Answers to test yourself questions

Topic 4

4.1 Oscilliations

1 a An oscillation is any motion in which the displacement of a particle from a fixed point keeps changing direction and there is a periodicity in the motion i.e. the motion repeats in some way.

b In simple harmonic motion, the displacement from an equilibrium position and the acceleration are proportional and opposite each other.

2 It is an oscillation since we may define the displacement of the particle from the middle point and in that case the displacement changes direction and the motion repeats. The motion is not simple harmonic however since there is no acceleration that is proportional (and opposite) to the displacement.

3 It is an oscillation since the motion repeats. The motion is not simple harmonic however since the acceleration is constant and is not proportional (and opposite) to the displacement.

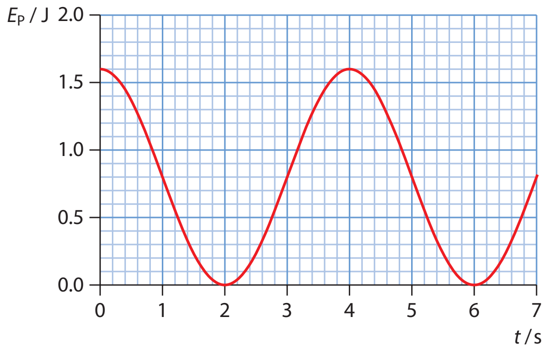
4 a The acceleration is opposite to the displacement so every time the particle is displaced there is a force towards the equilibrium position.

b The acceleration is not proportional to the displacement; if it were the graph would be a straight line through the origin.

5 a i It was not intended to ask about the mass – apologies!

ii The period is 8.0 s; the particle is at one extreme position at $t = 0$ and again at $t = 4.0$ s. This is half a period.

b

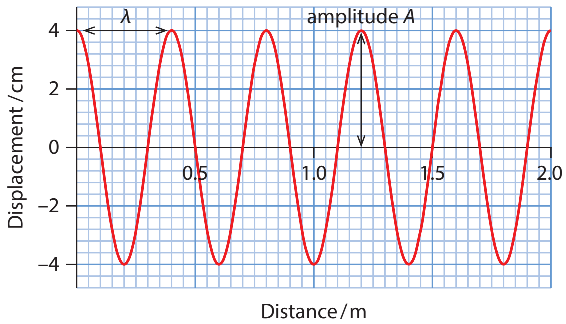


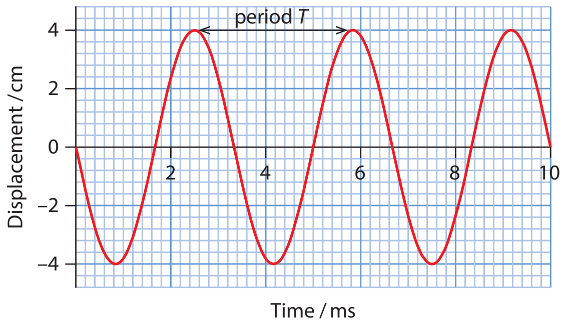
4.2 Travelling waves

6 The delay time between you seeing the person next to you stand up and you standing up and the number density of the people i.e. how many people per unit meter. For a fixed delay time, the closer the people are the faster the wave.

7 There is a disturbance that travels through the lie of dominoes just as a disturbance travels through a medium when a wave is present. You can increase the speed by placing them closer together. An experiment to investigate this might be to place a number of dominoes on a line of fixed length such that the dominoes are a fixed distance *d* apart. We must give the same initial push to the first domino (for example using a pendulum that is released from a fixed height and strikes the domino at the same place. We then measure time form when the first domino is hit until the last one is hit. Dividing the fixed distance by the time taken gives the speed of the pulse. We can then repeat with a different domino separation and see how the speed depends on the separation *d*.

8 **a** Wavelength – the length of a full wave; the distance between two consecutive crests or troughs
b Period – the time needed to produce one full oscillation or wave
c Amplitude – the largest value of the displacement from equilibrium of an oscillation
d Crest – a point on a wave of maximum displacement
e Trough – a point on a wave of minimum displacement





9 **a** In wave motion displacement refers to the difference in the value of a quantity such as position, pressure, density etc when the wave is present and when the wave is absent.
b In a transverse wave the displacement is at right angles to the direction of energy transfer, in a longitudinal it is parallel to the energy transfer direction.
c The falling stone imparts kinetic energy to the water at the point of impact and so that water moves. It will continue moving (creating many ripples) until the energy is dissipated.
d We must recall that the intensity of a wave is proportional to the square of the amplitude. The amplitude will decrease for two reasons: first, some energy is bound to be dissipated as the wave moves away and so the amplitude has to decrease. Second, even in the absence of any energy losses, the amplitude will still decrease because the wavefronts get bigger as they move away from the point of impact of the ripple. The energy carried by the wave is now distributed on a longer wavefront and so the energy per unit wavefront length decreases. The amplitude must then decrease as well.

10 **a** From left to right: down, down, up.
b From left to right: up, up, down.

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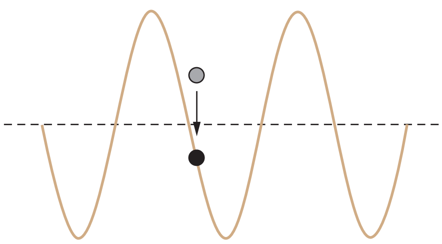
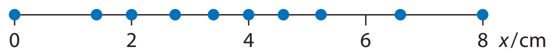


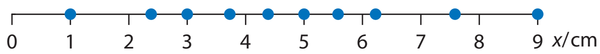
12 **a** $\lambda = \dfrac{v}{f} = \dfrac{330}{256} = 1.29$ m.

b $\lambda = \dfrac{v}{f} = \dfrac{330}{25 \times 10^{3}} = 1.32 \times 10^{-2}$ m.

13 a A wave in which the displacement is parallel to the direction of energy transferred by the wave.

b i 

ii At $x = 4.0$ cm

c i 

ii The compression is now at $x = 5.0$ cm.

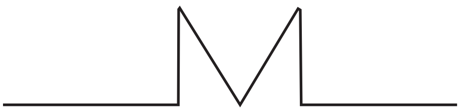
14 a $f = \dfrac{v}{\lambda} = \dfrac{340}{0.40} = 850$ Hz

b i A compression occurs at $x = 0.30$ m. Molecules just to the left of this point have positive displacement and so move to the right. Molecules just to the right move to the left creating the compression at $x = 0.30$ m.

ii By similar reasoning $x = 0.10$ m is a point where a rarefaction occurs.

4.3 Wave characteristics

15 Adding the pulses point by point gives the following diagram.



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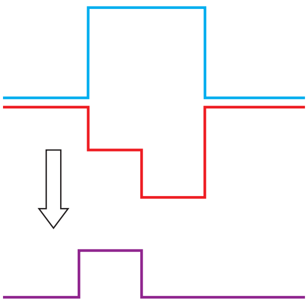
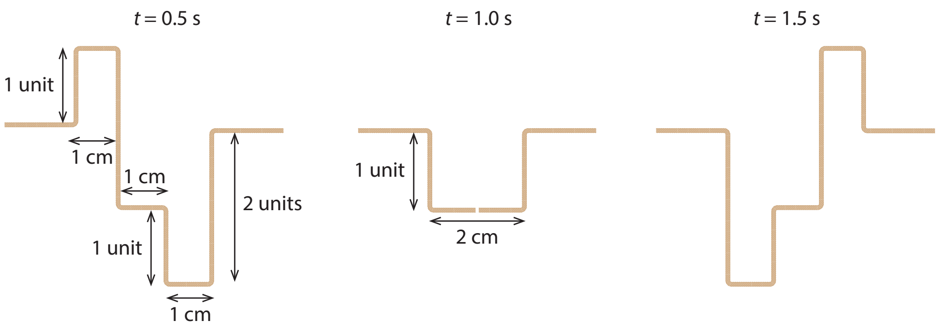
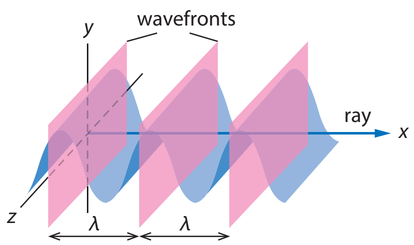


17 Adding the pulses point by point gives the following diagram.

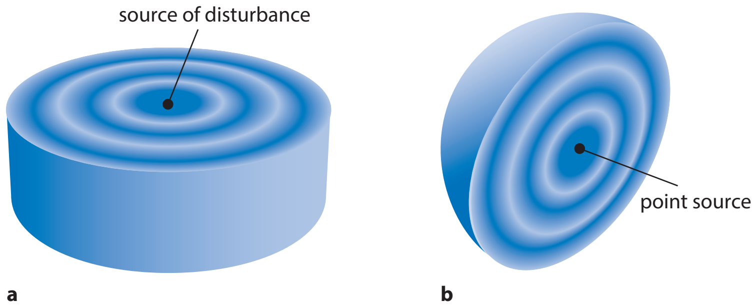


18 We add the pulses point by point. For example at $x = 0$ both waves have zero displacement and so we get zero displacement for the sum. At $x = 10$ cm, the blue pulse has $y = 0.50$ cm and the red pulse has $y = 0.75$ cm. The sum is 1.25 cm. At $x = 20$ cm, the blue pulse has $y = 0$ and the red pulse has $y = 1.0$ cm. The sum is 1.0 cm. At $x = 30$ cm, the blue pulse has $y = -0.50$ cm and the red pulse has $y = 0.70$ cm. The sum is 0.20 cm and so on.

19 a A wavefront is a surface on which all points have the same phase.



b A ray is the direction normal to wavefronts that corresponds to the direction of energy transfer.



20 **a** Polarised light is light in which the electric field oscillates on the same plane.

b Light can be polarised by passage through a polariser and by reflection off a n on-metallic surface.

21 In a polarised wave the displacement must be on the same plane. In a longitudinal wave the displacement is along the direction of energy transfer and so belongs to an infinity of planes at the same time. Hence it cannot be polarised.

22 **a** The light is not polarised. In the case of unpolarised light incident on an analyser, the intensity of the transmitted light would be half the incident intensity and so constant as required in the question.

b Since there is an orientation (call it X) of the analyser that makes the transmitted intensity zero, it follows that the incident light was polarised in a direction at right angles to the direction X.

c Since the intensity never becomes zero the light was not polarised. Since the intensity varies however, it follows that the incident light has unequal components in various directions so it is partially polarised.

23 **a** This relates the transmitted intensity I to the incident intensity I_0 when polarised light is incident and then transmitted through an analyser. The relation is $I = I_0 \cos^2\theta$ where θ is the angle between the transmission axis and the direction of the incident electric field.

b $\dfrac{I}{I_0} = \cos^2\theta = \cos^2 25° = 0.82$

24 **a** The light transmitted through the first polariser will be polarised in a given direction. The second polariser's axis is at right angles to this direction so the electric field has zero component along the axis of the second polariser. Hence no light gets transmitted.

b Light will be transmitted since now there will be a component of the electric field along the second polariser's axis.

c The situation is now identical to **a** and so no light goes through.

4.4 Wave behaviour

25 **a** From $1.00 \times \sin 38° = 1.583 \times \sin\theta_2$ we find $\sin\theta_2 = \dfrac{1.00 \times \sin 38°}{1.583} \Rightarrow \theta_2 = \sin^{-1} 0.3889 = 22.9°$.

b $n = \dfrac{c}{c_g} \Rightarrow c_g = \dfrac{c}{n} = \dfrac{3.0 \times 10^8}{1.583} = 1.9 \times 10^8 \text{ m s}^{-1}$

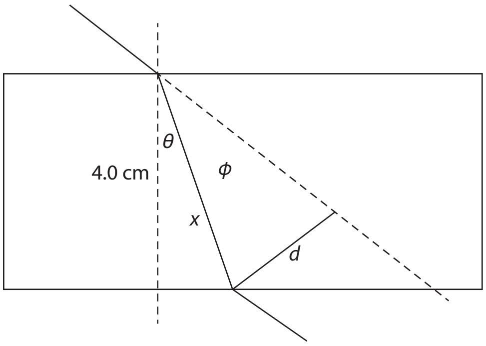
c The frequency in water is the same as that in air and so $\lambda_g = \dfrac{\lambda_a}{n} = \dfrac{6.8 \times 10^{-7}}{1.583} = 4.3 \times 10^{-7}$ m.

26 **a** $t = \dfrac{s}{c} = \dfrac{3.0}{3.0 \times 10^8} = 1.0 \times 10^{-8}$ s

b In this time, $1.0 \times 10^{-8} \times 6.0 \times 10^{14} = 6.0 \times 10^6$ full waves have been emitted. (Or, the wavelength is

$\lambda = \dfrac{3.0 \times 10^8}{6.0 \times 10^{14}} = 5.0 \times 10^{-7}$ m and in a length of 3.0 m we can fit $\dfrac{3.0}{5.0 \times 10^{-7}} = 6.0 \times 10^6$ full waves.)

27 First we find the angle of refraction (angle θ in the diagram).

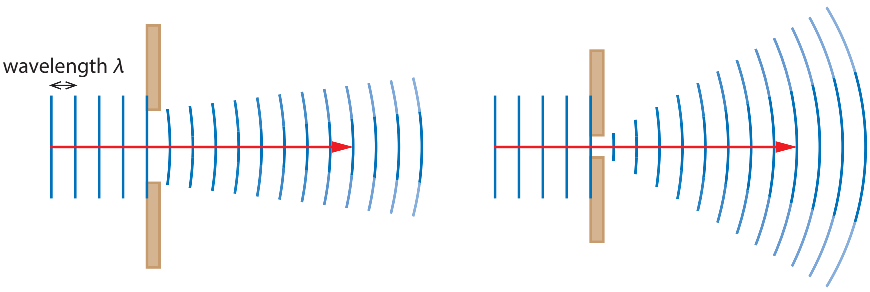


$1.00 \times \sin 40° = 1.450 \times \sin \theta$, hence $\theta = 26.3°$. This means that $x = \dfrac{4.0}{\cos 26.3°} = 4.46 \text{ cm}$.

Now $\varphi = 40°2 - 26.3° = 13.7°$ and so $d = 4.46 \times \sin 13.7° = 1.06 \text{ cm}$.

28 Let θ be the angle of incidence from air. The angle of refraction will be larger than θ and so as θ increases the angle of refraction will become $90°$ and so will not enter water. This happens when $\dfrac{\sin\theta}{340} = \dfrac{\sin 90°}{1500} \Rightarrow \theta = \sin^{-1}\dfrac{340}{1500} = 13.1°$.

29 The diagram must be similar to the one below.



30 There is no appreciable diffraction here; the wave continues straight through the opening.

31 There is poor reception because of destructive interference between the waves reaching the antenna directly and those reflecting off the mountain. The path difference is double the distance between the house and the mountain. The wave reflecting off the mountain will suffer a phase difference of π and so the condition for destructive interference is $2d = n\lambda$. The smallest d (other than zero) corresponds to $n = 1$ and so $d = 800$ m.

4.5 Standing waves

32 A standing wave is a special wave formed when two identical traveling waves moving in opposite directions meet and then superpose. This wave, unlike a traveling wave, has nodes i.e. points where the displacement is *always* zero. The antinodes, points where the displacement is the largest do not appear to be moving. A standing wave differs from a traveling wave in that it does not transfer energy and that the amplitude is variable. In a standing wave points in between consecutive nodes have the same phase whereas in a travelling wave the phase changes from zero to 2π after a distance of one wavelength.

33 A standing wave is formed when two identical traveling waves moving in opposite directions meet and then superpose.

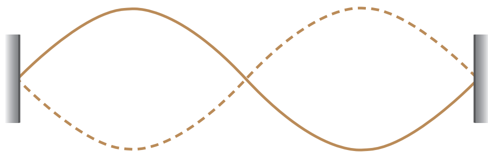
34 **a** A node is a point in the medium where the displacement is *always* zero.

b An antinode is a point in the medium where the displacement, at some instant, will assume its maximum value.

c Speed refers to the speed of the travelling waves whose superposition gives the standing wave.

35 **a** We must disturb the string with a frequency that is equal to the frequency of the second harmonic.

b



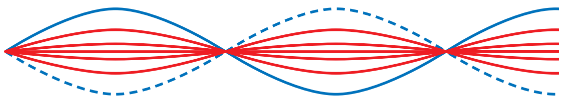
36 The wavelength of the wave will remain the same (and equal to twice the length of the string). Since the speed increases by $\sqrt{2}$ the frequency must do the same and so is 354 Hz.

37 The first harmonic has wavelength $2L$ (L is the length of the string) and the second a wavelength L. The ratio of the *frequencies* is then 2 since the speed is the same.

38 **a** The wavelength of the fundamental is $2L = 1.00$ m. The frequency is then $f = \frac{v}{2L} = 225$ Hz

b The sound produced by the vibrations of the string will have the same frequency i.e. 225 Hz and so the wavelength of sound will be $\lambda = \frac{c}{f} = \frac{340}{225} = 1.51$ m.

39



40 The wavelength of sound is $\lambda = \frac{c}{f} = \frac{340}{306} = 1.11$ m. Standing waves have wavelength given by $\lambda = \frac{4L}{n}$ with $n = 1, 3, 5, \ldots$. Therefore $\frac{4L}{n} = 1.11 \text{ m} \Rightarrow L = \frac{1.11 \times n}{4}$. This gives 0.28 m and 1.4m for $n = 3$ and $n = 5$.

41 **a** The wavelength is given by $\lambda = \frac{4L}{n} = \frac{0.800}{n}$ and also by $\lambda = \frac{c}{f} = \frac{c}{427}$. Hence

$\frac{c}{427} = \frac{0.800}{n} \Rightarrow c = \frac{427 \times 0.800}{n} = \frac{342}{n} \text{ m s}^{-1}$. The answer makes physical sense only if $n = 1$ (the first harmonic is established) in which case $c = 342 \text{ m s}^{-1}$.

b The next harmonic will have wavelength $\frac{4L'}{n} = 0.800 \Rightarrow L' = \frac{0.800n}{4} = 0.200n$. With $n = 3$ we get $L' = 0.600$ m.

42 **a** The wavelengths in the open tube are given by $\lambda = \frac{2L}{n}$. The frequencies of two *consecutive* harmonics are then $\left(f = \frac{c}{\lambda} = \frac{cn}{2L}\right)$, $300 = \frac{cn}{2L}$ and $360 = \frac{c(n+1)}{2L}$. This means that

$$\frac{360}{300} = \frac{\frac{c(n+1)}{2L}}{\frac{cn}{2L}} \Rightarrow \frac{n+1}{n} = 1.2 \Rightarrow n+1 = 1.2n \Rightarrow 0.2n = 1 \Rightarrow n = 5;$$ we have the fifth and sixth harmonics.

b We get $300 = \frac{340 \times 5}{2 \times L} \Rightarrow L = 2.833 \approx 2.8$ m.

43 The two harmonics have the same frequency and hence the same wavelength. The wavelength of the first harmonic in the open–open pipe is $\lambda = 2L_X$. The wavelength of the first harmonic in the closed–open pipe is $\lambda = 4L_Y$. Hence $2L_X = 4L_Y \Rightarrow \frac{L_X}{L_Y} = 2$.

44 With one step per second you shake the cup with a frequency of about 1 Hz. In the first harmonic mode the wavelength would be about twice the diameter of the cup i.e. 16 cm (we have antinodes at each end). This gives a speed of $v = 1 \times 16 = 16 \text{ cm s}^{-1}$.

45 **a** A standing wave is made up of two traveling waves. The speed of energy transfer of the traveling waves is taken to be the speed of the standing wave.

b From $y = 5.0\cos(45\pi t)$ we deduce that the frequency of oscillation of point P and hence also of the wave is $\frac{45\pi}{2\pi} = 22.5$ Hz. The wavelength is then $\lambda = \frac{v}{f} = \frac{180}{22.5} = 8.0$ m. Since the diagram shows a second harmonic this is also the length of the string.

c The phase difference is π and so $y = 5.0\cos(45\pi t + \pi) = -5.0\cos(45\pi t)$.

46 **a** The hit creates a longitudinal wave that travels down the length of the rod and reflects of the end. The reflected waves pushes the hammer back.

b $v = \frac{s}{t} = \frac{2.4}{0.18 \times 10^{-3}} = 1.3 \times 10^{4} \text{ m s}^{-1}$

c We assume free-free end points and so the wavelength is given by 2.4 m. The frequency is then

$$f = \frac{c}{\lambda} = \frac{1.3 \times 10^{4}}{2.4} = 5.6 \text{ kHz.}$$