Answers to test yourself questions

Topic 9

9.1 Simple harmonic motion

1 They are not simple harmonic because as shown in the textbook the restoring force whereas opposite to, is not proportional to the displacement away from the equilibrium position. If however the amplitude of oscillations is small the force does become approximately proportional to the displacement and the oscillations are then approximately simple harmonic.

2 a We notice that $x_0 \cos\left(\omega t - \frac{\pi}{2}\right) = x_0 \sin \omega t$ and so the phase is $-\frac{\pi}{2}$.

b At $t = 0$ the equation says that $x = x_0 \cos\phi$. The next time x assumes this value is at a time given by $x_0 \cos(\omega T + \phi) = x_0 \cos\phi$. Thus we must solve the equation $\cos(\omega T + \phi) = \cos\phi$. This means that the angles $\omega T + \phi$ and ϕ differ by 2π and so solutions are $\omega T + \phi = \phi + 2\pi \Rightarrow T = \frac{2\pi}{\omega}$

3 a At $t = 0$ we have $y = 5.0\cos(0) = 5.0$ mm.

b At $t = 1.2$ s we use the calculator (in **radian** mode) to find $y = 5.0\cos(2 \times 1.2) = -3.7$ mm.

c $-2.0 = 5.0\cos(2t) \Rightarrow 2t = \cos^{-1}\left(-\frac{2}{5}\right) = 1.98 \Rightarrow t = 0.99$ s.

d Use $v = \pm\omega\sqrt{x_0^2 - x^2}$. We know that $\omega = 2.0\ \text{s}^{-1}$. Therefore,
$6.00 = \pm 2.0\sqrt{25 - x^2} \Rightarrow 25 - x^2 = 9.0 \Rightarrow x = \pm 4.00$ mm.

4 a The equation is simply $y = 8.0\cos(2\pi \times 14t) = 8.0\cos(28\pi t)$.

b The velocity is therefore $v = -8.0 \times 28\pi \sin(28\pi t)$ and the acceleration is $a = -8.0 \times (28\pi)^2 \cos(28\pi t)$.
At $t = 0.025$ s we evaluate (in radian mode) $y = 8.0\cos(28\pi \times 0.025) = -4.7$ cm,
$v = -8.0 \times 28\pi \sin(28\pi \times 0.025) = -5.7\ \text{m s}^{-1}$ and $a = -8.0 \times (28\pi)^2 \cos(28\pi \times 0.025) = 3.6 \times 10^2\ \text{m s}^{-2}$.

5 The angular frequency is $\omega = 2\pi f = 2\pi \times 460 = 920\pi$. The maximum velocity is $\omega A = 920\pi \times 5.0 \times 10^{-3} = 14\ \text{m s}^{-1}$ and the maximum acceleration is $\omega^2 A = (920\pi)^2 \times 5.0 \times 10^{-3} = 4.2 \times 10^4\ \text{m s}^{-2}$.

6 a The equation of the string may be rewritten as $y = \left(6.0\sin(\pi x)\right)\cos(2\pi \times 520t)$ from which we deduce that the frequency of all points is 520 Hz and that the phase of all points is zero.

b From **a** the amplitude is $A = 6.0\sin(\pi x)$ and so is different for different points on the string.

c The maximum amplitude is obtained when $\sin(\pi x) = 1$, i.e. the maximum amplitude is 6.0 mm.

d The displacement is always zero at the ends of the string, in particular at the right end where $x = L$, the length of the string. The displacement is zero *all the time* when $6.0\sin(\pi x) = 0$ i.e. when $\pi x = \pi \Rightarrow x = 1.0$ m.

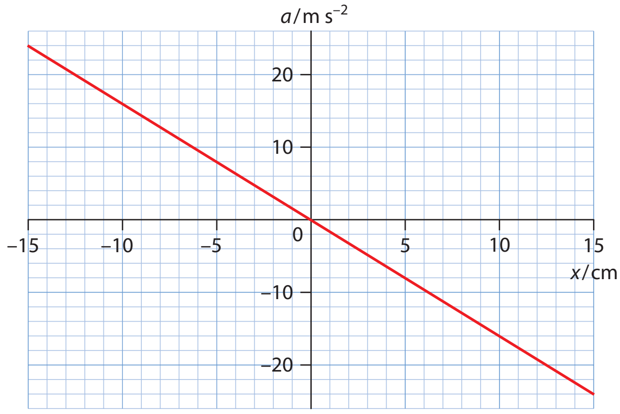
e When $x = \frac{3L}{4} = 0.75$ m the amplitude is $6.0\sin(\pi x) = 6.0\sin(0.75\pi) = 4.2$ mm.

7 a The area is approximately 0.50 cm (the exact value is 0.51 cm).

b This is the displacement from when the velocity is zero to when it is zero again i.e. from one extreme position until the other i.e. twice the amplitude.

c The period is 0.4 s and so the equation for displacement is $x = -0.25\sin\left(\frac{2\pi t}{0.4}\right) = -0.25\sin(5\pi t)$.

8 We need to graph the equation $a = -\omega^2 x$ where $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 12.57\ \text{s}^{-1}$. The slope would be $\omega^2 = 158\ \text{s}^{-2}$ or just 1.58 since we are plotting cm on the horizontal axis.



9 a The defining relation for SHM is that $a = -\omega^2 x$ which implies that a graph of acceleration versus displacement is a straight line through the origin with negative slope just as the given graph.

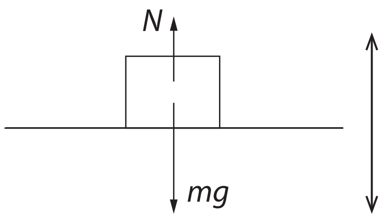
b The slope of the graph gives $-\omega^2$. The measured slope is $\frac{1.5}{0.10} = -15\ \text{s}^{-2}$ and so $\omega = \sqrt{15} = 3.873\ \text{s}^{-1}$. Thus the period is $T = \frac{2\pi}{3.873} = 1.6$ s.

c The maximum velocity is $\omega A = 3.873 \times 0.10 = 0.39\ \text{m s}^{-1}$.

d The maximum net force is $ma = m\omega^2 A = 0.150 \times 15 \times 0.10 = 0.225$ N.

e The total energy is $E_T = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}0.150 \times 3.873^2 \times 0.10^2 = 0.012$ J.

10 a The forces on the mass when the plate is at the top are shown below:



The net force is $mg - N = ma$. Since we have simple harmonic motion $a = \omega^2 x = 4\pi^2 f^2 x$ in magnitude, and the largest acceleration is obtained when $x = A$, the amplitude of the oscillation. The frequency is 5.0 Hz. The critical point is when $N = 0$. I.e. $g = 4\pi^2 f^2 A$ and so $A = \frac{g}{4\pi^2 f^2} = \frac{9.8}{4\pi^2 \times 25} = 0.0099$ m. The amplitude must not exceed this value.

b At the lowest point:

$$N - mg = ma = m4\pi^2 f^2 A$$

$$\Rightarrow N = mg + m4\pi^2 f^2 A$$

$$N = 0.120 \times 9.8 + 0.120 \times 4 \times \pi^2 \times 25 \times 0.0099$$

$$N = 2.35\ \text{N}$$

11 a The volume within the sphere of radius x is $\frac{4\pi x^3}{3}$ and that of the entire sphere is $\frac{4\pi R^3}{3}$ therefore the mass enclosed is the fraction $M\frac{x^3}{R^3}$.

b $F = G\frac{\frac{Mx^3}{R^3}m}{x^2} = \frac{GMmx}{R^3}$

c The acceleration of the mass is given by $ma = -\frac{GMmx}{R^3} \Rightarrow a = -\frac{GM}{R^3}x$ which is the condition for SHM with $\omega^2 = \frac{GM}{R^3}$.

d $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM}}$

e $T = 2\pi\sqrt{\frac{(6.4\times10^6)^3}{6.67\times10^{-11}\times6.0\times10^{24}}} = 5085\ s = 85$ min.

f From gravitation we know that $\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v^2 = \frac{GM}{R} = \left(\frac{2\pi R}{T}\right)^2 \Rightarrow T = 2\pi\sqrt{\frac{R^3}{GM}}$ as in **d.**

12 a When extended by an amount x the force pulling back on the body is $2kx$ and so

$ma = -2kx \Rightarrow a = -\frac{2k}{m}x$ and so $\omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times120}{2.0}} = 10.95\ \mathrm{s}^{-1}$ giving a period of

$T = \frac{2\pi}{\omega} = \frac{2\pi}{10.95} = 0.57$ s.

b With the springs connected this way, and the mass pulled to the side by small amount one spring will be compressed and the other extended. Hence the net force on the mass will still be $2kx$ so the period will not change.

13 a At the top the woman's total energy is gravitational potential energy equal to mgh where h is the height measured from the lowest position that we seek. At the lowest position all the gravitational potential energy has been converted into elastic energy $\frac{1}{2}kx^2$ and so $mgh = \frac{1}{2}kx^2$. Since $h = 15 + x$ we have that $mgh = \frac{1}{2}k(h-15)^2$.

We must now solve for the height h:

$$60\times10\times h = \frac{1}{2}\times220\times(h-15)^2$$

$$600h = 110(h^2 - 30h + 225)$$

$$110h^2 - 3900h + 24750 = 0$$

$$11h^2 - 390h + 2475 = 0$$

The physically meaningful solution is $h \approx 27$ m.

b The forces on the woman at the position in (a) are her weight vertically downwards and the tension in the spring upwards. Hence the net force is $F_{net} = T - mg = kx - mg = 220\times(27-15) - 600 = 2040$ N hence $a = \frac{F_{net}}{m} = \frac{2040}{60} = 34\ \mathrm{m\ s}^{-2}$.

c Let x be the extension of the spring at some arbitrary position of the woman. Then the net force on her is $F_{net} = T - mg = kx - mg$ directed upwards i.e. opposite to the direction of x. So $ma = -(kx - mg)$. The acceleration is not proportional to the displacement so it looks we do not have SHM. But we must measure displacement from an equilibrium position. This is when the extension of the spring is x_0 and $kx_0 = mg$. In other words call the displacement to be $y = x - x_0$. Then

$ma = -(k(y + x_0) - mg) = -ky - kx_0 + mg = -ky$ since $kx_0 = mg$. Hence we do have the condition for SHM. And so $a = -\frac{k}{m}y$ so that $\omega^2 = \frac{k}{m} \Rightarrow \omega = 1.91\ \mathrm{s}^{-1}$ and finally $T = \frac{2\pi}{\omega} = 3.28 \approx 3.3$ s.

d She will come to rest when the tension in the spring equals her weight i.e. when

$kx_0 = mg \Rightarrow x_0 = \frac{mg}{k} = \frac{60\times10}{220} = 2.7$ m. Hence the distance from the top is $15 + 2.7 = 17.7 \approx 18$ m.

e It has been converted to other forms of energy mainly thermal energy in the air and at the point of support of the spring.

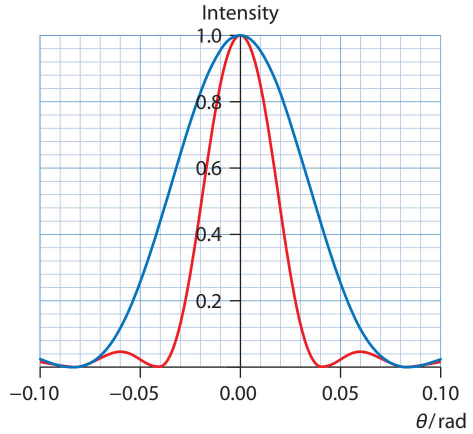
9.2 Single-slit diffraction

14 The diffraction angle is $\theta \approx \frac{\lambda}{b} = 0.333$ rad and so the angular width is double this i.e. 0666 rad or 38.2°.

15 The diffraction angle is $\theta \approx \frac{\lambda}{b} = \frac{6.00 \times 10^{-7}}{0.12 \times 10^{-3}} = 0.0050$ rad and so the angular width is double this i.e. 010 rad. The linear width is therefore $2d\theta = 2 \times 0.0050 \times 2.00 = 0.020$ m.

16 **a** The diffraction angle is about $\theta \approx \frac{\lambda}{b} = 0.0041$ rad and so $b \approx \frac{\lambda}{0.0041} \approx 24\lambda$.

b In **i** we have a smaller width and so a larger diffraction angle. In **ii** there will be no change since both wavelength and width halve. Notice however that if we were to pay attention to the vertical axis scale, with a smaller slit width less light would go through so in both cases the intensity would be less.



9.3 Interference

17 The separation is given by the booklet formula $s = \frac{\lambda D}{d} = \frac{680 \times 10^{-9} \times 1.50}{0.12 \times 10^{-3}} = 8.5$ mm.

18 The two flashlights are not coherent. This means that the phase difference between them keeps changing with time (very fast, on a time scale of nanoseconds). Thus, whatever interference pattern is produced at any moment in time, a different pattern will be produced a nanosecond later. Therefore all we can observe is an average of the rapidly changing patterns on the screen, i.e. no interference at all.

19 $d\sin\theta = n \times 680$ and $d\sin\theta = (n+1) \times 510$. Thus $n \times 680 = (n+1) \times 510 \Rightarrow 680n = 510n + 510 \Rightarrow n = 3$.

20 **a** The separation of the bright fringes is $s = \frac{3.1}{4} \times 10^{-3} = 0.775 \times 10^{-3}$ m. From $s = \frac{\lambda D}{d}$ we get

$$\lambda = \frac{sd}{D} = \frac{0.775 \times 10^{-3} \times 1.00 \times 10^{-3}}{1.2} = 6.46 \times 10^{-7} \approx 6.5 \times 10^{-7} \text{ m}$$

b The wavelength in water would be less (by a factor of 1.33) and so the distance would also be less.

21 **a** We must have $d\sin 20° = 1 \times \lambda$ and so $d = \frac{\lambda}{\sin 20°} = 2.92 \times \lambda$.

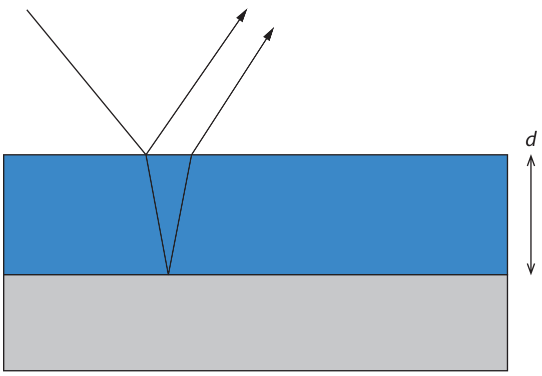
b From $s = \frac{\lambda D}{d}$, the distance between the bright fringes would double if d halves.

22 **a** Use $d\sin\theta = n\lambda$ with $d = \frac{1}{400}$ mm and $\lambda = 600.0$ nm to get:

n	θ
0	0.0°
1	13.89°
2	28.69°
3	46.05°
4	73.74°

b $n = 4$

23 There will be a phase change of π at both reflection points and so the condition for destructive interference (for normal incidence) is $2d = \left(k + \frac{1}{2}\right)\frac{\lambda}{n}$ where n is the refractive index of the coating.

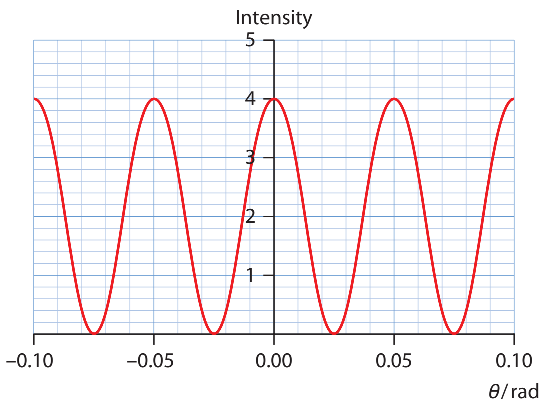


This gives $d = \left(k + \frac{1}{2}\right)\frac{\lambda}{2n} = \left(k + \frac{1}{2}\right)\frac{680}{2 \times 1.38}$ nm. The least thickness d is obtained for $k = 0$ and is $d = 123$ nm.

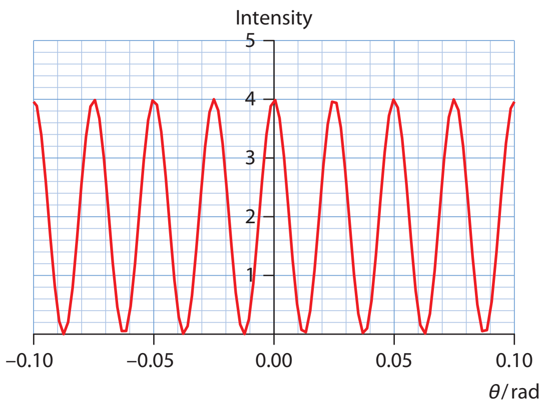
24 The reflected light must show constructive interference. There is a phase change only at the top reflection so the condition for constructive interference is $2d = \left(k + \frac{1}{2}\right)\frac{\lambda}{n}$ where n is the refractive index of the film. Then $d = \left(k + \frac{1}{2}\right)\frac{\lambda}{2n} = \left(k + \frac{1}{2}\right)\frac{550}{2 \times 1.33} = \left(k + \frac{1}{2}\right) \times 206.8$ nm. Possible values of d are then $d = 103$ nm, $d = 310$ nm etc.

25 **a** Coherent light means light where the phase difference between any two points on the same cross section of the beam is constant. Monochromatic light means light of the same wavelength.

b The first maximum is observed at $d \sin\theta = \lambda \Rightarrow \theta = \sin^{-1}\frac{7.0 \times 10^{-7}}{1.4 \times 10^{-5}} = 0.05$ rad and this gives the following graph.



c The angle in **b** would now halve to 0.025 rad giving the following graph.



26 **a** See, for example, Figure 9.19 in the coursebook.

b **i** The intensity increases, the maxima become thinner and there are secondary maxima

ii the intensity of the maxima stays the same but their separation increases.

9.4 Resolution

27 The angular separation of the points is $\theta_A = \dfrac{1\times10^{-2}}{10\times10^{3}} = 10^{-6}$ rad. The diffraction angle is $\theta_D = \dfrac{1.22\times600\times10^{-9}}{20\times10^{-2}} = 4\times10^{-5}$ rad. The objects will not be resolved since $\theta_A < \theta_D$.

28 **a** The angular separation is $\theta_A = \dfrac{1.4}{d}$ and the diffraction angle is $\theta_D = 1.22\times\dfrac{500\times10^{-9}}{5.0\times10^{-2}} = 1.22\times10^{-5}$ rad. For resolution we need $\theta_A \geq \theta_D$, i.e. $\dfrac{1.4}{d} \geq 1.22\times10^{-5}$ and so $d \leq \dfrac{1.4}{1.22\times10^{-5}} = 115$ km.

b It would decrease since the diffraction angle would get smaller.

29 **a** The diffraction angle is $\theta_D = 1.22\times\dfrac{5.0\times10^{-7}}{4.0\times10^{-3}} = 1.52\times10^{-4}$ rad and this is the smallest angular separation that can be resolved.

b With $\theta_D = \theta_A = 1.52\times10^{-4}$ we get $1.52\times10^{-4} = \dfrac{s}{3.8\times10^{8}} \Rightarrow s \approx 58$ km.

30 **a** The diffraction angle is $\theta_D = 1.22\times\dfrac{21\times10^{-2}}{76} = 3.4\times10^{-3}$ rad and this is the smallest angular separation that can be resolved.

b The angular separation of the two stars is $\dfrac{3.6\times10^{11}}{8.8\times10^{16}} = 4.1\times10^{-6} < \theta_D$ so the stars cannot be resolved.

31 The diffraction angle is $\theta_D = 1.22\times\dfrac{8.0\times10^{-2}}{300} = 3.3\times10^{-4}$ rad. The angular separation of two points on a diameter of Andromeda is $\dfrac{2.2\times10^{5}}{2.5\times10^{6}} = 0.088 > \theta_D$ so the telescope sees Andromeda as an extended object.

32 The diffraction angle is $\theta_D = 1.22\times\dfrac{5.5\times10^{-7}}{4.5\times10^{-3}} = 1.5\times10^{-4}$ rad. When this is about equal to the angular separation of the earth and the moon, i.e. $\theta_A = \dfrac{3.8\times10^{8}}{d}$, the objects will be resolved. This means $\dfrac{3.8\times10^{8}}{d} = 1.5\times10^{-4} \Rightarrow d = 2.5\times10^{12}$ m.

33 **a** The diffraction angle is $\theta_D = 1.22\times\dfrac{5.5\times10^{-7}}{2.4} = 2.8\times10^{-7}$ rad.

b It is free from atmospheric disturbances such as light pollution, turbulence in the air etc.

34 **a** From $d\sin\theta = n\lambda$, we get using the average wavelength of the two lines: $d = \dfrac{3\times589.29\times10^{-9}}{\sin 12°} = 8.5\times10^{-6}$ m.

b For resolution: $mN = \dfrac{\bar{\lambda}}{\Delta\lambda} \Rightarrow N = \dfrac{\bar{\lambda}}{m\Delta\lambda} = \dfrac{589.29\times10^{-9}}{3\times0.597\times10^{-9}} = 329$.

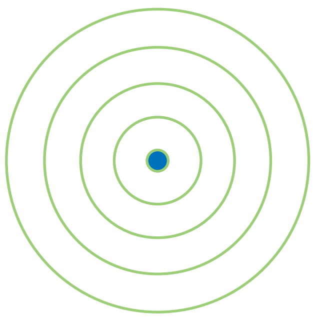
35 **a** From $\Delta\lambda = \dfrac{\bar{\lambda}}{mN} = \dfrac{550}{2\times3000} = 0.092$ nm.

b Increasing m and N both decrease $\Delta\lambda$ and so improve resolution. However the intensity of the light decreases with increasing m and so it is preferable to increase N instead.

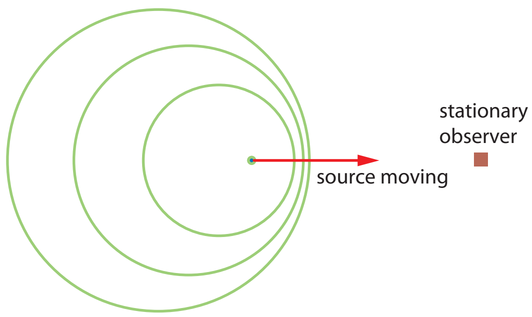
9.5 The Doppler effect

Note: Take the speed of sound in still air to be 340 m s^{-1}.

36



a



b

37 **a** This is a case of a source moving towards the observer and so

$$f = f_0 \frac{c}{c - v} = 500 \frac{340}{340 - 40} = 566.7 \approx 570 \text{ Hz.}$$

b **i** $\lambda = \frac{c}{f_0} = \frac{340}{500} = 0.68$ m

ii $\lambda' = \frac{c}{f} = \frac{340}{566.7} = 0.60$ m

38 **a** This is a case of a source moving away from the observer and so

$$f = f_0 \frac{c}{c + v} = 480 \frac{340}{340 + 32} = 438.7 \approx 440 \text{ Hz.}$$

b **i** $\lambda = \frac{c}{f_0} = \frac{340}{480} = 0.71$ m

ii $\lambda' = \frac{c}{f} = \frac{340}{438.7} \approx 0.78$ m

39 **a** This is a case of an observer approaching a stationary source and so the relevant formula is and so. $f = f_0\left(1 - \frac{v}{c}\right) = 512\left(1 - \frac{12}{340}\right) = 493.9 \approx 490$ Hz.

b **i** $\lambda = \frac{c}{f_0} = \frac{340}{512} = 0.66$ m

ii $\lambda' = \frac{c}{f} = \frac{340 - 12}{493.9} \approx 0.66 \text{ m} = \lambda$

40 **a** This is a case of an observer moving away from a stationary source and so the relevant formula is and so

$$f = f_0\left(1 + \frac{v}{c}\right) = 628\left(1 + \frac{25}{340}\right) = 674.2 \approx 670 \text{ Hz.}$$

b **i** $\lambda = \frac{c}{f_0} = \frac{340}{628} = 0.54$ m

ii $\lambda' = \frac{c}{f} = \frac{340 + 25}{674.2} \approx 0.54 \text{ m} = \lambda$

41 An observer on the approaching car will measure a higher frequency (f_1) than that emitted (f_0) because we have a case of the Doppler effect with an approaching source. The wave will then be reflected with frequency f_1. The car is now acting as an approaching source. The frequency received back at the source (f_2) will be higher than that emitted from the car. This is the case of a double Doppler effect.

42 The frequency received by the receding observer is (observer moving away) $f = f_0\left(1 - \frac{v}{c}\right)$. The wave is reflected backwards. The moving observer now acts as the source of the waves and the frequency emitted by this "source" is $f = f_0\left(1 - \frac{v}{c}\right)$ therefore, the original source now acts as a stationary observer and so the frequency it receives is now $f \frac{1}{\left(1 + \frac{v}{c}\right)} = f_0 \frac{\left(1 - \frac{v}{c}\right)}{\left(1 + \frac{v}{c}\right)}$. Hence $480 = 500 \frac{\left(1 - \frac{v}{c}\right)}{\left(1 + \frac{v}{c}\right)}$. Since $\frac{480}{500} = 0.96$ we have

$$0.96 = \frac{\left(1 - \frac{v}{c}\right)}{\left(1 + \frac{v}{c}\right)}$$

$$0.96 + 0.96\frac{v}{c} = 1 - \frac{v}{c}$$

$$1.96\frac{v}{c} = 0.04$$

$$\frac{v}{c} = 0.0204$$

$$v = 0.0204 \times 340 = 6.9 \text{ m s}^{-1}$$

Hint: You can put the equation $480 = 500\dfrac{\left(1-\dfrac{v}{c}\right)}{\left(1+\dfrac{v}{c}\right)}$ directly into the SOLVER of your graphics calculator (with $x = \dfrac{v}{c}$) and get the answer immediately without any of the tedious algebra above.

43 The frequency received by the stationary observer is (source moving towards) $f = \dfrac{f_0}{\left(1-\dfrac{v}{c}\right)}$. The wave is reflected backwards. The stationary observer now acts as the source of the waves and the frequency emitted by this "source" is $f = \dfrac{f_0}{\left(1-\dfrac{v}{c}\right)}$. The original source now acts as a moving observer and so the frequency it receives is now $f\left(1+\dfrac{v}{c}\right) = f_0\dfrac{\left(1+\dfrac{v}{c}\right)}{\left(1-\dfrac{v}{c}\right)}$. Hence $512 = 500\dfrac{\left(1+\dfrac{v}{c}\right)}{\left(1-\dfrac{v}{c}\right)}$. Since $\dfrac{512}{500} = 1.024$ we have

$$1.024 = \dfrac{\left(1+\dfrac{v}{c}\right)}{\left(1-\dfrac{v}{c}\right)}$$

$$1.024 - 1.024\dfrac{v}{c} = 1 + \dfrac{v}{c}$$

$$2.024\dfrac{v}{c} = 0.024$$

$$\dfrac{v}{c} = 0.01186$$

$$v = 0.01186 \times 340 = 4.0 \text{ m s}^{-1}$$

44 As far as the observer is concerned the velocity of the source is $v_S + v_0$ and the speed of the wave is $v_0 + c$. So using the formula of the stationary observer and an approaching source we have

$$f_0 = \dfrac{f_S}{1-\left(\dfrac{v_S+v_0}{c+v_0}\right)} = \dfrac{f_S}{\left(\dfrac{c+v_0}{c+v_0}\right)-\left(\dfrac{v_S+v_0}{c+v_0}\right)} = f_S\dfrac{c+v_0}{c-v_S}.$$

45 **a** The frequency emitted is f. The observer is moving away so he receives a frequency $f_R = f\dfrac{c-v}{c}$. This frequency is reflected from the object which now acts as a receding source. The frequency received back at the original source is then $f' = f_R\dfrac{c}{c+v} = \left(f\dfrac{c-v}{c}\right)\dfrac{c}{c+v} = f\dfrac{c-v}{c+v} = f\dfrac{\left(1-\dfrac{v}{c}\right)}{\left(1+\dfrac{v}{c}\right)}$.

b If $\dfrac{v}{c}$ is small then $f' \approx f\left(1-\dfrac{v}{c}\right)\left(1+\dfrac{v}{c}\right) \approx f\left(1-2\dfrac{v}{c}\right)$. Hence $\dfrac{\Delta f}{f} = \dfrac{2v}{c}$.

c **i** $\dfrac{\Delta f}{f} = \dfrac{2v}{c} \Rightarrow v = \dfrac{c\Delta f}{2f} = \dfrac{1500 \times 2.4 \times 10^3}{2 \times 5.00 \times 10^6} = 0.36 \text{ m s}^{-1}$

ii Because there is a range of speeds for the blood cells and the ultrasound is not incident normally on the cells.

46 $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c} \Rightarrow v = c\dfrac{\Delta\lambda}{\lambda} = 3.0 \times 10^8 \times \dfrac{0.17 \times 10^{-7}}{5.48 \times 10^{-7}} = 9.3 \times 10^6 \text{ m s}^{-1}$

47 **a** $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c} \Rightarrow v = c\dfrac{\Delta\lambda}{\lambda} = 3.0\times10^{8} \times \dfrac{3.0\times10^{-9}}{657\times10^{-9}} = 1.4\times10^{6}\ \text{m s}^{-1}$

b The speed derived in **a** is just the component of velocity along the line of sight, not the total velocity.

48 The speed of a point on the Sun's equator is $v = \dfrac{2\pi R}{T} = \dfrac{2\pi\times7.00\times10^{8}}{27\times24\times60\times60} = 1.89\times10^{3}\ \text{m s}^{-1}$.

The emitted frequency is $f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^{8}}{5.00\times10^{-7}} = 6.00\times10^{14}$ Hz. The shifts are then

$\dfrac{\Delta f}{f} = \dfrac{v}{c} \Rightarrow \Delta f = f\dfrac{v}{c} = \dfrac{6.00\times10^{14}\times1.89\times10^{3}}{3.00\times10^{8}} = 3.78\times10^{9}$ Hz.

49 **a** There is no shift since the velocity is at right angles to the direction of observation. The stars are neither approaching or moving away from the observer at that time.

b The speeds of the stars are $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c} \Rightarrow v = \dfrac{c\Delta\lambda}{\lambda} = \dfrac{3.00\times10^{8}\times0.08\times10^{-7}}{6.58\times10^{-7}} = 3.65\times10^{6}\ \text{m s}^{-1}$ and

$\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c} \Rightarrow v = \dfrac{c\Delta\lambda}{\lambda} = \dfrac{3.00\times10^{8}\times0.18\times10^{-7}}{6.58\times10^{-7}} = 8.21\times10^{6}\ \text{m s}^{-1}$.