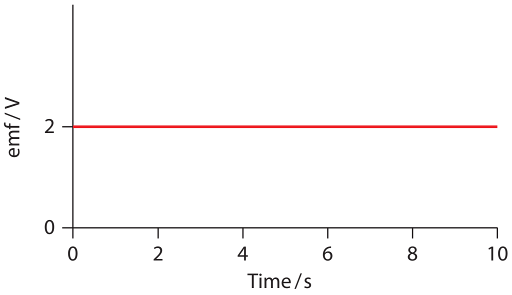
Answers to test yourself questions

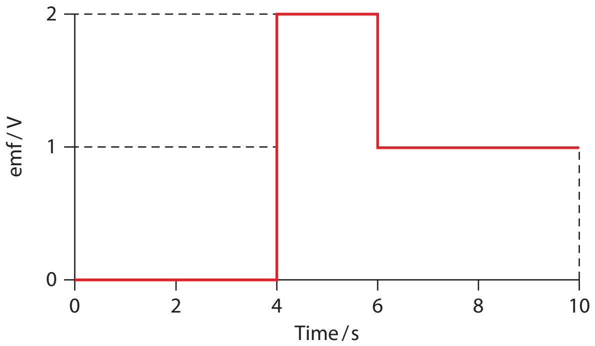
Topic 11

11.1 Electromagnetic induction

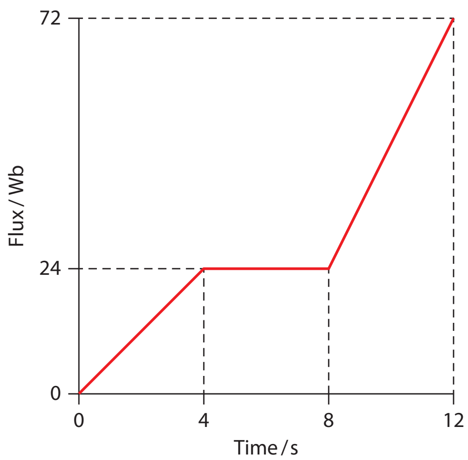
1 The flux is increasing at a constant rate so the induced emf is constant. It equals the slope which is 2.0 V giving the graph shown here.



2 The flux is not changing in the first 4 s so the induced emf is zero. In the next 2 s the slope and hence the emf is constant at 2 V. In the last 4 s the slope is 1 V. This gives the graph here.



3 **a** In the first 4 s the emf is constant at 6 V and so the flux is increasing at a constant rate. We have a straight line graph with slope 6. In the next 2 s the emf is zero which means that the flux is constant. Similarly, in the last 4 the emf is constant so the flux is increasing at a constant rate, i.e. the flux – time graph is a straight line with slope 12. A possible graph is shown here.



b The answer is not unique because there are many straight lines with the slopes given above – we just don't know the value of the flux, just its rate of change.

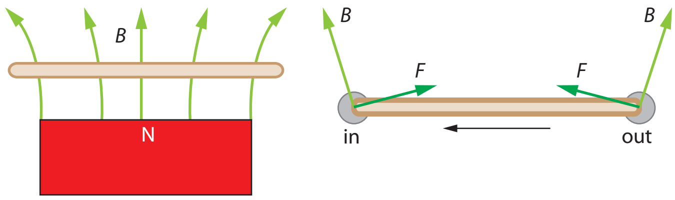
4 The magnetic field created by the outer solenoid is directed into the smaller coil. Since the current is increasing the flux is increasing. By Lenz's law the induced current must oppose the change i.e. it must decrease the flux. This can be done by having the induced current create a magnetic field directed out of the page. The current must then be counter-clockwise.

5 **a** Looking down from above the ring, we see that the magnetic field is directed towards us and the flux is increasing. So the induced current must produce a magnetic field directed away from us. By the right hand rule for the magnetic field direction, the current must be clockwise. As the ring moves away from the magnet we see magnetic field coming towards us and the flux is decreasing. So we must produce a current whose magnetic field comes towards us and so the current must be counter-clockwise. When the ring is half way down the length of the magnet the current must be zero.

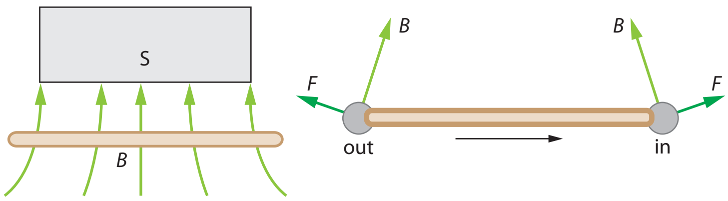
b The magnetic field we see now is directed away from us. So the induced current must create a magnetic field coming towards us and so the current is counter-clockwise. As the ring leaves, the flux is decreasing, the field is going away from us and the current must produce its own magnetic field away from us, i.e. the current is clockwise. Half way it must be zero.

6 This is answered in answer to question **5**.

7 **a** The diagram shows an edge – on view of the ring as it approaches the north pole of the magnet. The forces on two diametrically opposite points on the ring are as shown. The net force of these two forces is upwards. The same is true for any other two diametrically opposite points and so the net magnetic force on the ring is vertically upwards, making the ring fall slower than in free fall.



b As the ring moves away from the south pole the diagram is:



Therefore the net force is again upwards.

8 An electron in the rod is moving downwards and since the magnetic field is into the page the magnetic force on the electron will be directed to the left. Hence the left end will be negatively charged. The electrons that have moved to the left have left the right end of the rod positively charged.

9 **a** The magnetic field at the position of the loop is coming out of the page and is increasing. Hence the flux is increasing. To oppose this increase the induced current must produce a magnetic field into the page and so the current must be clockwise.

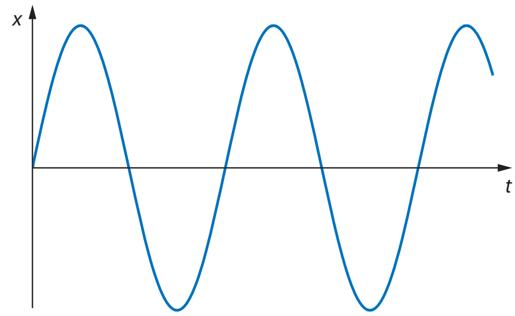
b The magnetic field at the position of the loop is coming out of the page and is decreasing. Hence the flux is decreasing. To oppose this decrease the induced current must produce a magnetic field out of the page and so the current must be counter-clockwise.

10 The induced emf is the rate of change of flux linkage i.e.

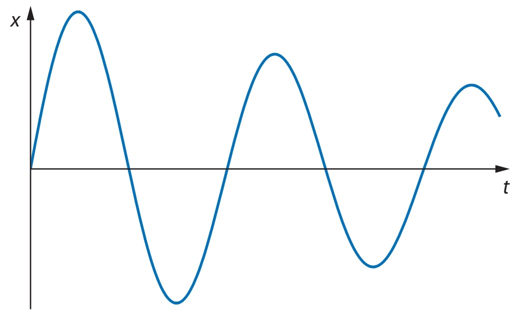
$$N\frac{\Delta\Phi}{\Delta t} = N\frac{\Delta B}{\Delta t}A = N\frac{\Delta B}{\Delta t}\pi r^2 = 200\times 0.45\times \pi \times 0.01^2 \approx 28\ \text{mV}.$$

11 The flux in the loop is increasing and so there will be an emf and a current in the loop. By Lenz's law, the magnetic field of the induced current will be directed out of the page hence the current will be counter – clockwise. The force on the movable rod is thus directed to the right. (Note: this could have been guessed since by moving the rod to the right we decrease the are and hence the flux. This is what must happen to oppose the change in flux which is an increase since the field is increasing.)

12 a i



ii



b i There is an induced emf but no current and so no force on the magnet. The oscillations are simple harmonic.

ii Now there is a current. As the magnet moves downward the flux in the coil is increasing. Assume the lower pole is a north pole. The induced current will produce a magnetic field upward and so there will be a retarding force on the magnet that will make the oscillations die out. A similar argument applies when the magnet moves upwards.

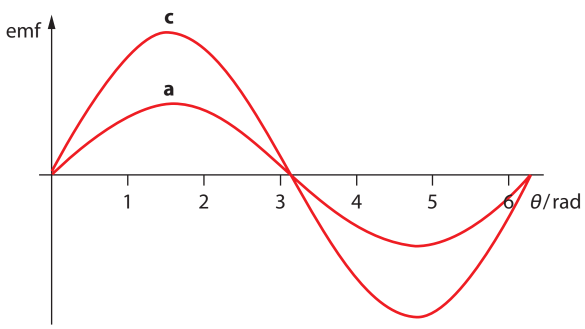
13 As the ring enters the region of magnetic field the flux will be increasing and so an emf will be induced in the ring. Since it is conducting a current will be established as well. The current must produce a field out of the page so as to oppose the increase in flux which means that the current is clockwise. On the lower part of the ring in the region of the field there will therefore be a magnetic force directed upward. Similarly there will be an upward force as the ring leaves the region of the magnetic field. In both cases the upward force delays the fsll of the ring so this ring land last.

11.2 Transmission of power

14 a The emf is the (negative) rate of change of the flux with time and so is a sine function with the same period as the graph for flux.

b This is a tricky question: we want the variation with angle and not time so the graph does not change.

c The maximum induced emf will now double since the rate of rotation doubles. But since we want the dependence on angle and not time the period of the graph will not change. This gives the graphs shown here.

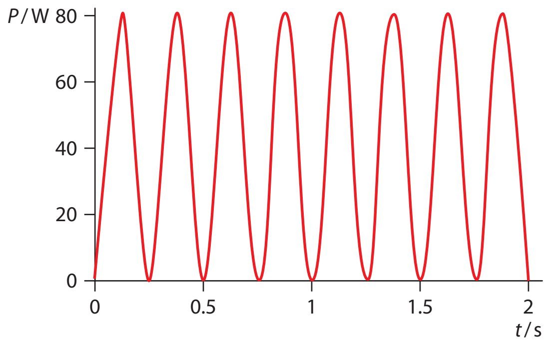


15 a The peak power is 20 W and so the average power is 10 W. Hence $\bar{P} = RI^2_{rms} \Rightarrow I_{rms} = \sqrt{\frac{10}{2.5}} = 2.0$ A.

b $R = \frac{V_{rms}}{I_{rms}} \Rightarrow V_{rms} = 2.0 \times 2.5 = 5.0$ V.

c The period is 1.0 s (there are two peaks within one period).

d At double the rotation speed the period will halve and the peak power will increase by a factor of 4 leading to the graph in the answers in the textbook. This is because at double the rotation speed the induced emf doubles and so the power (that depends on the square of the induced emf) increases by a factor of 4. This gives the graph shown here.



16 a From $\frac{V_P}{V_S} = \frac{N_P}{N_S}$ we get $\frac{220}{V_S} = \frac{500}{200} \Rightarrow V_S = \frac{200 \times 220}{500} = 88$ V. The frequency is unchanged so it stays 50 Hz.

b The power in the primary is $P = V_P I_P = 220 \times 6.0 = 1320$ W and so that in the secondary is $0.70 \times 1320 = 924$ W.

Hence the current is $\frac{924}{88} = 10.5$ A.

17 a The current produced is $I = \frac{P}{V} = \frac{300 \quad 10^6}{80 \quad 10^3} = 3750$ A. The power lost in the cables is then

$5.0 \times 3750^2 = 7.0 \times 10^7$ W, a fraction $\frac{7.0 \quad 10^7}{300 \quad 10^6} = 0.23$ of the power produced.

b At 100 kV, the current is $I = \frac{P}{V} = \frac{300 \quad 10^6}{100 \quad 10^3} = 3000$ A and the power lost is $5.0 \times 3000^2 = 4.5 \times 10^7$ W

representing a smaller fraction $\frac{4.5 \quad 10^7}{300 \quad 10^6} = 0.15$ of the total power produced.

18 The r.m.s. voltage is given by $V_{rms} = \frac{\omega NBA}{\sqrt{2}}$ and $\omega = 2\pi f = 100\pi \text{ s}^{-1}$.

Hence $B = \frac{V_{rms}\sqrt{2}}{\omega NA} = \frac{220\sqrt{2}}{100\pi \times 300 \times 0.20^2} = 0.0825$ T.

19 There is error in the question! The vertical axis is flux axis not power. The flux is given by

$\Phi = 10\sin(2\pi \frac{t}{0.9 \times 10^{-3}})$ and so the induced emf is $\varepsilon = \frac{10 \times 2\pi}{0.9 \times 10^{-3}} \cos(2\pi \frac{t}{0.9 \times 10^{-3}})$. The peak value is

$\varepsilon = \frac{10 \times 2\pi}{0.9 \times 10^{-3}} = 69813$ V and so the rms value is $\varepsilon_{rms} = \frac{69813}{\sqrt{2}} = 4.9 \times 10^4$ V.

20 a The current produced is $I = \frac{P}{V} = \frac{150 \quad 10^3}{1.0 \quad 10^3} = 150$ A. The power lost in the cables is then

$2.0 \quad 150^2 = 4.5 \quad 10^4$ W, a fraction $\frac{4.5 \quad 10^4}{150 \quad 10^3} = 0.30$ of the power produced.

b At 5000 V, the current is $I = \frac{P}{V} = \frac{150 \quad 10^3}{5.0 \quad 10^3} = 30$ A and the power lost is $2.0 \quad 30^2 = 1.8 \quad 10^3$ W representing a smaller fraction $\frac{1.8 \quad 10^3}{150 \quad 10^3} = 0.012$ of the total power produced.

21 The maximum power is $\frac{140^2}{24} = 816.7$ W. The average power is $\frac{816.7}{2} = 408 \approx 410$ W.

11.3 Capacitance

22 $C = \varepsilon_0 \frac{A}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0}$. Thus $A = \frac{1.0 \times 1 \times 10^{-2}}{8.85 \times 10^{-12}} = 1.1 \times 10^9\,\text{m}^2 = 1.1 \times 10^3\,\text{km}^2$. This is a huge area and shows that a 1 F capacitor is an enormous value for a capacitor.

23 The capacitance is $C = \varepsilon_0 \frac{A}{d} = 8.85 \times 10^{-12} \times \frac{0.25}{8.0 \times 10^{-3}} = 2.8 \times 10^{-10}$ F. The charge is then $Q = CV = 2.8 \times 10^{-10} \times 24 = 6.6 \times 10^{-9}$ C.

24 The charge on the fully charged capacitor is $Q = CV = 12 \times 10^{-6} \times 220 = 2.64 \times 10^{-3}$ C. The average current is then $\bar{I} = \frac{Q}{t} = \frac{2.64 \times 10^{-3}}{15 \times 10^{-3}} = 0.18$ A.

25 **a** $Q = CV = 20 \times 10^{-3} \times 9.0 = 180$ mC

b $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 20 \times 10^{-3} \times 9.0^2 = 810$ mJ

c $P = \frac{E}{t} = \frac{810 \times 10^{-3}}{50 \times \times 10^{-3}} = 16.2 \approx 16$ W

26 **a** $C_T = C_1 + C_2 = V = 120 + 240 = 360\,\mu\text{C}$

b The charges are: $Q_1 = C_1 V = 120 \times 10^{-6} \times 6.0 = 720\,\mu\text{C}$ and $Q_2 = C_2 V = 240 \times 10^{-6} \times 6.0 = 1440\,\mu\text{C}$.

c $E_1 = \frac{1}{2}C_1 V^2 = \frac{1}{2} \times 120 \times 10^{-6} \times 6.0^2 = 2160\,\mu\text{J} \approx 2.2 \times 10^{-3}$ J

and $E_2 = \frac{1}{2}C_2 V^2 = \frac{1}{2} \times 240 \times 10^{-6} \times 6.0^2 = 4320\,\mu\text{J} \approx 4.3 \times 10^{-3}$ J.

27 **a** $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2} = \frac{120 \times 240}{360} = 80\,\mu\text{F}$

b The charge on each capacitor is the same and equals $Q = C_T V = 80 \times 10^{-6} \times 6.0 = 480\,\mu\text{C}$.

c $E_1 = \frac{Q^2}{2C_1} = \frac{(480 \times 10^{-6})^2}{2 \times 120 \times 10^{-6}} = 960\,\mu\text{J}$ and $E_1 = \frac{Q^2}{2C_2} = \frac{(480 \times 10^{-6})^2}{2 \times 240 \times 10^{-6}} = 480\,\mu\text{J}$.

28 The capacitor has a charge of $Q = CV = 25 \times 10^{-12} \times 24 = 600$ pC. After connecting the charged capacitor to the uncharged charge will move form to the other until the voltage across each is the same. The total charge on both capacitors will be 600 pC by charge conservation.

a $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ and so $\frac{Q_1}{25} = \frac{Q_2}{75} \Rightarrow Q_2 = 3Q_1$. $Q_1 + Q_2 = 600\,\text{pC} \Rightarrow Q_1 + 3Q_1 = 600$ pC. Hence $Q_1 = \frac{600}{4} = 150$ pC and so $Q_2 = 450$ pC.

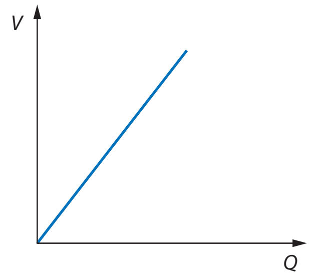
b Initially the energy stored was $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 25 \times 10^{-12} \times 24^2 = 7.2$ nJ. After the connection the total energy is $E_T = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{(150 \times 10^{-12})^2}{2 \times 25 \times 10^{-12}} + \frac{(450 \times 10^{-12})^2}{2 \times 75 \times 10^{-12}} = 1.8$ nJ. The difference is a "loss" of 5.4 nJ.

c The energy was dissipated as thermal energy in the connecting wires.

29 **a** $E=\frac{1}{2}CV^2=\frac{1}{2}\times 250\times 10^{-3}\times 12^2=18\text{ J}$

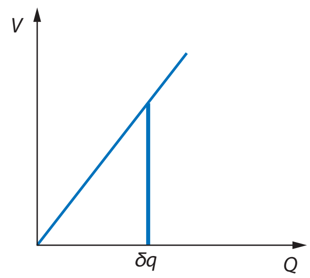
b The resistance of the lamp is found from: $P=\frac{V^2}{R}\Rightarrow R=\frac{V^2}{P}=\frac{12^2}{6.0}=24\ \Omega$ and the time constant for the circuit is $\tau=RC=6.0\,\text{s}$. This is, approximately, the time for which the current is appreciable enough to light the lamp.

30 **a** Since $V=\frac{Q}{C}$ the graph would be a straight line with positive slope through the origin:



To increase the charge on the capacitor by a small change of charge δq requires work $\delta q V$ to be done. This is represented by the area of the thin strip in the graph.

b



This work is stored as energy in the capacitor. Hence the total area is the total energy stored.

31 **a** and **c** The voltage across the capacitor is given by $V=V_0e^{-\frac{t}{RC}}=48e^{-\frac{0.20}{25\times10^{-5}\times15\times10^{3}}}=28.159\text{ V}$. The charge is then $Q=CV=25.0\times10^{-6}\times28.159=7.0\times10^{-4}\,\text{C}$.

b The voltage across the resistor is $48-28.159=19.841\,\text{V}$ and so the current is $I=\frac{V}{R}=\frac{19.841}{15\times10^3}=1.3\times10^{-3}\,\text{A}$.

32 **a** The "half-life" is 1.5 s and so from $\ln 2=\frac{T_{1/2}}{\tau}$ we find $\tau=\frac{T_{1/2}}{\ln 2}=\frac{1.5}{\ln 2}=2.16\text{ s}$.

b From $\tau=RC$ we find $R=\frac{\tau}{C}=\frac{2.16}{50\times10^{-6}}=43\text{ k}\Omega$.

33 **a** The charge stored on the capacitor initially is $Q=CV=250\times10^{-6}\times12=3.0\times10^{-3}\,\text{C}$. The charge t seconds later is $Q=Q_0e^{-\frac{t}{RC}}$ and the voltage is $V=V_0e^{-\frac{t}{RC}}$. Hence $\frac{Q}{V}=\frac{Q_0}{V_0}\Rightarrow Q=\frac{VQ_0}{V_0}=\frac{6.0\times3.0\times10^{-3}}{12}=1.5\times10^{-3}\text{ C}$.

b The current is $I=\frac{Q_0}{RC}e^{-\frac{t}{RC}}$ and again

$$\frac{I}{V}=\frac{Q_0}{RCV_0}\Rightarrow I=\frac{VQ_0}{RCV_0}=\frac{6.0\times3.0\times10^{-3}}{250\times10^{-6}\times75\times10^{3}\times12}=8.0\times10^{-5}\times10^{-3}\,\text{A}.$$

34 **a** $I=\frac{Q_0}{RC}e^{-\frac{t}{RC}}$ where the initial charge is $Q=CV=2.00\times10^{-6}\times9.0=1.80\times10^{-5}\,\text{C}$.

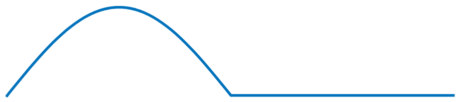
Then, $I=\frac{1.80\times10^{-5}}{5.00\times10^{6}\times2.00\times10^{-6}}e^{-\frac{1.00}{5.00\times10^{6}\times2.00\times10^{-6}}}=1.63\times10^{-6}\text{ A}$.

b We are asked to find the power in the resistor and so $P = RI^2 = 5.00 \times 10^6 \times (1.63 \times 10^{-6})^2 = 1.33 \times 10^{-5}$ W.

c This has to be the same as **b**.

35 Diode A is top left and diode B is top right!

a



A

B

b By using a higher value of the resistance or capacitance.