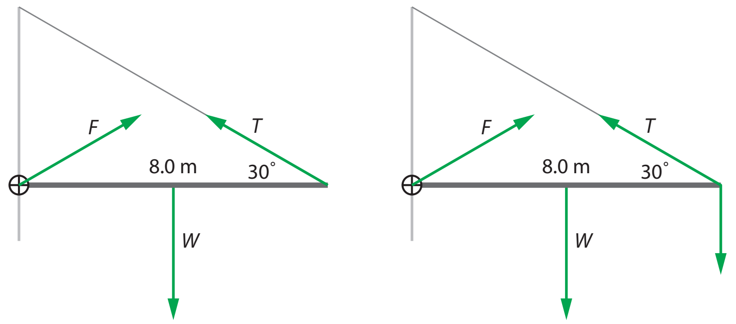
Answers to exam-style questions

Option B

1 ✓ = 1 mark

1 a i The forces are as shown in the left diagram:



The perpendicular distance between the axis and the line of the tension force is $L \sin 30°$. ✓

Rotational equilibrium by taking torques about an axis through the point of support gives:

$W \times \frac{L}{2} = TL \sin 30°$ ✓

Hence $W = T = 15$ kN. ✓

ii Translational equilibrium gives: $T \cos 30° = F_x$ and $T \sin 30° + F_y = 15$ kN. ✓

Hence $T \cos 30° = F_x = 12.99 \approx 13$ kN and $F_y = 7.5$ kN so that the magnitude of F is

$F = \sqrt{12.99^2 + 7.5^2} = 15$ kN. ✓

And the direction to the horizontal is $\theta = \tan^{-1} \frac{7.5}{12.99} = 30°$. ✓

b The critical case is when the worker stands all the way to the right. ✓

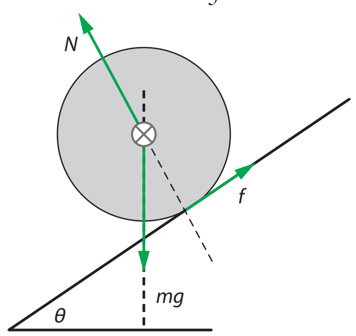
Rotational equilibrium in this case gives: $W \times \frac{L}{2} + mgL = TL \sin 30°$. ✓

Solving for the tension gives: $T = 16.7 \approx 17$ kN. ✓

2 a The forces are shown in the diagram and they are the weight of the cylinder, *mg*. ✓

The normal reaction, *N*, ✓

A frictional force *f*. ✓



b i Newton's second law for the translational motion down the plane is $Mg \sin \theta - f = Ma$. ✓

For the rotational motion by taking torques about the axis through the centre of mass is

$fR = \left(\frac{1}{2} MR^2\right)\alpha = \frac{1}{2} MRa$ ✓

$Mg \sin \theta - \frac{1}{2} Ma = Ma$ ✓

From which the result follows.

ii $f = Mg\sin\theta - Ma = 12\times 9.8\times \sin 30° - 12\times \frac{2}{3}\times 9.8\times \sin 30° = 19.6 \approx 20$ N ✓

c The rate of change of the angular momentum is the net torque. ✓
And this is $fR = 19.6\times 0.20 = 3.92 \approx 4.0$ Nm . ✓

3 a i When the ring makes contact with the disc and while it is sliding, it exerts a frictional force on the disc but the disc exerts equal and opposite force on the ring. ✓
Hence the net torque is zero and hence angular momentum is conserved. ✓

ii The initial angular momentum of the disc is $L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}\times 4.00\times 0.300^2\times 42.0 = 7.56$ Js . ✓

After the ring lands the total angular momentum is $L = \frac{1}{2}\times 4.00\times 0.300^2\times\omega + 2.00\times 0.300^2\times\omega$. ✓

Hence $\frac{1}{2}\times 4.00\times 0.300^2\times\omega + 2.00\times 0.300^2\times\omega = 7.56$ Js which gives $\omega = 21$ rad s^{-1} . ✓

iii The initial kinetic energy is $E_K = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\left(\frac{1}{2}\times 4.00\times 0.300^2\right)\times 42.0^2 = 158.76$ J. ✓

The final is $E_K = \frac{1}{2}\times\left(\frac{1}{2}\times 4.00\times 0.300^2\right)\times 21.0^2 + \frac{1}{2}\times(2.00\times 0.300^2)\times 21.0^2 = 79.38$ J leading to a loss of $79.38 \approx 79.4$ J . ✓

b i $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{21.0}{3.00} = 7.00$ rad s^{-2} ✓

ii $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\times 7.00\times 3.00^2 = 31.5$ rad ✓

Which is $\frac{31.5}{2\pi} = 5.0$ revolutions. ✓

iii $\Gamma = \frac{\Delta L}{\Delta t}$ ✓

$\Gamma = \frac{2.00\times 0.300^2\times 21.0}{3.00} = 1.26$ N m ✓

iv It is equal and opposite to that on the ring. ✓
Because the force on the ring is equal and opposite to that on the disc. ✓

c The change in the kinetic energy of the ring is $\frac{1}{2}\times(2.00\times 0.300^2)\times 21.0^2 = 39.69$ J . ✓

And so the power developed is $\frac{39.69}{3.00} = 13.2$ W. ✓

(This can also be done through $P = \Gamma\bar{\omega} = 1.26\times\frac{21.0}{2} = 13.2$ W .)

4 a i The temperature at B doubled at constant volume so the pressure also doubles at $p_B = 4.00\times 10^5$ Pa . ✓

ii From $pV^{\frac{5}{3}} = c$ and $pV = nRT$ we find $p^{-\frac{2}{3}}T^{\frac{5}{3}} = c'$. ✓

Hence $(4.00\times 10^5)^{-\frac{2}{3}}\times(600)^{\frac{5}{3}} = (2.00\times 10^5)^{-\frac{2}{3}}\times T_C^{\frac{5}{3}}$ leading to $T_C = 455$ K. ✓

iii The volume at B is $V_B = \frac{nRT_B}{p_B} = \frac{1.00\times 8.31\times 300}{2.00\times 10^5} = 1.246\times 10^{-2} \approx 1.25\times 10^{-2}$ m^3 . ✓

And so $\frac{V_B}{T_B} = \frac{V_C}{T_C} \Rightarrow V_C = V_B\frac{T_C}{T_B} = 1.246\times 10^{-2}\times\frac{455}{300} = 1.890\times 10^{-2} \approx 1.89\times 10^{-2}$ m^3 . ✓

b **i** $\Delta U_{AB} = \frac{3}{2}Rn\Delta T = +\frac{3}{2} \times 8.31 \times 1.00 \times 300$ ✓

$\Delta U_{AB} = +3739 \approx +3.74 \times 10^3$ J ✓

ii This happens from C to A: $W = -p\Delta V = 2.00 \times 10^5 \times (1.25 - 1.89) \times 10^{-2} = -1280$ J and the change in internal energy is $\Delta U_{AB} = \frac{3}{2}Rn\Delta T = \frac{3}{2} \times 8.31 \times 1.00 \times (300 - 455) = -1932$ J. ✓

Hence $Q = \Delta U + W = -1932 - 1280 = -3212 \approx -3.21 \times 10^3$ J. ✓

c Any heat engine working in a cycle cannot transform all the heat into mechanical work. ✓
And this engine rejects heat into the surroundings as it should. ✓

5 **a** **i** A curve along which no heat is exchanged. ✓

ii An adiabatic expansion involves a piston moving outwards fast. ✓
Hence molecules bounce back from the piston with a reduced speed and hence lower temperature. ✓

b The product pressure × volume is constant for an isothermal. ✓
This is the case for points A and C (product is 100 J). ✓
And the same is true for any other point on the curve, for example at $p = 2.00 \times 10^5$ Pa, $V = 0.50 \times 10^{-3}$ m^3. ✓

c **i** $\frac{V_A}{T_A} = \frac{V_B}{T_B} \Rightarrow T_B = T_A \frac{V_B}{V_A}$ ✓

$T_B = 300 \times \frac{0.38}{0.20} = 570$ K ✓

At C $T_C = 300$ K since AC is isothermal. ✓

ii Using data at A: $n = \frac{pV}{RT} = \frac{5.00 \times 10^5 \times 0.20 \times 10^{-3}}{8.31 \times 300}$ ✓

$n = 4.01 \times 10^{-2}$ ✓

d **i** Energy is transferred out of the gas along C to A. ✓
From $Q = \Delta U + W$ and $\Delta U = 0$ we find $Q = -160$ J. ✓

ii This happens from A to B: $W = 5.00 \times 10^5 \times (0.38 - 0.20) \times 10^{-3} = 90$ J and

$\Delta U = \frac{3}{2} \times 8.31 \times 4.01 \times 10^{-2} \times (570 - 300) = 135$ J. ✓

And so $Q = 135 + 90 = 225$ J. ✓

iii $W_{BC} = -\Delta U_{BC}$ (since BC is an adiabatic). ✓

And $\Delta U_{BC} = -\Delta U_{AB} = -135$ (since AC is an isothermal). ✓
OR
Since for the whole cycle $\Delta U = 0$, the net work is $Q_{in} - Q_{out} = 225 - 160 = 65$ J. ✓

And $W_{net} = W_{AB} + W_{BC} + W_{CA} \Rightarrow 128 = 90 + W_{BC} - 160 \Rightarrow W_{BC} = 198$ J. ✓

iv The efficiency is $e = \frac{W_{net}}{Q_{in}} = \frac{65}{225} = 0.290$. ✓

6 **a** For an adiabatic, $pV^{\frac{5}{3}} = c$ and since $V = \frac{nRT}{p}$. ✓

we find $p\left(\frac{nRT}{p}\right)^{\frac{5}{3}} = c$. ✓

Raising to the 3rd power gives $p^3\left(\frac{nRT}{p}\right)^5 = c^3$ and so the result. ✓

b **i** From $\frac{T^5}{p^2} = \text{constant}$ we find $\frac{320^5}{(2.0 \times 10^5)^2} = \frac{T^5}{(2.0 \times 10^6)^2}$. ✓

$$T = 320 \times \left(\frac{2.0\times10^6}{2.0\times10^5}\right)^{\frac{2}{5}} = 803.8 \approx 800\ \text{K}\ ✓$$

ii From $pV^{\frac{5}{3}} = \text{constant}$ we find. ✓

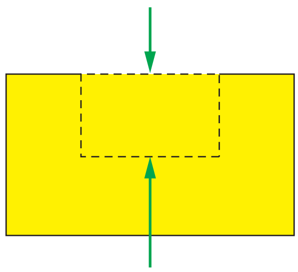
$$V = \left(\frac{2.0\times10^5}{2.0\times10^6}\right)^{\frac{3}{5}} \times 0.40 = 0.10\ \text{m}^3\ ✓$$

c The number of moles is $n = \frac{pV}{RT} = \frac{2.0\times10^5\times0.40}{8.31\times320} = 30.$ ✓

$$\Delta U = \frac{3}{2}Rn\Delta T = \frac{3}{2}\times8.31\times30\times(804-320)\ ✓$$

$\Delta U = 0.181\ \text{MJ}$ ✓

7 Delineate a rectangular region in the liquid whose top surface is the free surface of the liquid and has area A. ✓



Equilibrium demands that weight = net upward force. ✓
In other words that $\rho Ahg = pA - p_0A$. ✓
From which the result follows.

b **i** In free fall gravity "disappears" and so the pressure is just the atmospheric pressure. ✓

ii When the liquid accelerates upwards there is an additional force pushing the liquid upwards and so the pressure increases. ✓

c A body immersed in a fluid experiences an upward force that is equal to the weight of the displaced liquid. ✓

d **i** Equilibrium demands that $\rho_{\text{wood}}Vg = \rho_{\text{water}}\times0.75Vg$. ✓

Hence $\rho_{\text{wood}} = \rho_{\text{water}}\times0.75 = 750\,\text{kg m}$. ✓

ii Equilibrium demands that $\rho_{\text{wood}}Vg = \rho_{\text{oil}}\times0.82Vg$. ✓

Hence $\rho_{\text{oil}} = \frac{\rho_{\text{wood}}}{0.82} = \frac{750}{0.82} = 914.6 \approx 910\,\text{kg m}$. ✓

8 **a** **i** $p_0 + \rho gz = p_0 + \frac{1}{2}\rho v^2$ hence $v = \sqrt{2gz}$ ✓

$v = \sqrt{2\times9.8\times(220+40)} = 71.4 \approx 71\ \text{m s}^{-1}$ ✓

ii That the flow is laminar, ✓
and there are no losses of energy. ✓

b The flow rate is given by $Q = Av = \pi R^2 v$. ✓

Hence $Q = \pi\times(0.25)^2\times71.4 = 14\ \text{m}^3\text{s}^{-1}$ ✓

c **i** $p = p_0 + \rho gh = 1.0\times10^5 + 1000\times9.8\times40$ ✓

$p = 4.9\times10^5\ \text{Pa}$ ✓

ii The pressure is given by $p_0 + \rho gz = p + \frac{1}{2}\rho v^2$ where the speed can be found from the flow rate (i.e. the continuity equation) $\pi\times(0.65)^2\times v = 14.02 \approx 14\ \text{m}^3\text{s}^{-1}$. ✓

i.e, $v = 10.56 \approx 11\ \text{ms}^{-1}$. ✓

And hence

$$p = p_0 + \rho g z - \frac{1}{2}\rho v^2 = 1.0\times10^5 + 1000\times9.8\times40 - \frac{1}{2}\times1000\times10.56^2 = 4.36\times10^5 \approx 4.4\times10^5\ \text{Pa}$$ ✓

d The speed at depth h is $v = \sqrt{2gh}$. ✓

The flow rate is $Q = Av = \pi R^2\sqrt{2gh}$ and has to equal $0.40\ \text{m}^3\text{s}^{-1}$. ✓

Hence $h = \frac{1}{2g}\left(\frac{0.40}{\pi R^2}\right)^2 = \frac{1}{2\times9.8}\left(\frac{0.40}{\pi\times0.03^2}\right)^2 = 7.2\ \text{m}$. ✓

9 a The left side is connected to the holes in the tube past which the air moves fast. ✓

Hence the pressure there is low and the liquid is higher. ✓

b Call the pressure at the top of the left column p_L and that on the right p_R. Then

$p_L + \rho_{air} g z + \frac{1}{2}\rho v_L^2 = p_R + \rho_{air} g z + \frac{1}{2}\rho v_R^2$ and with $v_L = 0;\ v_R = v$, ✓

it becomes $v = \sqrt{\frac{2(p_L - p_R)}{\rho_{air}}}$. ✓

But $p_L - p_R = \rho g h$ which gives the result. ✓

c $v = \sqrt{\frac{2\rho g h}{\rho_{air}}} = \sqrt{\frac{2\times920\times9.8\times0.25}{1.20}}$. ✓

$v = 61.3 \approx 61\ \text{m s}^{-1}$. ✓

10 a Smooth streamlines. ✓

Closer together above the aerofoil. ✓

b i From $p_L + \rho g z + \frac{1}{2}\rho v_L^2 = p_U + \rho g z + \frac{1}{2}\rho v_U^2$ we obtain $\Delta p = p_L - p_U = \frac{1}{2}\rho v_U^2 - \frac{1}{2}\rho v_L^2$. ✓

Hence $F = A\Delta p = A(\frac{1}{2}\rho v_U^2 - \frac{1}{2}\rho v_L^2) = 8.0\times\frac{1}{2}\times1.20\times(85^2 - 58^2) = 18.53 \approx 19\ \text{kN}$. ✓

ii That the area above and below the foil are equal/that the flow is laminar. ✓

c The net upward force on the foil is about 16 kN and this is an estimate of the downward force on the fuselage. ✓

Ignoring effects of torque. ✓

d i The streamlines are no longer smooth but become eddy like and chaotic. ✓

ii Everywhere on the top side of the aerofoil and especially to the right. ✓

iii It will be drastically reduced. ✓

11 a In undamped oscillations the energy is constant and so the amplitude stays the same. ✓

In damped oscillations energy is dissipated and the amplitude keeps getting smaller. ✓

b i 8.0 s ✓

ii Correct readings of amplitudes. ✓

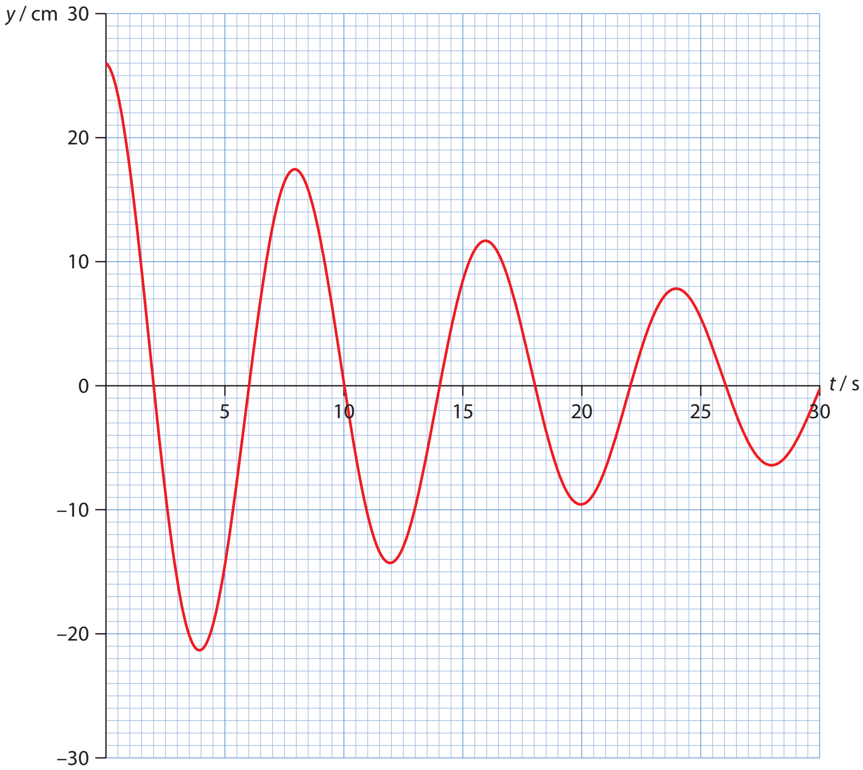
$Q = 2\pi\frac{26^2}{26^2 - 22^2}$ ✓

$Q \approx 22$ ✓

c i Amplitude reducing more every cycle. ✓

Period staying essentially unchanged/very slightly increases. ✓

ii It will decrease. ✓



d $Q = 2\pi \frac{5.0}{5.0 - 4.6}$ ✓

$Q \approx 79$ ✓

12 a All oscillating systems have their own natural frequency of oscillation. ✓

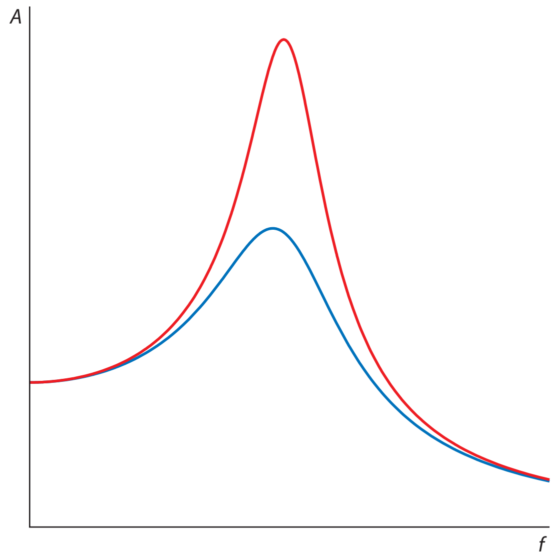
When a periodic external force is applied to the system the amplitude of oscillation will depnd on the relation of the external frequency to the natural frequency. ✓

The amplitude will be large when the frequency of the external force is the same as the natural frequency in which case we have resonance external. ✓

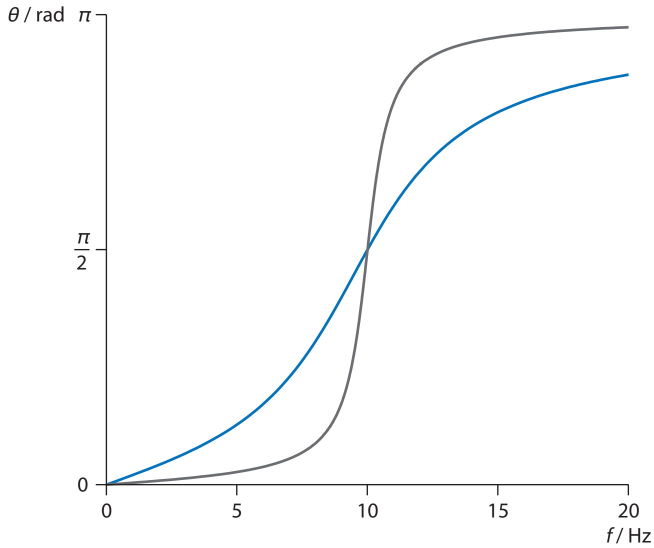
b Wider and lower curve. ✓

With peak shifted slightly to the right. ✓

See curve in blue.



c **i** Same intersection point. ✓
Less steep. ✓
See curve in blue.



ii 10 Hz ✓