Answers to test yourself questions

Topic 5

5.1 Electric fields

1 **a** $F = \frac{kQ_1Q_2}{r^2} = \frac{8.99\times10^9\times2.0\times10^{-6}\times4.0\times10^{-6}}{(5.0\times10^{-2})^2} = 28.8 \approx 29\text{ N}$

b The force would become 4 times as small.

i $F' = \frac{kQ_1Q_2}{(2r)^2} = \frac{F}{4}$

ii $F' = \frac{k2Q_1Q_2}{(2r)^2} = \frac{F}{2}$

iii $F' = \frac{k2Q_1\times2Q_2}{(2r)^2} = F$

2 The middle charge is attracted to the left by the charge on the left with a force of $F_1 = \frac{kqQ_2}{r^2} = \frac{8.99\times10^9\times2.0\times10^{-6}\times4.0\times10^{-6}}{(4.0\times10^{-2})^2} = 45\text{ N}$. It is attracted to the right by the charge on the right with a force of $F_2 = \frac{kqQ_2}{r^2} = \frac{8.99\times10^9\times2.0\times10^{-6}\times3.0\times10^{-6}}{(2.0\times10^{-2})^2} = 135\text{ N}$. The net force is thus $135-45=90\text{ N}$ directed towards the right.

3 Suppose we call the distance (in cm) from the left charge x. Then we need

$$\frac{8.99\times10^9\times2.0\times10^{-6}\times4.0\times10^{-6}}{x^2} = \frac{8.99\times10^9\times2.0\times10^{-6}\times3.0\times10^{-6}}{(6-x)^2}$$

$$\frac{4.0}{x^2} = \frac{3.0}{(6-x)^2}$$

This means that

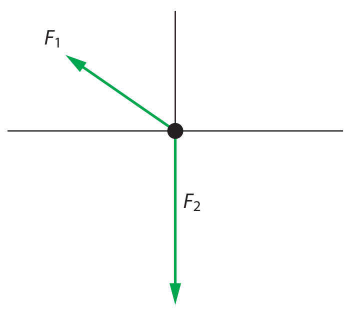
$$4.0(6-x)^2 = 3.0x^2$$

$$4(36-12x+x^2) = 3x^2$$

$$x^2-48x+144=0$$

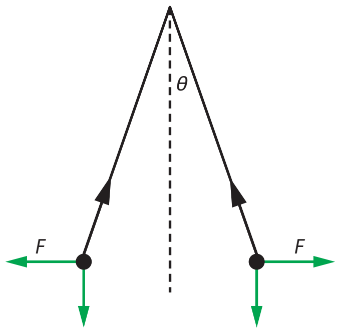
The solution is $x = 3.22$ cm.

4 The forces are as shown. The distance between the charge Q and the charge 2Q is 5.0 cm.



The magnitudes are $F_1 = \frac{kQ(2Q)}{d^2} = \frac{8.99 \times 10^9 \times 3.0 \times 10^{-6} \times 6.0 \times 10^{-6}}{(5.0 \times 10^{-2})^2} = 64.7$ N and $F_2 = \frac{kQQ}{d^2} = \frac{8.99 \times 10^9 \times (3.0 \times 10^{-6})^2}{(3.0 \times 10^{-2})^2} = 89.9$ N. We need to find the components of F_1: $F_{1x} = 64.7 \cos\theta = 64.7 \times \frac{4}{5} = 51.76$ N and $F_{1y} = 64.7 \sin\theta = 64.7 \times \frac{3}{5} = 38.82$ N. The components of the net force are: $F_x = -51.76$ N and $F_y = 38.82 - 89.9 = -51.08$ N. The net force has magnitude $F = \sqrt{51.76^2 + 51.08^2} = 72.7 \approx 73$ N and direction $180° + \arctan\frac{51.08}{51.76} = 224.6° \approx 225°$.

5 a A diagram is the following in which the angle θ of each string to the vertical is given by $\sin\theta = \frac{5}{85} \Rightarrow \theta = 3.37$.



We have that $T\cos\theta = mg$ and $T\sin\theta = F = \frac{kQ^2}{d^2}$ so that dividing side by side gives

$$\tan\theta = \frac{kQ^2}{mgd^2} \Rightarrow Q = \sqrt{\frac{mgd^2\tan\theta}{k}} = \sqrt{\frac{100 \times 10^{-6} \times 9.8 \times 0.1^2 \times \tan 3.37°}{8.99 \times 10^9}} = 8.0 \times 10^{-9} \text{ C}.$$

b This corresponds to $\frac{8.0 \times 10^{-9}}{1.6 \times 10^{-19}} = 5.0 \times 10^{10}$ electronic charges.

6 a Since the molar mass of water is 18 g per mole, a mass of 60 kg corresponds to $\frac{60 \times 10^3}{18} = 3333$ moles i.e. $3333 \times 6.02 \times 10^{23} = 2 \times 10^{27}$ molecules of water. A molecule of water contains 10 electrons (2 from hydrogen and 8 from oxygen) and so we have 2×10^{28} electrons in each person.

b The electric force is therefore $F_1 = \frac{kQ_1Q_2}{r^2} = \frac{9 \times 10^9 \times (2.0 \times 10^{28} \times 1.6 \times 10^{-19})^2}{(10)^2} = 9 \times 10^{26} \approx 10^{27}$ N, an enormous force.

c Assumptions include the use of Coulomb's law for objects that are not point charges, assuming the same distance between charges etc.

d We have neglected the existence of protons which gives each person a zero electric charge and hence zero electric force.

7 $E = \frac{F}{q} = \frac{3.0 \times 10^{-5}}{5.0 \times 10^{-6}} = 6.0 \text{ N C}^{-1}$

8 The magnitude of each of the fields produced at P is: $E = \frac{kQ}{r^2} = \frac{9.0 \times 10^9 \times 2.00 \times 10^{-6}}{\left(\sqrt{0.05^2 + 0.30^2}\right)^2} = 1.95 \times 10^5 \text{ N C}^{-1}$. The vertical components of the electric fields will cancel out leaving only the horizontal components. The horizontal component is $E_x = E\cos\theta = E\frac{d}{\sqrt{d^2 + \frac{a^2}{4}}} = 1.95 \times 10^5 \times \frac{0.30}{\sqrt{\frac{0.10^2}{4} + 0.30^2}} = 1.92 \times 10^5 \text{ N C}^{-1}$. The net field is then directed horizontally to the right and has magnitude $2 \times 1.92 \times 10^5 = 3.84 \times 10^5 \text{ N C}^{-1}$.

9 The two electric fields are $E_1 = E_2 = 1.95\times10^5\,\mathrm{N\,C^{-1}}$. Adding vectorially by taking components gives $E_x = 0$ and

$$E_y = 2\times1.95\times\times10^5\times\sin\theta = 2\times1.95\times10^5\times\frac{0.05}{\sqrt{\frac{0.10^2}{4}+0.30^2}} = 6.4\times10^5\,\mathrm{N\,C^{-1}}.$$

10 $I = nqAv = 8.5\times10^{28}\times1.6\times10^{-19}\times\pi(0.90\times10^{-3})^2\times3.6\times10^{-4} = 12.45\approx12\ \mathrm{A}$

11 **a** The current will be the same by conservation of charge.

b Since $I = nqAv$, we have that $A_1v_1 = A_2v_2$ and so $v_2 = \frac{A_1v_1}{A_2} = \frac{r_1^2v_1}{r_2^2} = \frac{1.0^2\times2.2\times10^{-4}}{2.0^2} = 5.5\times10^{-5}\ \mathrm{m\,s^{-1}}$.

12 From $I = nqAv$ we get $v = \frac{I}{nqA} = \frac{5.0}{5.8\times10^{28}\times1.6\times10^{-19}\times\pi\times(2\times10^{-3})^2} = 4.3\times10^{-5}\approx4\times10^{-5}\ \mathrm{m\,s^{-1}}$.

13 **a** One hour is $1\times60\times60 = 3600$ s and so $Q = It = 10\times3600 = 3.6\times10^4$ C.

b $N = \frac{Q}{e} = \frac{3.6\times10^4}{1.6\times10^{-19}} = 2.25\times10^{23}\approx2.2\times10^{23}$

14 **a** and **b** These points are inside the conducting sphere so the electric field is zero there.

c $E = \frac{kQ}{R^2} = \frac{8.99\times10^9\times4.0\times10^{-6}}{(15.0\times10^{-2})^2} = 1.6\times10^6\ \mathrm{N\,C^{-1}}$

d At 20 cm, $E = 1.6\times10^6\times\left(\frac{15}{20}\right)^2 = 9.0\times10^5\ \mathrm{N\,C^{-1}}$

5.2 Heating effect of electric currents

15 Electrons making up the current collide with lattice atoms and transfer some of their kinetic energy to these atoms. The average kinetic energy of the atoms increases and since temperature is proportional to the average kinetic energy of the atoms the temperature of the wire increases. The electric field keeps accelerating the electrons and so this process continues.

16 Doubling the length of the wire doubles the potential difference across its ends while the current stays the same.

Since $R = \frac{V}{I}$ the resistance doubles.

17 **a** Yes since the graphs are straight lines through the origin.

b The resistance for wire A is lower and so this wire corresponds to the lower temperature.

18 Since the resistance is constant, $\frac{6.0}{1.5} = \frac{V}{3.5}\Rightarrow V = 14$ V.

19 It obeys Ohm's law so the resistance is the same, 12 Ω.

20 $R = \frac{V}{I} = \frac{220}{15} = 15\ \Omega$

21 **a** $V = IR = 2\times4 = 8$ V across the first and $V = IR = 2\times6 = 12$ V across the second.

b There is no potential difference between B and C since there is no resistance between these points.

22 **a** The resistance is $R = \frac{V^2}{P} = \frac{220^2}{120} = 403\approx4.0\times10^2\ \Omega$.

b $403 = \frac{2.0\times10^{-6}\times L}{\pi\times(0.03\times10^{-3})^2}\Rightarrow L = 0.57$ m

23 **a** The two 4.0 Ω resistors are in series and are equivalent to 8.0 Ω. The lower two 2.0 Ω are equivalent to 4.0 Ω. The 8.0 Ω and the 4.0 Ω resistors are in parallel and are equivalent to $\frac{1}{R}=\frac{1}{8.0}+\frac{1}{4.0}\Rightarrow R=2.7\ \Omega$.

b The 6.0 Ω and 4.0 Ω resistors are in parallel and are equivalent to $\frac{1}{R}=\frac{1}{6.0}+\frac{1}{4.0}\Rightarrow R=2.4\ \Omega$. This and the other two are in series for an total of $R=2.0+2.4+8.0=12.4\ \Omega$.

c All three are in parallel for a total of $\frac{1}{R}=\frac{1}{3.0}+\frac{1}{3.0}+\frac{1}{3.0}\Rightarrow R=1.0\ \Omega$.

24 We have that $12=I(R_1+R_2)$ and $\varepsilon=IR_2$ where R_1 is the resistance of wire AC and R_2 the resistance of wire BC. Thus $\frac{\varepsilon}{12}=\frac{R_2}{R_1+R_2}$. But the resistances are proportional to the lengths and so $\frac{\varepsilon}{12}=\frac{54}{100}\Rightarrow\varepsilon=6.48$ V.

25 **a** Applying Kirchoff's laws to the two loops gives: $3.0=20(x+y)+30(x+y)$ and

$2.0=20(x+y)+30(x+y)+10y$. These simplify to

$3.0=50x+50y$

$2.0=50x+60y$

These are solved to give $10y=-1.0\Rightarrow y=-0.10$ A, and $x=0.16$ A.

b The potential differences are

$20\ \Omega: V=20(x+y)=20\times(0.16-0.10)=1.2$ V

$30\ \Omega: V=30(x+y)=30\times(0.16-0.10)=1.8$ V

$10\ \Omega: V=10y=10\times(-0.10)=-1.0$ V

26 Applying Kitchhoff's law: $9.0+3.0=4.0x$ so right away $x=3.0$ A. $3.0=-3.0(x-y)-2.0(x-y)=-5x+5y$. Hence $y=3.6$ A.

27 Applying Kitchhoff's law: $9.0=2.0x+5.0\times1.0\Rightarrow x=2.0$ A. In second loop $\varepsilon=3.0\times1.67+5.0\times1.0=10$ V.

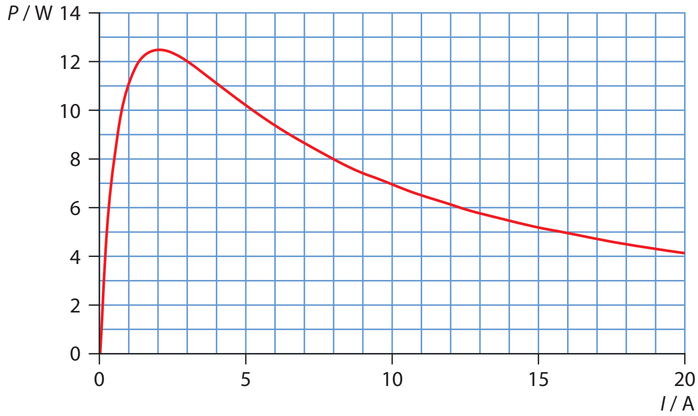
28 **a** From the graph, when the potential difference across each resistor is 1.5 V the current in X is about 2.68 A and in Y 1.55 A for a total current leaving the cell of 4.2 A.

b This has to be done by trial and error. The voltage across X plus that across Y must give 1.5 V. This is achieved for a current of about 1.1 A for which the voltages are 0.5 V and 1.0 V adding up to 1.5 V.

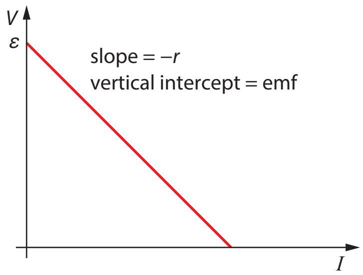
29 The top loop gives: $6.0=4.0x+3.2x\Rightarrow x=0.833$ A. The lower loop gives: $2.0=R\times0.8333x\Rightarrow R=2.4\ \Omega$. $\frac{L}{1}=\frac{2.4}{4}\Rightarrow L=0.60$ m.

5.2 Electric cells

30 Chemical energy in the top cell gets converted into thermal energy in the resistor, mechanical energy (and some thermal energy) in the motor (which in turn gets converted into gravitational potential energy as the load is being raised) and finally electrical energy that charges the lower battery.

31 

$R = \mathbf{2.0\ \Omega}$.

32 a 

b Since $V = \varepsilon - Ir$,

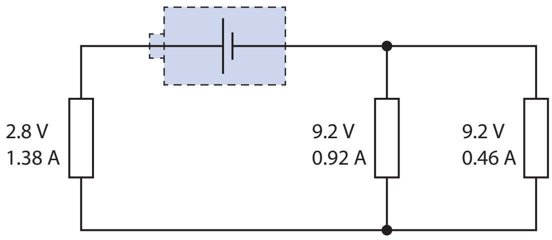
i the slope is the negative of the internal resistance of the battery

ii the vertical intercept is the emf of the battery

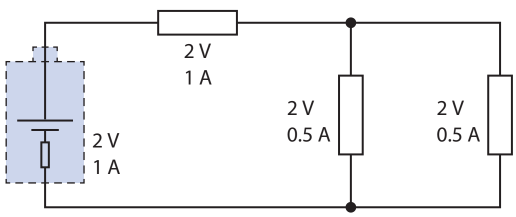
33 a The internal resistance is the slope and so equals $-r = \dfrac{2.4 - 9.6}{8.0 - 2.0} \Rightarrow r = 1.2\ \Omega$.

b Extending the line to find the vertical intercept gives an emf of about 12 V.

34 a The two parallel resistors are equivalent to $\dfrac{1}{R} = \dfrac{1}{10} + \dfrac{1}{20} \Rightarrow R = 6.67\ \Omega$. The total resistance of the circuit is then $R_T = 8.67\ \Omega$. The total current is then $I_T = \dfrac{12.0}{8.67} = 1.38\ \text{A}$. The potential difference across the 2.0 Ω resistor is $V = 1.38 \times 2.0 = 2.77 \approx 2.8\ \text{V}$. The potential difference across the parallel resistors is then $V = 12.0 - 2.77 = 9.2\ \text{V}$. So each of the two resistors get a current of $\dfrac{9.2}{10} = 0.92\ \text{A}$ and $\dfrac{9.2}{20} = 0.46\ \text{A}$.



b The two parallel resistors have a total of 2.0 Ω making a total circuit resistance of 6.0 Ω. The total current is then $I_T = \dfrac{6.0}{6.0} = 1.0\ \text{A}$. The internal resistance and the 2.0 Ω resistor get 1.0 A of current and the potential difference across each is 2.0 V. The potential difference across the parallel combination is 2.0 V and so each gets 0.50 A of current.



35 **a** and **b** Let r be the internal resistance and ε the emf. The total resistance when in parallel is $2.0 + r$ and so $3.0 = \dfrac{\varepsilon}{2.0+r}$. When in series the total resistance is $8.0 + r$ and so $1.4 = \dfrac{\varepsilon}{8.0+r}$. We must solve the system of equations

$$3.0 = \frac{\varepsilon}{2.0+r}$$

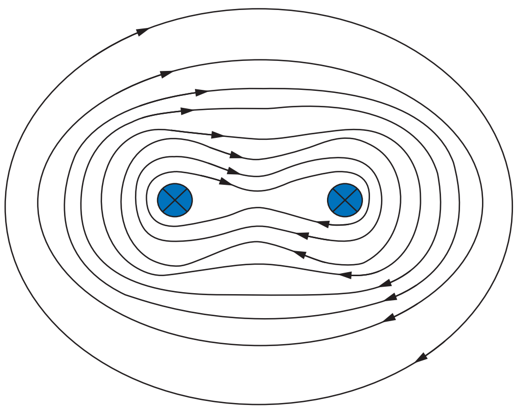
$$1.4 = \frac{\varepsilon}{8.0+r}$$

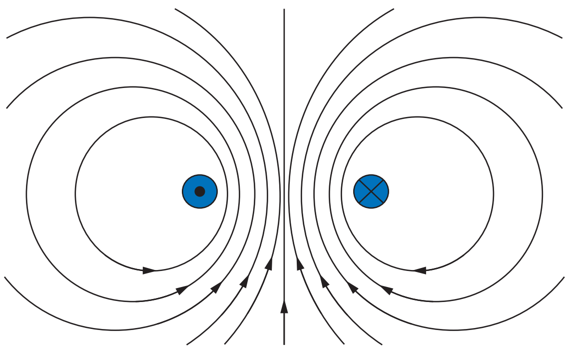
Dividing side by side $\dfrac{3.0}{1.4} = \dfrac{8.0+r}{2.0+r} \Rightarrow 2.14 = \dfrac{8.0+r}{2.0+r} \Rightarrow 4.28 + 2.14r = 8.0 + r \Rightarrow 1.14r = 3.17 \Rightarrow r = 3.25\ \Omega$ and so $\varepsilon = 5.25 \times 3.0 \approx 16$ V.

36 **a** The current everywhere is the same, call it x. Then $9.0 - 3.0 = 8x \Rightarrow x = 0.75$ A.
b The power in the top cell is $9.0 \times 0.75 \approx 6.8$ W and in the lower it is $-3.0 \times 0.75 \approx -2.2$ W.
c The power in the lower cell is negative implying that it is being charged.

5.4 Magnetic fields

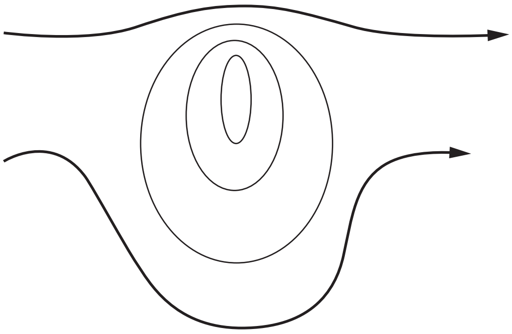
37





38 We must apply the right hand rule for force
a The magnetic field is into the page
b The force is into the page
c The magnetic field is out of the page
d The force is zero since the velocity is anti parallel to the field
e The force is zero since the velocity is parallel to the field

39



40 The magnetic field is directed into the page. In **a** the right hand rule (for a negative charge) gives a force downwards away from the wire. In **b** it gives a force to the right.

41 **a** the field is to the right and so the force is into the page
b the velocity is parallel to the field and the force is zero
c the force is towards the magnet (up the page)

42 **a** $eE = evB \Rightarrow B = \frac{E}{e} = \frac{2.4\times10^{3}}{2.0\times10^{5}} = 1.2\times10^{-2}$ T. The electric force is upwards so the magnetic force is downwards. Therefore the field must be into the page.
b The condition in **a** is independent of charge and mass so the proton will be undeflected as well.
c The electric force will stay the same but the magnetic force will double. Therefore the electron will be deflected downwards.

43 **a** There are equal and opposite forces at the poles of the magnet giving a net force of zero.
b The forces are opposite so they will rotate the magnet counterclockwise.

44 The force is $F = BIL\sin\theta = 5.00\times10^{-5}\times3000\times30.0\times\sin30° = 2.25$ N.

45 **Note:** This requires knowledge of circular motion (Topic 6).
a We have that $evB = m\frac{v^2}{r} \Longrightarrow v = \frac{eBr}{m}$. But $v = 2\pi f$ and so $2\pi rf = \frac{eBr}{m} \Rightarrow f = \frac{eB}{2\pi m}$.
b The mass is different and so the answer changes.

46 **a** The combined magnetic field from the two wires at point R must point downwards so as to cancel the uniform field. Since R is closer to Q, the field of Q is larger than the field from P. Hence the current in Q must go out of the page.
b If the current increases, the net field from P and Q increases as well, so that the total field at R is no longer zero. If we move closer to Q the field from Q will be much larger than the field from P and so their combined field will be downwards and much larger than external field. Hence the point has to move to the left.