points are:
A: $v_{Ax}=-4.0\ \text{m s}^{-1}$ and $v_{Ay}=0$
B: $v_{Bx}=+4.0\ \text{m s}^{-1}$ and $v_{By}=0$
C: $v_{Cx}=0$ and $v_{Cy}=4.0\ \text{m s}^{-1}$
Hence

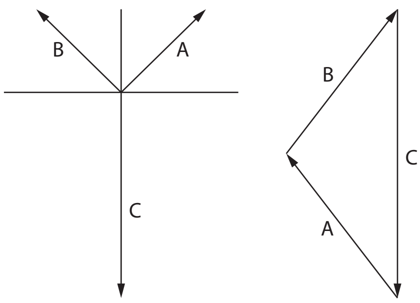
a From A to B the change in the velocity vector has components $v_{Bx}-v_{Ax}=+4.0-(-4.0)=8.0\ \text{m s}^{-1}$ and $v_{By}-v_{Ay}=0-0=0$.

b From B to C the change in the velocity vector has components $v_{Cx}-v_{Bx}=0-4.0=-4.0\ \text{m s}^{-1}$ and $v_{Cy}-v_{By}=4.0-0=4.0\ \text{m s}^{-1}$.

c From A to C the change in the velocity vector has components $v_{Cx}-v_{Ax}=0-(-4.0)=+4.0\ \text{m s}^{-1}$ and $v_{Cy}-v_{Ay}=4.0-0=4.0\ \text{m s}^{-1}$. The change in the vector from A to C is the sum of the change from A to B plus the change from B to C.

44 **A** $A_x=-10.0\cos 40°=-7.66$ and $A_y=-10.0\sin 40°=+6.43$
B $A_x=-10.0\cos 35°=-8.19$ and $A_y=-10.0\sin 35°=-5.74$
C $A_x=+10.0\cos 68°=+3.75$ and $A_y=-10.0\sin 68°=-9.27$
D $A_x=+10.0\cos(90°-48°)=+7.43$ and $A_y=-10.0\sin(90°-48°)=-6.69$
E $A_x=-10.0\cos(90°-30°)=-5.00$ and $A_y=-10.0\sin(90°-30°)=-8.66$

45 The vector we want is $\vec{C}=-(\vec{A}+\vec{B})$. The components of $\vec{A}$ and $\vec{B}$ are:
$A_x=6.0\cos 60°=+3.0$ and $A_y=6.0\sin 60°=+5.20$;
$B_x=6.0\cos 120°=-3.0$ and $A_y=6.0\sin 120°=+5.20$. Hence
$C_x=-(+3.0-3.0)=0$ and $C_y=-(+5.20+5.20)=-10.4$. The magnitude of the vector $\vec{C}$ therefore is 10.4 units and is directed along the negative y – axis.



46 **a** $A_x = 12.0\cos 20° = +11.28$ and $A_y = 12.0\sin 20° = +4.10$;
$B_x = 14.0\cos 50° = +9.00$ and $A_y = 14.0\sin 50° = +10.72$. Hence the sum has components:
$S_x = +11.28 + 9.00 = 20.28$ and $S_y = +4.10 + 10.72 = 14.82$. The magnitude of the sum is thus
$\sqrt{20.28^2 + 14.82^2} = 25.1$. Its direction is $\theta = \arctan\dfrac{14.82}{20.28} = 36.2°$.

b $A_x = 15.0\cos 15° = +14.49$ and $A_y = 15.0\sin 15° = +3.88$;
$B_x = 18.0\cos 105° = -4.66$ and $B_y = 18.0\sin 105° = +17.39$. Hence the sum has components:
$S_x = 14.49 - 4.66 = 9.83$ and $S_y = +3.88 + 17.39 = 21.27$. The magnitude of the sum is thus
$\sqrt{9.83^2 + 21.27^2} = 23.4$. Its direction is $\theta = \arctan\dfrac{21.27}{9.83} = 65.2°$.

c $A_x = 20.0\cos 40° = +15.32$ and $A_y = 20.0\sin 40° = +12.86$;
$B_x = 15.0\cos 310° = +9.64$ and $B_y = 15.0\sin 310° = -11.49$. Hence the sum has components:
$S_x = 15.32 + 9.64 = +24.96$ and $S_y = +12.86 - 11.49 = +1.37$. The magnitude of the sum is thus
$\sqrt{24.96^2 + 1.37^2} = 25.0$. Its direction is $\theta = \arctan\dfrac{1.37}{24.96} = 3.14°$.